MODELING TURBULENT BOTTOM BOUNDARY LAYER DYNAMICS

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ABSTRACT

This paper presents a modeling approach aimed at soving a complete hierarchy of turbulent bottom boundary layers which are often encountered in practical coastal and oceanographic engineering problems. The practical problem is extremely complex due to the presence and interaction of competing processes. A comprehensive model is thus needed to first provide fundamental understanding of a variety of turbulent bottom boundary layers before any simple model for the complex problem can be meaningfully constructed. This paper presents a comprehensive second-order closure model of turbulent transport and in addition, discusses some applications of the model to wave boundary layer, wave-current boundary layer, sediment-laden boundary layer and two-dimensional boundary layer. Example is provided to show how such a comprehensive model may be used to guide the development of a simple model.

I. INTRODUCTION

Flow in coastal waters is generally turbulent. A thorough understanding of the dynamics of turbulent bottom boundary layers is of paramount importance in the field of coastal and oceanographic engineering. For quantitative estimation of sediment transport rate and optimal design of structures in coastal waters, it is essential to first estimate the hydrodynamic forces acting on the ocean bottom which may be flat or complicated by the presence of bedforms or structures such as pipelines (Figure 1). The hydrodynamic forces can be produced by slowly varying currents (due to wind or tide) or short-period oscillatory currents (due to linear or nonlinear short-period waves) or a combination of both. When hydrodynamic forces exceed certain critical values, bottom sediments may be eroded and carried into the water column and, consequently, significant suspended sediment concentration and vertical concentration gradient may be produced thus leading to modification of the flow.

Simple Models or Comprehensive Models?

Dynamics of turbulent bottom boundary layers in coastal waters is extremely complex because of the coexistence of many competing mechanisms in turbulent flows: short-period wave, stratification, wave-

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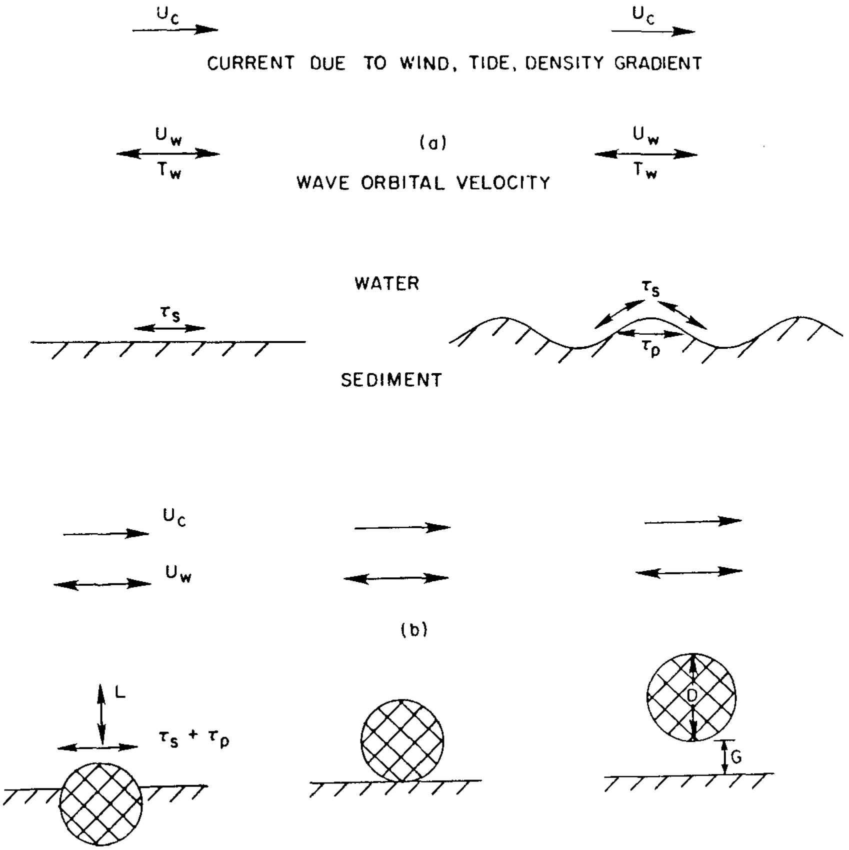

Figure 1 Bottom boundary layer flow over (a) flat and wavy bottom, and (b) flat bottom with pipelines.

current interaction, nonlinearity, and two-dimensional and even three-dimensional effects due to the presence of complex structures or bedforms. A comprehensive model capable of accurately resolving the complex turbulent boundary layer dynamics is urgently needed to provide physical insight and to solve practical engineering problems. Such a model must contain the proper physics of the major competing mechanisms and hence, cannot be necessarily "simple". Despite this fact, however, engineers faced with the complex practical problems in coastal sediment transport and structural design have generally favored the development and use of the so-called "simple" analytical or numerical models which, unfortunately, are often developed at the expense of compromising physics. Indeed, the literature is abound with such "simple" analytical and numerical models for some of the processes described above. For example, since the early eddy-viscosity model of Kajiura (1968), numerous similar eddy-viscosity models have been developed to simulate turbulent wave boundary layers. Eddy-viscosity models for wave-current interaction (e.g., Bakker and Van Doorn, 1978; Grant and Madsen, 1979) and stratified flow (e.g., Long, 1981) have also been attempted. While these models for individual processes could be calibrated to produce "good" results when sufficient data exist and under limiting conditions, these models generally fail to work when two or more of the competing mechanisms coexist or when little data exist and "prediction" is required. This is primarily due to the grossly simplified turbulence parameterization contained in these simple models, where the turbulence is typically assumed to be always in equilibrium with the mean flow and hence the turbulent transport is modeled as a local diffusive process only. Moreover, simple eddy-viscosity models developed for different processes cannot be readily superimposed for application to complex problems. Nevertheless, simple models continue to be developed and used. For example, forces on bottom pipelines have generally been estimated by using the simple but empirical Morison's equation (Sarpkaya and Isaacsen, 1981), which largely ignores the physics.

Modeling Approach

In order to develop meaningful simple analytical or numerical models for turbulent boundary layers and coastal sediment transport, one must start from comprehensive models which do not make ad-hoc assumptions about the processes described above. This paper describes such a comprehensive second-order closure model of turbulent transport which has been previously applied to simulate wave boundary layer (Sheng, 1982 and 1984) and wave-current interaction (Sheng, 1984). Such a comprehensive model, because of the added physics it contains, can simulate the individual processes without ad-hoc parameter tuning, and hence, is useful for understanding and predicting the complex turbulent boundary layer dynamics when many competing mechanisms coexist or when little data exist. In the long run, we can use the physical insight gained from these comprehensive models to provide guidelines for development of meaningful simple analytical or numerical models.

Model Validation

Previous eddy-viscosity models for wave boundary layers and current-wave boundary layers could only be calibrated against mean flow quantities. On the other hand, Sheng (1984) compared the results of a second-order closure model with both the mean and turbulent quantities within turbulent bottom boundary layers. This is a very important point—when modeling complex turbulent flow phenomena, model results must be compared with turbulence data. Comparison of model results against mean flow data alone and/or quantities of second-order importance (e.g., mass transport) is insufficient to judge the quality of the model. While the second-order mass transport may have some meaning in wave theory or laboratory study of wave boundary layer (Sleath, 1984), in practical situations the negligible mass transport velocity is easily overwhelmed by the tidal currents or wind-driven currents, hence, it is extremely hard to discern and is of questionable practical importance.

This Work

In addition to a brief review of the complete second-order closure model, this work also discusses two simplified versions of the model which are useful under certain conditions. Some recent applications of the second-order closure model to simulate wave boundary layers, wave-current boundary layers, sediment-laden bottom boundary layers, and two-dimensional effects are discussed.

II. MODELS OF TURBULENT FLOWS

Instead of presenting model equations which are only valid for a particular process of interest, this work will present the following general equations which are valid for turbulent flows where more than one of the processes (e.g., stratification, wave-current interaction, nonlineartiy, stratification, and 2-D or 3-D effects) are present. Anticipating applications to 1-D, 2-D, axisymmetric and even 3-D flow situations, the model equations are given in tensor notation.

Mean Equation

Reynolds-averaged equations for the mean flow quantities in an imcompressible fluid in the presence of rotation are:

$$\frac{\partial U_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial U_i}{\partial t} + \frac{\partial U_i U_j}{\partial x_j} = -\frac{\partial \overline{u_i u_j}}{\partial x_j} - \frac{1}{\rho_o}\frac{\partial p}{\partial x_j} - g_i \frac{\tilde{\rho}}{\rho_o} - 2E_{ijk}\Omega_j U_k + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial U_i}{\partial x_j}\right) \qquad (2)$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial U_j \Phi}{\partial x_j} = -\frac{\partial \overline{u_j \phi}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\kappa \frac{\partial \Phi}{\partial x_j}\right) \qquad (3)$$

where U_i and u_i are the mean and fluctuating velocity components, $\tilde{\rho}$ and ρ are the mean and fluctuating components of density, E is permutation tensor, Ω is earth's rotational speed, ν is the kinematic viscosity, κ is the kinematic diffusivity, and Φ and ϕ are the mean and fluctuating temperature (or salinity). An equation of state is needed to relate $\tilde{\rho}$ and Φ.

It is apparent that the above equations are not closed unless additional information is given for the second-order correlations $\overline{u_i u_j}$ and $\overline{u_i \phi}$. Models which provide information on $\overline{u_i u_j}$ and $\overline{u_j \phi}$ are called turbulence models.

Existing models for turbulent transport can be grouped into two categories: those which employ the turbulent (or eddy) viscosity/ diffusivity concept, and those which do not.

Eddy-Viscosity Models

The eddy-viscosity/diffusivity concept assumes that, in analogy to the viscous stresses and fluxes in laminar flow, the turbulent stresses and heat or mass fluxes are proportional to the mean velocity and temperature or concentration gradients, respectively. In tensor notation, the concept is:

$$-\overline{u_i u_j} = \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{q^2}{3}\,\delta_{ij} \tag{4}$$

$$-\overline{u_i \phi} = K_t \frac{\partial \Phi}{\partial x_i} \tag{5}$$

where ν_t and K_t are respectively the eddy viscosity and diffusivity, and $q^2 = u_i u_i$ is twice the turbulent kinetic energy (k). Unfortunately, ν_t and K_t are not fluid properties but depend strongly on the state of turbulence and may vary considerably across the flow, from time to time, and from one flow to another. Despite the simplicity of its concept, however, the precise value/formula for the eddy viscosity is not necessarily simple to determine. One often hears such remarks in a technical paper or presentation: "What kind of eddy viscosity is needed in order that the model results agree with data?" Thus, eddy viscosity is essentially a convenient "tuning parameter" for many people who do not wish to deal with turbulence in a rigorous fashion. One is cautioned that a "well tuned" eddy viscosity formula for one process/problem cannot be readily applied to another process/problem. In addition, the concept breaks down completely in flow regions where the flux is against the gradient of the transported quantity. Such counter-gradient-flux regions occur quite frequently in turbulent flows.

Second-Order Closure Models

This type of turbulence model uses transport equations for the turbulence stresses $u_i u_j$ and fluxes $u_i \phi$ such that the eddy-viscosity concept does not have to be introduced. The transport equations for a second-order closure model (e.g., Sheng, 1982), following a procedure outlined by Donaldson (1973), are presented in the following in terms of $u_i u_j$, $u_i \rho$ and $\rho\rho$:

$$\frac{\partial \overline{u_i u_j}}{\partial t} + U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} = -\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} - \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} - g_i \frac{\overline{u_j \rho}}{\rho_o} - g_j \frac{\overline{u_i \rho}}{\rho_o} \tag{6}$$

$$- 2E_{ikm}\,\Omega_k\,\overline{u_m u_j} - 2E_{jmk}\Omega_m\,\overline{u_k u_i}$$

$$+ v_c \frac{\partial}{\partial x_k}\left(q\Lambda \frac{\partial \overline{u_i u_j}}{\partial x_k}\right) - \frac{q}{\Lambda}\left(\overline{u_i u_j} - \delta_{ij} \frac{q^3}{3}\right) - \delta_{ij} \frac{2bq^3}{3\Lambda} + \nu \frac{\partial^2 \overline{u_i u_j}}{\partial x_k^2} - \frac{2a\nu \overline{u_i u_j}}{\Lambda^2}$$

$$\frac{\partial \overline{u_i \rho}}{\partial t} + U_j \frac{\partial \overline{u_i \rho}}{\partial x_j} = - \overline{u_i u_j} \frac{\partial \tilde{\rho}}{\partial x_j} - \overline{u_j \rho} \frac{\partial U_i}{\partial x_j} - \frac{g_i \overline{\rho\rho}}{\rho_o} - 2E_{ijk} \Omega_j \overline{u_k \rho} \tag{7}$$

$$+ v_c \frac{\partial}{\partial x_j} \left(q\Lambda \frac{\partial \overline{u_i \rho}}{\partial x_j}\right) - \frac{Aq}{\Lambda} \overline{u_i \rho} + \kappa \frac{\partial^2 \overline{u_i \rho}}{\partial x_j^2} + \frac{a\kappa \overline{u_i \rho}}{\Lambda^2}$$

$$\frac{\partial \overline{\rho\rho}}{\partial t} + U_j \frac{\partial \overline{\rho\rho}}{\partial x_j} = 2 \overline{u_j \rho} \frac{\partial \tilde{\rho}}{\partial x_j} + v_c \frac{\partial}{\partial x_j} \left(q\Lambda \frac{\partial \overline{\rho\rho}}{\partial x_j}\right) + \kappa \frac{\partial^2 \overline{\rho\rho}}{\partial x_j^2} - \frac{2bsq\overline{\rho\rho}}{\Lambda} \tag{8}$$

where the last 5 terms in Eq. (6), the last 4 terms in Eq. (7), and the last 3 terms in Eq. (8) are "modeled" terms. For example, in Eq. (6), the term containing v_c is a diffusion term, the term containing q/Λ is a tendency-toward-isotropy term, while the last 3 terms are dissipation terms with the last 2 terms vanishing at high Reynolds number. The model constants, $v_c = 0.3$, $b = 0.125$, $a = 3$, $A = 0.75$, and $s = 2.8$, are determined by comparing model simulations with a wide variety of critical laboratory experiments where only one or two of the modeled terms is dominant, and remain fixed for any new model application (Sheng, 1986a). Thus the model is sometimes termed an invariant model.

An equation for the turbulence macroscale Λ is needed to close the system of equations (1) through (8):

$$\frac{D\Lambda}{Dt} = 0.35\frac{\Lambda}{q^2} \overline{u_i u_j} \frac{\partial U_i}{\partial x_j} + 0.6bq + 0.3\frac{\partial}{\partial x_i}\left(q\Lambda \frac{\partial \Lambda}{\partial x_i}\right) - \frac{0.375}{q}\left(\frac{\partial q\Lambda}{\partial x_i}\right)^2 - \frac{0.8\Lambda}{q^2} g_i \overline{u_i \rho} \tag{9}$$

It is useful to derive an equation for q^2 from Eq. (6):

$$\frac{\partial q^2}{\partial t} + U_k \frac{\partial q^2}{\partial x_k} = -2\overline{u_i u_k} \frac{\partial U_i}{\partial x_k} - 2g_i \frac{\overline{u_i \rho}}{\rho_o} + v_c \frac{\partial}{\partial x_k}\left(q\Lambda \frac{\partial q^2}{\partial x_k}\right) - \frac{bq^3}{\Lambda} + \frac{\partial^2 q^2}{\partial x_k^2} - \frac{2a\nu q^2}{\Lambda^2} \tag{10}$$

Simplified Second-Order Closure Models

Equations (6), (7), (8) and (9) constitute the so-called "Reynolds stress model" and is the most complete second-order closure model. These dynamic equations contain much more physics of the turbulent flow than the eddy-viscosity model, or first-order closure model. Although the solution of these equations requires considerable computational effort, simplifications of the equations can sometimes be introduced in practical applications to reduce the computational effort. For example, if the time scale of mean flow is large compared to that of turbulence, "quasi-equilibrium" approximation may be introduced such that only Eqs. (9) and (10) are needed to represent the dynamics of turbulence, while Eqs. (6), (7) and (8) are replaced by a set of algebraic equations where the total derivative and diffusion terms are neglected. If, in addition, the turbulence does not change significantly over Λ, the "super-equilibrium" approximation is valid such that Eqs. (6), (7) (8) (9) are replaced by a number of algebraic equations. Such simplified second-order closure models are used to represent the vertical turbulence in the three-dimensional hydrodynamic model of coastal and estuarine circulation (Sheng, 1986b).

III. SECOND-ORDER CLOSURE MODELING OF WAVE BOUNDARY LAYERS

Wave Boundary Layer

The Reynolds stress model as presented above has been applied to simulate the wave boundary layer of Jonsson and Carlsen (1976). The one-dimensional version of the model was used and hence the equations become considerably simplified, since only vertical gradients appear in the equations. Boundary layer approximation was invoked and the free-stream oscillatory currents are balanced by a lateral pressure gradient which is imposed throughout the boundary layer. Results shown in Sheng (1982) and Sheng (1984) demonstrated that the Reynolds stress model was able to faithfully simulate the mean velocities at various phase angles and the phase lag of mean velocities at different vertical levels. In addition, comparison of the model results vs. computed Reynolds stress data indicated the lack of temporal resolution of mean velocity measurements.

Logarithmic Layer

Perhaps one of the most important model results is the explicit computation of the instantaneous logarithmic layer. It is a well known fact that the velocity distribution within a homogeneous and steady-state bottom boundary layer follows the logarithmic variation:

$$u = \frac{u_*}{k} \ln\left(\frac{z}{z_o}\right) \tag{11}$$

where k is the von-Karmen constant, u_* is the friction velocity and z_o is the roughness height, while the turbulent fluxes are more or less uniform within the layer. In the presence ot density stratification due to variation in temperature, salinity and/or suspended sediment concentration, the thickness of the logarithmic layer (or constant flux layer) may be significantly modified. In addition, the presence of a body force such as pressure gradient may also significantly alter the thickness of the logarithmic layer. Jonsson and Carlsen (1976) could not measure their turbulence directly and hence assumed a fixed logarithmic layer thickness of 6.3 cm. Results of the Reynolds stress model clearly indicate a much thinner logarithmic layer, as shown in Figure 2, which is modulated by the pressure gradient associated with the free-stream oscillatory flow. The instantaneous Reynolds stress at three levels (0.5 cm, 1 cm and 4 cm above the bottom) and two phase angles as shown agree well with model results.

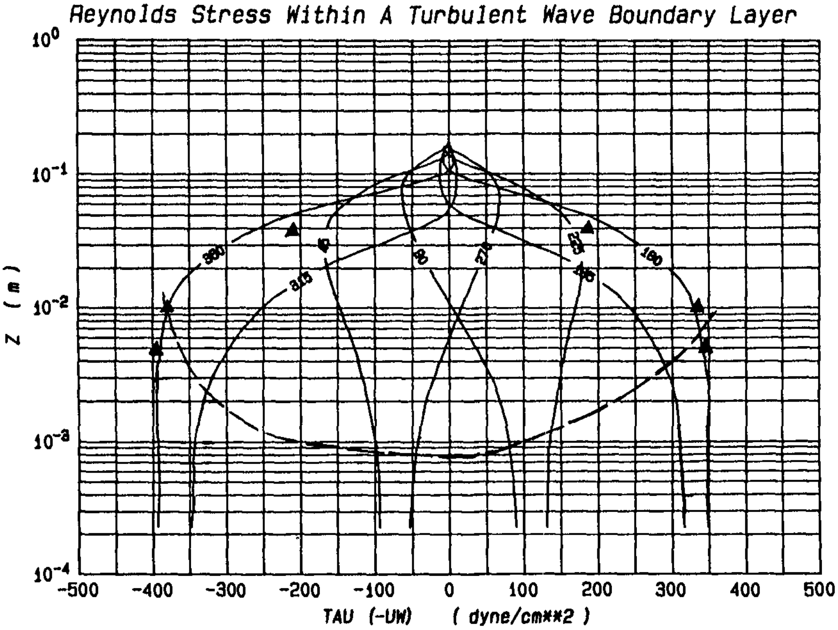

Figure 2 Reynolds stress within Jonsson and Carlsen's wave boundary layer. ——— : model; ▲'s: data; ----- indicates the approximate edge of the logarithmic layer.

Eddy Viscosity

The presence of the logarithmic layer is validated by the model, thus the eddy viscosity near the bottom can be found to scale with ku_*z, where z is the distance above the bottom:

$$\nu_t = \frac{\tau}{\partial u/\partial z} = \frac{\rho u_*^2}{u_*/kz} = \rho u_* kz \tag{12}$$

Since u_* is a function of time, ν_t must be a function of time and space. This explains why almost all of the previous eddy-viscosity models, which use either spatially varying or temporally varying eddy viscosity, fail to faithfully reproduce Jonsson and Carlsen's wave boundary layer data (Sheng, 1986a). However, Eq. (12) is not valid outside the logarithmic layer.

Another reason why eddy viscosity is a poor concept for studying turbulent boundary layers is visualized in Figure 3 which shows the eddy viscosity calculated from Jonsson and Carlsen's data via $\nu_t = \tau/(\Delta u/\Delta z)$. It is apparent that the eddy viscosity is often scattered and may become negative which is physically meaningless. The solid lines represent the eddy viscosity computed from the results of the Reynolds stress model via $\nu_t = \overline{ww}\,\Lambda/q$.

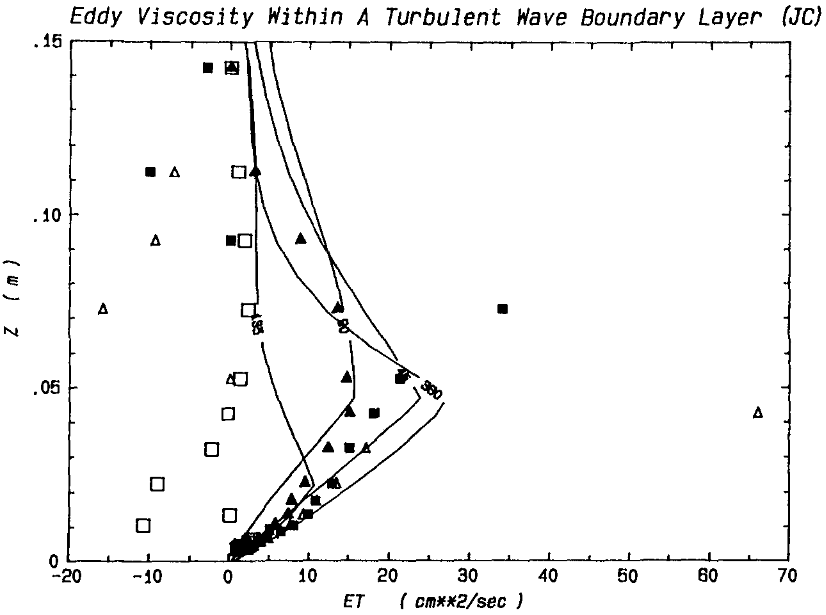

Figure 3 Eddy viscosity computed from Jonsson and Carlsen's data (Δ, , Δ,) and computed from the model results via $\overline{ww}\,\Lambda/q$.

Simplified Second-Order Closure Models for Wave Boundary Layer

Recently, the "quasi-equilibrium" and "super-equilibrium" versions of the second-order closure model were applied to simulate Jonsson and Carlsen's wave boundary layer (Sheng, 1987). The "quasi-equilibrium" results compared very well with data while the "super-equilibrium" results did not do as well. However, this in no way implies that the simplified second-order closure models are sufficient for simulating all turbulent wave boundary layers, since the state of turbulence may vary from one boundary layer to another and from one problem to another.

Other Wave Boundary Layers

Sheng (1984) simulated the turbulent bottom boundary layer underneath a cnoidal wave and found strong asymmetric behavior in the predicted mean velocities and Reynold stresses. The maximum Reynolds stress was found to be 50% higher than that produced by a simple sinusoidal wave boundary layer.

Another interesting wave boundary layer experiment was conducted by Hino, et al. (1983). It was reported that the measured turbulent intensities increased substantially while the flow changes from the acceleration stage to deceleration stage. Again, results of the Reynolds stress model agree well with data (Sheng, 1987).

IV. WAVE-CURRENT BOTTOM BOUNDARY LAYER

Kemp and Simons (1982) measured the mean and turbulent quantities within the turbulent bottom boundary layer underneath a wave propagating with the current. They found that for smooth bed, the turbulent stresses produced by the wave alone and the current alone can be superimposed to give the total stress for the wave-current case. For rough bed, however, the wave-current interaction is highly nonlinear and the turbulent stressed due to current and wave cannot be linearly superimposed. Although their experiments appear to contain 2-D and 3-D effects in the vicinity of the roughness elements (due to the relatively small ratio between particle displacement and roughness A_{bm}/k_s), it is interesting to compare the results of a 1-D Reynolds stress model against their data. As an example, comparison in terms of the maximum vertical varinace is shown in Figure 4, where the model results were for the case of WCR5. Notice the generally good agreement between data and model results, except in the vicinity of the roughness elements where 2-D effect is important.

V. SEDIMENT-LADEN BOTTOM BOUNDARY LAYER

The second-order closure model has been recently applied to simulate the effect of suspended sediment concentration on the boundary layer flow structure and to improve the determination of erosion/deposition relationships from laboratory experiments (Sheng, 1987).

VI. TWO-DIMENSIONAL EFFECT

Most bottom boundary layer models, including those cited above, are one-dimensional and hence cannot be expected to work when two dimensional effect due to bedforms or structures is present. DuToit and Sleath (1981) measured velocity measurements close to rippled beds in oscillatory flow and compared their data with three 1-D models (Kalkanis, 1964; Kajiura, 1968; Bakker, 1974). The agreement between data and model results was found to be rather poor. In 2-D flows, profile drag becomes important in addition to the skin friction drag. Vortex generation and shedding play an important role in the two-dimensional bottom boundary layer dynamics, but are simply not resolved in the one-dimensional models.

As an example, the 2-D version of the second-order closure model was used to simulate the flow and separation behind a backward facing step. As shown in Figure 5, the computed mean velocity and Reynolds stress distribution compare well with data at various downstream distances X behind the step with height H. The Reynolds stress distribution shown is in the vicinity of the flow reattachment point. It should be pointed out that for such a complex problem, a "simple" eddy-viscosity model simply does not exist.

The 2-D version of the Reynolds stress model can be applied to simulate the wave boundary layer or wave-current boundary layer over a wavy bottom. It should be pointed out that such model will be able to correctly resolve the nonlinear inertia terms, without treating it as "second-order" effect as in the 1-D model.

VII. CONCLUSIONS

The dynamics of turbulent bottom boundary layers in coastal waters is extremely complex because of the simultaneous presence of such competing processes as stratification, short-period wave, wave-current interaction, nonlinearity, and two-dimensional effects. Numerous "simple" (from the point of view of computational effort) models for individual processes (e.g., a pure wave boundary layer) have been developed in the past. Unfortunately, many of these models were developed with ad-hoc empiricism and hence one cannot superimpose the simple models for application to complex problems. In order to ultimately derive simple analytical or numerical models of the complex overall problem, a comprehensive model must be first constructed and used to gain physical insight and to provide guidance for formulating simpler models.

A comprehensive second-order closure model of turbulent transport is presented here. Model applications to a variety of bottom boundary layers (wave boundary layer, wave-current boundary layer, sediment-laden boundary layer, and two-dimensional boundary layers) are discussed. The model is capable of simulating the measured mean and turbulent quantities. Simplified versions of the second-order closure model are available and can be applied to some problems. An example was provided to demonstrate the fact that an eddy-viscosity model is meaningful only if it is derived based on sound understanding of turbulent flow.

For meaningful model validation, model results must be compared with measured turbulence data instead of mean flow data only.

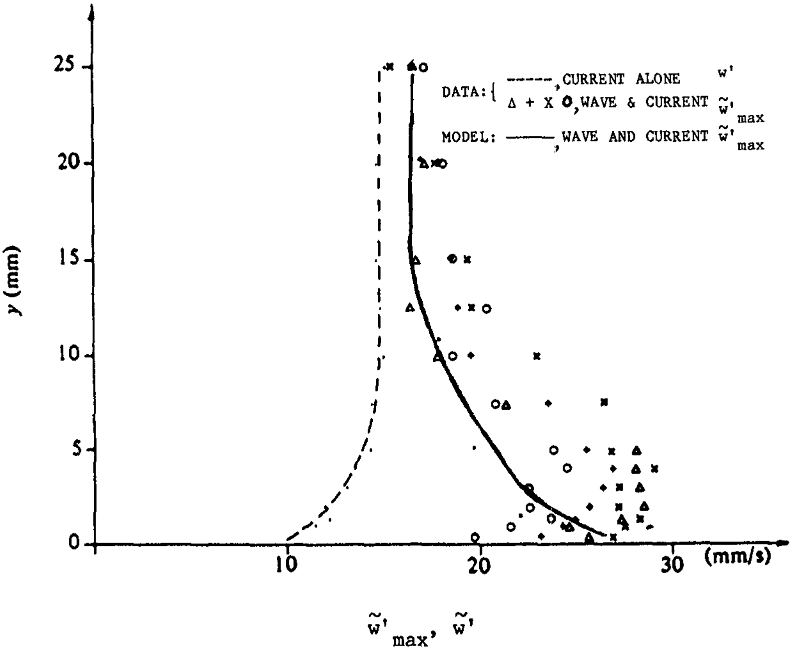

Figure 4 Vertical variance within 25 mm of bed-roughness apex in Kemp and Simons' 1982 wave-current experiment and computed by the Reynolds stress model.

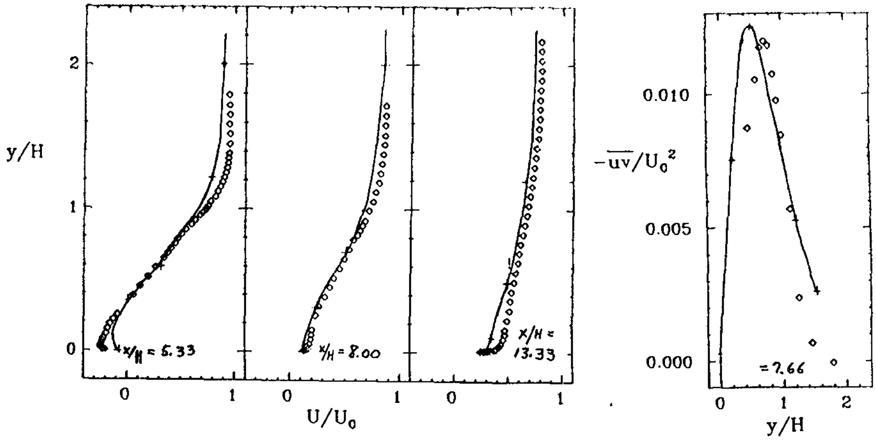

Figure 5 Mean velocity and Reynolds stress at several locations behind a backward facing step. His step height, U_0, is free stream velocity, and x and y are horizontal and vertical distances.

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