CHAPTER 63

Real-Time Wave Forecasting with adaptive ARMAX models

H.Wensink* and T. Schilperoort

Abstract

A method is described for improving the accuracy of the wave predictions which are required for the operational guidance of the shipping traffic in approach channels to harbours and to support offshore activities.
The method, as being described here, is based on the assimilation of wave physics and real-time hydrometeorological observations into a statistical time series model called ARMAX.

1 Introduction

In many practical situations operational wave prediction systems are needed, which have an accuracy that cannot be met by the prediction systems generally used in practice.
One way to improve existing methods is to integrate available hydrometeorological data and numerical wave prediction models into one prediction system.
This idea has been worked out by Poulisse and Schilperoort (1983), who investigated the potential of Kalman-filtering and adaptive time series (ARMAX) modelling for wave prediction purposes.

Schilperoort and Strating (1985) have described an adaptive modelling technique based on a correlation of wave observations, measured at different locations along the dominant propagation path of the wave energy.

More insight into the modelling of the wave processes and the development of new measuring instruments as well as the experience in an operational environment necessitated an improvement of the existing (ARMAX) method. These improvements are described by Wensink (1986a, 1986b and 1987).

The purpose of this paper is to describe an improved wave prediction system based on the adaptive time series modelling technique.
In paragraph 2 a short overview is given and the extensions of the wave prediction model are described. Paragraph 3 presents the results of the prediction system in an operational situation.
Finally in paragraph 4 the conclusions and some operational aspects are described.

2. The ARMAX system

The adaptive time series modelling technique as described by Schilperoort and Strating (1985) is a method that can be used for complex physical systems in which diffusion and advection plays an important role.

*Delft Hydrulics Lab, Delft, The Netherlands

The first parameter for which such a system has been developed is the 'low frequency wave energy', E_{10}, a parameter indicating the variance which is present in the wave spectra above the 10 s wave period.

This parameter has been chosen because of its pronounced influence on the motion of large ships and because of its low accuracy of prediction using existing prediction sytems.

The ARMAX prediction system for 'low frequency wave energy' is based on six elements:
- a data pre-processing module, in which, for each individual monitoring station, the raw hydrometeorological observations are transformed using simple wind-wave relationships;
- a data correction module;
  * in which, for each individual monitoring station the hydrometeorological data are reviewed and corrected for missing values
  * which determines the stations in the monitoring network to be coupled by ARMAX models;
- a correlation module, in which the transformed data of two neighbouring monitoring stations are correlated with each other using an ARMAX time series model
- a coupling module, in which a number of correlation modules is coupled, depending on the number of stations in the monitoring network and the quality of the input data;
- an identification module, in which the parameters of the ARMAX models used are estimated using an adaptive modified recursive least squares estimation method;
- a prediction module which generates the E_{10} predictions using the last identified ARMAX models;

2.1 Data pre-processing module

Wave fields not propagating in line with the monitoring stations affect the statistical relations between the observed 'low frequency wave energy' parameter (LFE) at two neighbouring monitoring stations and therefore reduce the maximum possible prediction accuracy.
To account for these secondary wave fields, at each individual monitoring Station i, crude estimates are made of the incoming E_{10} from Station (i-1), the locally generated E_{10} and the outgoing E_{10}, propagating to Station (i + 1).
These estimates are obtained by a direct processing of the online observed wind-velocity, wind-direction and directional wave spectra at that station using simple wind-wave relationships (WWR).
These physical relations include:

i) the calculation of the energy of growing sea, E_s and the growth stage, ξ, from the local wind data using a fetch limited exponential suturating growth curve and assuming a $\cos^2$-distribution function for the wind-sea directions, ϕ_s;

ii) the calculation of the average sea direction, $\bar{\Phi}_s$, from the wind data using a directional relaxation function according to Günther et al. (1981);

iii) the calculation of the parameters of an assumed spectrum model (Kruseman model) from the growth stage, using relationships according to Sanders et al. (1980);

iv) the calculation of the locally generated E_{10}, u_1^i, by integrating the spectrum model over the frequency range $0 < f < 0.1$ Hz;

v) the assumption of an exponential decay of u_1^i in the case of a vanishing or rapidly veering wind, with the decay rate being dependent on the wind and wave direction;

vi) the calculation of incoming E_{10}, u_2 by assuming a $\cos^{2s}\frac{\Theta}{2}$ directional model; the time delay Δ^{i-1}, is calculated by the first and second moment of the directional wave spectra.

vii) the calculation of the modulations of E_{10} generated by the tides in shallow water by using a frequency dependent dissipation function according to Lighthill (1978).

2.2 Data correction module

A prediction system based on real-time hydrometeorlogical observations requires reliable input data. This condition cannot always be fulfilled in an operational situation. Therefore the ARMAX system includes procedures to detect and correct missing data and outlayers.

These wind-wave correction procedures can only be used for missing or bad data over short time intervals. For long time intervals without reliable or without any wind-wave information, the data correction module selects only those stations which posesses reliable input data.

2.3 Correlation module

The ARMAX model structure is based on the assumption that the observed E_{10} at Station i at time k is due to both locally generated E_{10} at time k and the outgoing E_{10} from the preceding Station (i-1) at time $(k - \Delta^{i-1})$ i.e.

$$y^i(k) = u_1^i(k) + u_2^{i-1}(k - \Delta^{i-1}) \tag{1}$$

By introducing the time delay, Δ^{i-1}, in the u_2-contribution, the E_{10} propagation time is taken into account explicitly. However, the model given by equation (1) is much too simple to give an accurate description of the complex phenomena involved. For instance, features like energy dispersion and refraction are not considered.

The simple model equation (1) therefore has been generalized by defining the following ARMAX model

$$y^i(k) = \underbrace{a_1^i y^i(k-1) + \ldots + a_{p_i}^i y^i(k-p_i)}_{\text{autoregressive part}} +$$

$$+ \underbrace{b_o^i u_1^i(k) + \ldots + b_{q_i}^i u_1^i(k-q_i)}_{\text{local sea contribution}} +$$

$$+ \underbrace{c_o^i u_2^{i-1}(k-\Delta^{i-1}) + \ldots + c_{r_i}^i u_2^{i-1}(k-\Delta^{i-1}-r_i)}_{\text{distant swell contribution}} +$$

$$+ \underbrace{d_o^i w^i(k) + \ldots + d_{s_i}^i w^i(k-s_i)}_{\text{noise contribution}} + \underbrace{e_1^i \cdot 1}_{\text{bias term}} \tag{2}$$

The autogressive (AR) part acts like a filter for the input signals u_1, u_2 and w, which, for appropriate values of the AR-parameters, $a_1 \ldots a_p$, introduces a certain energy dispersion.
Variable growth rates and small variations in the time delays can be realized if the parameters, $b_o \ldots c_r$, are allowed to be time dependent. Energy dissipation occurs automatically for small values of $c_o \ldots c_r$. The noise components $w^i(k) \ldots w^i(k-s)$ serve to model the residuals of $y^i(k)$ which cannot be explained by the autoregressive and the exogeneous input. By formulating the noise contribution as a moving average process (MA), residual autocorrelations can be used to improve the model predictions. Finally, the bias term, e_1^i, is added to improve tracking of the E_{10} observations. The combination of autoregressive (AR), moving average (MA) and exogeneous inputs (X) explains the name ARMAX model.

2.4 Coupling module

Because of the structure of the ARMAX models, in which y^i is related explicity to u_2^{i-1}, it is easy to build a sequence of ARMAX models covering all the stations being considered. This is represented schematically in Figure 1, which is explained in more detail in the following sections.

2.5 Parameter estimation module

The ARMAX model (equation 2) can be used to predict the output y^i recursively from known inputs if the model parameters $a_1^i \ldots e_1^i$ and the model order (p_i, q_i, r_i, s_i) are known.
In the present system the model order has to be specified by the user. The specification of the parameter q_i and r_i requires an understanding of the physical processes involved. The specifications of the parameters p_i and s_i can be obtained from the hydrometeorological observations. Some criteria of best fit for the model order parameters p_i and s_i are described in Haykin (1986).

For a specified model order, the model parameters are estimated using a recursive adaptive estimation algorithm.
Recursive estimation is required because in an operational situation, there is a continuous data stream entering the prediction system, which must be processed on-line in order to avoid unnecessary delays.

In addition, the algorithm must be adaptive because the model (equation 2) is, as yet, too simple to give an accurate description of the E_{10} processes under all hydrometeorological situations. The ARMAX parameters $a_1^i \ldots e_1^i$ are therefore to be adapted to the hydrometeorological situation in order to garantee that the prediction is made with a model which best fit the actual wind and wave conditions.

The performance of a prediction system, which is based on time series modelling depends, to some extent, on the amount of information contained in the input data.

However, a prediction system, operating in real-time situations, must operate under all possible conditions.
Therefore techniques are incorporated to minimize the effect of redundant input data on the performance of the model (Wensink, 1986a). Some of this techniques are described by Goodwin and Sin (1984).

When hydrometeorological observations are used as the input for parameter estimation, the ARMAX system works in the identification mode. In this case there is no through-coupling of the various ARMAX elements. Instead, the model parameters are estimated separately for each individual element.

2.6 Prediction module

The ARMAX model parameters identified from the most recent observations always give the best description of the actual hydrometeorological situation; therefore, these must be used to generate the E_{10} predictions. As a consequence, the prediction system switches constantly from the identification mode to the prediction mode, and vice versa: In the identification mode E_{10} observations are used to estimate the ARMAX model parameter values whereas in the prediction mode these values are used to generate the E_{10} predictions, see Figure 1. In the prediction mode there is a through coupling, of the various elements. The prediction procedure is as follows, see Figure 1.

- Based on the observations from the most remote station, available up to the present time t, the u_2^o signal is calculated which then serves as an input for the first ARMAX element after a time delay Δ^o;
- the u_1 input for the first ARMAX element should then be known up to time $t + \Delta^o$. Since $t + \Delta^o$ is Δ^o time steps into the future, the u_1 calculations required are only possible when wind predictions are also available for the second station up to time $t + \Delta^o$;
- the identified first ARMAX element is then used to calculate recursively the E_{10} value y^1 at the second station up to t_1 time steps ahead, where t_1 corresponds to the maximum possible prediction time at the second station, Δ^o.
  The predicted value of E_{10} is subsequently used in the calculation of u_2^1 which serves as an input for the second ARMAX element after a time delay Δ^1;
- the u_1 input for the second ARMAX element should then be known up to the time $t + \Delta^o + \Delta^1$, etc.

Hence, the maximum possible prediction time $T_p(\max)$ at the final station is determined by the summation of the delays Δ^i.

3. Some results

The ARMAX system is fully in operation and processes the data of four stations in the North Sea Monitoring Network. These stations, AUK, K13, LEG and BG2, are located in line with the dominant swell direction in the southern part of the North Sea, see Fig. 2. A typical performance of the prediction system is illustrated in Figures 4 and 5, which, for the period between 30 december 1984 and 5 january 1985, compare the E_{10} observations and the 5-hours ahead predictions. Figure 3 presents the meteorologic situation on weather charts at that period (Wensink 86a,

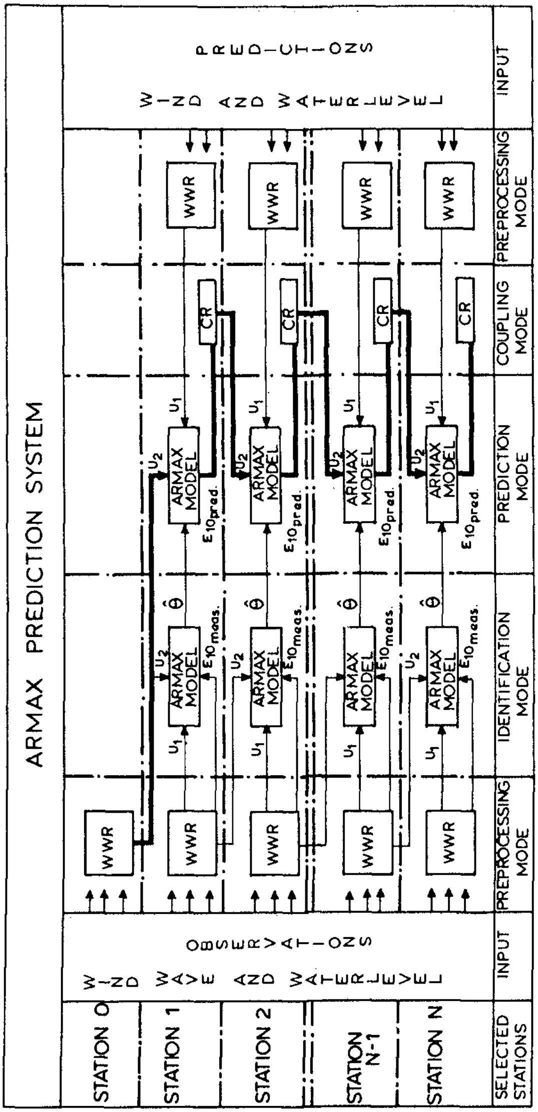

Fig. 1 Schematic representation of the ARMAX prediction system for N monitoring stations

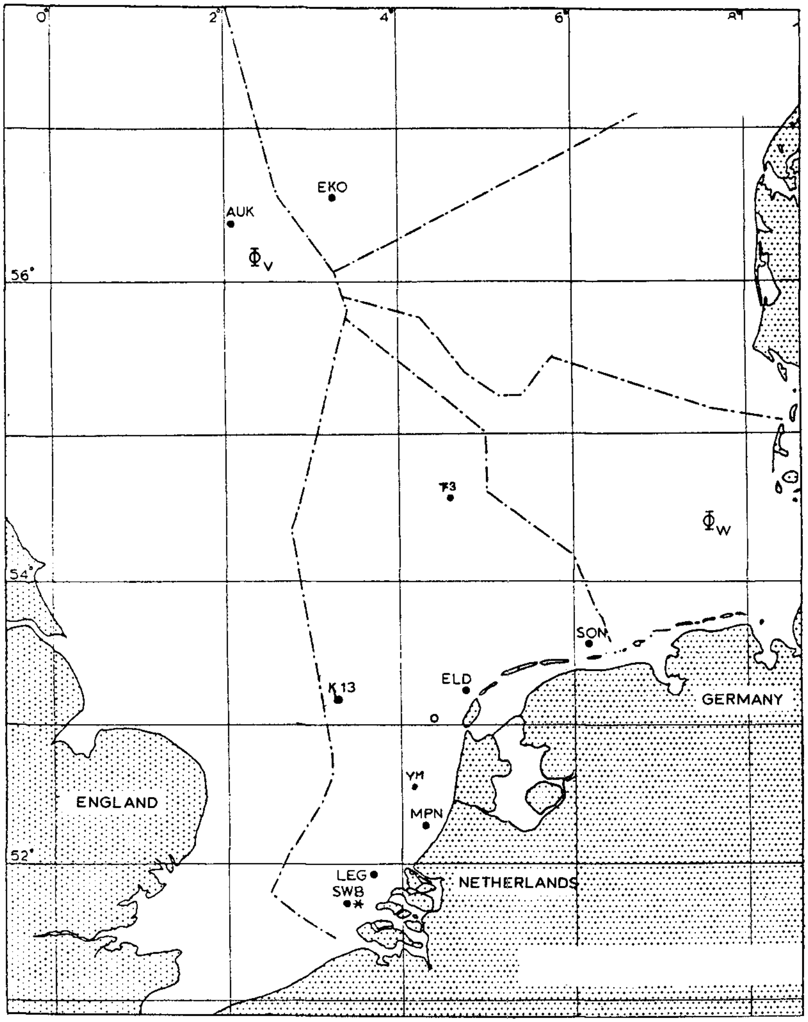

Fig. 2 The North Sea wave monitoring network

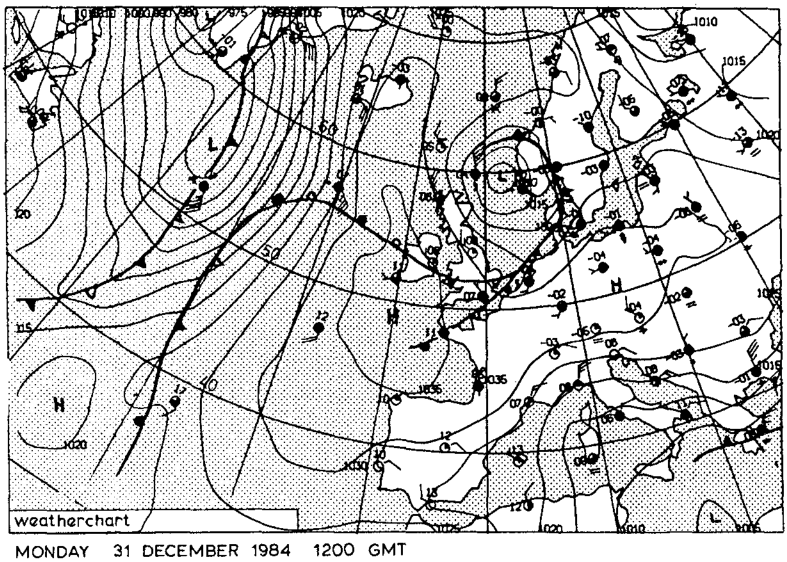

Fig. 3 Meteorological situation

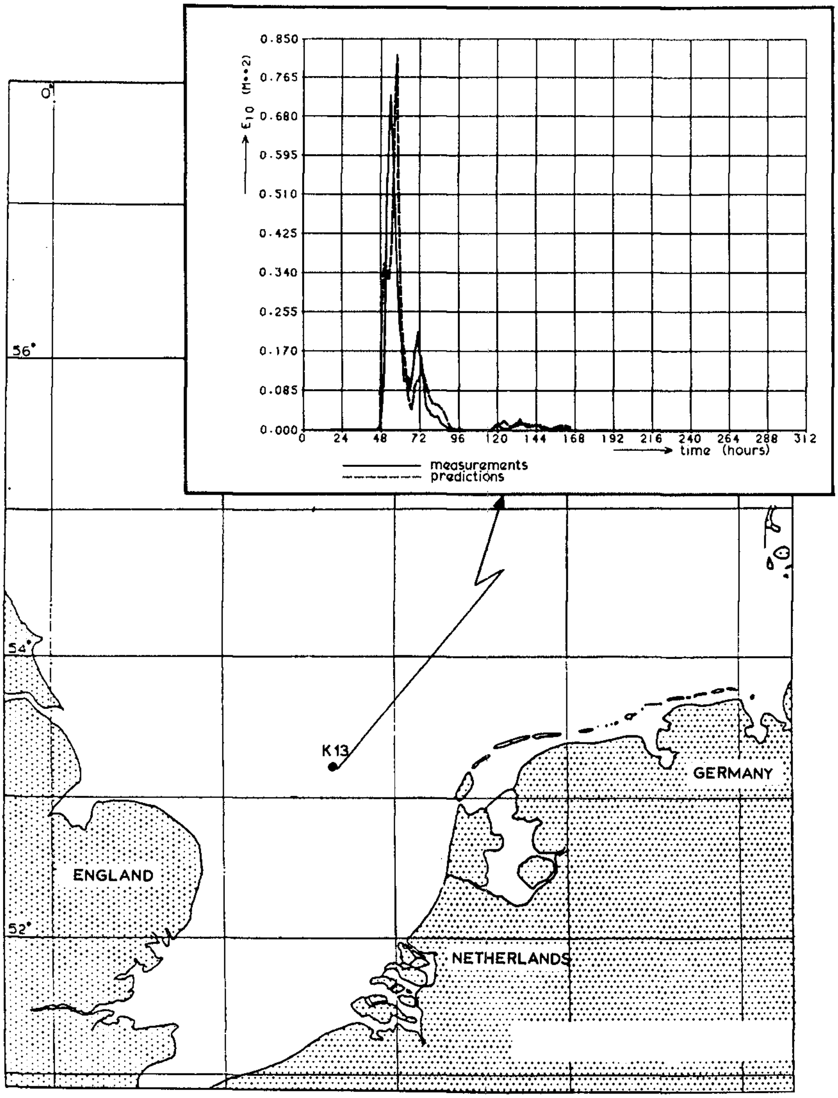

Fig. 4 The North Sea wave monitoring network

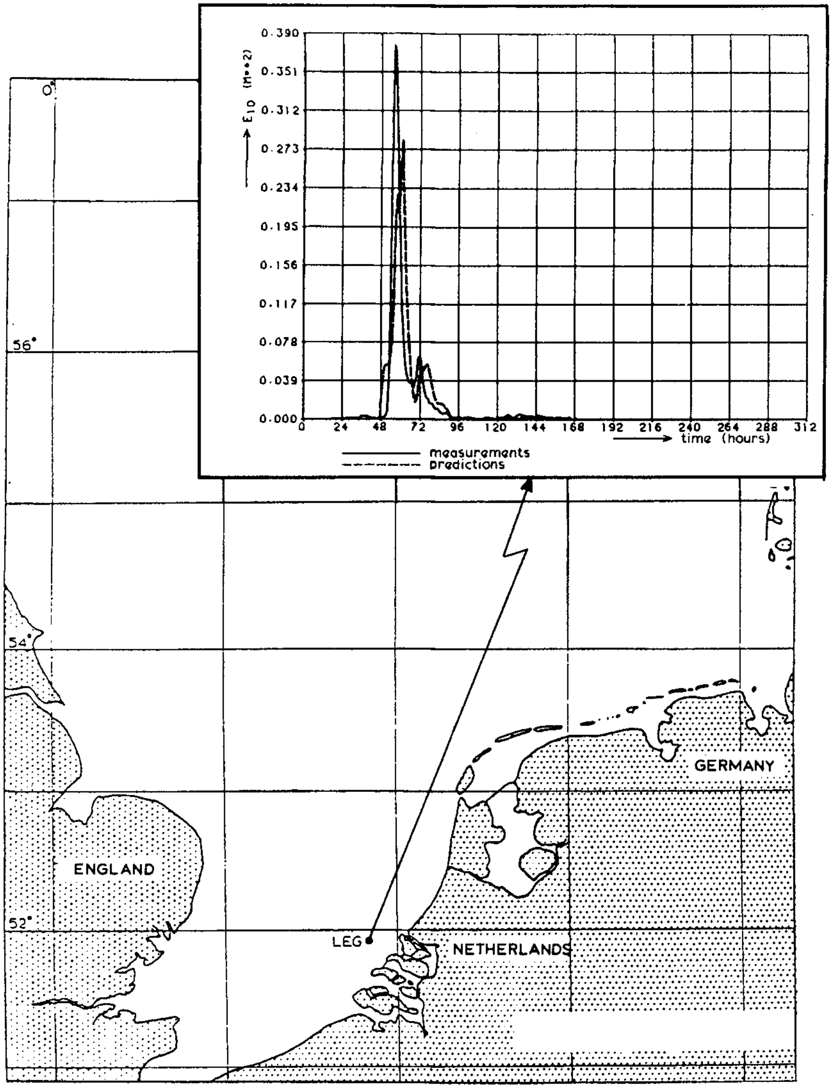

Fig. 5 The North Sea wave monitoring network

86b). Before the Norwegian coast a through of low pressure is moving to the South. On the west side of this through the wind speed increases from 2 m/s to 19 m/s. A combination of sea and swell is propagating into the North Sea. As can be seen in the Figures 4 and 5 the agreement is quite good. The differences between the predictions and observations are caused in part by an incorrect number of dispersion terms (r_1) and by an incomplete modelling of the wave physics.

4. Conclusions

The ARMAX system as described in this paper integrates, to a certain extent, observations and physical knowledge in one system. The system processes point measurements to predict wave energy at pre-selected locations.
Although the physical modelling as applied in the data pre-processing module is rather simple, the predictions are quite good.

The operational aspects of the prediction system can be summarized by:
- because of its size the system can be easily implemented on small computer systems
- because of its recursive parameter estimation algorithm the model has a short computing time
- because of its cascade structure it can be easily extended with more monitoring stations.

Because of its fast processing of new information the advantage of the model, as described here, is that in difficult meteorological conditions the ARMAX system gives immediate insight in the new hydrometeorological situation.

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