CHAPTER FORTY

RUN-UP OF RANDOM WAVES ON GENTLE SLOPES

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ABSTRACT

This paper investigates the following characteristics of run-up of random waves on gentle slopes experimentally: (1) the run-up wave energy spectrum, (2) the ratio of the number of run-up waves to that of incident waves, (3) the relationship between the representative run-up height, the deep-water wave steepness and the beach slope, (4) the run length of run-up heights, and (5) the effect of wave grouping on representative run-up heights.

Main results are summarized in the chapter of conclusions.

1. INTRODUCTION

Wave up-rush and back-rush on beaches are significant external forces for the change of beach profile near the shoreline. Besides, the height of wave run-up is important in determining for the height of coastal structures such as seawalls, dykes and surge barriers, and is the limit of onshore side for on-offshore and littoral sand transports.

A number of studies for run-up of random waves on steep slopes, as well as for run-up of monochromatic waves, have been performed, because front faces of traditional coastal structures are generally steep and accurate design data of wave run-up on steep slopes have been needed. Recently, seawalls and dykes with gentle slopes have been recommended, if possible, from the reason that sand beaches in front of the structures with steep slopes are apt to be lost during storms and from the reason of a good view and utilization of the beaches. However, studies for run-up of random waves on relatively gentle slopes like natural beaches are very few.

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When a beach slope becomes gentle, a period of one cycle of up-rush and back-rush of a wave becomes long and the interaction between a back-rush and a subsequent up-rush (or, overtaking and capturing of bores on beaches) becomes remarkable. The characteristics of run-up of random waves will depend on the time series of waves coming to the shoreline, and also on wave grouping of incident waves in deep water.

The purpose of this paper is to examine the following characteristics of run-up of random waves on gentle slopes experimentally by using random waves with two different groupiness factors defined by Funke and Mansard$^{1)}$, but with the same spectrum of Pierson-Moskowitz type: (1) the run-up wave energy spectrum, (2) the ratio of the number of run-up waves to that of incident waves, (3) the relationship between the representative run-up height, the deep-water wave steepness and the beach slope, (4) the run length of run-up heights, and (5) the effect of wave grouping on representative run-up heights.

2. EXPERIMENTS

A series of tests was carried out in a 50cm wide, 27m long and 75cm deep wave flume. At one of the ends of the wave flume, a random wave generator is installed. Beach slopes were selected as 1/5, 1/10, 1/20 and 1/30. The depth of water, h, in a uniform section of the flume was kept 45cm, but 43cm only in the case of a 1/30 beach slope. The outline of experimental apparatus is shown in Fig.1.

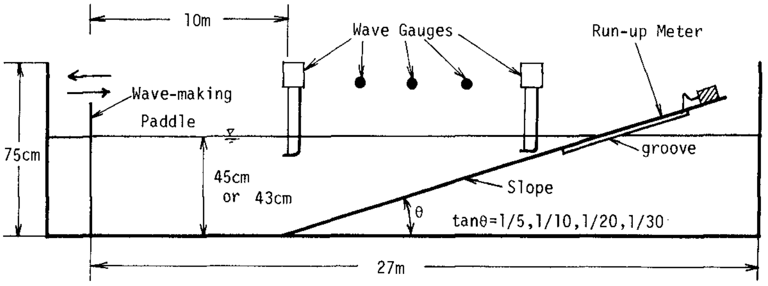


Fig.1 Experimental apparatus.

A wave gauge of capacitance type was used as a run-up meter, which was installed in a groove (3cm wide and 1cm deep) along a centerline of the slope to coincide the height of the capacitance wire (2.2mm diameter and 2m long) with the slope surface, see Photo.1. The run-up meter was calibrated by moving every 10cm along the slope surface. The calibration curve was closely approximated by a straight line. Additional calibrations of the run-up meter were done comparing

Photo.1 Run-up meter.

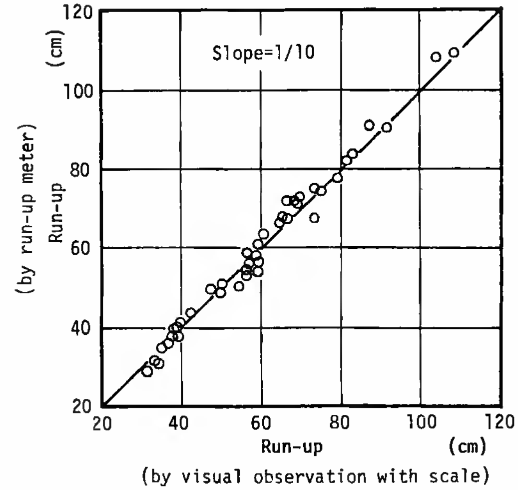


Fig.2 Comparison of run-up heights by run-up meter and by visual observation with scale.

run-up heights measured by the run-up meter and by a visual observation with a scale. Both run-up heights were nearly the same. An example of the comparison is shown in Fig.2, and the correlation coefficient of both run-up heights is about 0.99. We believe that the run-up meter used here is sufficient to measure run-up of random waves.

Random waves used here were simulated2) to have Pierson-Moskowitz type spectra with six kinds of peak frequencies of 0.4, 0.5, 0.6, 0.8, 1.0 and 1.2Hz, and with two kinds of groupiness factors of 0.74 (Case 1) and 0.53 (Case 2), based on the study of Funke and Mansard1). Electric signals for generating the random waves have been recorded by an analog data recorder. Three kinds of input levels of the electric signals were used to generate random waves with the peak frequency from 0.4 to 0.8Hz (in Cases 1 and 2) and two kinds were used for random waves with the peak frequency of 1.0Hz (in Cases 1 and 2) and 1.2Hz (in Case 1). A series of totally 30 tests were carried out for each beach slope.

Water surface variations were measured by using five wave gauges of capacitance type at water depths of 45 (40cm for a 1/30 beach slope), 20,15,10 and 5cm (W-1 ~ W-5).

Both of water surface variations and run-up variations (shoreline oscillations) were recorded simultaneously by an analog data recorder. The records were digitized by an A-D converter at the sampling interval of 0.04 second. The numbers of data are 40 000, 31 000, 26 500, 19 500, 15 500 and 12 500 for cases of random waves with peak frequencies of 0.4, 0.5, 0.6, 0.8, 1.0 and 1.2Hz. The numbers of individual waves at the wave gauge W-1 in each record are from 650 to 900.

3. EXPERIMENTAL RESULTS AND CONSIDERATIONS

3.1 Characteristics of incident waves

Table 1 shows representative quantities of waves measured at the wave gauge W-1 (40cm water depth), and the wave height and wave steepness in deep water, H_0 and H_0/L_0, calculated from the significant wave, for the experiments of a 1/30 beach slope. Individual waves are defined by the zero-up-cross method.

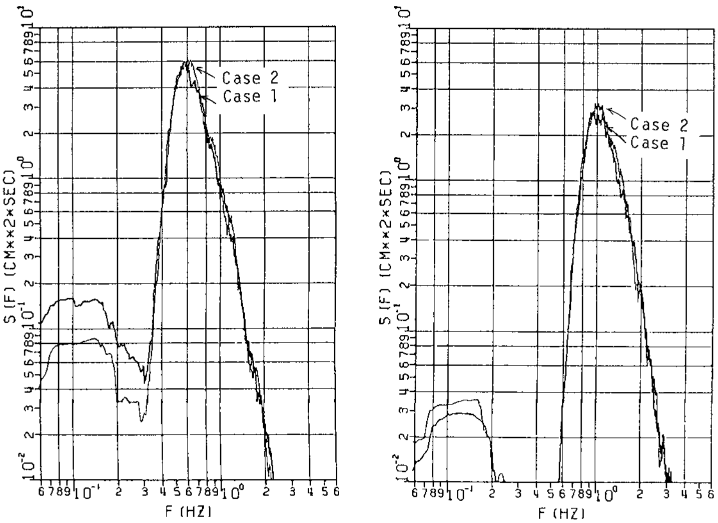
Fig.3 shows two examples of energy spectra of incident waves, in which (a) is for S30-Case 1-7 and S30-Case 2-7, and (b) for S30-Case 1-13 and S30-Case 2-13. The energy spectra of waves with the same case number of Cases 1 and 2 are almost the same except in the region of low frequency smaller than 0.3Hz. The difference of energy spectra in the low frequency region depends mainly on long waves generated by short waves with the different wave groupiness factors.

Fig.4 shows the groupiness factor, GF, against the deep-water wave steepness H_0/L_0. The average values of GF in Cases 1 and 2 are 0.73 and 0.57 respectively, while the expected values of GF are 0.74 and 0.53(2). Random waves with a small GF cannot be well reproduced in a wave tank when the deep-water wave steepness is larger than 0.03 (this corresponds to $kh>1.36$, k: the wave number), which is discussed in another paper3).

Statistical properties of random waves related to wave groups have been examined in the previous paper2).

Table 1 Representative quantities of incident waves in case of 1/30 beach slope.

Slope	Fp		H_0	H_0/L_0	H_{max}	$H_{1/3}$	H	T_{max}	$T_{1/3}$	T
1/30	0.4	S30-Case1- 1	4.685	0.0061	9.549	4.762	2.841	2.680	2.222	1.732
		2	3.560	0.0047	7.108	3.612	2.137	2.680	2.212	1.704
		3	2.641	0.0035	5.265	2.678	1.542	2.600	2.208	1.630
	0.5	4	6.085	0.0128	10.479	5.773	3.550	2.000	1.743	1.431
		5	4.761	0.0100	8.186	4.517	2.777	1.960	1.744	1.433
		6	3.447	0.0071	5.860	3.277	1.973	2.040	1.760	1.408
	0.6	7	7.664	0.0208	12.243	7.100	4.434	1.560	1.536	1.300
		8	5.905	0.0162	9.977	5.466	3.389	1.480	1.528	1.292
		9	4.748	0.0137	21.126	4.380	2.626	0.320	1.491	1.267
	0.8	10	9.993	0.0467	25.002	9.148	5.849	0.240	1.171	1.075
		11	7.830	0.0364	13.676	7.168	4.576	1.240	1.174	1.050
		12	5.720	0.0272	9.910	5.240	3.352	1.240	1.162	1.033
	1.0	13	6.576	0.0496	10.657	6.244	3.979	0.960	0.922	0.865
		14	5.357	0.0412	8.982	5.097	3.287	0.880	0.913	0.838
	1.2	15	6.839	0.0646	10.536	6.642	4.263	0.760	0.824	0.819
		16	5.881	0.0584	8.941	5.736	3.617	0.760	0.804	0.778
1/30	0.4	S30-Case2- 1	4.754	0.0059	7.000	4.872	3.176	1.880	2.278	1.846
		2	3.472	0.0042	5.158	3.565	2.312	1.880	2.290	1.828
		3	2.871	0.0035	4.251	2.945	1.872	1.880	2.282	1.785
	0.5	4	6.187	0.0117	9.302	5.949	3.906	1.440	1.843	1.527
		5	4.701	0.0090	7.287	4.514	2.918	1.400	1.833	1.497
		6	3.851	0.0073	5.939	3.692	2.390	1.400	1.840	1.502
	0.6	7	8.105	0.0223	12.088	7.502	4.955	1.280	1.526	1.356
		8	6.085	0.0165	9.017	5.639	3.717	1.280	1.538	1.351
		9	4.997	0.0134	7.313	4.634	3.054	1.240	1.546	1.349
	0.8	10	9.989	0.0464	14.869	9.143	5.953	1.120	1.174	1.063
		11	7.492	0.0360	10.991	6.867	4.483	1.200	1.155	1.030
		12	6.344	0.0308	8.534	5.817	3.835	1.120	1.149	1.035
	1.0	13	5.514	0.0434	9.069	5.259	3.369	0.920	0.902	0.828
		14	6.996	0.0528	10.701	6.617	4.260	0.960	0.919	0.865



(a) (b)

Fig.3 Energy spectra of incident waves at the wave gauge W-1 in case of 1/30 beach slope. (a) f_p=0.6Hz, (b) f_p=1.0Hz.

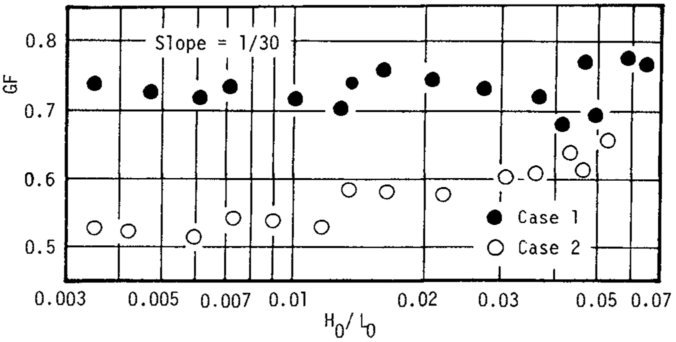


Fig.4 Groupiness factor of incident waves at the wave gauge W-1. Groupiness factor was defined by Funke and Mansard(1) to represent wave groupiness.

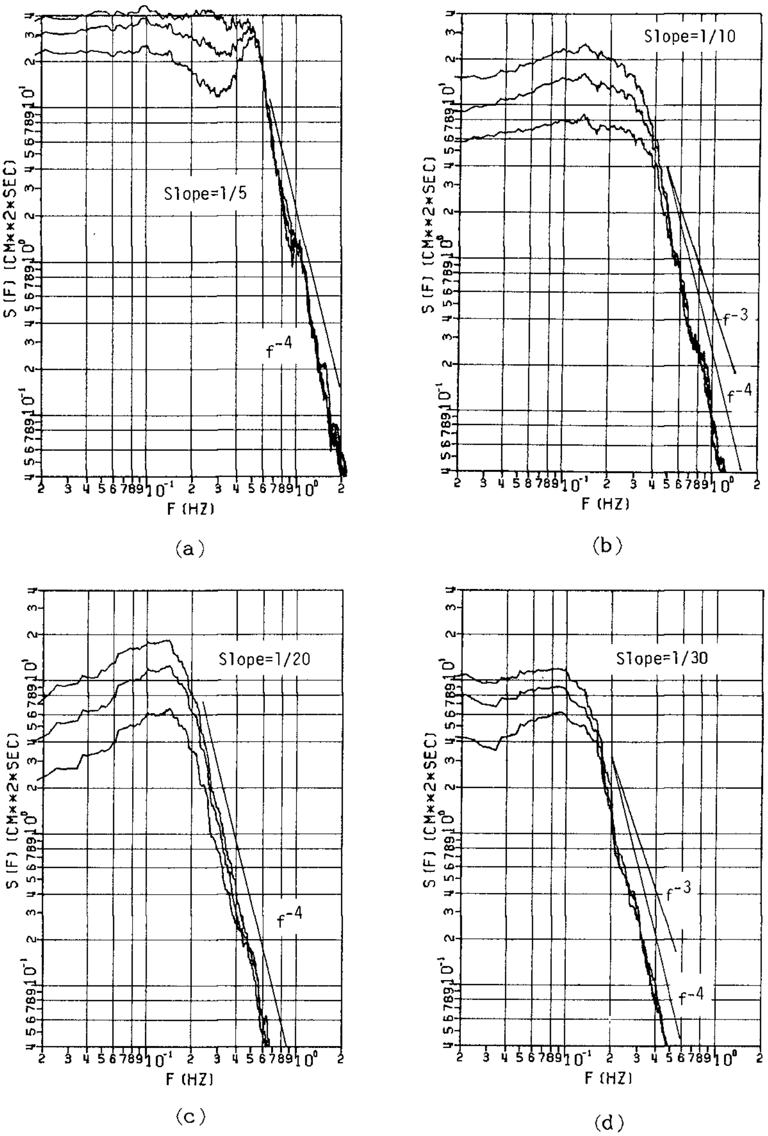
3.2 Energy spectrum of run-up waves (run-up variations)

Huntly, Guza and Bowen(4) remarked that run-up wave energy spectra have a universal 'saturation' form of f^{-4} dependence for the frequency region over which incident waves are large enough to break. Guza and Thornton(5) mentioned that run-up wave energy spectra in the saturation region show a f^{-3} dependence and spectral energy densities are independent of incident wave energies. The difference of both results is the f^{-4} or the f^{-3} dependence, which is considered as the effect of beach slopes by Guza et.al.

Fig.5 shows run-up wave energy spectra when the energies of incident waves with the peak frequency 0.6Hz (Case 1) are changed, for 1/5, 1/10, 1/20 and 1/30 beach slopes. It is seen from these figures that run-up wave energy spectra are saturated and show the f^{-4} dependence in the region of high frequency independently of incident wave energies and of beach slopes.

Fig.6 shows the relationship between the beach slope and the spectral energy density of run-up waves in the saturation region, in which values of the spectral energy density are plotted by the mean value and the standard deviation. It is found from this figure that the spectral energy density at a given frequency in the saturation region depends on the beach slope and is nearly proportional to $\tan^4\theta$.

These results of run-up wave energy spectra such as the f^{-4} and $\tan^4\theta$ dependence are the same as those estimated from the theoretical consideration by Huntly et.al(4) and Guza et.al(5).



(a) (b)

(c) (d)

Fig.5 Run-up wave energy spectra. (a) tanθ=1/5, (b) tanθ=1/10, (c) tanθ=1/20, (d) tanθ=1/30.

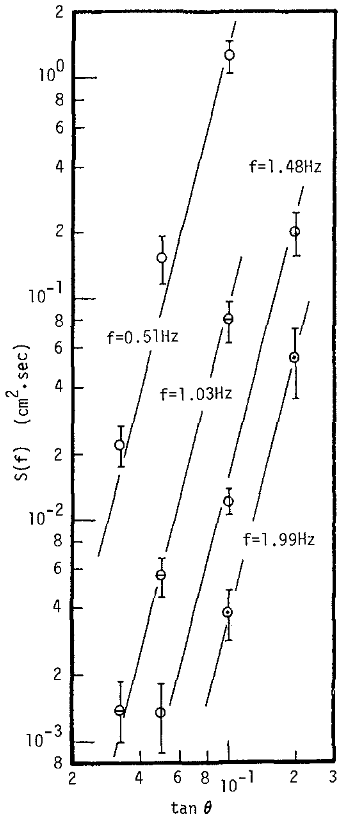


Fig.6 Relationship between beach slope and spectral energy density of run-up waves in saturation region.

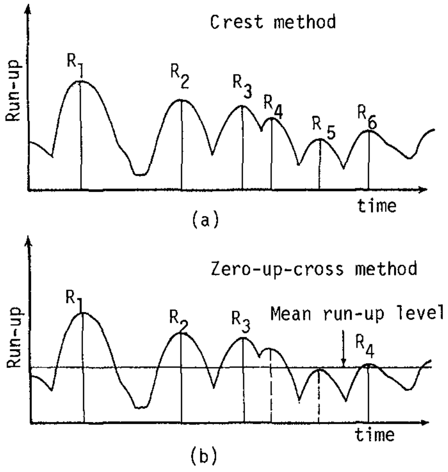


Fig.7 Definition method of run-up waves.

3.3 Individual wave analysis of run-up waves

(1) Difinition of an individual run-up wave

There are several defintion methods of an individual wave of run-up wave such as a zero-up-cross, a zero-down-cross, a crest-to-trough and so on. The number of individual run-up waves and the representative run-up heights change with the definition method. Fig.7 shows an example of definition of an individual wave, in which (a) is the crest method, (b) the zero-up-cross method. The number of run-up waves by the crest method is two waves more than that by the zero-up-cross method. In our experimental results, the ratio of the number of individual run-up waves defined by the zero-up-cross method to that by the crest method is from about 0.4 to 0.9 depending on the beach slope, and the ratio of significant run-up height is about 1.1 and the ratio of mean run-up height is about 1.2.

In the following sections, an individual run-up wave is defined by the crest method.

(2) Number of individual run-up wave

In the case of gentle slopes, it takes place that a wave (bore) coming to the shoreline cannot run up if the back-rush of a preceding wave (bore) is large and that a wave (bore) running up on the beach face is overtaken and captured by a subsequent wave (bore) before the wave (bore) reaches a maximum run-up height. Therefore, the number of run-up waves (the frequency of run-up waves) will be reduced compared to that of incident waves. Fig.8 shows the ratio α of the number of run-up waves to that of incident waves at the wave gauge W-1. The experimental data can be well arranged by the surf similarity parameter, $\xi(=\tan\theta/\sqrt{H_0/L_0})$, as shown in Fig.8. The ratio α becomes small as ξ becomes small (or the beach slope becomes gentle and the deep-water wave steepness becomes large). In this result, the effect of wave grouping is not clear.

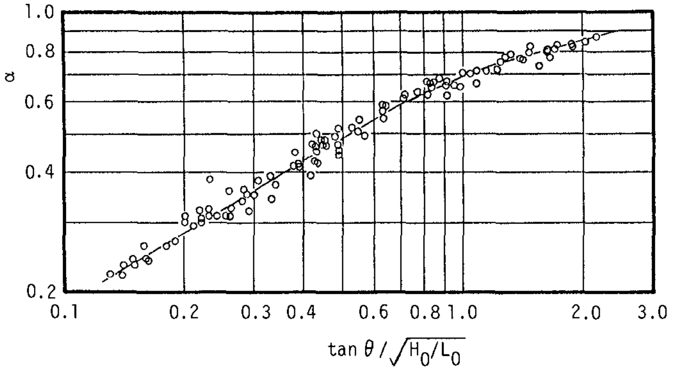


Fig.8 Ratio of the number of run-up waves to that of incident waves.

(3) Representative run-up heights

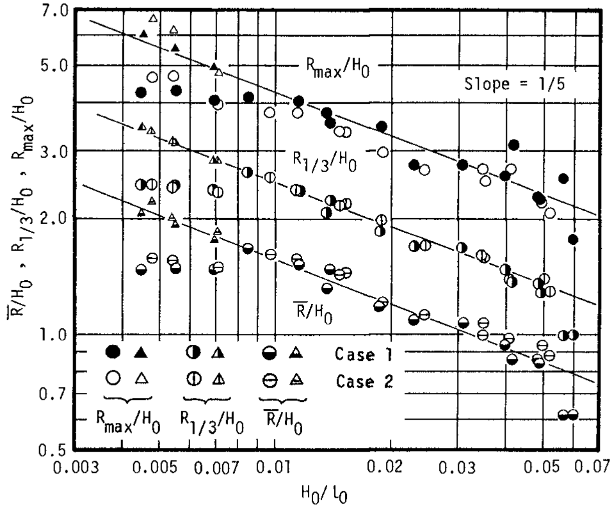
According to the dimensional analysis, the height of run-up, R, on an impermeable, smooth slope is expressed as follows:

$$\frac{R}{H_0}=f\left(\frac{H_0}{L_0},\tan\theta,\frac{h_0}{H_0}\right), \tag{1}$$

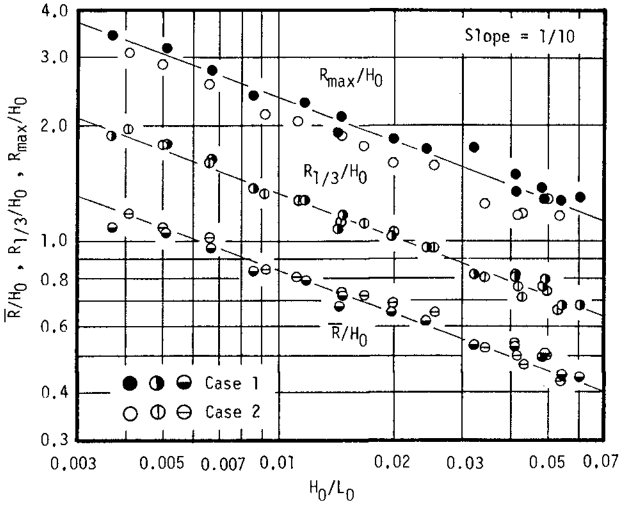
in which, h_0 is the water depth at the toe of a slope. When h_0/H_0 is larger than 3.0, the effect of h_0/H_0 is negligible6) and Eq.(1) becomes

$$\frac{R}{H_0}=f\left(\frac{H_0}{L_0},\tan\theta\right). \tag{2}$$

Hunt7) proposed the following equation for run-up heights of monochromatic waves based on the experimental data:

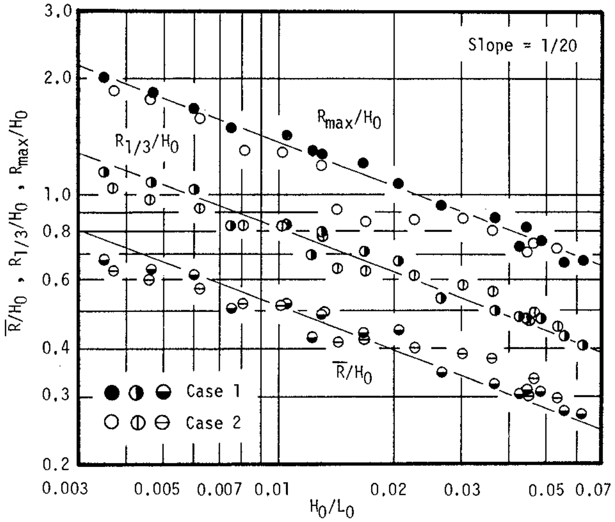


(a)

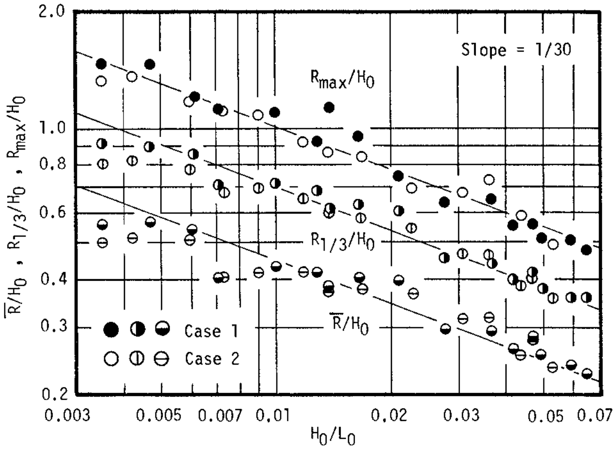


(b)

Fig.9 Relationship between representative run-up heights and deep-water wave steepness. (a) $\tan\theta=1/5$, (b) $\tan\theta=1/10$, (c) $\tan\theta=1/20$, (d) $\tan\theta=1/30$.



(c)



(d)

Fig.9 (c) (d) (Continued).

$$\frac{R}{H_0}=\tan\theta/\sqrt{H_0/L_0}\,. \qquad (3)$$

In this paper, representative run-up heights of random waves are examined based on Eq.(2), in which H_0 and L_0 are taken as the significant wave height and wave length in deep water (see Table 1).

Fig.9 shows the relationship between the representative run-up heights (R_{max} : maximum run-up height, $R_{1/3}$: significant run-up height, $\overline{R}$: mean run-up height) and the deep-water wave steepness. Straight lines were drawn parallel to each other in all figures, so as to represent the tendency of the experimental data of which the deep-water wave steepness is larger than 0.008. The gradient of the lines is -0.37, which is a little different from -0.5 by Hunt. In the cases of smaller deep-water wave steepness than 0.005 for 1/20 and 1/30 beach slopes, the values of $R_{1/3}/H_0$ and $\overline{R}/H_0$ are a little smaller than the values by the straight lines. Especially for the 1/5 beach slope, all representative run-up heights are considerably smaller than the values by the straight lines in the region of smaller deep-water wave steepness than 0.008. Although we used the same electric signal to generate random waves, the significant wave height at the wave gauge W-1 in the case of the 1/5 beach slope was large compared with the results in other gentler slopes 1/10, 1/20 and 1/30 due to the wave reflection at the slope. The larger value of H_0 causes the smaller values of R_{max}/H_0, $R_{1/3}/H_0$ and $\overline{R}/H_0$. When we use the mean value of H_0 obtained in other gentler slopes, the run-up heights are plotted with triangular shape symbols (in Fig.9(a)) represented well by the straight lines.

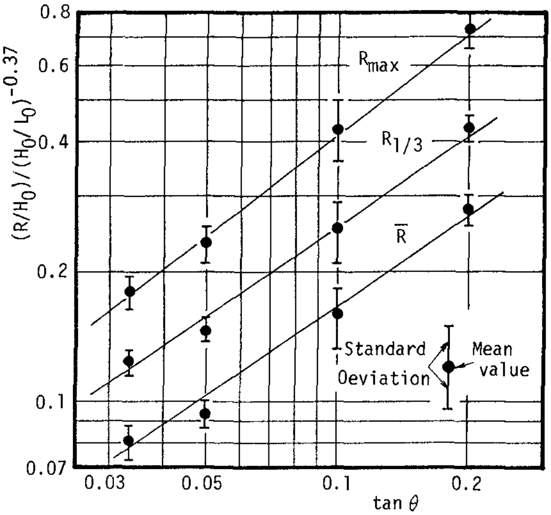


Fig.10 Relationship between run-up heights and beach slope.

The maximum run-up height in Case 1 of which the groupiness factor is about 0.73 is about 10% larger than that in Case 2 of which the groupiness factor is about 0.57. However, there is little effect of wave grouping on other representative run-up heights of $R_{1/3}/H_0$ and $\overline{R}/H_0$.

Fig.10 shows the relationship between the representative run-up heights and the beach slope tanθ, in which experimental data are plotted by the mean value and the standard deviation. It is seen from this figure that the representative run-up heights increase as the beach slope becomes steep. The gradients of the straight lines are 0.69 for the significant and mean run-up heights, and 0.77 for the maximum run-up height.

(4) Experimental formula for run-up height of random waves

Here, we formulate the run-up height by the following two equations:

$$\frac{R}{H_0}=a(\tan\theta)^b(H_0/L_0)^c, \tag{4}$$

$$\frac{R}{H_0}=d(\tan\theta/\sqrt{H_0/L_0})^e. \tag{5}$$

The coefficients in Eqs.(4) and (5) are determined by the least squares method, based on the experimental data of which the deep-water wave steepness is larger than 0.008 for the 1/5 beach slope and larger than 0.005 for the 1/20 and 1/30 beach slopes.

Table 2 shows the result, in which σ denotes the standard deviation of the difference between the experimental value and the formula. The standard deviation in Table 2(b) is almost the same as that of Table 2(a). We propose the simple experimental formula for run-up heights of random waves, represented by the surf similarity parameter of Eq.(5).

(5) Run length of run-up heights

The highest wave in a wave record does not appear alone but usually appears with several high waves. Such a phenomenon was firstly studied by Goda[8]. On the other hand, the highest run-up appears singly very often. Table 3 shows the distribution of run length of run-up heights exceeding the significant run-up height and the mean run-up height, based on all 120 test runs containing 54 244 run-up waves. The mean length of runs of run-up heights is 1.18 for $R>R_{1/3}$ and 1.95 for $R>\overline{R}$, which is smaller than the mean length of runs of wave heights (1.42 for $H>H_{1/3}$ and 2.37 for $H>\overline{H}$). The mean length of conditional runs of run-up heights containing the highest run-up is 1.15 for $R>R_{1/3}$ and 1.78 for $R>\overline{R}$ (2.37 for $H>H_{1/3}$ and 4.82 for $H>\overline{H}$). The highest run-up occurs when the preceding back-rush is small, and the subsquent run-up height is reduced by the back-rush of the highest run-up. Therefore, the highest run-up is apt to appear alone.

(6) Effect of wave grouping on run-up heights

It was demonstrated by Carstens, Torum and Traetteberg[9] and Johnson, Mansard and Ploeg[10] that random waves with large wave grouping cause larger wave run-up and therefore severer damage to rubble mound breakwaters than waves with poor wave grouping. Van Ooschot[11] reported that random waves with a large spectral width parameter cause larger wave run-up than waves with a small spectral width parameter, which means random waves with poor wave grouping

Table 2 Experimental formulas for run-up heights of random waves.

(a) $\frac{R}{H_0}=a(\tan\theta)^b(H_0/L_0)^c$

	Crest Method				Zero-up-cross Method			
	a	b	c	σ	a	b	c	σ
R_{max}	2.565	0.783	−0.367	0.151	2.565	0.783	−0.367	0.151
$R_{1/3}$	1.308	0.696	−0.361	0.095	1.375	0.686	−0.363	0.088
$\bar{R}$	0.946	0.697	−0.330	0.072	0.992	0.668	−0.356	0.079

(b) $\frac{R}{H_0}=d\left(\frac{\tan\theta}{\sqrt{H_0/L_0}}\right)^e$

	Crest Method			Zero-up-cross Method		
	d	e	σ	d	e	σ
R_{max}	2.319	0.771	0.159	2.319	0.771	0.159
$R_{1/3}$	1.378	0.702	0.094	1.497	0.695	0.086
$\bar{R}$	0.878	0.688	0.074	1.085	0.678	0.077

Table 3 Distribution of run length of run-up heights.

Run Length	Ordinary Runs		Runs containing R_{max}	
j	$R>R_{1/3}$	$R>\bar{R}$	$R>R_{1/3}$	$R>\bar{R}$
1	5396	6449	102	58
2	800	3162	18	36
3	138	1630		16
4	1	777		6
5	13	373		1
6	1	174		1
7		86		
8		36		
9		14		
10		12		
11		3		
mean	1.18	1.95	1.15	1.78

(120 test runs, containing 54,244 run-up waves)

cause larger wave run-up. In our results, the maximum run-up height in Case 1 (large wave grouping) is about 10% larger than that in Case 2 (poor wave grouping). However, there is little effect of wave grouping on the significant and mean run-up heights.

Fig.11 shows the groupiness factor at the wave gauge W-5 in 5cm water depth, which shows the degree of wave grouping before waves run up. It is seen from this figure that the values of *GF* in cases 1 and 2 become small compared with the results shown in Fig.4 and the difference of wave grouping between in Cases 1 and 2 diminishes due to wave breaking in the surf zone.

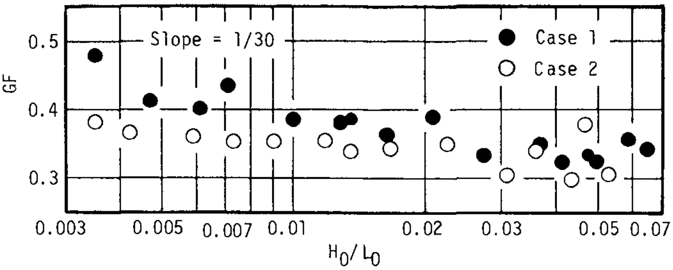


Fig.11 Groupiness factor at the wave gauge W-5 after wave breaking.

The change of wave grouping in shallow water has been investigated by the authors[12),13),14)].

In the experiments by Carstens et.al and Johson et.al, it seems that since the slope of a breakwater is very steep such as 1:1.25 and 1:1.5, the properties of wave grouping are not changed on the slope and the effect of wave grouping on wave run-up and stability of the breakwater was remarkable.

CONCLUSIONS

This paper examined the characteristics of wave run-up of random waves on gentle slopes experimentally by using random waves having the same spectrum and two different groupiness factors. Main results of this paper are summarized as follows:

1) Run-up wave energy spectra show a f^{-4} dependence in the region of high frequency (saturation region) independently of incident wave energies and beach slopes, and also show a $\tan^4\theta$ dependence in the saturation region.

2) The ratio of the number of run-up waves to that of incident waves at the deepest water (45cm or 40cm in our experiments) becomes small as the beach slope becomes gentle and the deep-water wave steepness becomes large. The experimental data can well be arranged by the surf similarity parameter as shown in Fig.8. Reducing of the number of run-up waves is caused by overtaking and capturing of bores in the surf zone and on the beach face, and by large back-rushes suppressing

following running up waves.

3) A linear relation is obtained when two pairs of representative run-up heights vs. deep-water wave steepnesses and representative run-up heights vs. beach slopes are plotted on a log-log scale. We propose the simple experimental formula for run-up heights of random waves represented as a function of the surf similarity parameter by Eq.(5).

4) The highest run-up of each test run appears singly very often because the highest run-up occurs when the preceeding back-rush is small, and the subsequent run-up height is reduced by the back-rush of the highest run-up.

5) The maximum run-up height of random waves in Case 1 of which the groupiness factor is about 0.73 is 10% larger than that in Case 2 of which the groupiness factor is 0.57 on the average. However, there is little effect of wave grouping on other representative run-up heights $R_{1/3}/H_0$ and $\bar{R}/H_0$. This is due to the fact that the difference of wave grouping of random waves between in Cases 1 and 2 becomes small in very shallow water by wave breaking in the surf zone in the case of gentle beach slopes.

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