CHAPTER TWENTY NINE

A Nonlinear Model of Irregular Wave Run-up Height and Period Distributions on Gentle Slopes

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ABSTRACT

This paper discusses the probability distributions of irregular wave run-up height and period on gentle slopes.

Assuming that the long period component appeared on the run-up oscillation corresponds to the incident envelope wave period, a nonlinear model to estimate the probability distributions of run-up heights and periods is proposed.

Laboratory experiments on gentle slopes of 1/15,1/30 and 1/40, and field measurements on a natural sandy beach with swash slopes of 1/6 to 1/14 were performed to examine the proposed model. The proposed model is shown to agree with the experiments.

1. INTRODUCTION

The elucidation of irregular wave run-up mechanism on a gently sloping beach is one of the important Coastal Engineering problems. The run-up oscillation presents a shoreward boundary condition for the onshore-offshore sediment transport which results in the change of beach profile, and a precise prediction of the swash oscillation, especially run-up heights give a significant information for the construction site of dikes, beach nourishment works,etc.,

So far, investigations have been performed to verify the nature of the swash oscillation on gentle slopes from field measurements[1)-4)] and laboratory experiments[5)-9)]. Most of the foregoing researches have pointed out that the swash oscillation has long period components which are not included in an incident wave outside the surf-zone, and the power level in the low frequency range of the swash oscillation becomes larger than that of the incident individual wave.

Some theoretical models[10)-13)] to predict a probability distribution of run-up heights have been proposed. All the foregoing models, however,

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assumed that run-up heights of individual waves in an irregular wave train indicate the values following a formula proposed for a regular wave, such as Hunt's equation[14]. Therefore, the long period component observed in field and laboratory measurements has not been taken into consideration. In this sence, the foregoing models should be called a " linear model ". Then, a " nonlinear model " considering the long period components is expected to be settle down.

With this situation, the present paper is intended to propose a nonlinear model to evaluate the probability distribution of the run-up height and period of irregular waves on gently sloping beaches, from engineering purposes in mind.

First of all, the nature of wind wave-generated run-up oscillations on a sandy beach is discussed. Secondly, two-component composite waves showing a strong wave grouping are treated and the effect of wave groupiness on the run-up oscillation is revealed in laboratory flume. Lastly, the irregular wave-induced swash oscillation is investigated in an indoor wave tank. Based on the experimental fact that the long period on the swash oscillation is highly correlated with the incident envelope wave period, the probability distribution of swash oscillation periods is proposed to be given by a combined distribution of the incident individual wave and their envelope wave periods. By extending the idea, the probability distribution of run-up height is also proposed to be given by a combined distribution of the incident individual waves and their envelope waves-induced run-up height distributions. The proposed model is shown to agree with experiments.

2. FIELD MEASUREMENT

2 -1. Measurements

Field measurements were conducted at Kanaiwa Beach, Ishikawa Pref., in Honshu Island, during Sept., 1981 and Nov., 1983. The Kanaiwa Beach which faces to the Japan Sea is moderately well sorted sandy beach and the sand has a diameter of 0.5 mm and the coefficient of permeability of the beach is 0.124 cm/s .

A typical beach face topography is shown in Fig.1. The local beach slope where the run-up oscillation was measured was between 1/14 and 1/6. The slope in the surf-zone and just offshore was more gentle (< 0.01). Bar-trough topography was in general weak or absent, and the beach bottom counter was relatively uniform in the longshore direction (see Fig.2).

The change of tidal level at Kanaiwa Beach was so small enough to be constant during 45 min measurement. The swash oscillation was measured at three different locations (O, R and L in Fig.2), where the distances between R and O, and O and L were about 300 m. Most of the run-up oscillation data were collected at location O.

The run-up meter was a capacitance wire, and its length was 50 m and the diameter was 2 mm, and it was supported about 0.5 cm above the bed. Sand level changes required raising and lowering the support wood stakes. The run-up meter was calibrated before and after each data acquisition by shorting the wire at three different locations.

Nanao Harbor Construction Office at 1st Harbor Construction Bureau of Transportation Ministry has been measuring regularly 20 min wave data at 2 hours intervals at the depth of 20 m with the ultrasonic-type wave gauge

(see Fig.2). The tidal level has been also measured by the Office at the adjacent Kanazawa Port. In this research, the wave and tidal data measured by the Nanao Horbor Construction Office were used as an incident wave and an initial stillwater level, respectively.

The run-up oscillation was recorded on a magnetic tape with 45 min. The wave and run-up oscillation were cut discretely at 1 Hz intervals and a power spectrum was calculated by FFT method with 1024 points. The incident individual and envelope waves were defined by the zero-upcrossing method. On the other hand, the individual wave of run-up oscillation was defined here both by the zero-upcrossing and trough-to-trough methods. This is due to the following reason. The zero-upcrossing method is very objective, but it sometimes fails to define small waves which donot cross the mean water level. On the other hand, the trough-to-trough method is subjective, in general, and it can define noise-like small waves. Thus, both definitions have its own merits and demerits. Since there have been little discussions on which definition is better, this paper uses the two definitoins.

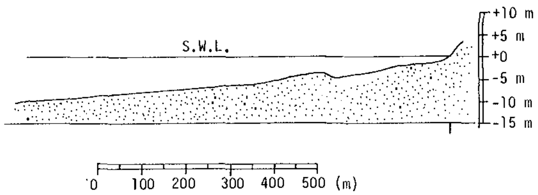


Fig.1 Typical beach topography

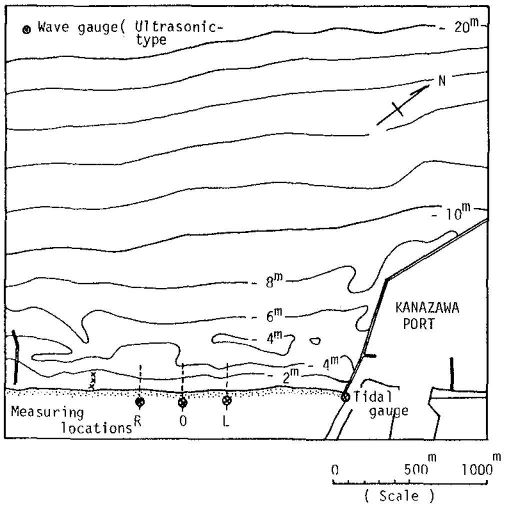


Fig.2 Measuring locations

3-2. Results and Discussions

Fig.3 is one example indicating an incident wave profile at the depth of 20 m and the corresponding run-up oscillation measured at the location O. It can be pointed out that the run-up oscillation has flat crests and sharply edged troughs in general.

Fig.4 shows an effect of the swash zone slope S on the non-dimensional period of run-up oscillation $\bar{T}_R/\bar{T}_I$, where $\bar{T}_R$ is the mean period of the run-up oscillation and $\bar{T}_I$ is the mean period of incident individual waves. The mean period of run-up oscillation defined by the trough-to-trough method as well as by the zero-upcrossing method becomes longer with decreasing of the swash zone slope S. $\bar{T}_R/\bar{T}_I$ defined by the zero-upcrossing method is about two times larger than those defined by the trough -to-trough method. The reason is that the former method neglects short period waves which do not cross the mean water level, on the other hand the latter method defines noise-like short period waves.

Thus, the swash oscillation cycle becomes longer than the incident individual wave periods on gentle slope such as 1/14 in our field measurement. The mechanics of generation of the long period component have been pointed out as follows[1)-8)].

(1) marked decay of wave energy around a spectral peak frequency by wave breaking(see Fig.5),
(2) amplification of long period components by nonlinear wave-wave interactions in the surf-zone,
(3) nolinear wave-wave inter-action on the swash zone,i.e. large bores overtaking and cap-turing smaller ones in the up-rush and downrush process,
(4) edge wave or standing long long wave, et..

Therefore, a new model con-sidering the long period compo-nent for predicting the swash oscillation is needed to be established.

Fig.5 indicates that the power spectral shape(A) of the run-up oscillation in low fre-quency range resembles approx-imately to that of the incident envelope wave(A'). On the other hand, the spectral shape(B) of the run-up oscillation in high frequency range(> 0.1 Hz) can be seen to correspond to that of the incident individual wave (B'). From Fig.4 and Fig.5, we assume here that the probability distribution of run-up oscilla-tion $P_{rR}(T_R)$ is , for 1st approxi-mation, given by a combined distribution of the incident individual wave period distri-bution P_{rI} (T_I) and the incident

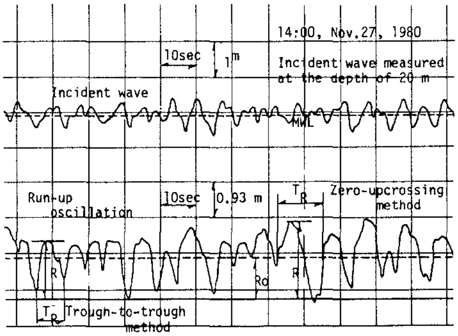


Fig.3 Incident wave and swash oscillation

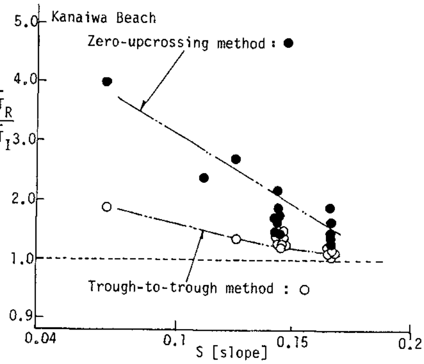


Fig.4 Effect of swash beach slope on period of run-up oscillation

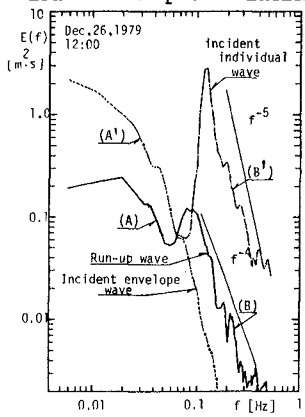


Fig.5 Power spectra of incident individ-ual and envelope waves and run-up wave

envelope wave period distribution $P_{rE}(T_E)$, as expressed by Eq.(1).

$$P_{rR}(T_R) = \alpha P_{rI}(T_I) + \beta P_{rE}(T_E)$$
$$\alpha + \beta = 1 \qquad (1)$$

In Eq.(1), α and β are weight coefficients. In case of $\alpha = 1$, the swash oscillation periods equal the incident individual wave periods and there is no long period component. On the other hand, $\beta = 1$ presents a situation where only long period components appear on the run-up oscillation. Therefore, β is an index showing a degree of nonlinear interaction in the surf and swash zones.

Fig.6 shows that the probability distribution of run-up oscillation period can be estimated by Eq.(1) for moderate values of α and β . The value of β for the zero-upcrossing wave is generally larger than that for the trough-to-trough wave.

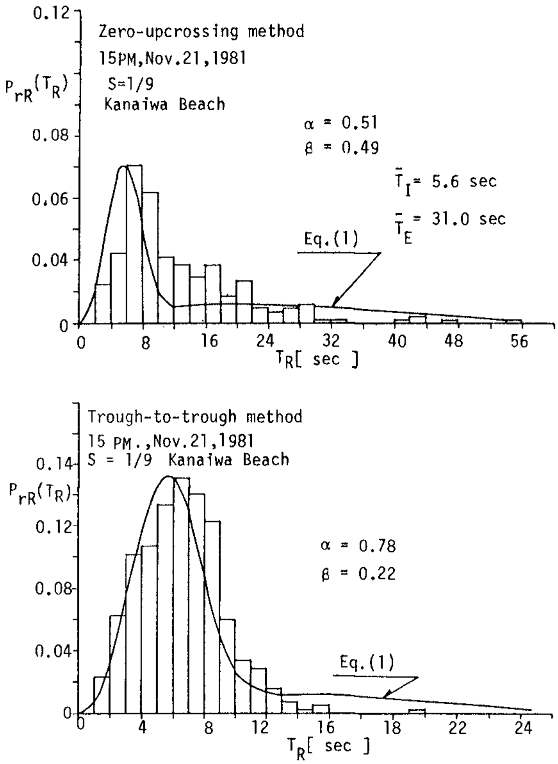


Fig.6 Probability distribution of run-up oscillation period (Comparison between measurements and Eq.(1))

3. TWO-COMPONENT COMPOSITE WAVE

3-1. Equipment and Procedure

Experiments were conducted to examine the effect of wave groupiness on run-up heights and periods on gentle slopes. In the experiments, an indoor wave tank in 0.75 m width, 0.9 m height and 30 m length was used. At one end of the wave tank, a uniform slope was installed. The slopes of 1/40, 1/30 and 1/15 were used in this experiment. At the other end of the tank, a flap-type wave generator was set, which is controlled by an electric dynamic shaker. The electric dynamic shaker is activated by an electric voltage input. Throughout the experiments, the stillwater depth was kept constant (73 cm) in front of the wave board.

Two-component composite waves were generated by composing two monochromatic waves ([H_1, T_1] and [H_2, T_2]) with use of a multi-wave composer, where H and T are the wave height and period, respectively. Wave periods of T_1 and T_2 were changed from 0.67 sec to 1.47 sec and were chosen to be almost equal each other in order to produce grouping or beat waves. H_1 and H_2 were also determined to have almost an equal value. The envelope wave period T^* and height H^* were 3.1 sec to 47.0 sec and 2.0 cm to 15.8 cm, respectively. T^* and H^* are indicated in Fig.7(c). The number of experimental waves were about 70.

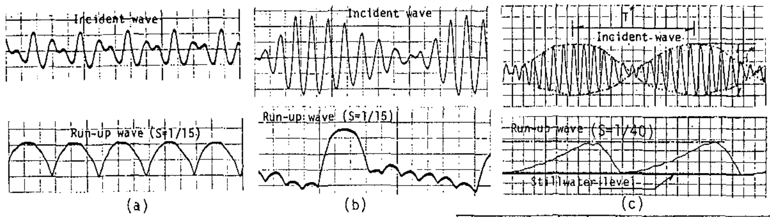
The incident water surface profile was measured by a capacitance-type wave gauge, and a time history of the run-up oscillation was measured by the run-up measuring instrument[13] which was set along the surface of slope bed. All the measured wave profiles were recorded onto magnetic tapes in about 120 sec.

Incident individual and envelope waves were defined by the zero-up-crossing method which is more objective than the trough-to-trough method. The run-up wave which is discussed in this paper is *a time history of run-up wave front* (swash oscillation) measured by the run-up measuring instrument. The run-up oscillation waves were defined by the zero-upcrossing and trough-to-trough methods.

3-2. Results and Discussions

(1) Time history of run-up oscillation

Fig.7 shows some typical examples of time histories of incident two-component composite waves and their corresponding run-up oscillations. As indicated in Fig.7(a),(b) and (c), incident individual wave periods do not, in general, appear on a time history of the run-up oscillation with decreasing of the bottom slope. And, the period of run-up oscillation becomes to close to the incident envelope wave period T^* with decreasing the beach slope(Fig.7(c)). This is very different from the case of the regular wave as in Fig.7(d). The physical explanation of this seems to be following. Since the individual wave heights change gradually and regularly with the beat period, the nonlinear wave-wave interaction,e.g. big waves overtaking and capturing smaller waves in the run-up and run-down process occur regularly and smoothly in the swash zone, which smooths out variation generated by incoming incident individual wave components. Therefore, it can be understood that the time history of the swash oscillation shows a change of the mean water level. Its oscillation period comes to close to the incident envelope wave period with decrease of the bottom slope.



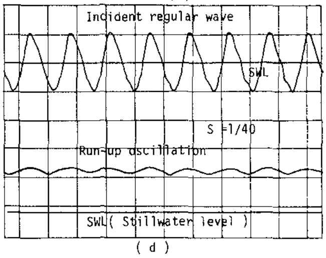


Fig.7 Incident waves and run-up oscillations of two-component composite and regular waves

(2) Wave run-up height

The run-up height is defined by a vertical distance of the run-up oscillation crest from the stillwater level. Based on a dimensional consideration, the run-up height R is dominated by 6 physical quantities,

$$R = F\ (T^*, H^*, T_1, h, g, S) \qquad (2)$$

here, h is the stillwater depth, g is the gravitational acceleration, and S is the beach slope. From Eq.(2), a non-dimensional run-up height R/H^* is governed by 4 factors;

$$R/H^* = F\ (H^*/L_o^*,\ \ T^*/T_1,\ h/L_o^1,\ S) \qquad (3)$$

where, $L_o^* = gT^{*2}/2\pi$ and $L_o^1 = gT_1^2/2\pi$.

The run-up height R of the two-component composite wave increases with increasing of H^* and T^*, in general, as indicated in Fig.8. This is very similar to the case of the regular wave[13]. The run-up height of two-component waves are generally higher than those of the regular waves. Putting it in another way,a grouping wave is apt to produce higher run-up height than the regular wave which does not show a grouping wave. The same thing can be said from Fig.9, where the wave height and period of the regular wave are equal to the maximum wave period and height among the incident individual waves. This experimental fact would imply that the run-up height of irregular waves becomes possibly higher than that of the regular wave.

By the way, the non-dimensional run-up height of the two-component composite wave R/H^* decreases as the incident envelope wave steepness H^*/L_o^* becomes larger, as indicated in Fig.10. In our experiments, the effect of h/L_o^1 on R/H^* was little (the figures are not presented in this paper.). From Fig.10, the non-dimensional run-up height R/H^* can be approximated by the following equations.

$$R/H^* = 0.05\ (H^*/L_o^*)^{-0.1} \qquad \text{for} \quad S = 1/40$$

$$\left.\begin{array}{lll} R/H^* = S/\sqrt{H^o/L^o} & \text{for } H^o/L^o \geq 0.05 & \\ \quad = 0.15 & \text{for } H^o/L^o < 0.05 & \text{for } S=1/30 \\ \text{where, } H^o/L^o = 0.766\,(T^*/T_1)^{2.18}\,(H^*/L_o^*) & & \\ R/H^* = S/\sqrt{H^o/L^o} & \text{for } H^o/L^o \geq 0.022 & \text{for } S=1/15 \\ \quad = 0.45 & \text{for } H^o/L^o < 0.022 & \\ \text{where, } H^o/L^o = 0.309\,(T^*/T_1)^{2.18}\,(H^*/L_o^*) & & \end{array}\right\} \quad (4)$$

Eq.(4) will be used for calculating run-up heights caused by the incident envelope waves in an irregular wave train in the next section.

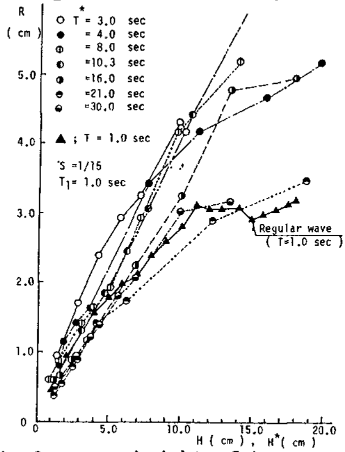


Fig.8 Run-up height of two-component composite waves (T1=1.0 sec)

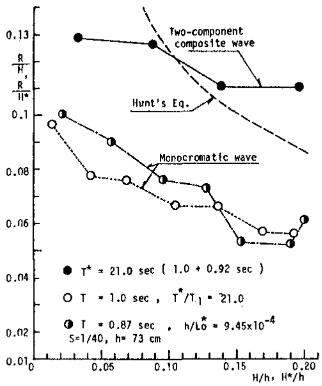


Fig.9 Comparison between run-up heights of regular and two-component composite waves

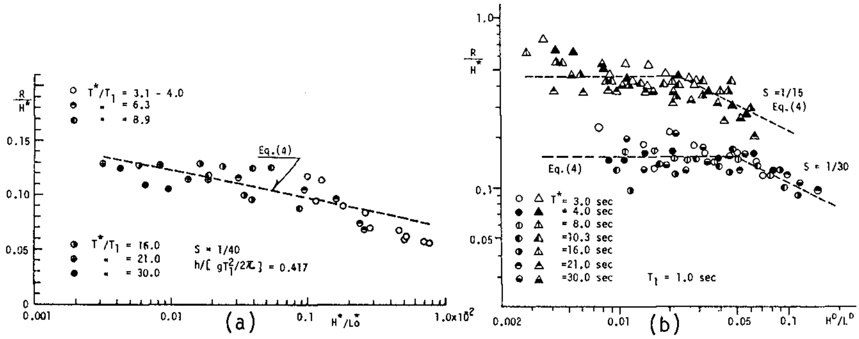


Fig.10 Non-dimensional run-up height of two-component composite wave

4. IRREGULAR WAVE

4-1. Equipment and Procedure

Experiments were carried out by using the same wave tank and instruments used for two-component composite wave experiments.

Irregular waves were generated to have a Bretschneider spectrum. In this work, 11 random waves(W.-1 - W.-11) were produced, and their power spectral and statistical quantities are given in Table 1. Time histories of incident individual and envelope waves and run-up waves were cut discretely, respectively, at each 0.05 sec, 0.5 sec, and 0.2 sec intervals, and the corresponding power spectra were calculated by BT-method with data of 4000 and freedom of 40, data of 6000 and freedom of 60 and data of 2400 and freedom of 30, respectively.

Incident individual and envelope waves were defined by the zero-upcrossing method. The run-up waves were defined both by the zero-upcrossing and trough-to-trough methods.

Table 1. Experimental waves

Wave	$H_{1/3}$	$\bar{H}$	$T_{1/3}$	$\bar{T}$	Q_p	Ho/Lo	S
W.- 1	3.5	2.2	2.2	1.9	2.17	0.005	1/40
W.- 2	5.0	3.3	2.1	1.9	2.22	0.008	
W.- 3	5.0	3.2	1.8	1.5	2.32	0.011	
W.- 4	7.4	4.9	1.3	1.2	2.18	0.027	
W.- 5	8.9	5.7	1.2	1.0	1.89	0.043	
W.- 6	6.0	3.9	0.8	0.7	1.48	0.060	
W.- 7	2.8	1.8	1.2	1.0	1.47	0.013	1/30 and 1/15
W.- 8	3.9	2.6	1.1	0.9	1.55	0.021	
W.- 9	8.0	5.1	1.2	1.1	1.94	0.037	
W.-10	7.9	5.1	1.0	0.9	1.98	0.051	
W.-11	10.3	6.8	1.1	1.0	2.08	0.062	

$H_{1/3}$; significant wave height, $\bar{H}$; mean wave height, $T_{1/3}$; significant wave period, $\bar{T}$; mean wave period, Q_p; spectral peakedness, Ho/Lo; equivalent deep water wave steepness, S; beach slope

Unit: cm for $H_{1/3}$ and $\bar{H}$, and sec for $T_{1/3}$ and $\bar{T}$

4-2. Results and Discussions

(1) Time history and power spectrum of run-up oscillation

Fig.11 shows time histories of run-up oscillation. Different from the regular wave, the irregular wave-caused run-up oscillation contains long period components which are not included in the incident wave at the wave board. Therefore, this fact shows that the assumption " *run-up heights of individual waves in an irregular wave train indicate the values following the formula, such as Hunt's formula, deduced for the regular wave* " is not established on the gentle slopes of 1/15, 1/30 and 1/40.

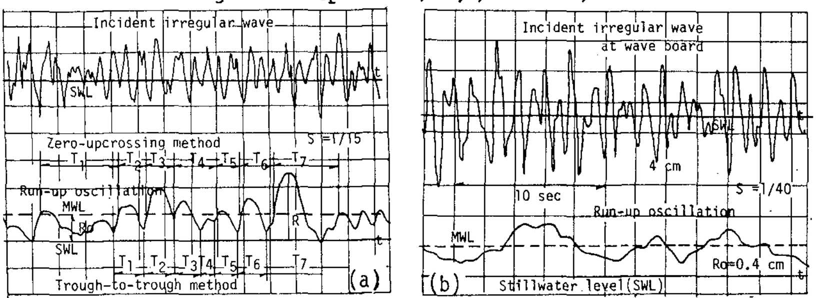


Fig.11 Time histories of incident wave and run-up oscillation

Fig.12(a) shows one example of power spectra of run-up oscillations. The remarkable feature of run-up oscillation is a marked increase of power density in a low frequency range than the spectral peak frequency of the incident individual wave. A predominant power peak is always recognized to appear at a little lower frequency side than the power spectral peak frequency of the incident individual wave. This is due to the reason of rapid decay of the wave energy around the spectral peak frequency by wave breaking and nonlinear wave-wave interactions in the swash and surf zones, etc..

The power spectral shape of run-up oscillation in a low frequency band is very similar to that of the incident envelope wave, different from the spectral shape of the incident individual wave. This would imply that the long period components which are supposed to be caused by nonlinear interactions including wave-wave interaction can be estimated from incident envelope wave periods. Typical examples are the cases of two-component composite waves discussed in the previous section.

Therefore, it may be possible to assume that the power spectrum of run-up oscillation is constructed by the two components.i.e. the individual wave and its envelope wave power spectra.

By the way, as indicated in Fig.12(a) and (b), the spectral slope of the run-up wave on high frequency range is proportional to f^{-4} (f; frequency). This fact coincides with experiments of Sutherland et al.[6].

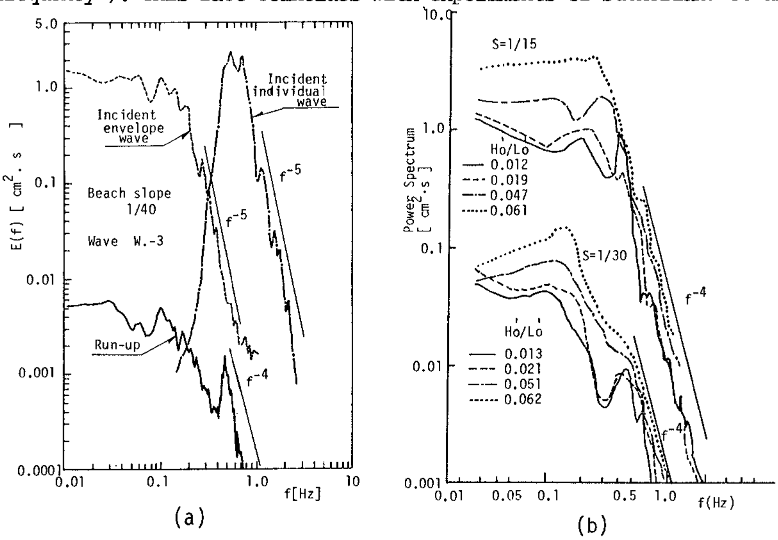
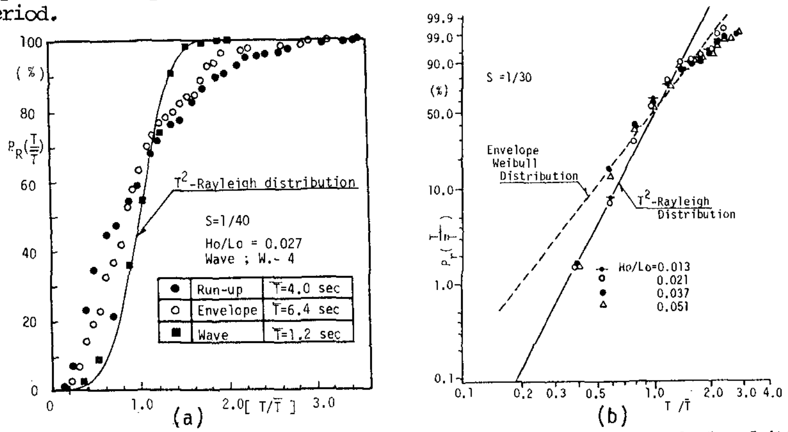


Fig.12 Power spectra of run-up oscillation on gentle slopes

(2) Distribution of run-up wave period

The distribution of run-up wave periods (run-up oscillation periods) is different from T^2-Rayleigh distribution which is usually observed for the incident individual wave, as indicated in figure 13. The run-up wave periods distribute comparatively widely from short periods to long periods, and the distribution becomes to have two peaks with increasing of individual wave steepness Ho/Lo, as in Fig.14. The second peak period appears in the period domain corresponding to the incident envelope wave period.



(Zero-upcrossing method S=1/40) (Trough-to-trough method S=1/30)

Fig.13 Cumulative distribution of period of run-up oscillation ($\bar{T}$; mean wave period)

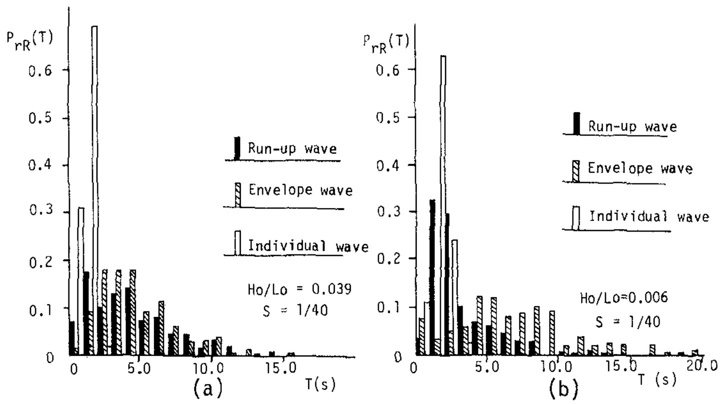


Fig.14 Probability distribution of period of run-up oscillation (Waves defined by zero-upcrossing method)

As already cited in this paper, the foregoing theoretical distributions for run-up heights are strongly based on the assumption of "*Equivalency*". The assumption is, however, inconsistent with experimental facts described above.

Since the period of long period components of run-up waves almost equals incident envelope wave periods and the spectral shape of swash oscillation in low frequency resembles that of the incident envelope wave, this paper uses the following assumption. That is, the long periods are appearances of incident envelope wave periods, and the occurrence frequency of short periods(individual wave periods) and long periods(envelope wave periods) on the run-up oscillation is probabilistic,because of extreme complexity of nonlinear wave-wave intreaction in the surf and swash zones.

Following the above-mentioned assumption, the probability density function of run-up wave period, $P_{rR}(T)$ is defined here,like the field measurement, by a following combined probability density function,

$$\left.\begin{aligned} P_{rR}(T) &= \alpha P_{rI}(T) + \beta P_{rE}(T) , \\ &= \alpha\left[\frac{m_{I1}}{n_{I1}} T^{m_{I1}-1} \exp\left(-\frac{1}{n_{I1}} T^{m_{I1}}\right)\right] + \beta\left[\frac{m_{E1}}{n_{E1}} T^{m_{E1}-1} \exp\left(-\frac{1}{n_{E1}} T^{m_{E1}}\right)\right] , \\ \alpha + \beta &= 1 , \end{aligned}\right\} \quad (5)$$

here,$P_{rI}(T)$ and $P_{rE}(T)$ are the probability density functions of the incident individual and envelope wave periods, respectively. Since thare are no established expressions for $P_{rI}(T)$ and $P_{rE}(T)$, they are given by Weibull probability density functions. In Eq.(5), n and m are, respectively, a scale and shape parameters, α and β are weight coefficients whose meaning is the same as in Eq.(1), and subsuffixes R,I and E indicate quantities regarding the run-up,incident individual and envelope waves. The subsuffix 1 shows quantities concerning incident wave periods.

The mean of the largest 1/k wave period,$T_{1/k}$ is defined by Eq.(6).

$$\bar{T}_{1/k} \int_{T_{1/k}}^{\infty} P_{rR}(T)\, dT = \int_{T_{1/k}}^{\infty} T\, P_{rR}(T)\, dT \quad (6)$$

From Eqs.(5) and (6), $\bar{T}_{1/k}$ is expressed by

$$\bar{T}_{1/k} = \frac{\left[\alpha\, n_{I1}^{1/m_{I1}}\, \Gamma\left(\frac{1}{m_{I1}} + 1, \frac{1}{n_{I1}} T_{1/k}^{m_{I1}}\right) + \beta\, n_{E1}^{1/m_{E1}}\, \Gamma\left(\frac{1}{m_{E1}} + 1, \frac{1}{n_{E1}} T_{1/k}^{m_{E1}}\right)\right]}{\left[\alpha \exp\left(-\frac{1}{n_{I1}} T_{1/k}^{m_{I1}}\right) + \beta \exp\left(-\frac{1}{n_{E1}} T_{1/k}^{m_{E1}}\right)\right]} \quad (7)$$

where, $\Gamma\left(\frac{1}{m_{j1}} + 1, \frac{1}{n_{j1}} T_{1/k}^{m_{j1}}\right)$, j=I and E, are incomplete Gamma functions.

Figure 15 shows some comparisons between calculations and experiments. As indicated in Fig.15, agreement of the estimated values using Eq.(5) with experimental values is good for moderate values of α and β in both waves defined by the zero-upcrossing and trough-to-trough methods. The run-up oscillation period defined by the zero-upcrossing method is, in general, longer than that defined by the trough-to-trough method. The reason is, as mentioned already, that the zero-upcrossing method cannot define small period waves which do not cross the mean water level, on the other hand, the trough-to-trough method is apt to define or pick up noise-like short period waves.

Following comparions between calculations and experiments, it seems that correspondence between estimated values and experiments for the zero-upcrossing method-defined wave is better than that for the trough-to-trough method-defined wave .

In calculating Eq.(5), the scale and shape factors of n and m were determined,by a least square method,to fit best the incident experimental wave. α and β were also decided by the least square method in order for the estimated value of Eq.(5) to agree with experimental distributions.

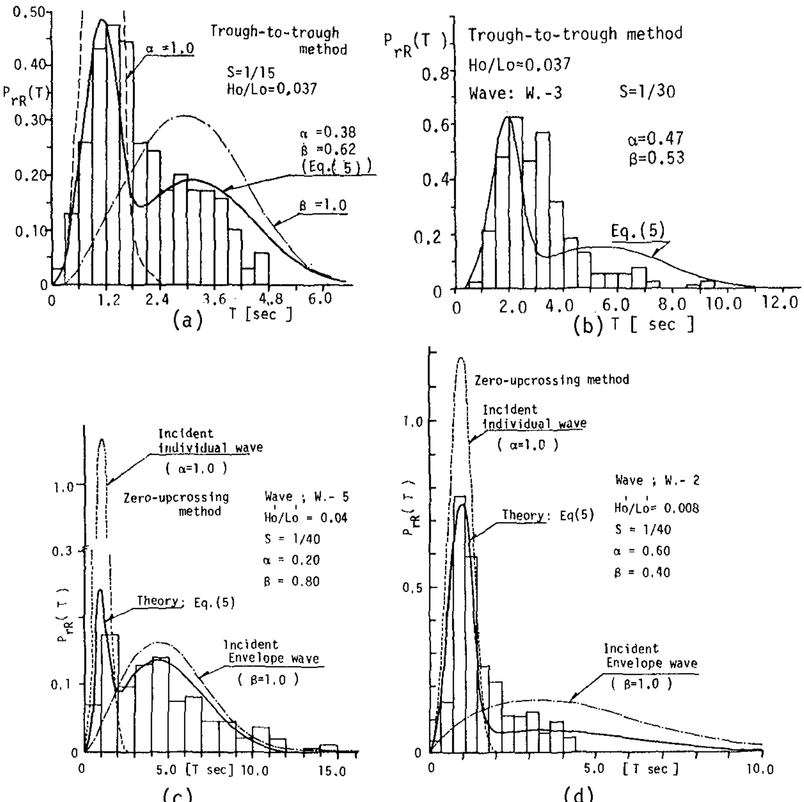


Fig.15 Some comparisons between measurements and calculations (probability distribution of run-up wave period)

The values of β thus determined are shown in Fig.16, where Ho/Lo is the incident significant wave steepness (for an individual wave) in deep water depth and S is the beach slope. The value of β which implys a degree of nonlinear interaction increases with increasing of Ho/Lo and decreasing of S. That is, the gentler the beach slope and the steeper the wave steepness become, the more long period components are generated. Due to the shortage of experimental values, we cannot propose an emperical expression for β in relation to Ho/Lo and S. Further elaborate experiments are needed to set up an experimental formular for β, which will be done in the near future by the present authors.

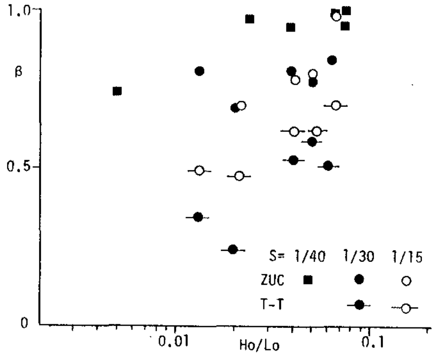


Fig.16 Variation of β with Ho/Lo and S (T-T;Trough-to-trough method, ZUC;Zero-upcrossing method)

(3) Distribution of run-up heights

The distribution of run-up heights is, like that of run-up wave period, assumed to be given by a combined distribution of the incident individual wave-induced run-up height distribution and the incident envelope wave-induced run-up height distribution.

The probability density function $P_{rR}(R)$ of run-up height distribution is then given by Eq.(8a) and Eq.(8b).

$$P_{rR}(R) = \alpha P_{rI}(R) + \beta P_{rE}(R) \qquad (8a)$$

(for trough-to-trough method)

$$P_{rR}(R) = \alpha \frac{P_{rI}(R)}{\int_{Ro}^{\infty} P_{rI}(R)\, dR} + \beta \frac{P_{rE}(R)}{\int_{Ro}^{\infty} P_{rE}(R)\, dR} \qquad (8b)$$

(for zero-upcrossing method)

where, α and β are weight coefficients determined by Eq.(5), and Ro is the vertical distance of the mean water level from the stillwater level.

$P_{rI}(R)$ is the probability density function of run-up heights induced by the incident individual waves,and Eq.(9) proposed by the authors[13] is used for $P_{rI}(R)$.

$$\left.\begin{aligned} P_{rI}(R) &= K_1(R)\int_0^{\infty} K_2(R)\, dt \\ K_1(R) &= [2P(S)\, h^{Q(S)}\, (g/2\pi)^{1/2-Q(S)}]\,[\frac{m_{I2}}{n_{I2}}\,\frac{m_{I1}}{n_{I1}}\, R^{2m_{I2}-1}] \end{aligned}\right\} \quad (9)$$

$$\left.\begin{aligned} P(S) &= 4.56 \times 10^{-2} \times S^{-0.133} \\ Q(S) &= 5.85 \times 10^{-3} \times S^{-1.246} \quad (\text{ for } 1/10 \leq S \leq 1/40 \text{ }) \\ a(S) &= 2(\ 1/2 - Q(S)\ ) \end{aligned}\right\}$$

In Eq.(9), subsuffixes 1 and 2 indicate quantities regarding the incident wave period and height, respectively.

$P_{rE}(\ R\ )$ is the probability density function of run-up heights produced by the incident envelope wave. Eq.(4) is adopted here as $P_{rE}(\ R\ )$, for the 1st approximation. In case of S=1/40, $P_{rE}(\ R\ )$ is expressed by Eq.(10).

$$\left.\begin{aligned} P_{rE}(\ R\ ) &= K_3(\ R\ )\int_0^{\infty} K_4(\ t\ )\ dt \\ K_3(\ R\ ) &= \frac{b(S)}{1-C(S)}\,\frac{m_{E1}}{n_{E1}}\,\frac{m_{E2}}{n_{E2}}\,R^{d(S)} \\ d(\ S\ ) &= (\ m_{E2} + 2C(S) - 1\ )/\ (\ 1 - C(S)\ ) \\ K_4(\ t\ ) &= t^{e(S)}\exp[\ -\frac{1}{n_{E1}}\,t^{m_{E1}} - \frac{1}{n_{E2}}\,R^{j(S)}\ t^{-2j(S)C(S)}\ ] \\ b(S) &= 0.137,\ C(S) = 0.100,\ e(S) = m_{E1} - 1 - [\ 2C(S)m_{E2}/(1-C(S))\,] \\ j(S) &= m_{E2}/(\ 1-C(S)\ ) \end{aligned}\right\} \quad (10)$$

In cases of S=1/15 and S=1/30, we can deduce similar but a little different expressions, which are not presented in this paper because of limited pages.

By the way, using the same definition as in deriving Eq.(7), the mean of the largest 1/k run-up height $\bar{R}_{1/k}$ is expressed by

$$\bar{R}_{1/k} = \frac{\alpha\int_{R_{1/k}}^{\infty} R\,P_{rI}(\ R\ )\ dR + \beta\int_{R_{1/k}}^{\infty} R\,P_{rE}(\ R\ )\ dR}{\alpha\int_{R_{1/k}}^{\infty} P_{rI}(\ R\ )\ dR + \beta\int_{R_{1/k}}^{\infty} P_{rE}(\ R\ )\ dR} \quad (11a)$$

(for trough-to-trough method)

$$\left.\begin{aligned} \bar{R}_{1/k} &= \frac{\alpha'\int_{R_{1/k}}^{\infty} R\,P_{rI}(\ R\ )\ dR + \beta'\int_{R_{1/k}}^{\infty} R\,P_{rE}(\ R\ )\ dR}{\alpha'\int_{R_{1/k}}^{\infty} P_{rI}(\ R\ )\ dR + \beta'\int_{R_{1/k}}^{\infty} P_{rE}(\ R\ )\ dR} \\ \alpha' &= \alpha/\int_{Ro}^{\infty} P_{rI}(R)\ dR\ , \quad \beta' = \beta/\int_{Ro}^{\infty} P_{rE}(R)\ dR \end{aligned}\right\} \quad (11b)$$

(for zero-upcrossing method)

Thus, as described above, the probability density function of run-up wave heights $P_{rR}(R)$ can be calculated by putting Eq.(9) and Eq.(4) or Eq.(10)(in case of S=1/40) into Eq.(8a) or Eq.(8b).

Fig.17 shows some comparisons between calculated values and experiments. Run-up waves in Fig.17 are all defined by the trough-to-trough method. The correspondence of the calculated values to the experimental values is, in general, good on the slopes of 1/15 and 1/30. In particular, quantitative agreement of calculations with experiments is recognized on the slope of 1/15 (see Fig.17(a) and (b)). On the beach slope of 1/30, however, the discrepancy between calculated vales and measurements is sometimes observed for steep waves such as W.-10 (see Fig.17(d)) and W.-11 listed in Tabel 1. This may be partly caused by the reason that Eq.(4) is not the best formula for estimating the envelope wave-induced run-up heights.

In calculating Eq.(8a), m_{I1}, m_{I2}, m_{E1}, m_{E2}, n_{I1}, n_{I2}, n_{E1} and n_{E2} were all determined by the least square method with use of incident wave statistics measured in our experiments.

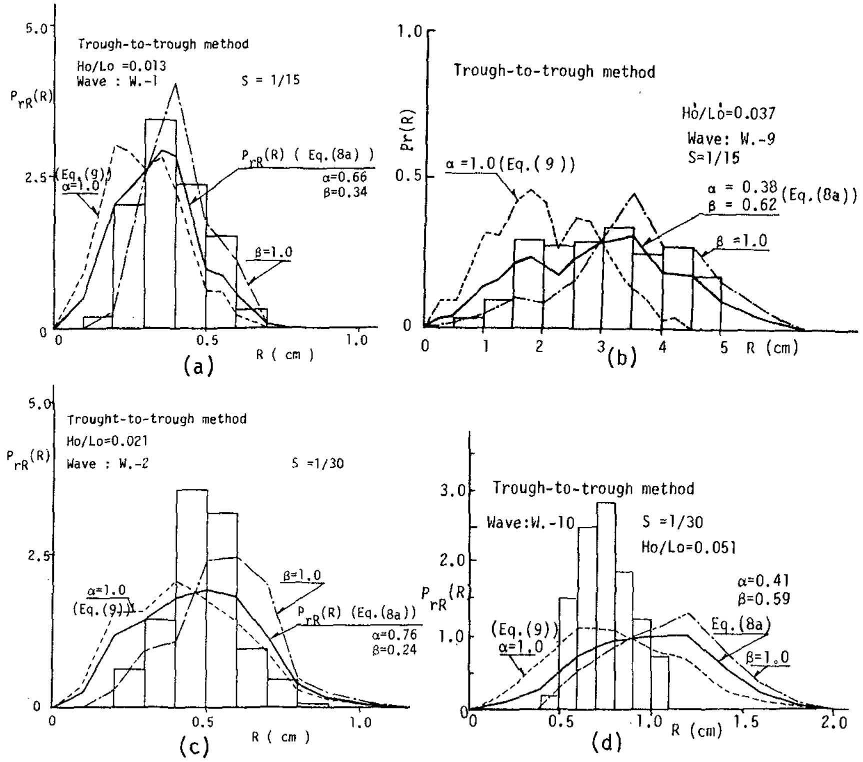


Fig.17 Comparisons between calculations and experimental values (Trough-to-trough method)

Fig.18 shows some comparisons between measurements and calculations. In Fig.18, waves are defined by the zero-upcrossing method. The agreement of calculated values with experimental values is also good, like the case for waves defined by the trough-to-trough method. However, in some cases on the slope of 1/15; the calculated values of W.-10 and W.-11 do not correspond well to the experiments. The reason may be the same as that described for waves defined by the trough-totrough method.

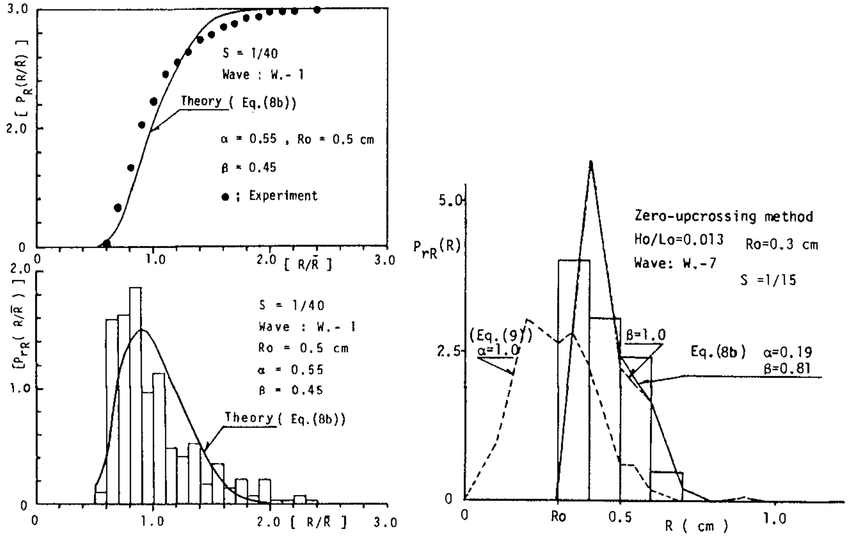


Fig.18 Comparison between measurements and calculations (Zero-upcrossing method)

In the calculations, m_{I1}, m_{I2}, m_{E1}, m_{E2}, n_{I1}, n_{I2}, n_{E1}, and n_{E2} were all the same values as those for waves defined by the trough-to-trough method. Ro was decided as the mean value over 8 minutes.

(4) Extreme distribution of run-up height and period

The estimation of the maximum values of run-up height R_{max} and period T_{max} is usefull for engineering purposes.

The maximum run-up wave period T_{max} is defined by

$$N (1 - P_r(T < T_{max})) = 1 \tag{12}$$

In Eq.(12), N is the number of wave , $P_r(T < T_{max}$) is the cumulative probability that T does not exceed T_{max}.

From Eq.(5) and Eq.(12), T_{max} can be determined by solving the following equation.

$$\alpha \exp\left(- \frac{1}{n_{I1}} T_{max}^{m_{I1}} \right) + \beta \exp\left(- \frac{1}{n_{E1}} T_{max}^{m_{E1}} \right) = \frac{1}{N} \qquad (13)$$

The same argument is applied to the maximum run-up height R_{max}. The maximum run-up height R_{max} is obtained by solving the following equations.

$$\alpha \int_{R_{max}}^{\infty} P_{rI}(R)\,dR + \beta \int_{R_{max}}^{\infty} P_{rE}(R)\,dR = 1/N \qquad (14a)$$

(for trough-to-trough mehtod)

$$\alpha' \int_{R_{max}}^{\infty} P_{rI}(R)\,dR + \beta' \int_{R_{max}}^{\infty} P_{rE}(R)\,dR = 1/N \qquad (14b)$$

(for zero-upcrossing method)

One example of calculations is given in Fig.19. As indicated in Fig.19, the correspondence of the calculation using Eqs.(5)-(14) to the experiment is not bad.

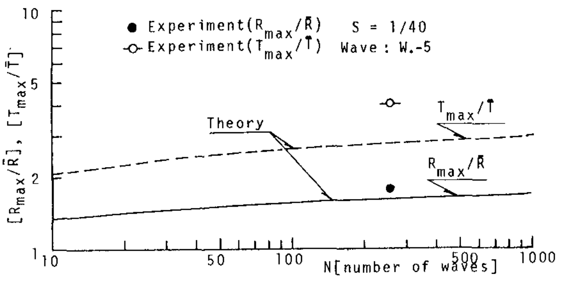


Fig.19 Nondimensional maximum run-up height and period (Zero-upcrossing method)

5. CONCLUDING REMARKS

In this paper, the distributions of the run-up height and period of irregular waves on gentle slopes of 1/15,1/30 and 1/40 are mainly discussed from engineering viewpoint. The theoretical distributions of run-up heights and periods which consider long period components generated by nonlinear wave-wave interactions in the surf and swash zones are proposed.

The theoretically estimated values are seen to agree with experiments. The proposed model is, however, largely based on the assumption that the long period components can be represented by the incoming envelope wave periods. It should be stressed that the energetic investigations on mechanics of long period wave generation by wave-wave interactions in relation to the incident wave characteristics are needed to develope more elaborate theoretical distributions for the run-up oscillation.

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