MODELLING OF SEA-BED EVOLUTION UNDER WAVES ACTION

by

Y. COEFFE (1) and PH. PECHON (2)

ABSTRACT

This paper presents a numerical model computing sea-bed evolution due to longshore currents, which has been developed at the Laboratoire National d'Hydraulique.

This model has been applied to a schematic semi-circular bay. The computation of sand sea-bed modifications reveals two main tendencies : a marked accretion in the "up-stream" zone of the bay and a marked erosion in the "down-stream" zone of the bay, as far as longshore currents are concerned.

1. INTRODUCTION

The action of waves is the predominant factor of sea-bed changes in the surf zone. The currents induced by breaking waves can be rather important, and their action is reinforced by the turbulent effects which appear in the breaking zone.

The model presented is able to predict longshore currents and sand transport field, and can deduce sea-bed evolutions under wave action for any bottom shape.

A mathematical model to compute longshore currents has been available at the Laboratoire National d'Hydraulique since 1977. In the numerical solution, energy transfers are simulated by introducing additional terms, derived from the Longuet-Higgins formulation, in the long wave equations.

---

(1) Head of the Maritime Hydraulics Division, Laboratôire National d'Hydraulique, E.D.F, Chatou, France.

(2) Research engineer, Laboratoire National d'Hydraulique, E.D.F, Chatou, France

This model has been applied to the simplified case of a semi-circular bay, the refraction of a monochromatic wave being computed with an other model, to estimate wave height, crest incidence and shoaling number in the domain. Recently, the introduction of a solid transport formula into the numerical model made it possible to estimate changes of the sea-bottom resulting from the longshore currents. Bijker formula has been selected because it takes into account the coupling of the average current over a vertical line and the wave orbital velocity. It is used to calculate bed and suspended load transport.

Once the quantities of transported materials are known, erosion and accretion can be calculated. The interaction between the modification of the sea-bed and wave induced currents is taken into account by adjusting the wave propagation after each significant change of the bathymetry and a new longshore current field is then deduced.

The modelling principles are summed up on figure 1. The numerical model includes at first a wave propagation model, then a current model and a bed evolution model ; this paper presents each part and the application to the schematic bay.

2. WAVES PROPAGATION MODEL

A classical pure refraction model is used, assuming linear wave theory, neglecting diffraction effects and using Snell's law.

The breaking of waves is considered. The method used here is to follow a wave orthogonal and to test in each point if breaking is occuring or not. The Battjes' criterion is used : the wave breaks at a place of depth d if the wave height H verifies

$$H \geqslant jd$$

where j is an experimental coefficient, function of bottom slope and local wave steepness, which determines the wave breaking evolution.

So, wave height incidence and shoaling number are estimated over the entire domain of interest.

This model has been applied to the schematic semi-circular bay, the characteristics of which being (fig 2) :

- radius of the bay = 260 m
- off-shore depth = 7,5 m

The off-shore wave conditions considered are :

- monochromatic wave
- height : 3 m
- period : 10s

The refraction diagram is shown on figure 3.

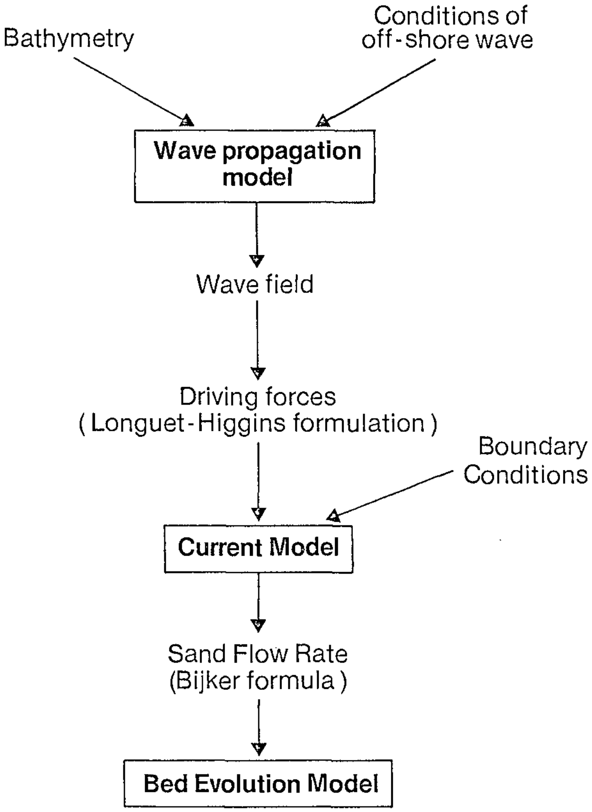

Fig.1 Modelling Principles

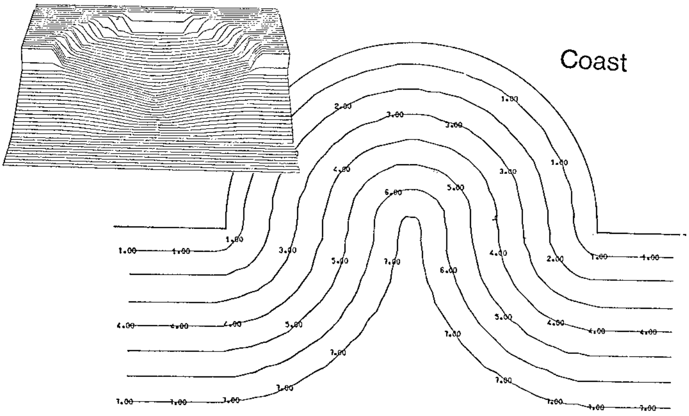

Fig.2 Initial Bathymetry of the Bay

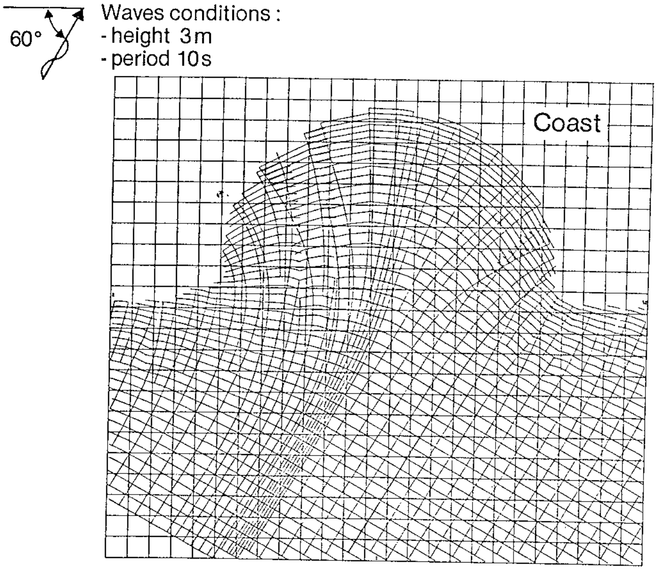

Fig.3 Refracted Waves Diagram

3. LONGSHORE CURRENT MODEL

3.1. Description

The assumptions of the longshore current model are :

- incompressible fluid
- linear Stokes waves
- the vertical component of the current is neglected
- for the calculation of waves currents, the variations of sea surface elevation are neglected compared to the total depth.
- the mass current is assumed to be constant over the depth
- wind action is neglected

With these assumptions, the Navier-Stokes equations can be integrated over the total depth and time averaged over a wave period. That leads to long waves equations in which appears a radiation stress tensor $\bar{S}$, representing the extra terms coming from averaging non linear terms involving velocities

The wave driving forces can be defined as follows :

$$\vec{\tau} = - \frac{1}{\rho} \operatorname{div} \bar{S}$$

Using Longuet-Higgins formulation, the tensor $\bar{S}$ is written :

$$\bar{S} = E \begin{bmatrix} (2n - \frac{1}{2}) \cos^2\alpha + (n - \frac{1}{2}) \sin^2\alpha & n \sin\alpha \cos\alpha \\ n \sin\alpha \cos\alpha & (2n - \frac{1}{2}) \sin^2\alpha + (n - \frac{1}{2}) \cos^2\alpha \end{bmatrix}$$

where $E = \frac{1}{8} \rho g H^2$ is the wave energy

n is he shoaling number

α is the wave incidence

The bottom friction has been assumed proportional to the squared velocity, taking into account the orbital velocity in the following form :

$$\vec{\tau}_b = - \frac{\rho g}{C_s^2} \vec{u} \, || \vec{u} + \vec{u}_{orb} ||$$

where $\vec{u}$ = mean velocity

$$\vec{u}_{orb} = \frac{2H}{T} \frac{1}{\operatorname{sh} (2\pi \frac{d}{CT})} \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix}$$

3.2. Application

The longshore current model has been applied to the semi-circular bay. Using the propagation model results, driving stresses are calculated. Then the wave height field is smoothed (fig. 4) to get a more regular radiation stress tensor. Then induced currents are computed.

The boundary conditions along the left side of the grid are coming from the application of the model to a rectilinear shore, with a bathymetry similar to the incoming part of the semi-circular bay. For this last case, the water flow obtained at the left boundary is transfered at the right one in order to get the stationary solution of an infinite shore.

At the right open boundary, a free exit of the current is imposed.

The obtained velocity pattern is shown on figure 5. Strong velocities appear in the breaking zone and an eddy takes place in the right part of the bay. The maximum velocities reach about 1,4 m/s, in the rectilinear part.

4. SEDIMENT TRANSPORT CALCULATION

4.1. Transport formula

A sediment transport formula was added to the current model. The chosen transport law is Bijker's formula which takes into account the coupling of the averaged current and the wave orbital velocity. The mean bed shear resulting of the combination of waves and longshore current is hold responsible for the stirring up of the material. Once the material is stirred up, it is transported by the normal current. For the bed load part this transport law is written as :

$$T_b = 5D\,(\mu\tau c/\rho)^{1/2}\quad e^{-0,27}\;\frac{\Delta D\rho g}{\mu\ \tau_r}$$

Δ : relative apparent density

D : grain size

μ : ripple coefficient

τ_c : bed shear due to mean current

τ_r : bed shear due to mean current and waves

Tb : bed load transport.

The suspended load is evaluated by assuming a logarithmic velocity distribution

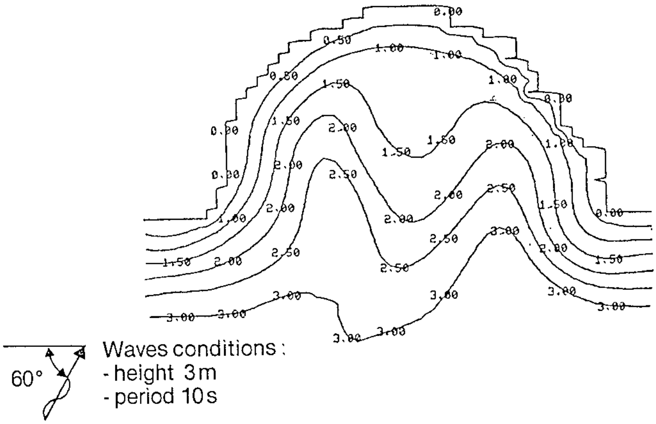

Fig.4 Contour-lines of Smoothed Wave Height

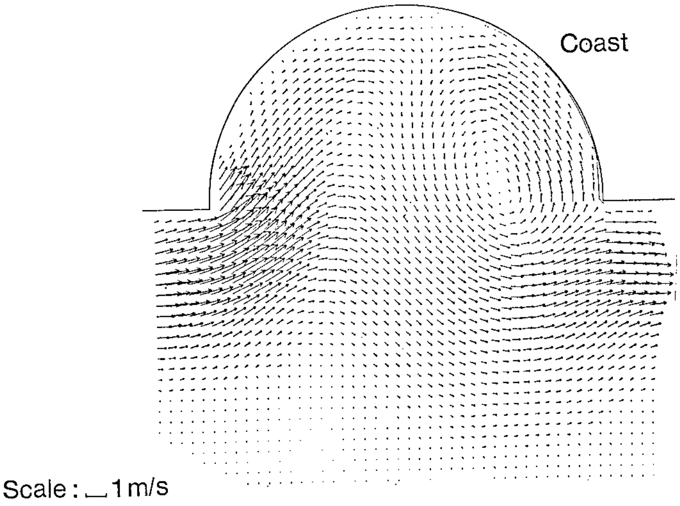

Fig.5 Velocity Field

The concentration c(z) is written as :

$$\frac{C(z)}{Ca} = \left[\frac{h - z}{z} \times \frac{a}{h - a}\right]^{Z} \text{ in which } Z = W/Kv_*$$

Where W = fall velocity of the particles

K = Karman constant

v^* = stress velocity due to wave and mean current

C_a = concentration of suspended load immediately above the bed

4.2. Application

The transport flow rate resulting from the velocity field in the bay is presented on figure 6, for a 0,5 mm grain size.

The total sand transport going through a section of the left rectilinear shore near the bay has been compared with the CERC formula and the formula of Larras and Bonnefille :

- formula of the CERC :

$$Q = 6{,}5\ 10^{-2}\ H^{5/2} \cos^{1/4} \alpha \sin 2\ \alpha$$

- formula of Larras et Bonnefille :

$$Q = K(H/L,\ D)\ \frac{H^3}{T} \sin \frac{7\alpha}{4}$$

where H is the wave height, α is the wave incidence, T is the period and D is the sand diameter.

The transport flow evaluated with the first formula is equal to 0,8 m^3/s whereas the second one gives 0,1 m^3/s. The difference between these two valuations comes from the difficuties of mesuring sand transport flow in the surf zone. The computation gives for this section (fig 7) Q = 0,3 m^3/s which is in the range of these formulae.

5. EVOLUTION OF THE SEA-BOTTOM

5.1. Description of the method

Continuity equation applied to the sediment transport gives the bed evolution :

$$\frac{\partial Z_F}{\partial t} + \operatorname{div} \vec{T} = 0$$

where $\vec{T}$ is the sediment transport flow rate, and Z_F the bed level.

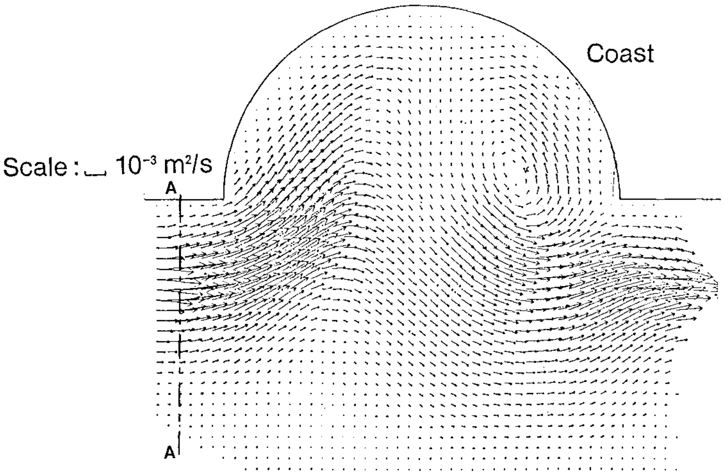

Fig.6 Sediment transport field

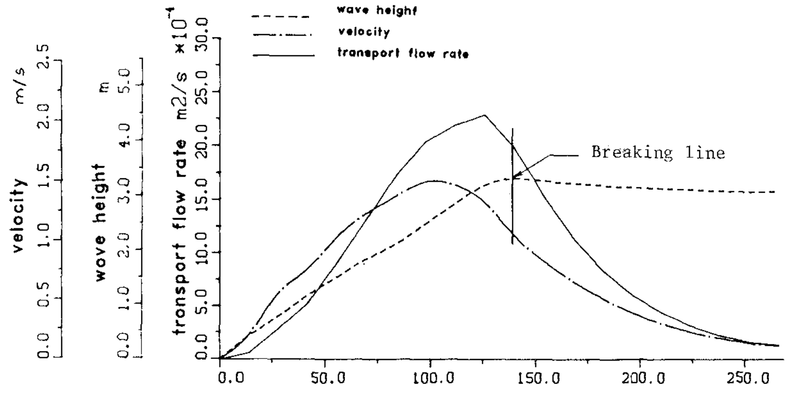

Fig.7 Velocity,wave height and transport distribution along A—A section

The bed evolution modifies progressively the waves propagation and the current field. It is very important to take this coupling into account, and to update the longshore currents according to new water depth. However the sedimentological time scale being much greater than the velocity time scale, the sea-bottom and water flow changes are calculated by doing two loops as shown on the following diagram :

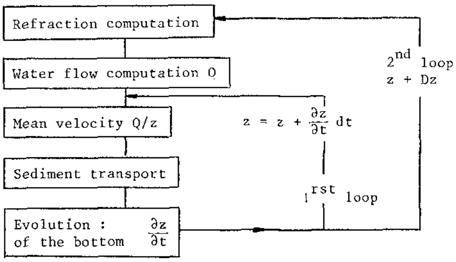

Fig.8 Modelling principles of bed evolution calculation

The first loop takes into account the interaction of the velocity field and the sea-bottom evolution for a constant water flow field, by the following way :

- sand transport calculation using Bijker's formula
- calculation of $\frac{\partial Z_F}{\partial t} = -\,\mathrm{div}\,\vec{T}$
- modification of sea-bottom $\frac{\partial Z_F}{\partial t}$ dt. The time step dt is chosen so that the water depth is changed of 5 % at most in the domain.
- adjustement of the velocity, considering a constant water flow rate and return to the first step.

When the modifications of the bathymetry are big enough, the waves characteristics in the domain are re-evaluated. The new driving forces and the induced longshore currents are computed again with the wave propagation model.

5.2. Results

In the case of the semi-circular bay, the comparison between the initial topography and the topography after 100 hours storms reveals the main following tendencies (fig 9) :

- large accretion in the left side of the bay due to incoming littoral transport
- large erosion of the beach in the right side of the bay due to reconstitution of littoral transport
- erosion of the area close to the left cape at the entrance of the bay and accretion just behind the cape
- general filling of the beach of the bay.

These changes agree with some natural tendencies but the obtained values need to be checked over real cases or physical model experiments.

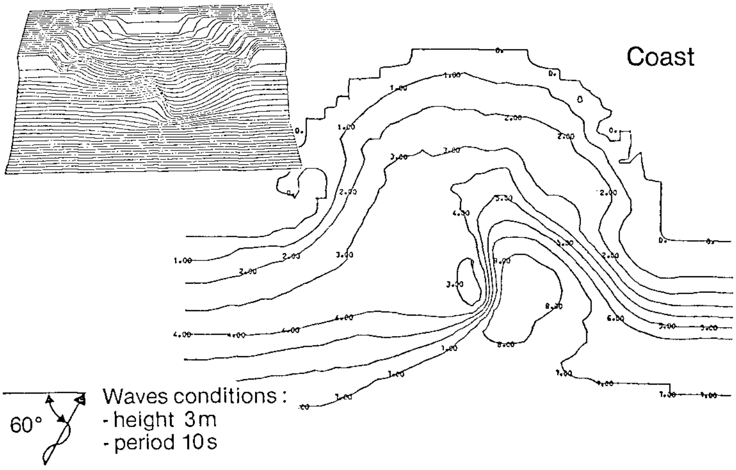

Fig.9 Bathymetry after 100 hours storm

6. CONCLUSION

The first results obtained by the mathematical model show that the modelling of sea bed evolution under the action of waves is no more a domain restricted to physical models.

The presented bidimensional model is able to compute longshore currents induced by breaking waves and to simulate bathymetry modifications, considering the interaction between sea-bed changes and velocity field. The application to a schematic semi-circular bay has given reasonnable results in agreement with known natural tendancies. However the developement of this model requires more accurate comparisons with scale model results or real cases. This work will be planned in the near future.

7. AKNOWLEDGEMENT

This study has been done as part of a joint research programm of EDF-LNH and the "Ministère de la mer".

8. REFERENCES

BIJKER, E.W. (1968). Littoral drift as function of waves and current. 11e Coastal Engineering Conference. London.

BIJKER, E.W. (1967). Some considerations about scales for coastal models with movable bed. Pub n° 50 of the Delft Hydraulic Laboratory.

FRIJLINK, H.C. (1952). Discussion des formules de débit solide de Kalinske, Einstein et Meyer-Peter et Muller. Deuxième journée de l'hydraulique. Transport hydraulique et décantation de matériaux solides. Grenoble.

SABATON, M, HAUGUEL, A. (1978). A numerical model of Longshore Currents. Rapport Electricité de France n° HE/42/78.34