EFFECT OF WAVE-CURRENT INTERACTION ON THE WAVE PARAMETER

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1. ABSTRACT

The interaction of a gravity wave with a steady uniform current is described in this paper. Numerical calculations of the wave length change by different non-linear wave theories show that errors in the results computed by the linear wave theory are less than 10 percent within the range of $0.15 \leq d/L_S \leq 0.40$, $0.01 \leq H_S/L_S \leq 0.07$ and $-0.15 \leq U/C_S \leq 0.30$. Numerical calculations of wave height change employing different wave theories show that errors in the results obtained by the linear wave theory in comparison with the non-linear theories are greater when the opposing relative current and wave steepness become larger. However, within range of the following currents such errors will not be significant. These results were verified by model tests. Nomograms for the modification of wave length and wave height by the linear wave theory and Stokes' third order theory are presented for a wide range of d/L_S, H_S/L_S and U/C. These nomograms provide the design engineer with a practical guide for estimating wave lengths and heights affected by currents.

2. INTROOUCTION

With increasing human activities in both the coastal and immediate offshore region, the problem of wave-current interaction has been evaluated by a number of researchers. From an engineering practice point of view, the effect of wave current interaction on the wave parameters must be known. Previous research involved different assumptions and considerations. Several researchers employed the linear wave theory combined with the idea of conservation of energy flux (5,12,13), or combined with the idea of conservation of wave action flux (8,9); others employed the non-linear wave theory (6,15). The problem concerning the interaction of waves and currents in an inlet has also been studied (3,7). Additional research (11,18) describes the change of velocity distribution due to the interaction of waves with currents.

One purpose of this study was to evaluate the difference between the change in wave parameters when employing either the equation of conservation of wave energy flux or the equation of conservation of wave

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action flux. The results were evaluated with reference to practical engineering applications. Another purpose of this study was to determine the error in the calculated wave parameter by employing both the linear wave theory and non-linear wave theories (Stokes' third order wave theory was mainly considered). A two-dimensional case was considered and both the following and the opposing currents were analyzed.

3. THEORETICAL CONSIDERATIONS

Evaluation of change in the wave parameters may be obtained in two steps: a) evaluation of the change in wave length, and b) evaluation of the change in wave height. In both cases, the energy dissipation in wave propagation was neglected.

3.1 Change in Wave Length

The linear wave theory was considered first. The influence of non-linearity of waves is evaluated in the latter part of this section. The wave celerity in the current may be calculated as follows:

$$C_a = U + C_r \quad \ldots \quad (1)$$

where

$$C_r = \sqrt{g/k_r \, Th\, k_r d} \quad \ldots \quad (2)$$

Equation 1 can be rewritten as

$$\frac{L}{T_a} = U + \frac{L}{T_r} \quad \ldots \quad (3)$$

From Equations 1, 2 and 3, the following equation can be obtained

$$\frac{L}{L_s} = \frac{C}{C_s} = \frac{1}{(1 - \frac{U}{C})^2} \frac{Th\ k\ d}{Th\ k_s d} \quad \ldots \quad (4)$$

and $L = L_a$, $C = C_a$.

In engineering applications, the value of $C = C_a$ is unknown, therefore, the ratio U/C is also unknown. Thus the following formula may be used

$$C_r/C_s = \sqrt{L/L_s \ Thkd/Thk_s d} = \frac{1}{1 - \frac{U}{C}} \frac{Thkd}{Thk_s d} \quad \ldots \quad (5)$$

Combining equations 6 and 7, the relationship between L/L_S with U/C_S and $k_S d$ can be obtained from

$$L/L_s = C/C_s = f(\frac{U}{C_s}, \frac{d}{L_s}) \quad \ldots \quad (6)$$

and

$$\frac{U}{C_s} = \frac{U}{C}\frac{C}{C_s}$$

3.2 Change of Wave Height

The process of the interaction of progressive waves with a steady uniform current (the following or the opposing current only) may be evaluated as follows: (a) in the first step only currents and waves propagate separately, (b) in the second step the combined current and waves interact and finally a steady motion of wave-current combination propagates as shown in Figure 1. If we neglect the change in the current profile because of the wave-current interaction, the current energy flux will remain the same before and after the interaction process.

To-date, two concepts, i.e. the conservation of wave energy flux (12,13) and the conservation of wave action flux (1,8) were employed. However, since there is no major difference between the two methods, the principle of conservation of wave action flux was selected for the analysis.

The idea of conservation of wave action flux was first suggested by Garrett and evaluated by Bretherton and Garrett (1). Jonsson (8) described a practical application for the case of interaction of waves with a steady uniform current. In the present case (as shown in Fig. 1) the following equation may be used

$$\frac{d}{dx}\left\{\frac{E}{\omega_r} C_{ga}\right\} = 0 \qquad (7)$$

Equation 7 indicates that the wave action flux before and after interaction must be the same, i.e.

$$\frac{E}{\omega_r} C_{ga} = \frac{E_s}{\omega_s} C_{gs} \qquad (8)$$

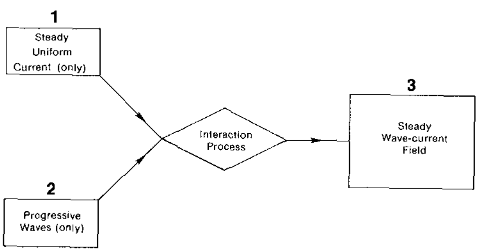

Fig. 1. Wave-current interaction

where

$$\omega_r = \omega_a - kU = \omega_s - kU \quad (9)$$

$$C_{gs} = \tfrac{1}{2} C_s A_s \quad (10)$$

$$C_{ga} = U + C_{gr} = U + \tfrac{1}{2} C_r A \quad (11)$$

$$C_{gr} = \tfrac{1}{2} C_r A \quad (12)$$

$$A = 1 + \frac{2kd}{Sh2kd} \quad (13)$$

$$A = 1 + \frac{2k_s d}{Sh2k_s d} \quad (14)$$

Thus Equation 8 can be rewritten as follows

$$\frac{E}{E_s} = (1 - \frac{U}{C}) \frac{L_s}{L} \frac{A_s}{A} (1 + \frac{U}{C} \frac{2-A}{A})^{-1} \quad (15)$$

Accordingly, the wave height change can be determined by the following equations:

3.2.1 Linear wave theory

$$\frac{H}{H_s} = (1 - \frac{U}{C})^{\frac{1}{2}} (\frac{L_s}{L})^{\frac{1}{2}} (\frac{A_s}{A})^{\frac{1}{2}} (1 + \frac{U}{C} \frac{2-A}{A})^{-\frac{1}{2}} = R \quad (16)$$

When non-linear wave theory is employed, Equation 12 is no longer correct. Generally, the error in using this eouation instead of the correct one for non-linear waves will not be greater than 6 percent. Thus as a simplification for the non-linear wave analysis, Equations 12 and 15 may be used and will not produce a significant error. The results by Skjelbreia (14), Tsuchiya and Yasuda (17) were employed in calculating the wave energy.

3.2.2 Skjelbreia's Stokes' Third-Order Wave Theory (Old)

Wave energy E may be calculated by the following equation

$$E = \frac{\gamma H^2}{8} \{1 + \pi^2 (\frac{H}{L})^2 [B + \frac{3}{(32)(256)} \pi^2 (\frac{H}{L})^2 F]\} \quad (17)$$

where

$$B = \tfrac{1}{2}(\frac{1}{Th\frac{2\pi d}{L}})^2 (1 + \frac{3}{1+2(sh\frac{2\pi d}{L})^2})^2 + \frac{9}{32} \frac{Ch\frac{4\pi d}{L}}{(Sh\frac{2\pi d}{L})^6} \quad (18)$$

and

$$F = 3\left(\frac{1+8(Ch\frac{2\pi d}{L})^6}{(Sh\frac{2\pi d}{L})^6}\right) + \tfrac{1}{2}\,\frac{Th\frac{2\pi d}{L}\;Sh\frac{12\pi d}{L}\;(11-2Ch\frac{4\pi d}{L})^2}{(Sh\frac{2\pi d}{L})^{14}} \quad \ldots\ldots\ldots (19)$$

Accordingly, wave energy in still water, E_S, can be calculated similarly as

$$E_s = \frac{\gamma H_s^2}{8}\left\{1 + \pi^2\left(\frac{H_s}{L_s}\right)^2\left[B_s + \frac{3}{(32)(256)}\pi^2\left(\frac{H_s}{L_s}\right)^2 F_s\right]\right\} \quad \ldots\ldots (20)$$

where B_S and F_S may be obtained from a similar formula as B and F in Equations 18 and 19 (changing L to L_S only). The wave height change can then be calculated as follows

$$\frac{H}{H_s} = R\,\frac{\left\{1+\pi^2\left(\frac{H_s}{L_s}\right)^2\left[B_s + \frac{3}{(32)(256)}\pi^2\left(\frac{H_s}{L_s}\right)^2 F_s\right]\right\}^{\frac{1}{2}}}{\left\{1+\pi^2\left(\frac{H}{L}\right)^2\left[B + \frac{3}{(32)(256)}\pi^2\left(\frac{H}{L}\right)^2 F\right]\right\}^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots (21)$$

where R is determined by Equation 16. This equation must be solved by iteration.

3.2.3 Tsuchiya's Stokes' Third-Order Wave Theory (New)

Energy flux, W, can be determined by the following equation

$$W = \frac{1}{8}\gamma H^2 C\left\{\tfrac{1}{2}A + \left(\frac{\pi H}{L}\right)^2 G\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

where A is defined by Equation 15, and G is

$$G = \frac{Ch^2 2kd + 3Ch2kd + 2}{16Sh^4 kd} + \frac{3}{4}Sh^2 kd + \frac{9(2kd + Sh2kd)}{64Sh^7 kdChkd}$$

$$+ \frac{3(Chkd + Ch3kd)}{8Sh^4 kdChkd} + \frac{kdThkd + Sh^2 kd}{2Sh^4 kd} \quad \ldots\ldots\ldots\ldots\ldots (23)$$

Since

$$W = \tfrac{1}{2}EC_r\left(1 + \frac{2kd}{1 + Sh^2 kd}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Thus wave energy may be determined as follows

$$E = \frac{1}{8}\gamma H^2\left\{1 + 2\left(\frac{\pi H}{L}\right)^2\frac{G}{A}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

and wave energy in still water E_s is

$$E_s = \frac{1}{8}\gamma H_s^2 \{1 + 2(\frac{\pi H_s}{L_s})^2 \frac{G_s}{A_s}\} \qquad (26)$$

where G_s can be determined similarly as G from Equation 23, and the wave number k is changed to k_s in the case of still water. As a result, the wave height change can be calculated as follows

$$\frac{H}{H_s} = R \frac{\{1+2(\frac{\pi H_s}{L_s})^2 \frac{G_s}{A_s}\}^{\frac{1}{2}}}{\{1+2(\frac{\pi H}{L})^2 \frac{G}{A}\}^{\frac{1}{2}}} \qquad (27)$$

where R is determined by Equation 16. Equation 28 must also be solved iteratively. When the non-linear wave theory is used, the wave celerity is determined not only by the parameter of kd, but also by the wave steepness H/L. Due to the interaction of wave and current, the wave steepness H/L can be changed, as the wave length determined by the non-linear wave theory L_N is not the same as that determined by the linear theory L, i.e., $L_N \neq L$. The value of L_N/L can be determined as follows

3.2.4 Employing Skjelbreia's Method

$$\text{Wave celerity } C_N = (\frac{g}{k}\,\text{Thkd})^{\frac{1}{2}} \{1 + (\frac{\pi H}{L})^2 \frac{14+4\text{Ch}^2 2kd}{16\text{Sh}^4 kd}\} \qquad (28)$$

$$\text{and } L_N/L = 1 + \frac{1}{16}(\pi\frac{H}{L})^2 \frac{14+4\text{Ch}^2 2kd}{\text{Sh}^4 kd} = N_1 \qquad (29)$$

where wave steepness H/L is the value under wave-current interaction.

$$\text{Thus, } L_N/L_s = \frac{L_N}{L} \cdot \frac{L}{L_s} = N_1 \frac{L}{L_s} \qquad (30)$$

3.2.5 Employing Tsuchiya's Method

$$\text{Wave celerity } C_N = (\frac{g}{k}\,\text{Thkd})^{\frac{1}{2}}\{1 + \frac{1}{16}(\pi\frac{H}{L})^2 \frac{1}{\text{Sh}^4 kd}(8\text{Ch}^4 kd - 4\text{Ch}^2 kd + 5)\}$$

$$= (\frac{g}{k}\,\text{Thkd})^{\frac{1}{2}} N_2 \qquad (31)$$

and

$$L_N/L = N_2 = \{1 + \frac{1}{16}(\pi\frac{H}{L})^2 \frac{1}{\text{Sh}^4 kd}(8\text{Ch}^4 kd - 4\text{Ch}^2 kd + 5)\} \qquad (32)$$

Thus

$$L_N/L_s = \frac{L_N}{L} \cdot \frac{L}{L_s} = N_2 \frac{L}{L_s} \quad \ldots\ldots\ldots\ldots\ldots (33)$$

Coefficients N_1 and N_2 show the influence of non-linearity of waves.

4. DATA ANALYSIS AND RESULTS

4.1 The Results of Modification of Wave Length by the Linear Wave Theory Due to Wave-Current Interaction

Using Equations 4, 5 and 6 different combinations of dimensionless parameters U/C (or U/C_S), and d/L_S may be calculated. The results are shown in Figures 2 and 3. As shown in Figure 2, it is clear that the wave length change L/L_S with the relative current U/C is greater in deep water than in shallow water.

4.2 Modification of Wave Length by Non-Linear Wave Theory and Error Estimates Due to Linear Theory

Numerical calculations of the wave length were made employing Stokes' third order wave theory (14,17). The ratio of wave length computed by the non-linear wave theory with that computed by the linear wave theory is given in Table I. It can be seen that the wave length calculated by the non-linear wave theory is usually greater than that calculated by the linear theory. Within the range of $0.15 \leq 0.40$, $0.01 \leq H_S/L_S \leq 0.07$ and $-0.15 \leq U/C_S \leq 0.30$, the wave lengths computed by the linear wave theory are approximately 10 percent less than those computed by Stokes' third-order wave theory. Table II shows a comparison of the results obtained by the linear wave theory with data obtained by the non-linear wave theories and with model test data. It appears that the results obtained employing different methods are quite similar. Also a comparison of the wave length (obtained by the linear wave theory) with Hales' and Herbich's model tests data (2) are given in Table III; the agreement is considered quite good.

4.3 Modification of Wave Height

The results obtained with the linear wave theory (Equation 17) are shown in Figure 4.

The results given by Skjelbreia's Stokes' third-order wave theory for different wave steepnesses (i.e. $H_S/L_S = 0.01 \sim 0.07$) (Equation 26) are shown in Figures 5-8.

The results shown by Tsuchiya's modification of Stokes' third-order wave theory are presented in Figures 9-12 for wave steepness $H_S/L_S = 0.01 \sim 0.07$.

A comparison of the results employing the linear theory (Figure 4) with that of Skjelbreia's method (Figures 5 through 8) is shown in Table IV. The following observations were made:

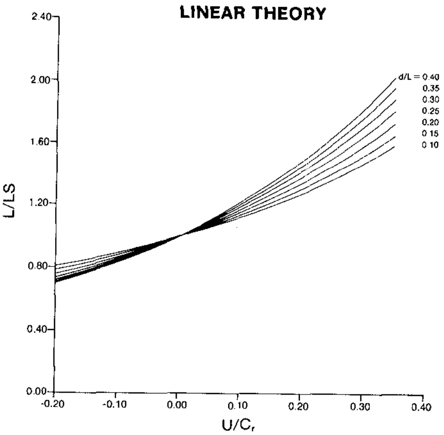

Fig. 2. Wave length change computed by linear wave theory (L/L_S versus U/C)

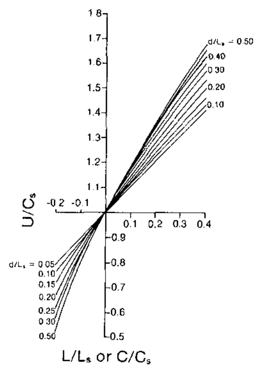

Fig. 3. The wave length change computed by linear wave theory (L/L_S versus U/C_S)

Table I. The Ratio of Wave Length L_N/L***

$\frac{d}{L_s}$	$\frac{H_s}{L_s}$	U/C_s = -0.15 (New*) Stokes III	(Old**) Stokes III	-0.10 (New) St.III	(Old) St.III	0.00 (New) St.III	(Old) St.III	0.10 (New) St.III	(Old) St.III	0.20 (New) St.III	(Old) St.III	0.30 (New) St.III	(Old) St.III
0.20	0.01	1.002	1.002	1.001	1.002	1.001	1.002	1.001	1.001	1.001	1.001	1.001	1.002
	0.03	1.010	1.019	1.008	1.016	1.008	1.014	1.007	1.013	1.008	1.013	1.010	1.017
	0.05	1.025	1.048	1.024	1.042	1.021	1.037	1.021	1.034	1.023	1.038	1.029	1.050
	0.06	1.034	1.064	1.032	1.057	1.031	1.054	1.031	1.049	1.035	1.056	1.044	1.077
	0.07	1.043	1.082	1.042	1.074	1.041	1.072	1.043	1.067	1.049	1.078	1.065	1.117
0.30	0.01	1.002	1.002	1.002	1.002	1.001	1.001	1.001	1.001	1.001	1.001	1.000	1.001
	0.03	1.007	1.018	1.006	1.014	1.005	1.010	1.004	1.007	1.004	1.006	1.004	1.006
	0.05	1.013	1.043	1.013	1.035	1.014	1.026	1.013	1.020	1.013	1.017	1.013	1.018
	0.06	1.017	1.058	1.018	1.047	1.020	1.037	1.020	1.028	1.020	1.025	1.021	1.026
	0.07	1.022	1.072	1.023	1.060	1.027	1.049	1.029	1.037	1.029	1.034	1.032	1.035
0.40	0.01	1.001	1.002	1.001	1.002	1.001	1.001	1.001	1.001	1.001	1.001	1.000	1.000
	0.03	1.004	1.018	1.004	1.014	1.005	1.009	1.004	1.006	1.004	1.005	1.004	1.004
	0.05	1.008	1.043	1.009	1.034	1.013	1.024	1.015	1.017	1.015	1.013	1.015	1.012
	0.06	1.010	1.057	1.013	1.045	1.018	1.034	1.023	1.024	1.018	1.019	1.025	1.017
	0.07	1.013	1.071	1.016	1.058	1.024	1.045	1.033	1.032	1.039	1.026	1.041	1.023

*Skjelbreia's result.
**Tsuchiya's result.
***L_N is the wave length calculated from nonlinear wave theory; L is the wave length obtained from linear wave theory.

Table II. A Comparison of Wave Length Computed Using Figure 2 With Test Oata and Computed Values From Non-Linear Wave Theories

d(m)	wave height H(m)	T (sec)	U (m/s)	$L_{exp.}$ (m)	d/L_s	U/C_s	$(L/L_s)_{cal}$	L_{cal}(m)	wave length(%) error $=\frac{L_{cal}-L_{exp}}{L_{exp}}\times 100$
0.587	0.021	1.25	-0.13	2.05	0.26	-0.067	0.88	1.98	-3.4
Note exp. by G.P. Thomas									
0.570	0.024	1.25	-0.20	1.82	0.26	-0.11	0.81	1.81	-0.5
Note exp. by G.P. Thomas									
30.5	15.2	10.0	+0.90	152.5	0.22	+0.07	1.10	151	-1.0
Cal. by Oalrymple									
10	2.5	12.0	+1.0	130	0.09	+0.11	1.09	126	-3.0
Cal. by T.S. Hedges*									

*T.S. Hedges used cnoidal wave theory.

Table III. A Comparison of the Wave Length Computed Using Figure 2 With the Model Test Oata of Hales and Herbich

d(m)	L_{mea}(m)	U(m/s)	d/L_s	U/C_s	$(L/L_s)_{cal}$ (from Fig. 2)	L_{cal}	wave length error (%) = $\frac{L_{cal}-L_{mea}}{L_{mea}} \times 100$
0.098	0.79	+0.277(flood	0.17	0.34	1.41	0.80	+1.2
	0.94	" current)	0.14	0.32	1.37	0.96	+2.0
	1.16	"	0.11	0.30	1.32	1.20	+3.4
	1.39	"	0.09	0.30	1.32	1.44	+3.6
	1.58	"	0.08	0.29	1.31	1.63	+3.2
	2.00	"	0.06	0.28	1.30	2.06	+3.0
	2.25	"	0.055	0.28	1.28	2.29	+1.8
0.098	0.69	+0.143(flood	0.17	0.17	1.21	0.69	0
	0.83	" current)	0.14	0.16	1.20	0.84	+1.2
	1.04	"	0.11	0.16	1.18	1.07	+2.9
	1.26	"	0.09	0.16	1.18	1.29	+2.3
	1.44	"	0.08	0.15	1.16	1.45	+0.7
	1.79	"	0.06	0.15	1.15	1.83	+2.2
	2.02	"	0.055	0.15	1.15	2.06	+2.0
0.098	0.50	-0.076(ebb	0.17	-0.09	0.88	0.50	0
	0.62	" current)	0.14	-0.09	0.88	0.62	0
	0.82	"	0.11	-0.08	0.91	0.82	-1.0
	1.00	"	0.09	-0.08	0.91	0.99	0
	1.14	"	0.08	-0.08	0.91	1.14	0
	1.49	"	0.06	-0.08	0.92	1.46	-2.0
	1.65	"	0.055	-0.08	0.92	1.65	0
0.098	0.42	-0.165(ebb	0.17	-0.20	0.70	0.40	-4.7
	0.53	" current)	0.14	-0.19	0.74	0.52	1.9
	0.71	"	0.11	-0.18	0.78	0.71	0
	0.88	"	0.09	-0.18	0.79	0.86	-2.3
	1.03	"	0.08	-0.17	0.80	0.99	-3.9
	1.32	"	0.06	-0.17	0.82	1.30	-1.4
	1.50	"	0.055	-0.17	0.82	1.47	-2.0

Table III. (continued)

d(m)	L_{mea}(m)	U(m/s)	d/L_s	U/C_s	$(L/L_s)_{cal}$ (from Fig. 2)	L_{cal}	wave length error (%) $= \frac{L_{cal}-L_{mea}}{L_{mea}} \times 100$
0.445	0.65	+0.265(flood	1.07	0.33	-	-	-
	1.22	" current)	0.47	0.22	1.39	1.31	+7.4
	1.72	"	0.32	0.19	1.29	1.77	+2.9
	2.41	"	0.22	0.16	1.20	2.46	+2.1
	3.04	"	0.17	0.15	1.18	3.06	+0.7
	4.07	"	0.12	0.14	1.16	4.15	+1.9
0.445	0.54	+0.134(flood	1.07	0.17	-	-	-
	1.10	" current)	0.47	0.11	1.20	1.13	+2.7
	1.57	"	0.32	0.09	1.14	1.56	-0.6
	2.23	"	0.22	0.08	1.11	2.26	+1.3
	2.84	"	0.17	0.08	1.09	2.83	-0.4
	3.81	"	0.12	0.07	1.08	3.86	+1.3
0.445	0.37	-0.055(ebb	1.07	-0.07	-	-	-
	0.92	" current)	0.47	-0.05	0.92	0.87	-5.4
	1.34	"	0.32	-0.04	0.94	1.29	-3.7
	1.95	"	0.22	-0.03	0.95	1.94	-0.5
	2.51	"	0.17	-0.03	0.97	2.51	0
	3.48	"	0.12	-0.03	0.97	3.47	-0.3
0.445	0.29	-0.168(ebb	1.07	-0.21	-	-	-
	0.76	" current)	0.47	-0.14	0.73	0.68	-10.5
	1.18	"	0.32	-0.12	0.80	1.09	-7.6
	1.80	"	0.22	-0.10	0.84	1.72	-4.4
	2.29	"	0.17	-0.09	9.88	2.29	0
	3.20	"	0.12	-0.09	0.89	3.18	-0.6

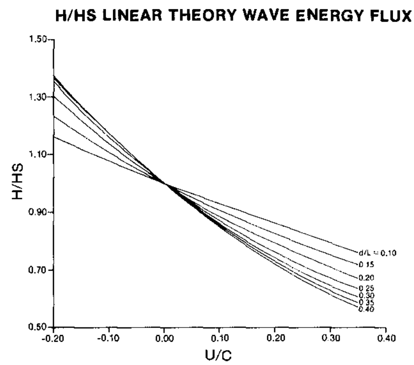

Fig. 4. The wave height change computed by linear wave theory

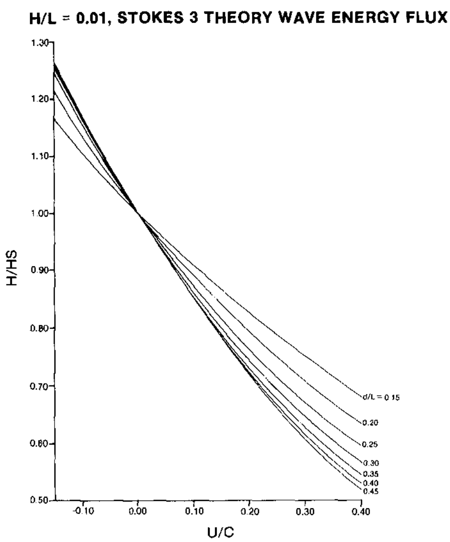

Fig. 5. The wave height change by Stokes (old) third-order wave theory for H_S/L_S=0.01

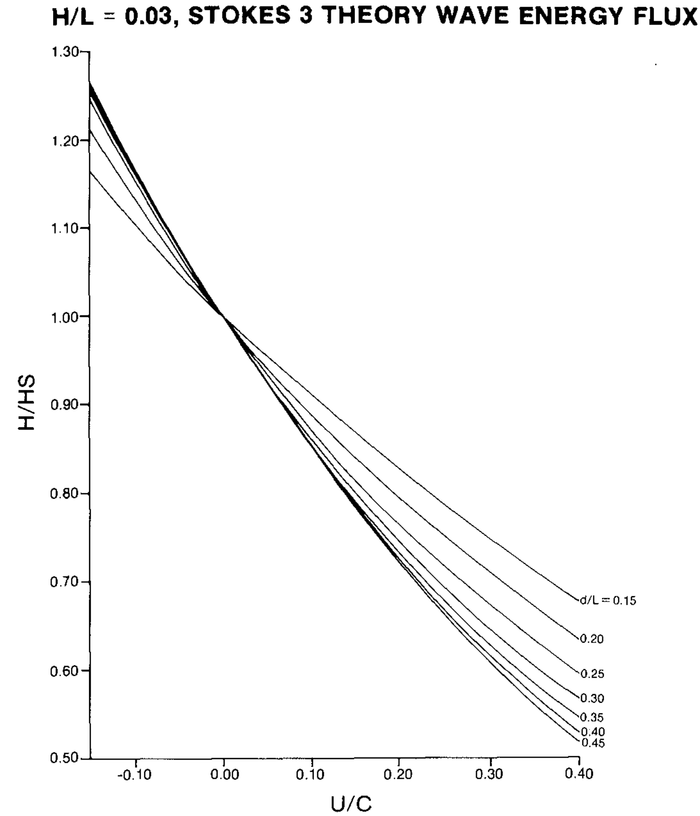

Fig. 6. The wave height change by Stokes (old) third-order wave theory for $H_s/L_s = 0.03$

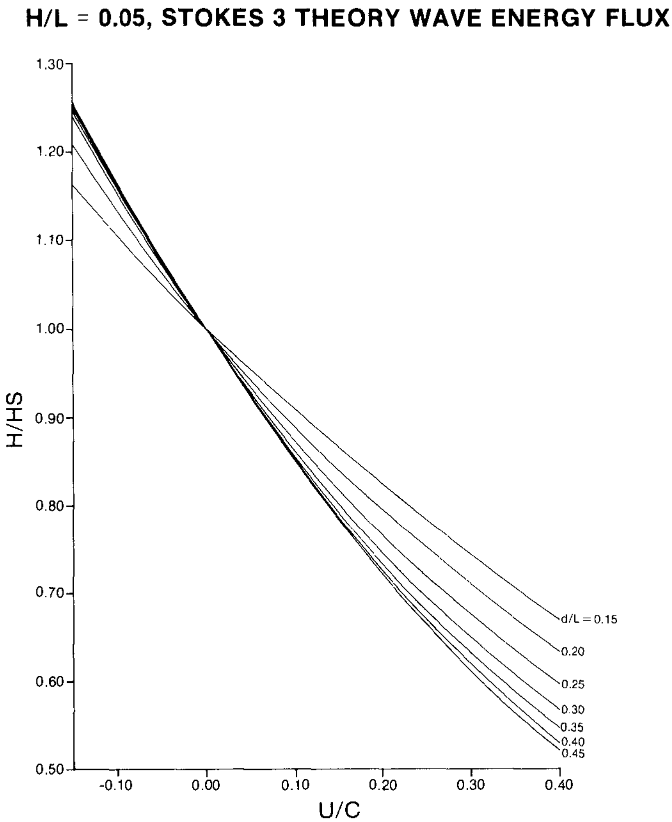

Fig. 7. The wave height change by Stokes (old) third-order wave theory for $H_s/L_s = 0.05$

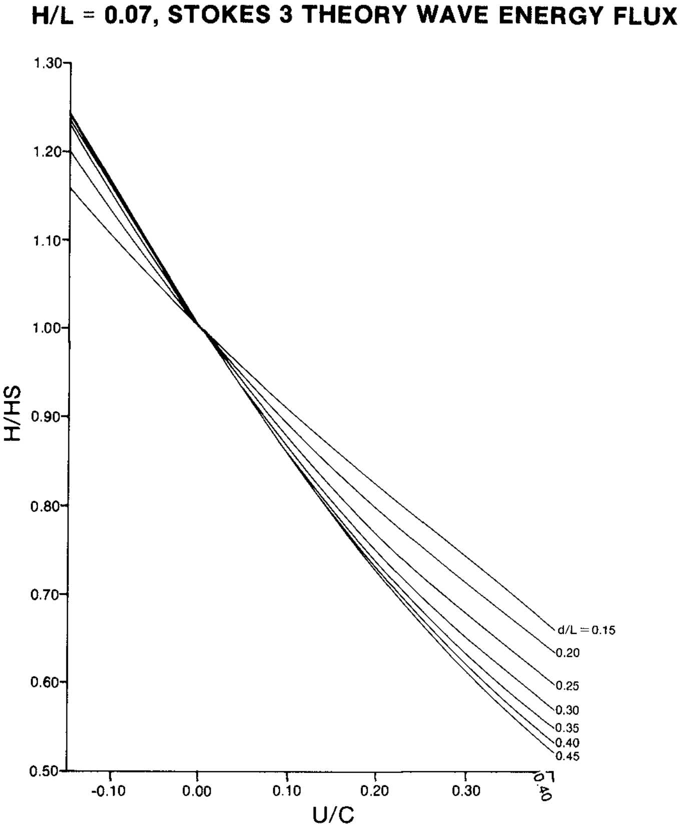

Fig. 8. The wave height change by Stokes (old) third order wave theory for H_S/L_S=0.07

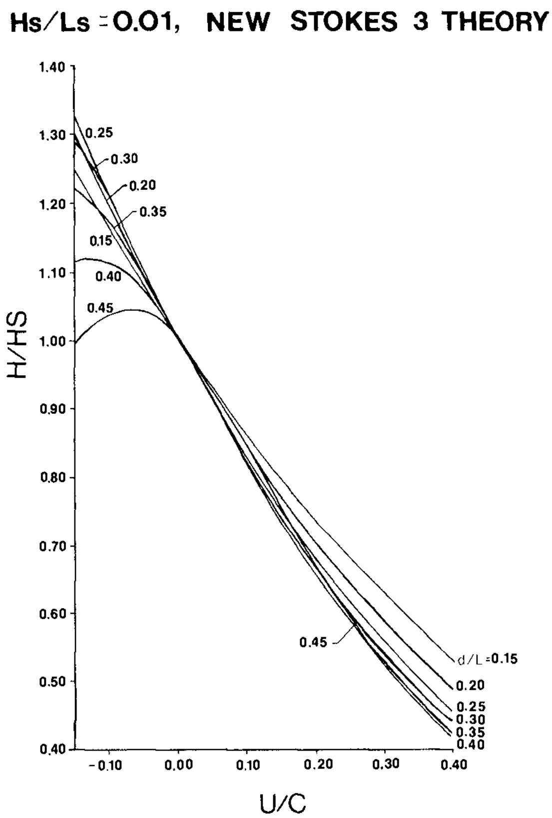

Fig. 9. The wave height change by Stokes (new) third-order wave theory for H_S/L_S=0.01

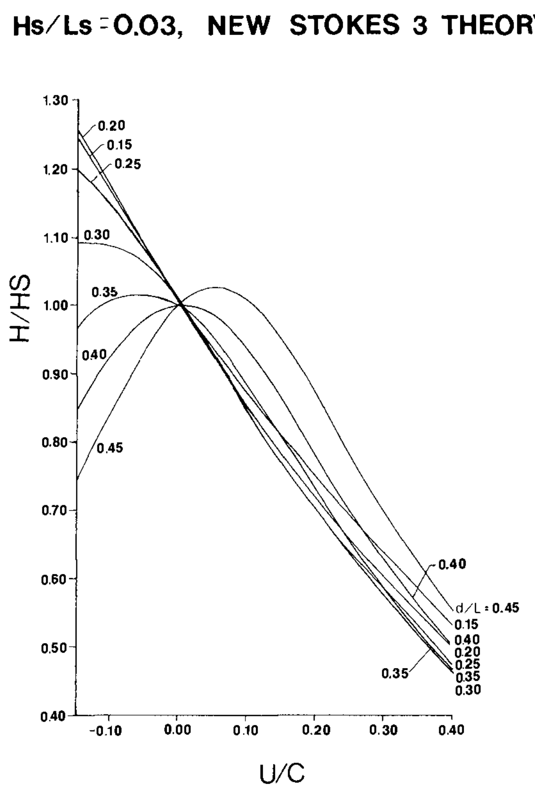

Fig. 10. The wave height change by Stokes (new) third order wave theory for H_s/L_s=0.03

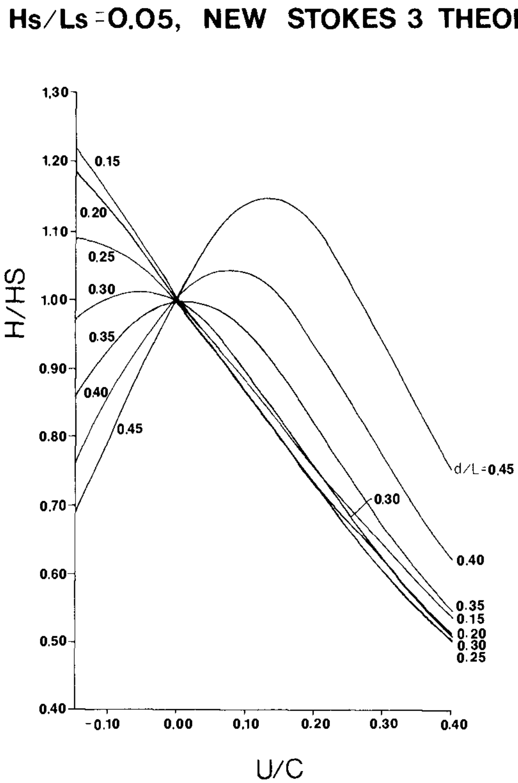

Fig. 11. The wave height change by Stokes (new) third order wave theory for H_s/L_s=0.05

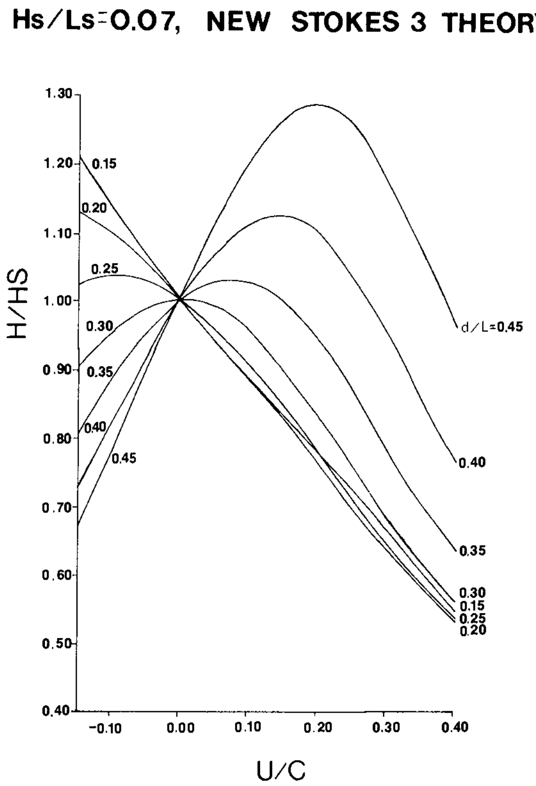

Fig. 12. The wave height change by Stokes (new) third order wave theory for $H_s/L_s=0.07$

Table IV. Wave Height Change H/H_s

$\frac{d}{L_s}$	$\frac{H_s}{L_s}$	U/C = -0.15 Airy	-0.15 (Old) Stokes 3	-0.10 Airy	-0.10 (Old) Stokes 3	0.00 Airy	0.00 (Old) Stokes 3	0.10 Airy	0.10 (Old) Stokes 3	0.20 Airy	0.20 (Old) Stokes 3	0.30 Airy	0.30 (Old) Stokes 3
0.20	0.01	1.305	1.300	1.193	1.190	1.000	1.000	0.841	0.841	0.708	0.708	0.593	0.593
	0.03		1.264		1.167		1.000		0.838		0.709		0.595
	0.05		1.210		1.129		1.000		0.834		0.709		0.597
	0.06		1.182		1.108		1.000		0.830		0.710		0.599
	0.07		1.154		1.086		1.000		0.827		0.710		0.601
0.30	0.01	1.359	1.353	1.228	1.224	1.000	1.000	0.815	0.815	0.664	0.664	0.541	0.541
	0.03		1.310		1.197		1.000		0.813		0.665		0.542
	0.05		1.245		1.154		1.000		0.810		0.666		0.545
	0.06		1.213		1.130		1.000		0.807		0.667		0.547
	0.07		1.182		1.106		1.000		0.805		0.669		0.550
0.40	0.01	1.357	1.351	1.229	1.225	1.000	1.000	0.806	0.806	0.646	0.646	0.515	0.515
	0.03		1.306		1.197		1.000		0.804		0.647		0.517
	0.05		1.240		1.153		1.000		0.802		0.649		0.520
	0.06		1.207		1.129		1.000		0.800		0.650		0.522
	0.07		1.174		1.104		1.000		0.798		0.652		0.524

(1) In the following current, errors in wave height change by the linear wave theory are minimal in comparison with the results obtained by the non-linear wave theories.

(2) When there is an opposing relative current and the wave steepness is sufficiently large, errors in wave height employing the linear theory will be greater than errors obtained employing the non-linear theories.

(3) When the relative current velocity U/C increases, the change of wave height by Tsuchiya's theory differs from the above results. Using his method, under deep water conditions the wave height change H/H_S increases with the increase of U/C and reaches a peak value, after which the value of H/H_S decreases with the increase of U/C.

4.4 Comparison of Numerical Solution Experimentally Obtained Results

Table V shows a comparison of the results of the wave height change by computation (Equation 21) with the experimental data. It can be seen that the computed values agree reasonably well with experimental data. The mean error is less than 8 percent.

Table V. Comparison of Calculated Values With Authors' Test Data

d/L_s	H_s/L_s	U/C	H/H_S Experimental Data	H/H_S Calculated Value
0.093	0.048	0.083	0.97	0.89
0.093	0.048	0.161	0.87	0.85
0.054	0.012	0.075	0.87	0.91
0.092	0.030	0.151	0.86	0.85
0.067	0.017	0.157	0.90	0.80
0.244	0.075	0.078	0.89	0.84
0.244	0.075	0.177	0.64	0.71
0.250	0.043	0.080	0.73	0.84
0.134	0.013	0.069	0.88	0.88
0.141	0.035	0.070	0.92	0.88
0.271	0.073	0.136	0.77	0.71
0.094	0.019	0.122	0.88	0.81
0.183	0.021	0.148	0.85	0.73
0.159	0.063	0.145	0.70	0.81

Table VI shows a comparison of the results of the wave height change by numerical calculation (Equation 21) with the results of Hales' and Herbich's empirical formula (2). Their formulas are as follows:

Wave height change due to the following current:

$$\frac{H}{H_s} = 0.90760 - 0.98801 \frac{U}{C_s} + 0.21123 \left(\frac{U}{C_s}\right)^2 + 0.00164 \frac{L_s}{d} + 0.00006 \left(\frac{L_s}{d}\right)^2$$

$$+ 0.88017 \frac{H_s}{L_s} + 1.05971 \left(\frac{H_s}{L_s}\right)^2 - 0.00312 \frac{U}{C_s} \left(\frac{L_s}{d}\right) + 0.88371 \frac{U}{C_s} \left(\frac{H_s}{L_s}\right)$$

$$+ 0.24931 \frac{H_s}{L_s} \left(\frac{L_s}{d}\right) \quad \ldots\ldots (34)$$

Table VI. Comparison of Calculated Values With Hales' and Herbich's (H&H)

$\frac{d}{L_s}$	$\frac{H_s}{L_s}$	$U/C_s = 0.10$		0.20		0.30	
		H&H	Cal.	H&H	Cal.	H&H	Cal.
0.20	0.01	0.841	0.865	0.748	0.781	0.659	0.745
	0.03	0.886	0.862	0.795	0.780	0.689	0.743
	0.05	0.932	0.859	0.843	0.780	0.726	0.741
	0.07	0.979	0.853	0.891	0.779	0.764	0.737
0.30	0.01	0.834	0.848	0.742	0.765	0.653	0.736
	0.03	0.871	0.847	0.780	0.764	0.687	0.735
	0.05	0.929	0.844	0.820	0.763	0.724	0.732
	0.07	0.976	0.840	0.860	0.760	0.762	0.727
0.40	0.01	0.832	0.845	0.738	0.765	0.650	0.763
	0.03	0.865	0.843	0.773	0.764	0.686	0.734
	0.05	0.898	0.842	0.808	0.762	0.723	0.731
	0.07	0.933	0.838	0.844	0.759	0.761	0.727

Since Hales' and Herbich's model test data were discrete during the opposing current, a comparison of numerical calculations with their empirical equations was not made. From Table V it can be seen that the wave height change H/H_s decreases with the increase of relative depth d/L_s both by the theoretical method and by Hales' and Herbich's formula. It appears that the wave steepness variable has a greater influence in Hales' and Herbich's empirical formula than in the theoretical equation. On the average, the difference in the results given by these two methods is within a range of 5 to 7 percent.

5. CONCLUSIONS

1. There is no dominant difference in the results obtained for the change of wave length due to the wave-current interaction by using either the linear or the non-linear wave theories. Within the range of $0.15 \le d/L_s \le 0.40$, $0.01 \le H_s/L_s \le 0.07$ and $-0.15 \le U/C_s \le 0.30$, errors by the linear wave theory are less than 10 percent as compared with the results obtained employing the non-linear wave theories.

2. Numerical calculations of wave height change by different wave theories indicate that the errors resulting from the use of the linear wave theory in comparison with errors resulting from the non-linear theories are greater when the opposing relative current

and wave steepness both become larger. However, within the range of the following currents such errors will be minimal.

3. The important factors influencing the change of wave parameter are relative current velocity U/C (or U/C_S), relative water depth d/L_S and wave steepness H_S/L_S. The relative current velocity U/C is the most important parameter. In case of the wave-current interaction, not only the wave parameter is changed but also the velocity distribution of steady flow with depth is changed. In this paper the change of steady surface velocity due to wave-current interaction is neglected. This should be considered in further research.

4. For engineering purposes, the nomogram provided in this paper may be used to estimate the change of wave parameter due to wave-current interaction. Figure 3 and Figures 5 through 8 are recommended for the calculation of wave length and wave height changes, respectively.

APPENDIX I - REFERENCES

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APPENDIX II - NOTATION

The following symbols are used in this paper:

C = wave celerity;
Ch = hyperbolic cosine;
d = water depth;
E = wave energy;

H = wave height;
K = wave number;
L = wave length;
Sh = hyperbolic sine;
Th = hyperbolic tangent;
T = wave period;
U = surface current velocity;
ω = angular frequency.

Subscripts:

a = apparent value by the observer;
g = group velocity;
r = relative value of wave to current;
s = value in still water.