SOME RECENT RESULTS FOR WAVE INDUCED MOTIONS OF A SHIP IN SHALLOW WATER

P. Å. Madsen*   I. A. Svendsen*   C. Michaelsen*

ABSTRACT

The paper describes an analytical approach to the problem of wave induced oscillations of a long ship in water with a depth which is only slightly larger than the draught of the ship. The problem is linearized (i.e. small amplitude motions and waves) and the water flow induced by the incident waves, and by the motion of the ship is determined. This also yields results for the forces in the equations of motion for the ship. These equations can be solved analytically, but the paper concentrates on giving numerical results for the solutions. Results are also given for the hydrodynamic masses and movements of inertia, and for the damping due to radiation of wave energy.

1. INTRODUCTION

The problem of ship motions in water of limited depth has been approached by several authors previously. The difficulty is the presence of the sea bed. Wilson (1958) represented this by empirically increasing the hydrodynamic mass and the damping of the ship, and Wendel (1950) determined the hydrodynamic mass of a hull section theoretically by conformal mapping. More successful is the integral equation method which consists in transforming the boundary value problem of the flow equations into an integral equation assuming the waves to be small amplitude sinusoidal (see e.g. Bai and Yeung (1974) and van Oortmerssen (1976)). Whereas this method works well for deep water conditions, the numerical computations require an increasingly large number of integration points as the depth decreases towards the draught of the ship. Yet van Oortmerssen is able to present results for hydrodynamic masses at a draught to depth ratio as large as 0.95.

Recently Andersen (1979) used a finite element technique to solve the problem numerically, but in principle had the same difficulties in describing the flow close to and underneath the ship when the draught of the ship is close to the depth of water. The largest value of the draught to depth ratio for which results are given is 0.67. Both van Oortmerssen and Andersen, however, can treat the full three-dimensional problem. The same does in principle Tuck (1970) who uses a slender body approach for which he gives results for added mass and damping coefficients.

*Inst. of Hydrodyn. and Hydraulic Engrg. (ISVA), Tech. Univ. Denmark, Build. 115, DK-2800 Lyngby. — Mail should be sent to: I. A. Svendsen.

The method presented in this paper is based on an approximation which makes it well suited to treat exactly the cases of very small underkeel clearance. In fact, the smaller the underkeel clearance, the better is the approximation. It is, however, essentially based on the assumptions of two-dimensional motion and hence restricted to application to a long ship and the heave, roll and sway motion of such a ship. Actually we will see that as the sway motion for a ship in very shallow water is strongly influenced by three-dimensional effects (the water can flow around the ends of the ship as well as beneath it) the results for sway are the least accurate. This is further discussed in § 5. The paper formulates the problem which originates from the above mentioned idea (§ 2). The equations are linearized, and the solution for the water flow is then determined by dividing the problem into subproblems which are described briefly, and examples of the solutions are given (§ 3). In § 4 we discuss the wave induced heave motion and in § 5 the roll and sway motions. Finally, the results for hydrodynamic masses and moments of inertia are given in § 6 and results for the wave generation (i.e. the damping coefficients) are presented in § 7.

2. BASIC ASSUMPTIONS AND EQUATIONS FOR THE WATER MOTION

The motion of the ship is caused by the pressure forces generated by the water motion. To determine this we consider the two-dimensional situation of a ship in beam seas. The hull is approximated by a rectangular shape (Fig. 1, which also shows the notation) and the basic assumptions are then

(i) In the clearance $R (\ll h)$ the flow velocity is entirely horizontal (i.e. $w \ll u$) and uniform over R. This is assumed to apply at sections 1-1 and 2-2, and it implies static pressure variation under the ship.

(ii) The motion under the ship is considered separately and matched directly to the flow outside the gab by requiring continuity in pressure (averaged over R) at 1-1 and 2-2.

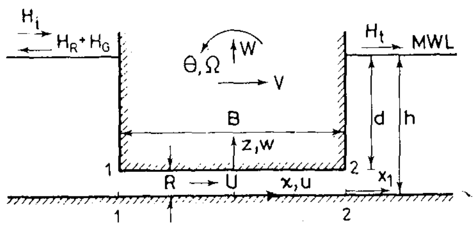

Fig. 1 Definition sketch.

a. *The water motion beneath the hull*

With reference to (i) the continuity equation for the flow beneath the hull is written

$$\frac{\partial Q}{\partial x} = - \frac{\partial R}{\partial t} \qquad \text{with} \qquad Q \equiv \int_0^R u\,dz = u\,R \tag{1}$$

Since

$$R = R_0 + z_0(t) + \theta(t)\, x \tag{2}$$

where R_0 represents the equilibrium position, z_0 and θ the heave and roll motions, respectively, we get

$$\frac{\partial R}{\partial t} = z_{0_t} + \theta_t x \equiv W(t) + \Omega(t)\, x \tag{3}$$

where $W(t)$ and $\Omega(t)$ represent the velocity of the heave motion and the angular velocity of the roll, respectively, of the ship. Substituting (3) into (1) and integrating yields Q and hence U as a function of W and Ω

$$U(x,t) = -\frac{1}{R}\{Wx + \Omega\frac{x^2}{2} + A(t)\} \tag{4}$$

where the arbitrary function $A(t)$ represents the net flux trough the gap caused by different (wave) pressures at the two sides of the hull.

The horizontal component of the equation of momentum becomes

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{5}$$

The pressure $p^+(x,z,t)$ generated by the wave and ship motion is extracted from p by the definition

$$p(x,z,t) = p^+(x,t) + \rho\, g(h-z) \tag{6}$$

where p^+ according to (i) is independent of z.

Substitution of (4) and (6) into (5) then yields (after integration) the following relation between the pressure p^+ in the gap beneath the hull and the roll and heave motion described by Ω and W

$$\begin{aligned}-\frac{1}{\rho}p^+(x,t) = {} & \frac{1}{R^2}\left\{\frac{\Omega^2}{8}x^4 + \frac{1}{2}W\Omega x^3 + \frac{1}{2}(W^2+\Omega A)x^2 + WAx + \frac{1}{2}A^2\right\}\\ & + \frac{1}{2}\Omega^2\int x^3/R^2\,dx + \frac{3}{2}W\Omega\int x^2/R^2\,dx + (W^2+\Omega A)\int x/R^2\,dx\\ & + WA\int R^{-2}\,dx - W_t\int x/R\,dx - \frac{1}{2}\Omega_t\int x^2/R\,dx - A_t\int R^{-1}\,dx\\ & - p_0^+(t)/\rho \qquad\qquad -\frac{B}{2}\le x\le\frac{B}{2}\end{aligned} \tag{7}$$

where the arbitrary function $p_0(t)$ represents the pressure $x=0$, and the integrals may be evaluated by substituting (2) for R.

Eq. (7) is the general result for the pressure beneath the hull, expressed in terms of the heave and roll motions of the ship, and A(t) and p_0^+. We can find A(t) and p_0^+ and determine the water pressures on the side of the hull and then substitute this into the equations of motion for the hull we get three simultaneous, ordinary (but non-linear) second order differential equations for the heave, roll and sway motions. These equations can in principle be solved by straightforward numerical procedures.

To obtain analytical solutions, however, we will here assume that the vertical component of the ship motion causes only small changes in R (which will be true for most situations in practice anyway). Then

(7) in the first approximation can be shown to reduce to the far simpler

$$\frac{1}{\rho}\,p^+(x,t) = W_t\,\frac{x^2}{2R} + \Omega_t\,\frac{x^3}{6R} - \frac{A_t}{R}\,x + p_0^+(t)/\rho \qquad (8)$$

The only real problem left is then to determine the pressure p_0^+, the function A and the pressures on the side of the hull. All these quantities are closely related to the water motion at the side of the hull (i.e. $|x| \geq B/2$).

b. The water motion at the side of the hull

This water motion is caused by a combination of the incoming waves, their reflection from the hull and transmission through the gap beneath the ship hull to the other side, and by the waves generated by the heave, roll and sway motion of the ship.

The following additional assumptions are used:

(iii) We neglect all energy dissipation and boundary layer effects and assume potential flow everywhere.
(iv) Both incoming and generated waves are assumed of small amplitude.

Thus the problem of wave motion at the side of the ship is linear too and we may consider the different components separately. Introducing the velocity potential ϕ (with $\vec{v} = (u,w) = -\nabla\phi$) the problem is to solve the Laplace equation

$$\nabla^2 \phi = 0 \qquad (9)$$

at each side of the ship satisfying the following boundary conditions (here formulated for the right hand side of the ship)

$$\phi_{tt} + g\,\phi_z = 0 \qquad \text{at} \quad z = h \qquad 0 < x_1 < \infty \qquad (10)$$

(with $x_1 = x - B/2$, see Fig. 1)

$$\phi_z = 0 \qquad \text{at} \quad z = 0 \qquad 0 < x_1 < \infty \qquad (11)$$

$$\phi_x = \begin{cases} -U_2 & 0 < z < R \\ \Omega(t)\,(z-m) - V & R < z < h \end{cases} \quad x_1 = 0 \qquad (12)$$

where $U_2 = U(x = B/2, t)$ in (4), and V is the velocity of the sway motion of the ship. m is the height of the point of rotation above the sea bed.

Finally at $x_1 \to \infty$ a radiation condition applies. This boundary value problem for the flow at $|x| > B/2$ is illustrated in Fig. 2.

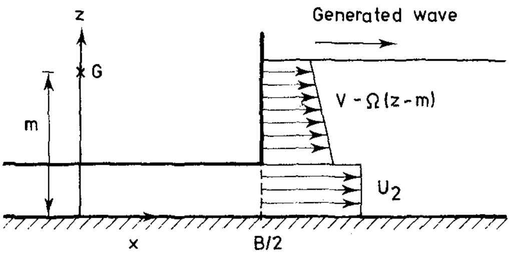

Fig. 2
Boundary value problem for flow at the side of the hull.

The total motion in the linear problem then consists of the following components:

1. The standing wave composed of the incoming plus a (formally) fully reflected wave. This motion occurs only at one side of the hull (assumed to be the left side in Fig. 1). (ϕ_W, H_W, p_W^+ etc.)
2. The motion generated under a fixed ship by the above mentioned standing wave. On the right hand side this represents the waves transmitted through the opening beneath the hull. On the left hand side it represents the reduction in reflection due to the transmission. (ϕ_G, H_G, p_G^+ etc.)
3. The waves generated by the sway motion of the ship. (ϕ_S, H_S, p_S^+ etc.)
4. The waves generated by the heave motion of the ship. (ϕ_H, H_H, p_H^+ etc.)
5. The waves generated by the roll motion of the ship. (ϕ_R, H_R, p_R^+ etc.) where we have

$$\phi = \phi_W + \phi_G + \phi_S + \phi_H + \phi_R \text{ ; etc.} \tag{13}$$

It should be mentioned that it is not strictly necessary for a solution to split the problem into the components described above. The solution may, of course, well be obtained by solving the complete boundary value problem described by (9) - (12). The splitting is convenient, however, since the results are rather complicated and since the effect of the individual components of the total motion has to be separated anyway later on in the equations of motion for ship.

3. DESCRIPTION OF FLOW COMPONENTS AND RESULTS

In the following we give some results for the pressures induced on the ship in each of the flow situations mentioned above. As the calculations are rather tedious it will be impossible at the space available to bring details of the derivations.

A more detailed description of the solution for the heave generated flow was given by Svendsen (1968) (who in fact considered the case of large amplitudes). For roll (point 5 in the list above) the flow problem was analysed by Svendsen et al. (1977 a), and the reflection - transmission problem (points 1 and 2) was discussed in Svendsen et al. (1977 c).

The method of solution is to write the velocity potential at $|x| \geqq B/2$ for each of the flow components

$$\phi_i = e^{i(\omega t + \psi_i)} \sum_n X_{in}(x) \; Z_{in}(z) \tag{14}$$

and assume that in (4) and (12) U_2, W, Ω and V vary as $e^{i(\omega t + \gamma_i)}$.

a. The reflection - transmission problem

The water flow around a fixed ship in waves is sketched in Fig. 3. The wave pattern consists of the standing wave which yields the pressure

$$p_W^+ = \rho \, g \, H_i \frac{\cosh kz}{\cosh kh} \cos \omega t \qquad \text{at} \quad x = -B/2 \tag{15}$$

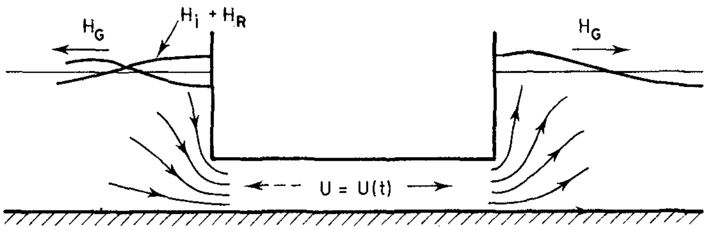

Fig. 3 Description of wave induced flow around a fixed ship.

and the waves generated by the water flow under the hull. This flow is induced by a combination of the standing wave pressure given by (15) and the pressure from the waves generated by the flow itself. Therefore the solution to the problem involves determination of the velocity U_G beneath the hull, and since the body of water has a certain inertia (finite width of the ship) U_G is not in phase with p_W^+ and neither are the waves generated.

To illustrate the nature of the results we find for U_G

$$U_G = \frac{H_i\ \omega}{a^2 + b^2}\ e\,(b\ \cos\omega t + a\ \sin\omega t) \tag{16}$$

where

$$e = \frac{g}{\omega^2\ B}\ \frac{\sinh kR}{kR\ \cosh kh} \tag{17}$$

$$a = 1 + 8\,h\ B_W/B\ ; \qquad b = 8\,h\ A_W/B \tag{18}$$

and

$$A_W = \frac{\sinh^2 kR}{(kR)^2\,(\sinh 2kh + 2kh)}\ \frac{R}{h} \tag{19}$$

$$B_W = \sum_n \frac{\sin^2 \sqrt{\lambda_n}\,R}{(\sqrt{\lambda_n}\,R)^2\,(\sin 2\sqrt{\lambda_n}\,h + 2\sqrt{\lambda_n}\,h)}\ \frac{R}{h} \tag{20}$$

The parameter k is the wave number which in general satisfies the dispersion relation

$$\omega^2 - g k\ \tanh kh = 0 \tag{21}$$

where ω is the frequency of the incident and the generated waves, and $\sqrt{\lambda_n}$ similarly satisfies the equation

$$\omega^2 + g\sqrt{\lambda_n}\ \tan\sqrt{\lambda_n}\,h = 0 \tag{22}$$

Eq. (21) has one root whereas (22) has infinitely many roots.

The total pressures p_{WG} under the hull corresponding to the combination of the standing wave and the generated waves may then be written

$$p_{WG}^+(x,t) = 8\,\rho\,\omega\,h\left\{A_W\,U + B_W\ U_t/\omega\right\}\frac{x}{B}$$
$$+\,\rho\,g\,H_i\ \frac{\sinh kR}{kR\ \cosh kh}\ \cos\omega t\left(\frac{1}{2} - \frac{x}{B}\right) \qquad -\frac{B}{2} \le x \le \frac{B}{2} \tag{23}$$

and along the sides of the hull ($x = \pm\ B/2$) we get (in addition to p_W in (15))

$$p_G^+\left(\frac{B}{2},z,t\right) = - p_G^+\left(-\frac{B}{2},z,t\right)$$
$$= 4\,\rho\,\omega\,h\left\{A\,U\,\cosh kz + \sum b_n \frac{U_t}{\omega}\cos\sqrt{\lambda_n}\,z\right\} \tag{24}$$

where

$$A = \frac{\sinh kR}{kR(\sinh 2kh + 2kh)} \tag{25}$$

$$b_n = \frac{\sin\sqrt{\lambda_n}R}{\sqrt{\lambda_n}\,R(\sin 2\sqrt{\lambda_n}h + 2\sqrt{\lambda_n}h)} \tag{26}$$

Notice that in this case where U is independent of x we find (in accordance with (the linearized version of)(5)) that p^+ varies linearly with x in the gap underneath the ship ($-B/2 < x < B/2$). It should also be mentioned that since the pressure due to (24) varies with z, which p^+ under the ship by assumption does not (see (6)), the continuity in pressures at 1-1 and 2-2 (Fig. 1), has been imposed on the mean values over R.

Figs. 4 and 5 show the variation of the pressures given by (23) (at $x = B/2$) and (24), respectively. We see that for most situations the assumption of p^+ constant over k, which led to averaging p^+ over R for the flow outside the gap, is actually quite reasonable, in particular for long waves.

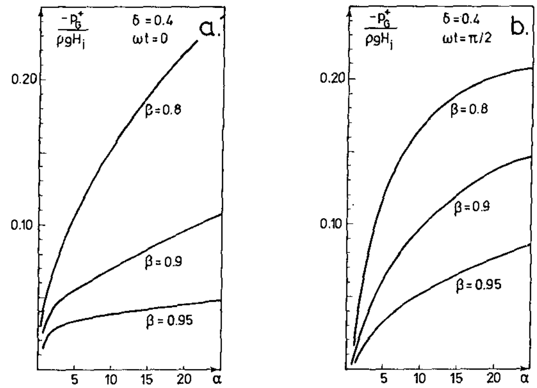

Fig. 4 Pressure variation at 2-2 versus $\alpha = g/\omega^2 h$ and $\beta = d/h$ for a fixed ship

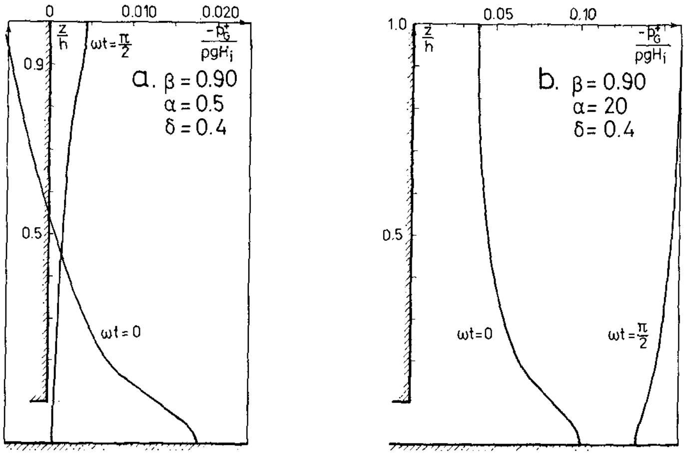

Fig. 5 Pressure variation along the side of a fixed ship.

b. Flow and pressures induced by sway

The instantaneous flow pattern generated by a forced sway motion is shown in Fig. 6. Qualitatively this flow resembles the motion generated by the standing wave system described above, but quantitatively the problem is closer related to the flow arpund a piston type wave generator with a large leak at the bottom. The difference is that in our problem the width of the ship is significant which (as mentioned above) makes the inertia of the water volume beneath the hull an important factor. It causes both a change in amplitude of the waves generated, and a phase shift relative to the motion of the ship.

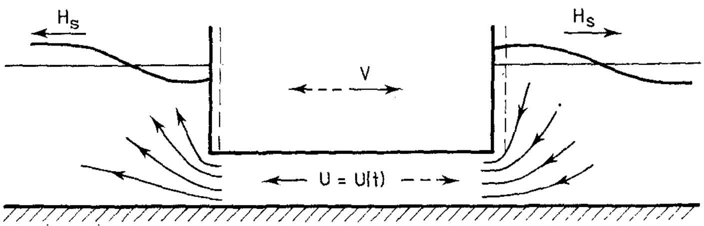

Fig. 6 Description of flow induced by a sway motion.

In the end the pressures on the hull corresponding to the flow will be integrated to forces which enter the equations of motion, and in principle we may envisage both horizontal and vertical forces, and moments about the center of gravity to originate from each component of motion considered. Since each of these forces or movements represents the reaction on the ship from the surrounding water due to the motion in question and hence depends on that motion (e.g. the sway), it is crucial for the solution of the equations for the motion of the ship to express the magnitude of the above mentioned forces and moments in terms of the velocity and acceleration associated with the component of motion in question. Thus for sway the solution for the pressures on the hull is written

$$p_S^+ = f_1\left(g/\omega^2 h, \frac{d}{h}, \frac{d}{B}, x\right) V + f_2\left(g/\omega^2 h, \frac{d}{h}, \frac{d}{B}, x\right) V_t/\omega \qquad -\frac{B}{2} \le x \le \frac{B}{2} \quad (27)$$

where V and V_t are instantaneous velocities and accelerations, respectively, in the sway motion.

Both Ω and W are zero for pure sway, so we see from (8) that for sway the pressure under the ship must vary linearly with x. The variation at x = B/2 (sect. 2-2) is shown in Fig. 7. The pressures are shown at two different times, namely at the time when $V_t = 0$ (Fig. 7 a) and at the time when V = 0 (Fig. 7 b). Thus the two figures give the coefficients of each of the two terms in (27). Along the side the pressure variation is a cosh kz superimposed by the classical cos $\sqrt{\lambda_n}$ z terms also present in (24), and the same averaging of the pressures over R has again been applied in the matching procedure.

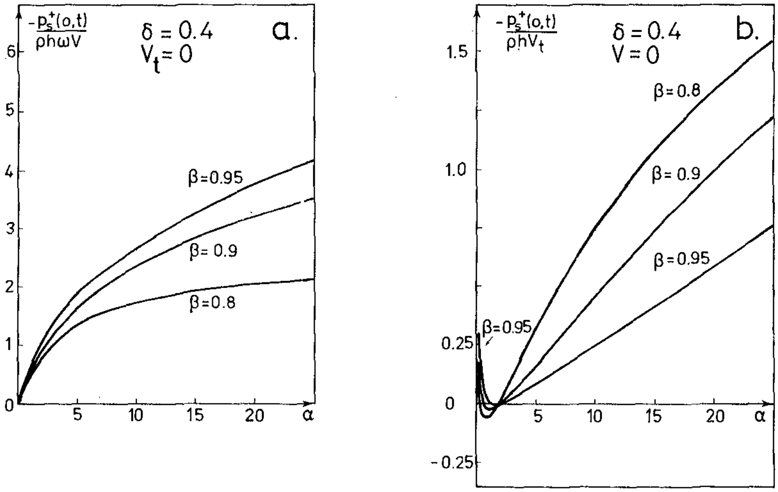

Fig. 7 Pressure variation at 2-2 versus $\alpha = g/\omega^2 h$ and $\beta = d/h$ for sway induced flow.

c. Flow and pressures induced by heave

The results for this case can be derived directly from Svendsen (1968) by linearizing with respect to W, the velocity in the heave motion. The flow is described in Fig. 8.

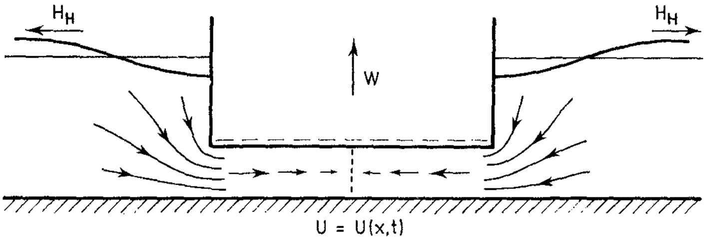

Fig. 8 Description of flow induced by a heave motion.

As can also be inferred from (8) the pressure under the ship varies as x^2, because for symmetry reasons we must have $A(t) = 0$. Here we show figures with the pressure variation with x under the ship (Fig. 9) and the variation at sect. 2-2 (Fig. 10), in both cases at two different times, namely when $W_t = 0$ (Fig. 10 a) and when $W = 0$ (Fig. 10b). Writing again the pressures as

$$p_H^+ = g_1 W + g_2 W_t/\omega \tag{28}$$

this means that Fig. 10 shows the two coefficients in (28) at sect. 2-2.

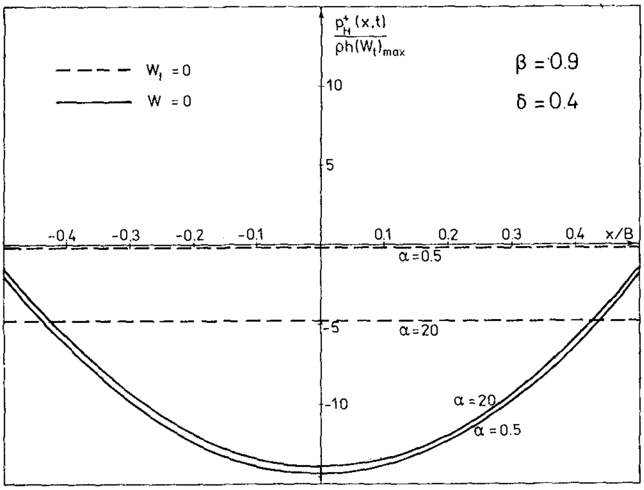

Fig. 9 Pressure variation under the ship induced by a heave motion.

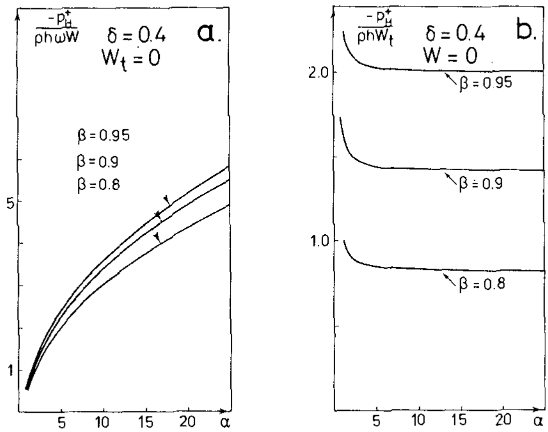

Fig. 10 Variation of pressure at 2-2 for heave induced flow.

d. Flow and pressures induced by roll

This represents by far the most complicated flow conditions. The situation is shown in Fig. 11. Whereas the flow outside the gap is more or less the same as in the previous cases we see that under the ship the direction of the flow changes at two points. The flow is illustrated quantitatively in Fig. 12 where the discharge Q is shown versus x for different phases. Two cases are shown: a typical deep water situation (Fig. 12 a) and a long wave situation (Fig. 12 b). Since $Q = UR$ we see from (4) that Q varies as x^2. The situation is somewhat more complicated, however, because the waves generated are in antiphase which causes A(t) to be nonzero which again means there is a phase difference between U and the angular velocity Ω of the roll motion (as for sway). This is particularly clear in Fig. 12 b where $U \neq 0$ when $\Omega = 0$ and the points of zero velocity are changing with time.

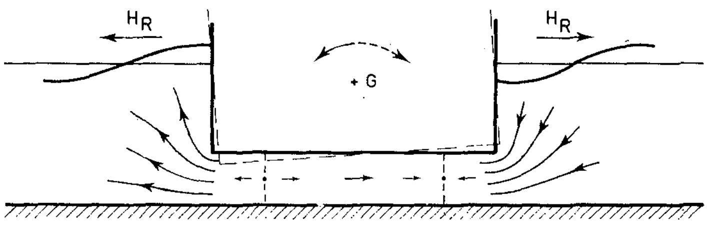

Fig. 11 Description of flow induced by a roll motion.

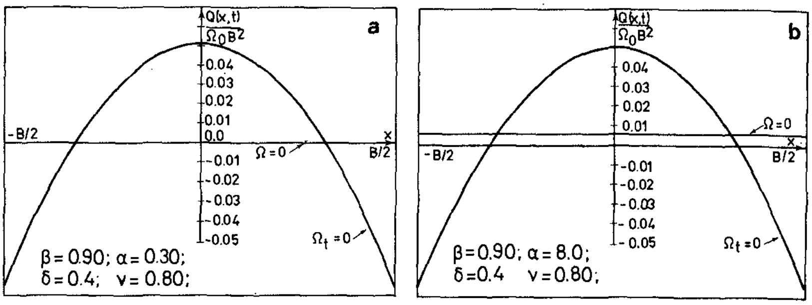

Fig. 12 Discharge Q under the ship induced by a roll motion.

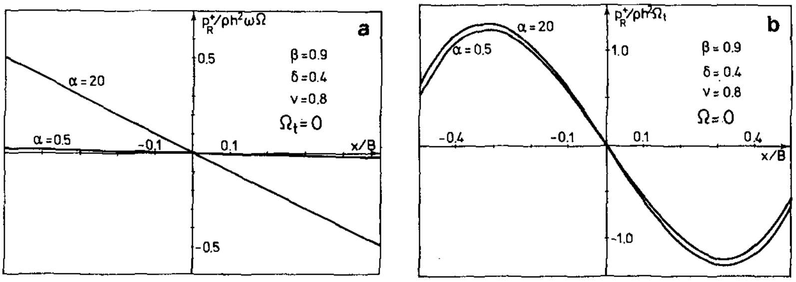

Fig. 13 Pressure variation under the ship induced by a roll motion.

Fig. 13 shows the corresponding pressures which according to (8) vary as x^3 with an x component due to A(t).

As in the previous cases the result for the pressure along the side of the ship contains one term varying as $\cosh kz$ and an infinite number of terms varying as $\cos \sqrt{\lambda_n}\, z$ (where $\sqrt{\lambda_n}$ are the solutions to (22)) which are necessary to satisfy the boundary conditions for the flow at $|x| = B/2$. All the results for roll are rather complicated and we shall omit them here.

4. WAVE INDUCED HEAVE MOTION

The equations of motion for the ship are the three components of Newton's second law corresponding to horizontal and vertical projection, and moment about an axis perpendicular to the x-z-plane through the center of gravity G.

It turns out that neither roll nor sway produce total pressure forces on the ship which has vertical components. Similarly heave does not produce net horizontal forces or forces that have a moment about G. Consequently the equation for heave (vertical projection) does not contain terms proportional to V, V_t, Ω or Ω_t and the other two equations have no terms with W or W_t. This means that heave is not coupled to the other two degrees of freedom, and the equation for wave induced heave may be solved independently of the equations for roll and sway. This problem of wave induced heave was described in some detail by Svendsen et al. (1977 b).

When the forces are integrated as mentioned previously we find for the heave motion $z_0(t)$ the equation

$$M_H \frac{d^2 z_0}{dt^2} + S_H \frac{dz_0}{dt} + g\, z_0/d = g \frac{H_i}{2\,d} \frac{\sinh kR}{kR \cosh kh} \cos \omega t \tag{29}$$

where

$$M_H = 1 + \frac{2\,B}{d} B_w + \frac{B^2}{12\,R\,d} \tag{30}$$

$$S_H = \frac{2\,B}{d} A_w \omega \tag{31}$$

In (30) the first term (1) represents the mass of the ship. The other two terms in (30) originate from the components of the water pressures which are proportional to W_t. Hence these two terms represent the hydrodynamical mass for the heave motion.

The coefficient S_H represents the damping of the oscillation which is linear, i.e. proportional to W. The energy dissipation described by this W-term is physically caused by the generated waves which carry energy away from the ship.

Even though the problem has been linearized it still contains the leading or dominating effect of the presence of the bottom. From (29) we can deduce that this effect emerges as a quite significant change in the resonance frequency, relative to the value we would have in deep water. The reason is that the hydrodynamic mass increases rapidly when the underkeel clearance decreases (as we shall see later).

In Fig. 14 is shown the resonance or natural frequency ω_e (undamped) nondimensionalized as $g/(\omega_e^2\, h\, \beta)$. The deep water value for ω_e shown in the figure has been determined using the results for the hydrodynamic mass in deep water given by Andersen (1978).

The equation (29) may be solved analytically, and the result becomes

$$\left.\begin{aligned} &\frac{z_0(t)}{H_i} = \frac{\alpha(\alpha - \beta M_H)}{2(\beta^2 S_H^2 + (\alpha - \beta M_H)^2} \frac{\sinh kR}{kR \cosh kh} \frac{\cos(\omega t + \phi)}{\cos \phi} \\ &\text{where} \\ &\phi = \arctan \left(\frac{-\beta S_H}{\alpha - \beta M_H} \right) \\ &\text{and} \\ &\alpha \equiv g/\omega^2 h \qquad \beta = d/h \end{aligned}\right\} \tag{32}$$

Numerical results for the amplitude of (32) are shown in Fig. 15 for three different values of β. We notice that somewhat unexpected perhaps the amplitudes generally increase as the underkeel clearance decreases.

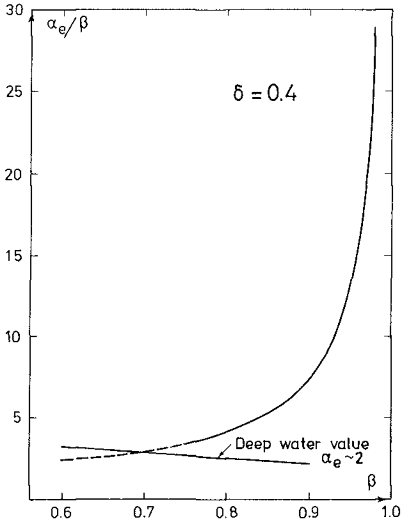

Fig. 14 The resonance frequency ω_e for heave versus $\beta = d/h$

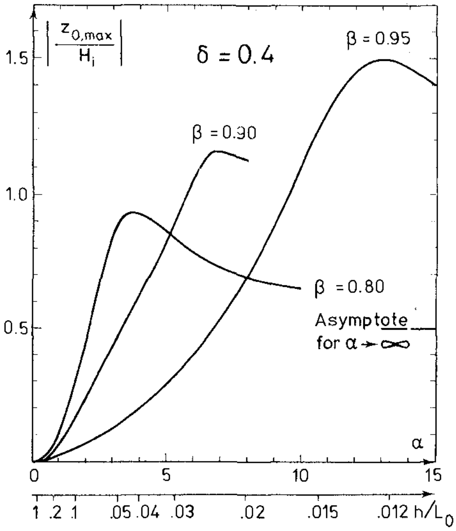

Fig. 15 The amplitudes of wave generated heave motion versus $\alpha = g/\omega^2 h$

5. WAVE INDUCED ROLL AND SWAY MOTION

In contrast to heave the presence of a sway motion will automatically generate a roll motion and vice versa. These two motions are even in this simple model coupled, and the corresponding two equations of motion consequently contain terms with both independent variables (simultaneous differential equations). This naturally complicates matters a great deal, but since the equations for a given incident wave are ordinary second order differential equations with constant coefficients an analytical solution is in principle straightforward. The algebra and the results are just rather complicated, so we only present the equations and numerical results for the solution. The equations may be written

$$a_1 \frac{d^2\xi}{dt^2} + a_2 \frac{d\xi}{dt} + a_3\,\xi = a_4 \frac{d^2\theta}{dt^2} + a_5 \frac{d\theta}{dt} + a_6\,\theta + a_7 \cos\omega t + a_8 \sin\omega t \tag{33}$$

$$b_1 \frac{d^2\theta}{dt^2} + b_2 \frac{d\theta}{dt} + b_3\,\theta = b_4 \frac{d^2\xi}{dt^2} + b_5 \frac{d\xi}{dt} + b_6\,\theta + b_7 \cos\omega t + b_8 \sin\omega t \tag{34}$$

where $\xi(t)$ represents the sway motion and $\theta(t)$ the roll.

The coefficients (a_i, b_i) are here functions of the parameters of the problem. Of those we have already mentioned $\delta = d/B$, $\beta = d/h$ and $\alpha = g/\omega^2 h$.

However, other parameters enter the problem too. Firstly, the coefficient a_3 which represents the restoring force in the sway motion can only be nonzero if we introduce e.g. mooring lines. This brings two new parameters into the problem: $k_f/\rho g h$, the nondimensional stiffness of the moorings (pr. m length of the ship), and l/h the nondimensional height of the mooring point above the center of gravity (see Fig. 16). These parameters actually occur in a_3, b_3, a_6 and b_6.

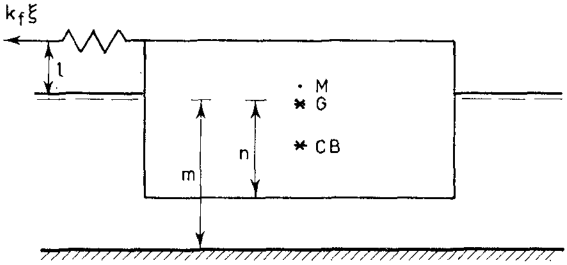

Fig. 16 The geometry of moorings and ship's stability.

Secondly, the roll motion of course depends on the stability of the ship, i.e. on the height n of the center of gravity above the bottom of the ship (Fig. 16). In the presentation of the numerical results n is nondimensionalized as $\nu = n/d$.

Quite obviously it is not possible numerically to map the variation of the results with all these parameters. We have therefore chosen to show the variation with what we consider the most interesting two, namely α and β, and in doing so we keep the other parameters fixed at chosen values.

Thus from stability considerations (ships usually have a metacentric height of a few metres) we have chosen $\gamma = 0.8$. For $k_f/\rho g h$ it may be shown that even a set of the stiffest steel wires cannot bring $k_f/\rho g h$ above about 0.1, and we will see that the stiffest moorings are the most interesting for our conclusions. So we show results for $k_f/\rho g h = 0.1$ only. Further we use (a little unrealistically) $l = 0$, but it may be inferred from the results that this is not so important. Finally we choose $\delta = 0.4$ which is a realistic value particularly for bulky ships as tankers or bulk carriers.

Fig. 17 then shows results for the resonance periods at different β. Since there are two degrees of freedom there are also two resonance frequencies. Each of these represents a free oscillation in which both roll and sway occur but locked together phasewise and with a fixed ratio between their amplitudes.

We notice that one of the resonance frequencies is much smaller than the other. The associated mode of resonant oscillation consists predominantly of sway. This shows that even for the stiffest moorings we can think of ($k_f/\rho g h = 0.1$) the resonance period for the sway-type mode of oscillations corresponding to very long period waves. In storm waves such waves merely occur as subharmonics. The phenonmenon is quite well-known, as in harbours or model experiments with harbours particularly sway (and surge) motions often occur at a subharmonic frequency.

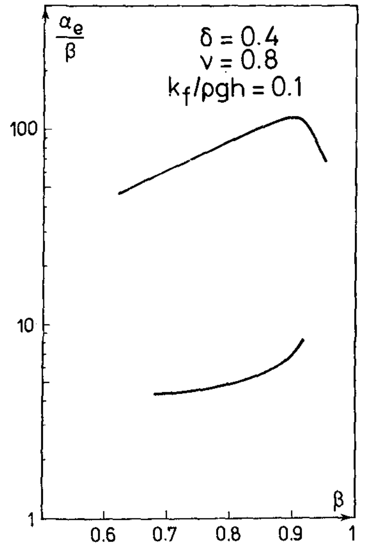

Fig. 17 Resonance frequencies for coupled roll and sway.

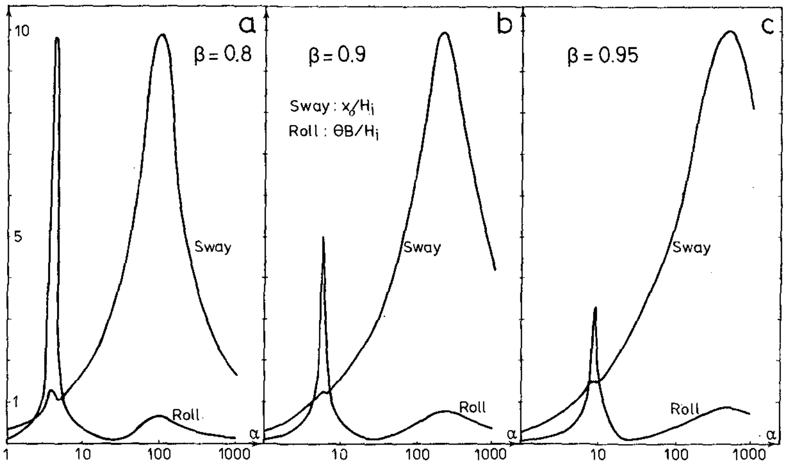

Fig. 18 The amplitudes of wave generated roll and sway.

It should be added, however, that in particular for sway the three-dimensional effects may be quite important. In a three-dimensional model the water can escape both beneath the ship and around the ends. The latter possibility is not included in our two-dimensional model, and in particular for a small underkeel clearance this will yield too large values of the hydrodynamical mass and consequently too large values of the resonance period.

Results for the wave induced amplitudes are given in Fig. 18 versus α for three different values of β.

6. HYDRODYNAMIC MASSES AND MOMENTS OF INTERTIA

As was already indicated in the description of wave induced heave, one of the by-products of the solution for the water flow is the hydrodynamic masses for heave and sway and the hydrodynamic moment of inertia for roll.

These quantities represent that part of the pressure forces which is proportional to the acceleration in the motion considered. Numerical results are given in Fig. 19. The values given are relative to the ships mass or moment of inertia.

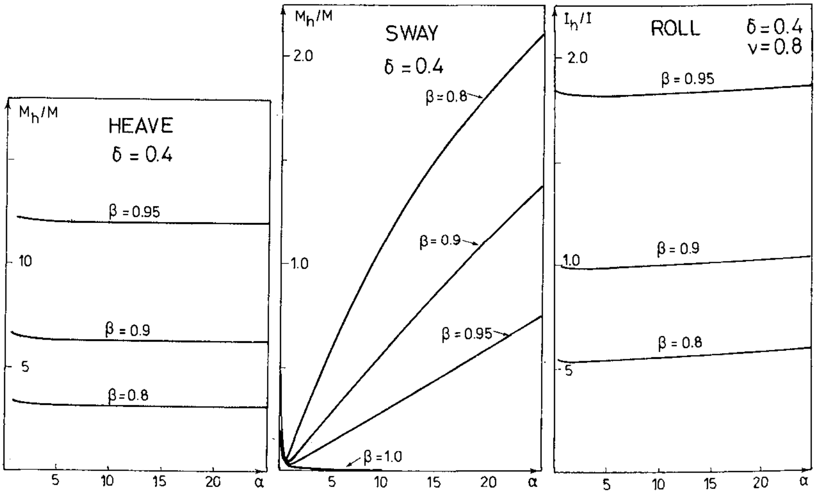

Fig. 19 Hydrodynamic masses for heave and sway, and moment of inertia for roll.

We see that in particular the hydrodynamic mass of heave (Fig. 19 a) is many times the ship's own mass and depends strongly on β, the relative draught of the ship.

The hydrodynamic moment of inertia found for roll is small — of the same order of magnitude as the ships own moment of inertia. Yet the result depends very much on β.

Both for heave and roll the results are virtually independent of α, the nondimensional wave period. This indicates that the major contribution in these cases comes from the flow underneath the ship which is consistent with the fact that when these two motions occur water is necessarily squeezed in and out of the gap.

This is not the case with sway (Fig. 19 c) where the water motion in the gap is driven entirely by the pressure from the waves generated which means that the hydrodynamic mass results from a combination of the e^{-x}-terms in the wave solution and the phase shift in that solution caused by the flow under the ship.

7. RESULTS FOR THE WAVE GENERATION

The oscillating ship acts as a wave generator, and the radiation of energy due to the waves represents the damping in the oscillations. It is therefore of interest to look at the height of the waves generated.

Fig. 20 shows numerical results for these waves for each of the three motions in question. In each of the translatory cases the wave height has been related to the amplitude of the motion, for roll to twice the amplitude θ_0 B at x = B/2.

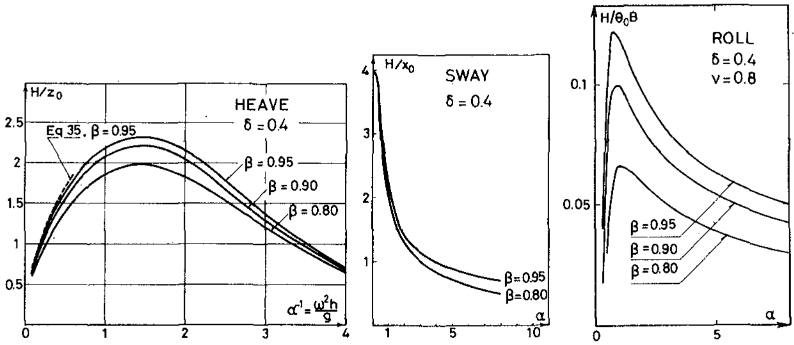

Fig. 20 Height of the generated waves.

We see that in particular at a sway motion the ship is a very efficient wave generator. We also notice that the difference in wave height at sway for different values of β = d/h indicates the influence of the flow beneath the ship (i.e. the 'leakage' in the wave generator problem).

In contrast to the less frequency dependent damping at sway, both heave and roll show a maximum damping at intermediate frequencies. The reason is that in these two cases the waves are generated almost entirely by the water flow in and out of the gap. In the case of deep water waves (small α) this is a very inefficient way of generating waves, hence the small heights. For very long waves (large α) the height of the waves will decrease to the limit given by a purely 'hydraulic' model which e.g. for heave yields

$$\frac{H}{z_0} = \alpha^{-1/2}\,\frac{B}{h} \tag{35}$$

This result is also shown in Fig. 20 for $\beta = 0.95$.

It should be added that of course energy is conserved also in the case of a fixed ship even though it seems as if a standing wave (to the left in Fig. 1) — which yields no net energy flux towards the ship — causes generation of progressive waves which on both sides of the ship represents an energy flux away from the ship. The reason for this paradox is that the sum of the standing wave and the wave propagating towards the left at $x < -B/2$ actually has an energy flux towards the right which is exactly the energy transmitted through the gap and away from the ship through the wave at $x > B/2$.

8. CONCLUDING REMARKS

As mentioned in § 2 the method presented in this paper is based on a number of assumptions. The validity of some of these were already discussed by Svendsen (1968) who showed by comparison with experiments for heave that the pressure underneath the hull is quite accurately described by the present model. He also found that for a certain phase interval of each wave period the pressure close to 1-1 and 2-2 may (for β close to one) be influenced by the separation of the outgoing flow from the hull of the ship (jet-like flow). However, this deviation from the assumption (iii) will only have a minor effect on the total forces. Finally the results in Fig. 5 show that the patching of the average of the pressures at 1-1 and 2-2 will be a very good approximation except perhaps for quite short wave periods.

Thus in general we must expect that the model will yield rather accurate results for the two-dimensional situations it covers.

9. REFERENCES

Andersen, P. (1978) Wave induced motions of and loads on a ship in restricted water (in Danish). Inst. Ocean Engrg., Tech. Univ. Denmark, Ph.D. thesis, 122 pp.

Andersen, P. (1979) Ship motions and sea loads in restricted water depth. Ocean Engrg., 6, pp. 557-569.

Bai, K.J. and R.W. Yeung (1974) Numerical solutions to free surface flow problems. Proc. 10th O N R Symposium.

Oortmerssen, G. van (1976) The motion of a ship in shallow water. Ocean Engrg., 3, pp. 221-255.

Svendsen, I.A. (1968) On the forces induced on a rectangular cylinder by a forced heave motion with draught-depth ratio close to unity. J. Hydr. Res., 6, 4, pp. 335-360.

Svendsen, I.A., P. Madsen and C. Michaelsen (1977 a) The rolling of a ship in shallow water. Inst. Hydrodyn. and Hydr. Engrg., Tech. Univ. Denmark, Prog. Rep. 42, pp. 21-28.

Svendsen, I.A., P.Å. Madsen and C. Michaelsen (1977 b) Wave induced heave motion of a ship in shallow water. Inst. Hydrodyn. and Hydr. Engrg., Tech. Univ. Denmark, Prog. Rep. 43, pp. 23-30.

Svendsen, I.A., P.Å. Madsen and C. Michaelsen (1977 c) Wave reflection and transmission from a ship in shallow water. Inst. Hydrodyn. and Hydr. Engrg., Tech. Univ. Denmark, Prog. Rep. 44, pp. 27-34.

Tuck, E.O. (1970) Ship motions in shallow water. J. Ship Res., 14, pp. 317-327.

Wendel, K. (1950) Hydrodynamische Massen und hydrodynamische Massenträgheitsmoment. Jahrbuch der Schiffbautechnischen Gesellschaft, 44.

Wilson, B.W. (1958) The energy problem in the mooring of ships exposed to waves. Proc. Princeton Conf. on Berthing and Cargo Handling at Exposed Locations, Princeton Univ., New Jersey.