CHAPTER 90

SCALE RELATIONS FOR EQUILIBRIUM BEACH PROFILES

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ABSTRACT

The scale relation for modeling natural beach profiles in the laboratory and selecting size of model sediments is examined. The results are shown for relating scale law to the dimensionless fall velocity and the model law for selecting sand size is proposed.

INTRODUCTION

There are many practical problems related to coastal engineering, which may be solved by using movable bed scale models of the coastal zone. The scale model must obey the laws of sediment transport in order to obtain satisfactory results. However, sediment transport in the coastal zones is so complex that the mechanism is not fully understood.

Some authors have attempted to find a model law relationship for equilibrium beach profiles. Yalin[1)] derived a scale law for the offshore zone using a bed velocity based on laminar boundary conditions. Brebner, Kamphuis and Paul[2)] have performed extensive experimental tests on movable-bed models using light weight sediment, and Le Méhauté[3)] presented a scaling law for coastal movable-bed models in the breaker zones.

The purpose of this study is finally to determine the scale law relationship for coastal movable-bed model. As first step, this paper concerns itself with the derivation of proper scale laws for modeling of equilibrium beaches.

DIMENSIONAL ANALYSIS AND SCALE RELATIONS

The depth of beach profiles, h may be expressed as a function of

$$H_o,\ T,\ D,\ \rho,\ \rho_s,\ g,\ \nu,\ x$$

where H_o the deepwater wave height, T the period of waves, D the representative diameter of sediments, ρ the fluid density, ρ_s the sediment density, i_o the initial slope of the beach, g the acceleration of gravity, ν the kinematic viscosity of fluid, and x the horizontal distance measured from the initial shoreline as shown in Figure 1.

The beach profile may be expressed as

$$h = f\ (\ x,\ H_o,\ T,\ D,\ \rho,\ \rho_s,\ i_o,\ g,\ \nu\ ) \cdots\cdots\cdots(1)$$

By using the relation of $L_o = gT^2/2\pi$ and neglecting the fluid viscosity,

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Equation (1) may be rewritten in the dimensionless form as

$$\frac{h}{L_o} = f\left(\frac{x}{L_o}, \frac{H_o}{L_o}, \frac{H_o}{D}, \frac{\rho_s}{\rho}, i_o \right) \dots\dots\dots\dots\dots\dots (2)$$

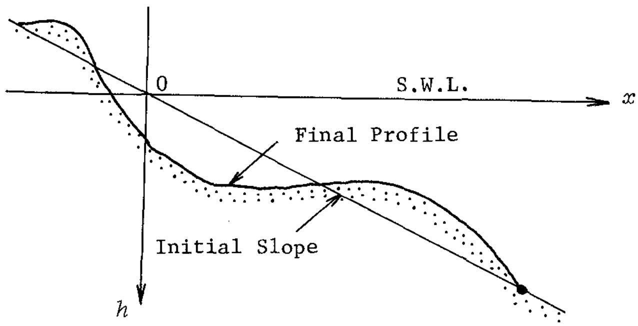

Figure 1 Definition sketch

The author[4] previously indicated that the deepwater wave steepness H_o/L_o and the ratio of the deepwater wave height to the median diameter of sediment H_o/D_{50} are important parameters governing generation of longshore bars. It appears that these two parameters also are useful to describe the equilibrium beach profiles formed in the simplified conditions of onshore-offshore sediment motion due to wave action.

From Equation (2), the following scale laws may be derives as

$$\left.\begin{array}{l} n_h = n_{L_o},\ n_x = n_{L_o},\ n_{i_o} = 1 \\ n_{H_o} = n_{L_o},\ n_{D_{50}} = n_{H_o},\ n_\rho = n_{\rho s} \end{array}\right\} \dots\dots\dots\dots\dots\dots (3)$$

where n is the scale expressed as model value over prototype one. Equation (3) implies that the horizontal and the vertical scales of beach profiles must be identical with those of wave motion, that is, the models cannot be distorted, and that $n_\rho = n_{\rho s} = 1$, i.e. the density of model sediment must be selected so as to be identical with that of prototype one, since it is necessary to use water in model, while particle sizes must be scaled down geometrically.

By taking the effect of the fluid viscosity into account, the another expression of the beach profiles was presented by Brebner and others[2] as

$$\frac{h}{L_o} = f\left(\frac{x}{L_o}, \frac{H_o}{L_o}, \frac{H_o}{D_{50}}, \frac{D_{50}\sqrt{gH_o}}{\nu}, \frac{\rho_s}{\rho}, i_o \right) \dots\dots\dots (4)$$

As pointed out by them, the effect of $D_{50}\sqrt{gH_o}/\nu$ is negligible when the value of $D_{50}\sqrt{gH_o}/\nu$ is large enough to ensure a turbulent flow around the grain in both prototype and model. However, it will be shown that the effect of the Reynolds number cannot be negligible in the scale of laboratory. On the other hand, scale laws on H_O/L_O and $D_{50}\sqrt{gH_O}/\nu$ yield conflicting values for n_{D50}, i.e. the former is $n_{D50} = n_{H_O}$ and the latter $n_{D50} = n_{H_O}^{-0.5}$. Therefore, both conditions of similarity cannot be satisfied concurrently.

Dean[5] presented a model for the shift from a storm to a swell profile and found that the dimensionless parameter $\pi w_f/gT$, where w_f is the fall velocity of grains, is important in governing the equilibrium beach profiles. Therefore, an equation of the equilibrium beach profiles is expressed as

$$\frac{h}{L_o} = f\left(\frac{x}{L_O}, \frac{H_O}{L_O}, \frac{\pi w_f}{gT}, i_o\right) \quad\cdots(5)$$

From Equation (5), the following scale laws may be derived as

$$\left.\begin{aligned} n_{H_O} &= n_{L_O}, \ n_{w_f} = n_T = n_{L_O}^{1/2} \\ n_h &= n_{L_O}, \ n_x = n_{L_O}, \ n_{i_O} = 1 \end{aligned}\right\} \quad\cdots(6)$$

The fall velocity of a grain w_f is expressed as

$$w_f = \left\{\frac{4}{3}\left(\frac{\rho_s}{\rho} - 1\right)\frac{gD}{C_D}\right\}^{1/2} \quad\cdots(7)$$

where C_D is the drag coefficient of sphere grain and a function of Reynolds number w_fD/ν. If it is assumed that

$$C_D = \alpha\left(\frac{w_f D}{\nu}\right)^{-m}, \quad\cdots(8)$$

the fall velocity w_f is given by

$$w_f^{2-m} = \frac{4}{3\alpha}\left(\frac{\rho_s}{\rho} - 1\right)\frac{g}{\nu^m} D^{1+m} \quad\cdots(9)$$

where α and m are constants. Substituting Equation (9) into dimensionless parameter $\pi w_f/gT$, this quantity may be rewritten as follows:

$$\frac{\pi w_f}{gT} = \beta\left(\frac{\rho_s}{\rho} - 1\right)^{1/(2-m)}\left(\frac{D_{50}\sqrt{gH_O}}{\nu}\right)^{m/(2-m)}\left(\frac{H_O}{L_O}\right)\left(\frac{H_O}{D}\right)^{1/2} \quad\cdots(10)$$

where $\beta = \sqrt{\pi/2}\cdot(4/3\alpha)^{1/2-m}$.
If $D = D_{50}$, the dimensionless parameter is a function of $(\rho_s/\rho)-1$, H_O/L_O, H_O/D_{50} and $D_{50}\sqrt{gH_O}/\nu$. Therefore it is evident that the application of Equation (5) for the scale law coordinates the contradiction mentioned above.

EXPERIMENTAL EQUIPMENT

Laboratory experiments of the equilibrium beach profiles were made using a steel wave tank 20m long, 0.5m wide and 0.6m deep at the Hydraulic Laboratory, Tottori University. Waves were generated by a flatter type generator with a 2 HP electric motor. Incident wave heights were measured by a capacitance type wave gage which was installed at the part of constant water depth, h = 0.4m.

Two kinds of the well-sorted sand were used in the experiments and the median diameters of these sands were 0.3mm and 0.6mm, respectively.

All tests were carried out in the initial slope of 1:10, for sufficient time to form an equilibrium beach. The equilibrium beach profiles were measured with a point gage along the flume centerline.

EXPERIMENTAL RESULTS

(a) Scale relations based on Equation (3)

Figure 2 shows that the different model beach profiles are compared

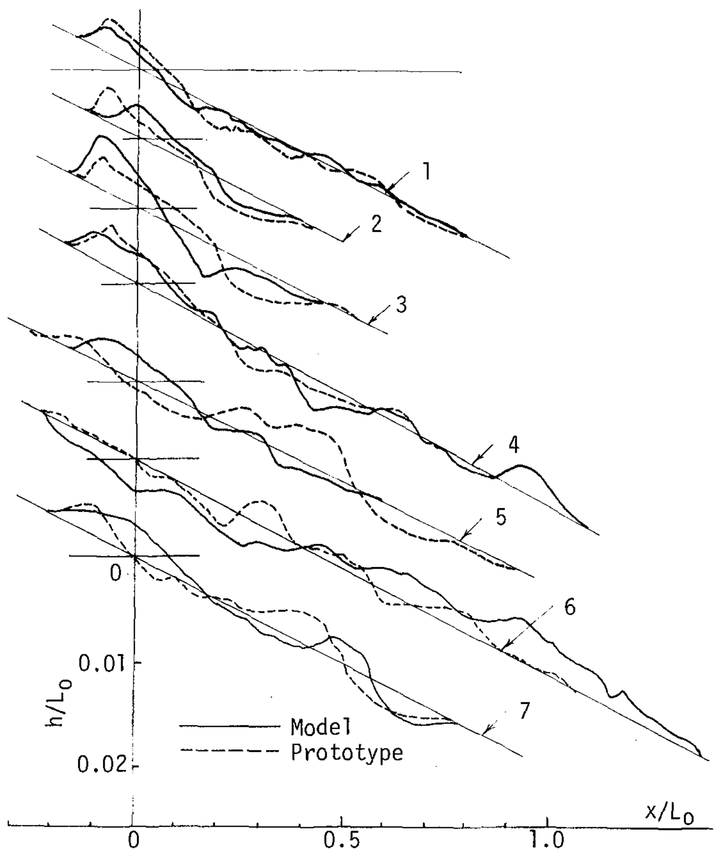

Figure 2 Comparisons of dimensionless profiles between prototype and model, based on Eq.(3)

Table 1 Test conditions and scale relations

NO.	H_0 (cm)	T (sec)	D_{50} (mm)	H_0/L_0	H_0/D_{50}	n_{H_0}	$n_{D_{50}}$
P-1 M-1	9.00 4.78	2.54 1.85	0.34 0.17	0.0089	264 281	1/1.88	1/2
P-2 M-2	2.68 1.32	1.04 0.74	0.60 0.30	0.0155	44	1/2	1/2
P-3 M-3	5.38 2.70	1.46 1.03	0.60 0.30	0.0163	90	1/2	1/2
P-4 M-4	10.98 5.50	2.06 1.46	0.60 0.30	0.0165	183	1/2	1/2
P-5 M-5	6.15 3.08	1.13 0.80	0.60 0.30	0.0309	103	1/2	1/2
P-6 M-6	13.24 6.62	1.60 1.13	0.60 0.30	0.0332	221	1/2	1/2
P-7 M-7	5.40 2.70	0.90 0.64	0.60 0.30	0.0427 0.0423	90	1/2	1/2

with their corresponding prototype beach profiles, and Table 1 indicates the test conditions and the scale relations represented by Equation (3). Since the tests were carried out using natural sand in both prototype and model, the scale was selected in values of $n_{H_0} = n_{L_0} = n_{D50} = 1/2$, in order to avoid the use of the cohesive range for the model sand.

The comparisons in the test results indicate a closer similarity between prototype and model of foreshore berm in No. 4, but significant differences still exist beyond wave breaking zone. Furthermore considerable differences are observed for the other test results. Therefore, it is evident that the scale laws based on Equation (3) cannot be adopted.

It seems that the differences between comparable profiles depend upon the influence of fluid viscosity. The values of H_0/L_0 and H_0/D_{50} are same for each comparable profiles, but the model value of $D_{50}\sqrt{gH_0}/\nu$ cannot be equivalent to prototype one for each test. From the test results of No. 4, 5 and 6, it is apparent that the lower limit of model beaches, which is related to beginning sand movement due to wave motion, makes a considerable difference from that of prototype beaches. This fact shows that the difference is related to Reynolds number $D_{50}\sqrt{gH_0}/\nu$.

(b) Scale relations based on Equation (6)

Figure 3 shows model and prototype beach profile data plotted in the dimensionless form x/L_O versus h/L_O, and Table 2 indicates the test conditions and the scale ratios of H_O and D_{50}. These results were obtained by using the scale laws represented by Equation (6). The comparisons between the test results indicate a closer similarity. From these figures, however, a few differences of profiles between prototype and model are observed. The reason is due to a few difference of the value of $\pi w_f/gT$ as shown in Table 2. Therefore, it should be noted that the dimensionless parameter $\pi w_f/gT$ must be severely preserved in model tests.

Figure 4 shows that the various model beach profiles are compared with a prototype beach profile (DP) in order to clarify the effect of

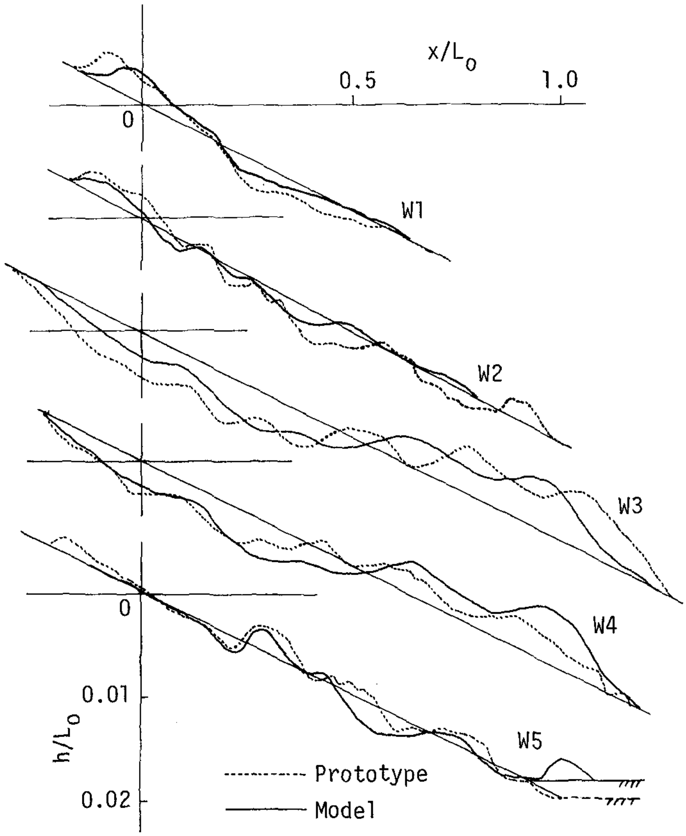

Figure 3 Comparisons of dimensionless profiles between prototype and model, based on Eq. (6)

Table 2 Test conditions and scale ratios

NO.	H_0 (cm)	T (sec)	D_{50} (mm)	H_0/L_0	$\frac{\pi w f}{gT}$	n_{H_0}	$n_{D_{50}}$
W1-P W1-M	10.98 1.32	2.06 0.74	0.60 0.30	0.0166 0.0155	0.0132 0.0153	1/8.32	1/2
W2-P W2-M	5.50 2.10	1.64 0.90	0.30 0.17	0.0165 0.0166	0.0085 0.0071	1/2.62	1/1.76
W3-P W3-M	5.83 2.42	1.07 0.70	0.28 0.17	0.0326 0.0312	0.0099 0.0092	1/2.41	1/1.65
W4-P W4-M	6.62 2.42	1.13 0.70	0.30 0.17	0.0332 0.0312	0.0088 0.0092	1/2.74	1/1.76
W5-P W5-M	13.24 6.61	1.60 1.12	0.60 0.48	0.0332 0.0337	0.0196 0.0215	1/2.00	1/1.25

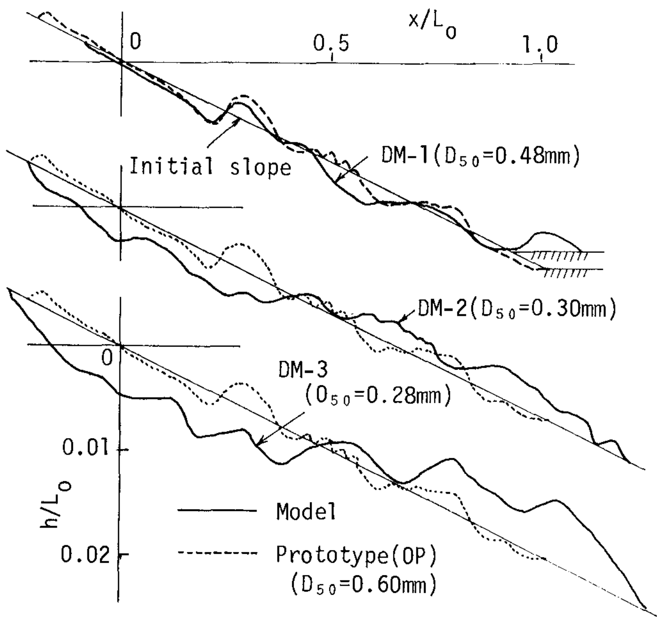

Figure 4 Comparisons of influence of sand size

Table 3 Test conditions for Figure 4

NO.	H_0 (cm)	T (sec)	D_{50} (mm)	H_0/L_0	$\frac{\pi w_f}{gT}$	n_{H_0}	$n_{D_{50}}$
DP	13.24	1.60	0.60	0.0332	0.0196		
DM-1	6.61	1.12	0.48	0.0337	0.0215	1/2	1/1.25
DM-2	6.62	1.13	0.30	0.0332	0.0088	1/2	1/2
DM-3	5.83	1.07	0.28	0.0326	0.0099	1/2.27	1/2.14

size of model sediment. From this figure, it is evident that the upper model profile (DM-1) is coincident with the prototype one (DP), but the middle (DM-2) and the lower (DM-3) model profiles do not reproduce the prototype profiles correctly. In the upper case of Figure 4, it is apparent that the value of n_{D50} is less than that of n_{H_O}. From these test results, sand size used in model may be chosen so as to reproduce prototype beach profiles.

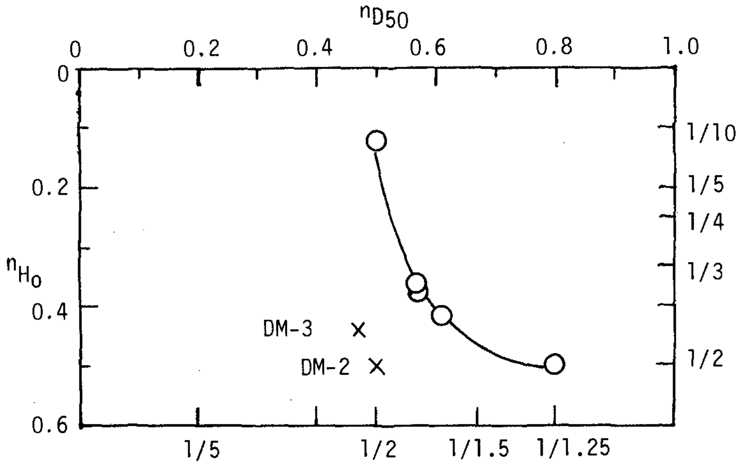

Figure 5 Relationship between n_{HO} and n_{D50}

Figure 5 indicates the relationship between n_{HO} and n_{D50} to be recommended when using the sand in model. In this figure, it is apparent that the test results of DM-2 and 3 do not reproduce the prototype beach profiles.

(c) Possibility of distorted movable-bed models

Figure 6-a indicates dimensionless comparison between equilibrium beach profiles obtained by Watts[6] (prototype) and Iwagaki and Sawaragi[7] (model), and their test conditions are presented in Table 4. From Table 4, it is evident that the horizontal and the vertical scales with respect to wave motion are $n_{H_O} = 1/4$ and $n_{L_O} = 1/8$ respectively, and $n_{D_{50}} = 1/2.14$. Therefore, $N_\omega = n_{H_O}/n_{L_O} = 2$, where N_ω is the model distortion of wave motion. On the other hand, model scale of distortion with respect to beach profile, N is expressed as

$$N = n_x/n_h \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

where n_x and n_h are the horizontal and the vertical scales of the beach profiles, respectively.

Figure 6-b shows that the model beach profile is coincident with the prototype one, if $N = N_\omega^{0.7}$. The relationship among N, N_ω and n_{D50} is not clear and therefore no general conclusion can be drawn as to validity of the distorted models. However, it is recognized that it is possible to use the distorted models in practical purpose.

CONCLUSIONS

The following conclusions may be derived from the laboratory test results developed in this paper:

1) Two dimensional equilibrium beach tests, based on the scale laws preserving the parameters H_O/L_O and H_O/D_{50}, indicate that it is impossible to obtain closer similarity between prototype and model beach profiles.

2) Closer similarity between the profiles are obtained when dimensionless fall velocity is preserved and the sediment size may be modeled by Figure 5 when using the sand in model.

3) Based on the experimental results, it is found that the coastal movable-bed model may be distorted. However, no general conclusion can be drawn as to the validity of the distorted models.

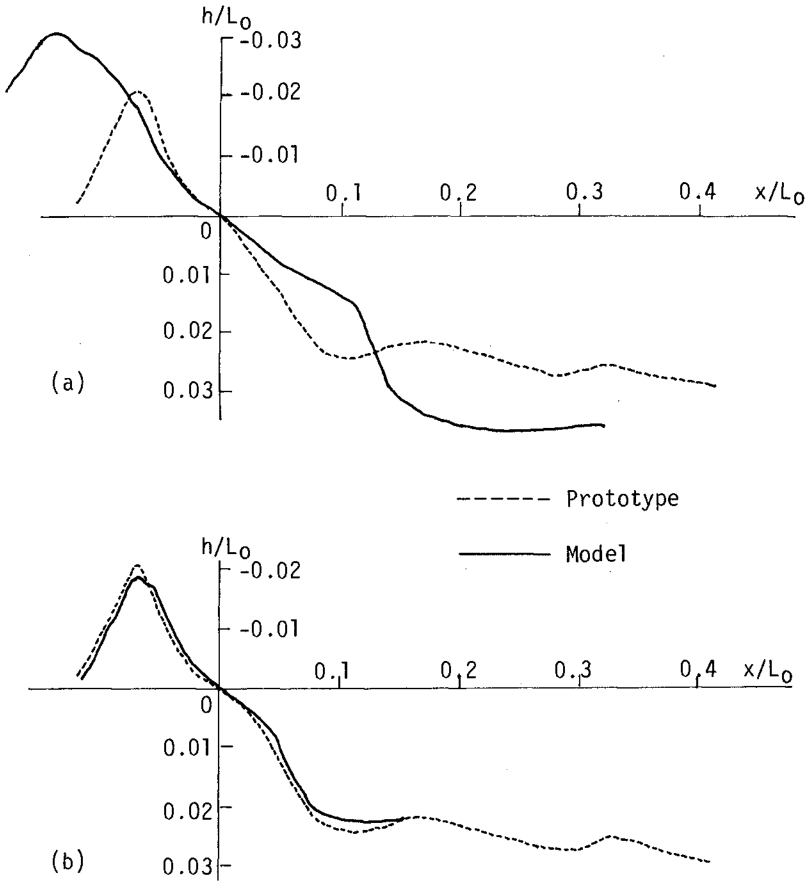

Figure 6 An example of distorted model profile

Table 4 Test conditions for distorted model

	H_0(cm)	T(sec)	D_{50}(mm)	i_0
prototype	12.66	2.68	3.44	1/20
model	3.17	0.95	1.61	1/10

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