CHAPTER 170

NEGATIVELY BUOYANT SLOT JETS

by

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Abstract

The behavior of negatively buoyant slot jets discharged from near the bottom of stagnant and flowing receiving-water environments of uniform density was studied in laboratory experiments. In particular, the locations of the maximum height of rise and maximum horizontal displacement were determined as well as the centerline dilutions at those points. Experiments in stagnant environments were conducted for discharges at angles of 30°, 45°, 60°, and 90° from the horizontal and with discharge densimetric Froude numbers of 10, 20, 30, 40 and 50. For flowing environments, discharges at angles of 45° and 90° for the same range of densimetric Froude numbers were directed into uniform currents with discharge to ambient velocity ratios of 15 and 25. The reattachment eddy formed between the jet and bottom boundary was found to have significant effects on the slot jet trajectories and dilutions. Dilutions at the maximum height of rise and maximum horizontal displacement were less than those predicted for slot jets far removed from the influence of the bottom boundary. The strictly two-dimensional discharges studied provide lower bounds on the dilutions to be expected for the cases of slots of finite length or merged multiport diffuser discharges in which the reattachment eddy is not as well developed.

Introduction

Negatively buoyant discharges are those with densities greater than the density of the receiving-water environment into which they are injected. The residual effluents from coastal desalting plants are brines which are negatively buoyant relative to the source waters. The warm surface water used to provide the heat of vaporization in LNG plants is cooled and negatively buoyant upon return to the surface waters. Industrial effluents, particularly those containing acids or acid-salts, are sometimes negatively buoyant despite having temperatures greater than those of the receiving water.

Slot jets or two-dimensional jets are created by the discharge of an effluent into a receiving-water body from a single orifice of slot

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geometry or by the lateral merging of adjacent round jets from a multiport diffuser. Negatively buoyant slot jets sink due to the gravity force acting on them. Because sites for the continuous disposal of negatively buoyant effluents are usually located offshore, near-surface discharges are not often possible, and the discharge structure is located on or near the bottom. Consequently, the effluent is discharged upward into the receiving water at some angle from the horizontal. The initial jet momentum carries the effluent upward so that mixing occurs before the gravity force brings the diluted effluent back to the bottom as indicated in Figure 1. The characteristics of a negatively buoyant slot jet of particular interest are the geometric coordinates and dilutions at the maximum height of rise of the jet and at the point of bottom impingement. The former is important to assure that the jet remains completely submerged, and the latter is of concern in the assessment of the impact on benthic communities.

The results of laboratory experiments with turbulent negatively buoyant slot jets discharged into stagnant and flowing receiving waters of uniform density are reported. For these experiments the discharge is two-dimensional; that is, the slot extends completely across the receiving-water body. Also the jet remains fully submerged below the free surface. Attention, in these experiments, was focused on the jet characteristics at the maximum height of rise and the region of bottom impingement. The applicability of the results to slot discharges of finite length and to merged jets from multiport diffusers is discussed.

Review of Round Negatively Buoyant Jet Behavior

Although no published data on the behavior of negatively buoyant slot jets are known, work has been reported on the behavior of negatively buoyant round jets in both stagnant and flowing receiving water environments of uniform density.

Turner[1] performed experiments with negatively buoyant round jets discharged vertically upward into stagnant environments and determined an expression for the maximum height of rise of such jets. Zeitoun et al.[2] performed similar experiments for negatively buoyant round jets discharged at a variety of angles from the horizontal and for several discharge conditions. They found that the dimensionless maximum height of rise of the jet, Y_m/D, and the centerline dilution at that point, $S(Y_m)$, could be expressed as functions of the discharge densimetric Froude number, F_r, and the discharge angle, θ_o, as follows:

$$\frac{Y_m}{D} = C_1 F_r , \tag{1}$$

and

$$S(Y_m) = C_2 F_r , \tag{2}$$

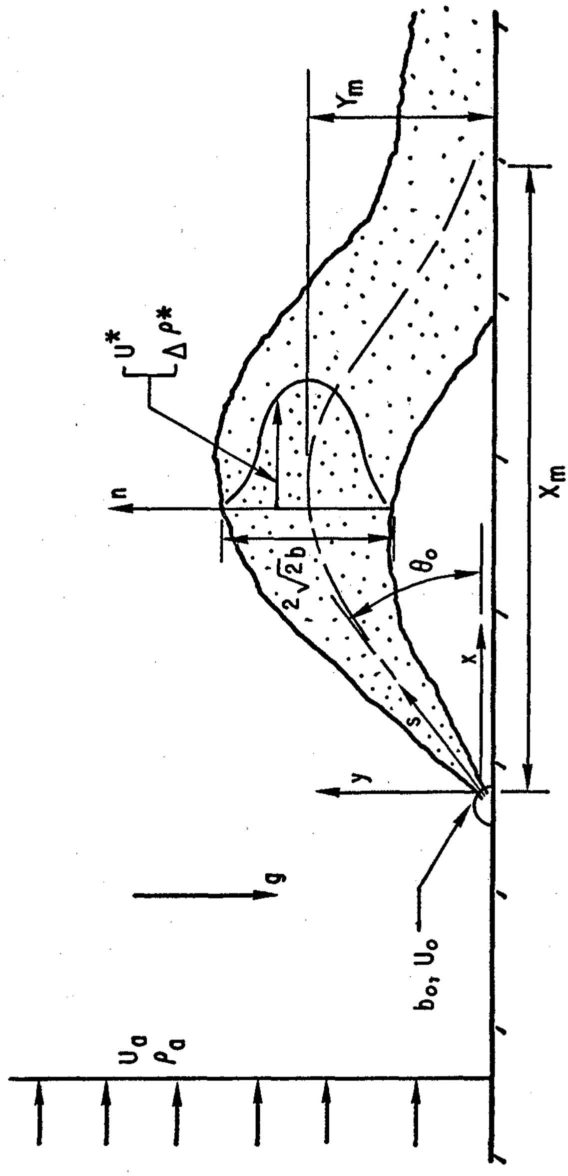

Fig. 1. Schematic Diagram of a Negatively Buoyant Slot Jet

where

$$F_r = \frac{U_o}{\left(\frac{\Delta\rho_o}{\rho} g\ D\right)^{1/2}}$$

Y_m = maximum height of rise above the discharge

D = discharge diameter

$\frac{\Delta\rho_o}{\rho}$ = discharge density difference divided by a reference density

$S(Y_m)$ = centerline dilution at Y_m

C_1, C_2 = coefficients which are constant for a given angle of discharge, θ_o.

The results of the experiments of Zeitoun et al. for θ_o values of 30°, 45°, 60°, and 90° indicate that, for a given discharge angle and no interaction with the free surface, the maximum height of rise and the dilution at that point increase monotonically with increases in the discharge Froude number (as indicated in Eq. (1) and (2)).

Morton[3] and Abraham[4] have presented analyses of the case of a negatively buoyant round jet discharged vertically upward which employ the integral-similarity approach often used in analyses of positively buoyant jets. Predictions for this particualr case are difficult to match with experimental results because the jet falls back on itself and the similarity assumption becomes tenuous. Zeitoun et al.[2] applied an integral-similarity analysis with a variable entrainment coefficient to the non-vertical discharge case and found reasonably good agreement between predictions and their experimental data for trajectories and dilutions.

Experiments on negatively buoyant round jets discharged upward into flowing environments of uniform density and velocity have been reported by Holly and Grace[5] for a vertical discharge and by Pincince and List[6] for a discharge inclined at 60° from the horizontal and in the direction of flow. Anderson et al.[7] conducted experiments for a variety of discharge angles, discharge densimetric Froude numbers, and ratios between the discharge and crossflow velocities. The crossflow acted to bend the jet trajectory downward which resulted in a decrease in the dilution at the maximum height of rise over the equivalent case in a stagnant environment. The dilution in the region of bottom impingement, however, was found to increase due to the elongation of the trajectory over that in a stagnant case. Anderson et al. were able to effect agreement between an integral-similarity model and the experimental results by varying the entrainment coefficient with the discharge densimetric Froude number and velocity ratio.

Slot-Jet Experiments

The experiments for negatively buoyant slot jets in stagnant and flowing environments were conducted in a flume 5.5 m long, 0.33 m wide, and 0.46 m deep. The jet was discharged from a slot in a small (1.27 cm radius) half-cylinder mounted on the flume bottom and extending across the entire width of the flume. Slots of different opening sizes and angles to the horizontal were employed. The receiving-water depth was maintained large enough to avoid any interaction of the jet with the free surface. The receiving water flow was turbulent, in the cases of flowing environments, and the negatively buoyant jets were all turbulent with a minimum discharge Reynolds number (based on slot width) of about 2000.

The discharge effluent was a saltwater solution and the receiving water was fresh with the discharge density and flow rate adjusted to produce the densimetric Froude number desired. Since a saltwater solution was used as the effluent, conductivity probe measurements in the jet were used to infer concentrations and density and thus to determine dilution values. Dye injected into the discharge and a grid on the transparent flume wall were used for photographic records of the jet trajectory. The conductivity measurements were concentrated in the regions of the maximum height of rise and the bottom impingement of the plume. Receiving-water velocities for the flowing cases were measured with a hot-film anemometer and appeared to be quite uniform over the cross-section, except for small regions near the side and bottom boundaries.

Detailed descriptions of the experimental equipment and techniques are given in Reference 8.

Results of Slot-Jet Experiments — Stagnant Environment

Dimensional analysis of the discharge of a negatively buoyant slot jet into a stagnant environment indicates that the dimensionless trajectory coordinates, y/B and x/B, and centerline dilution, S, are functions of the slot discharge densimetric Froude number, F_s, and the angle of discharge from the horizontal, θ_o, where,

$$F_s = \frac{U_o}{\left(\frac{\Delta\rho_o}{\rho} g\, B\right)^{1/2}}$$

and

B = discharge slot width.

Experiments were performed in which the Froude number had values of 10, 20, 30, 40, and 50 for each of the discharge angles of 30°, 45°, 60°, and 90°. The results of these experiments are given in terms of the maximum height of rise of the jet centerline above the discharge, Y_m, the maximum horizontal displacement of the jet centerline away from the discharge in the region of bottom impingement, X_m, and the centerline dilutions at those points, $S(Y_m)$ and $S(X_m)$, respectively.

The data for the normalized maximum height of rise as a function of Froude number for discharge angles 45°, 60°, and 90° are shown in Figure 2. The data for $\theta_o = 30°$ are not shown as those discharges interacted so quickly with the bottom that trajectory and dilution measurements were difficult. With results similar to those for negatively buoyant round jets, the normalized maximum height of rise varied directly with the discharge Froude number. Contrary to the data of Zeitoun et al. for round jets, for which the $\theta_o = 60°$ case produced the largest value of the coefficient C_1 in Eq. (1), the slot jet data indicate that that coefficient increases monotonically as θ_o increases from 0° to 90°. Figure 3 indicates that the normalized maximum horizontal displacement increases with discharge Froude number for a given θ_o and that $\theta_o = 45°$ results in the greatest horizontal displacement.

The dissimilarity between round and slot jet behavior becomes even more apparent from the centerline dilution data. The centerline dilution at the maximum height of rise is given as a function of Froude number and discharge angle in Figure 4. The dilution increases monotonically with discharge Froude for all angles, similar to round jet experience in Eq. (2), for Froude numbers less than about 30. For discharge Froude numbers greater than about 30, the dilution at the height of maximum rise decreases with increases in Froude number. Similar behavior was found for the dilution at the maximum horizontal displacement, shown in Figure 5. The decrease of the dilution for increases in discharge Froude number appears to be due to the reattachment eddy formed between the sinking slot jet and the bottom boundary and to reentrainment by the jet. The reattachment eddy, shown schematically in Figure 6, acts to pull the jet toward the bottom because of the pressure field established and to enhance reentrainment of the discharged effluent into the jet. The entrainment of this diluted effluent, instead of fresh receiving water, decreases the dilution and decreases the local jet densimetric Froude number.

Analytical-Model Predictions for the Stagnant Case

The effects of the reattachment eddy are manifested in the attempt to predict negatively buoyant slot jet behavior with a model which ignores the presence of the bottom boundary. An integral-similarity model for an unbounded, stagnant receiving water with uniform density was employed. The governing equations were similar to those developed by Fan and Brooks[9] for positively buoyant slot jets for the cases of the discharge conditions which existed in the experiments. The entrainment coefficient and spreading coefficient were set at 0.16 and 0.89, respectively, and the predictions were corrected for the zone of flow establishment.

Figures 7 and 8 indicate the model predictions of trajectory and dilutions at the maximum height of rise and the maximum horizontal displacement and the experimental values for these parameters for two discharges inclined at 60° from the horizontal. Both cases show that

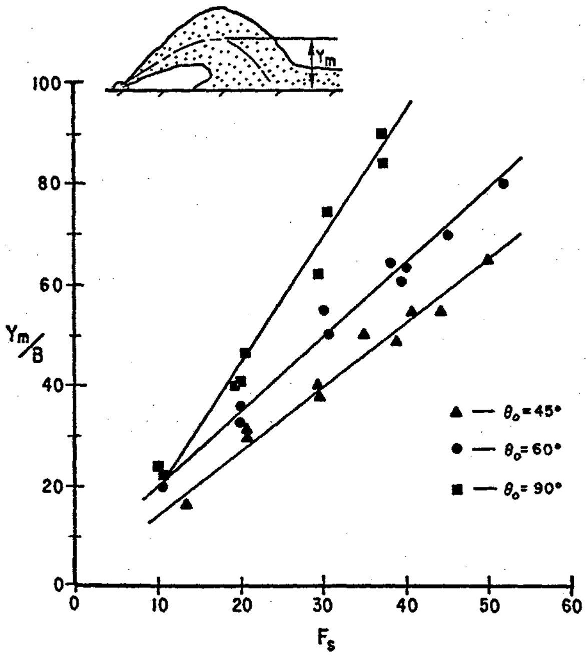

Fig. 2. Normalized Maxium Height of Rise for Stagnant Environment

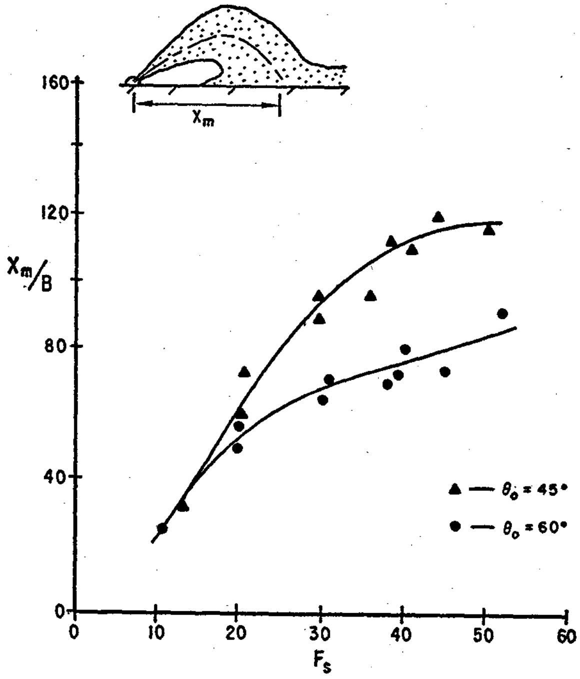

Fig. 3. Normalized Maxium Horizontal Displacement for Stagnant Environment

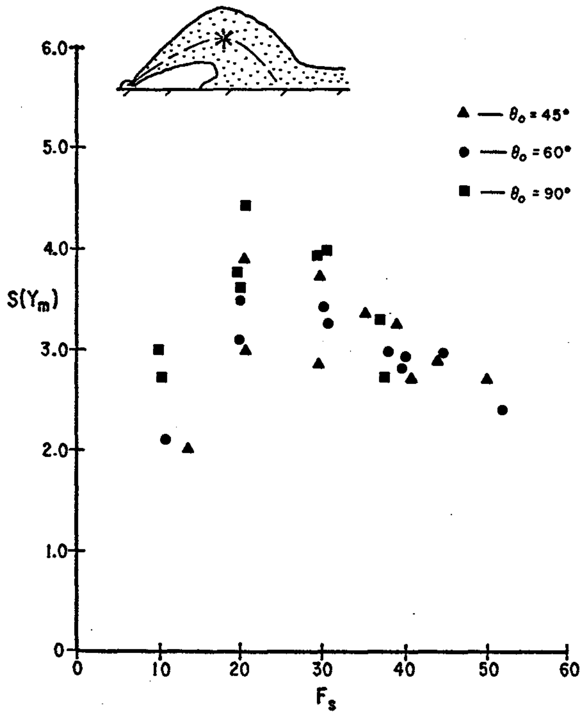

Fig. 4. Centerline Dilution at Y_m for Stagnant Environment

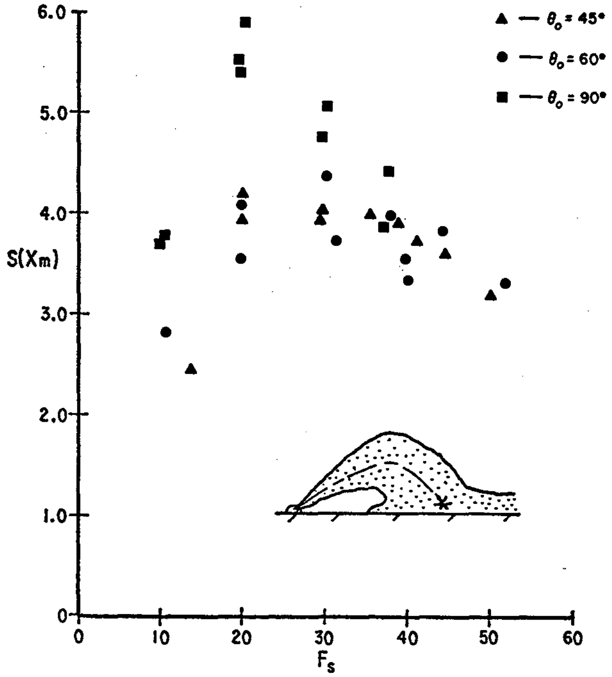

Fig. 5. Centerline Dilution at X_m for Stagnant Environment

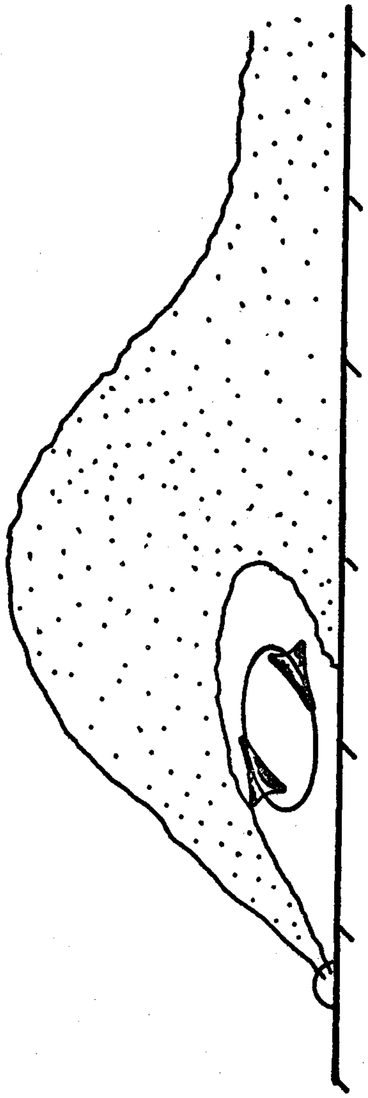

Fig. 6. Schematic Diagram of Reattachment Eddy

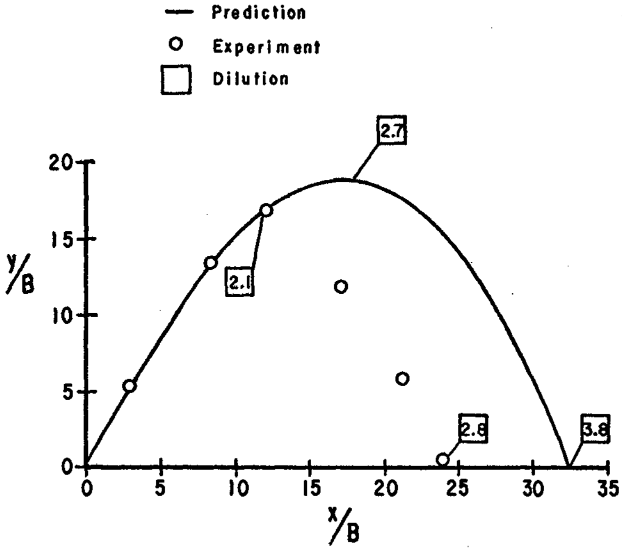

Fig. 7. Comparisons of Predictions and Measurements of Trajectory and Dilution for $\theta_o = 60°$ and $F_s = 10.4$

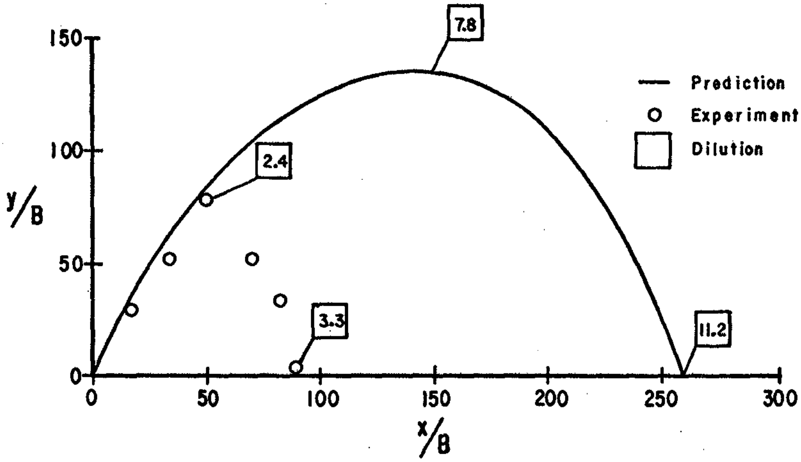

Fig. 8. Comparison of Predictions and Measurements of Trajectory and Dilution for $\theta_o = 60°$ and $F_s = 52.4$

the experimental trajectories were shorter and lower and that the measured dilutions were lower than the predictions. The disparity between predictions and measurements becomes greater for larger discharge Froude numbers (Figure 7 versus Figure 8). Additional comparisons[8] indicate that the model for an unbounded receiving water, in general, predicts that the dilution at the maximum height of rise is a monotonically increasing function of Froude number without the decreases in dilution found in the experimental data (Figure 2). A large discharge densimetric Froude number implies relatively large discharge momentum, and consequently, for such a case, more intense reattachment eddies than for the case of a small discharge Froude number.

The failure of the model to predict the negatively buoyant slot jet behavior is directly related to the fact that the bottom boundary and its effects on the flow are neglected in the model. Although the values of the entrainment coefficient and the spreading coefficient are not well-established for the unbounded case, sensitivity studies with variations in these parameters indicated that agreement between predictions and experimental data could not be forced. This simply confirms the fact that the proximity of the bottom boundary is the important feature governing the jet behavior.

Results of Slot-Jet Experiments — Flowing Environment

Dimensional analysis for the case of a deeply-submerged negatively buoyant slot jet in a flowing environment yields an additional governing parameter, K, the velocity ratio,

where $K = \frac{U_o}{U_a}$,

and U_a = uniform ambient velocity.

Thus, the governing parameters for the behavior of negatively buoyant slot jets discharged into flowing environments are F_s, θ_o, and K. Experiments were performed for the cases of discharge angles of 45° and 90° for the range of F_s used for the stagnant cases (10 to 50) for K values of 15 and 25.

The normalized maximum height of rise and the normalized maximum horizontal displacement for the $\theta_o = 45°$ case are shown as functions of F_s and K in Figures 9 and 10, respectively. The stagnant case ($K = \infty$) is shown for reference. Figure 9 shows that the jet is bent downward by the ambient current as expected. Figure 10 indicates that the relatively smaller ambient current (K = 25) acts to extend the jet downstream farther than in the stagnant case, yet the larger current (K = 15) apparently intensifies the reattachment eddy for $F_s > 20$ and lessens the downstream extent of the impingement. The dilutions at Y_m and X_m for $\theta_o = 45°$ are shown in Figures 11 and 12, respectively. The decrease in dilution for large Froude numbers found for the stagnant case appears again in the flowing cases. The bending downward of the jet trajectory due to the

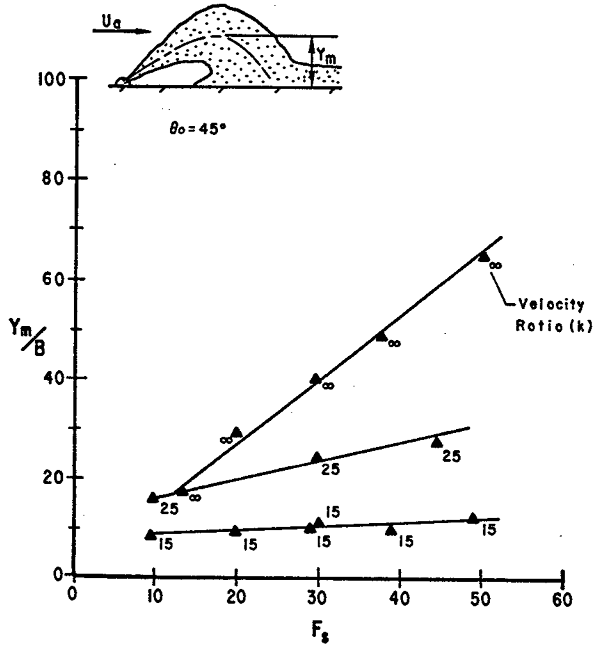

Fig. 9. Normalized Maximum Height of Rise for Flowing Environment, $\theta_o = 45°$

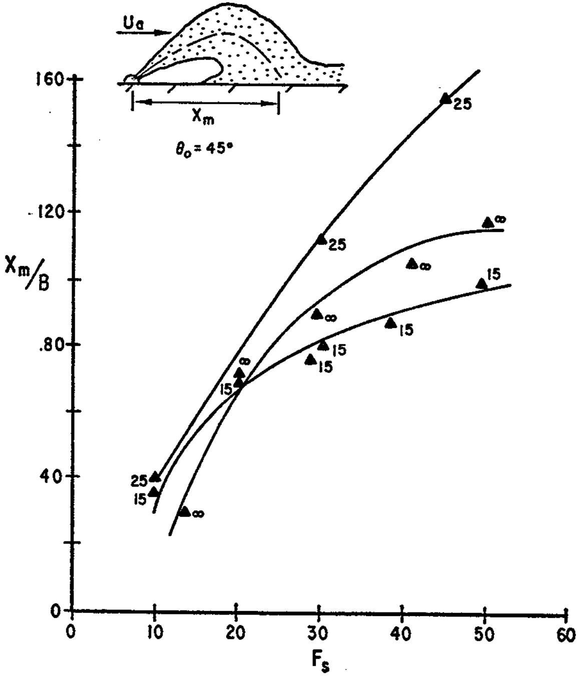

Fig. 10. Normalized Maximum Horizontal Displacement for Flowing Environment, $\theta_o = 45°$

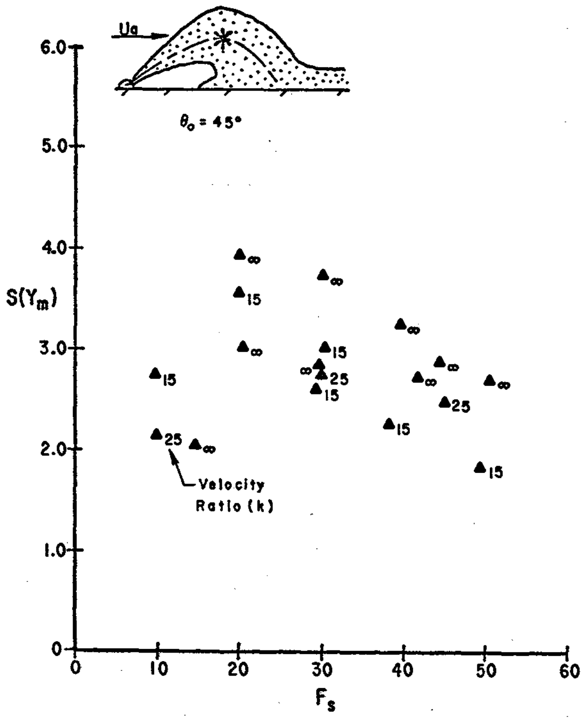

Fig. 11. Centerline Dilution at Y_m for Flowing Environment, $\theta_o = 45°$

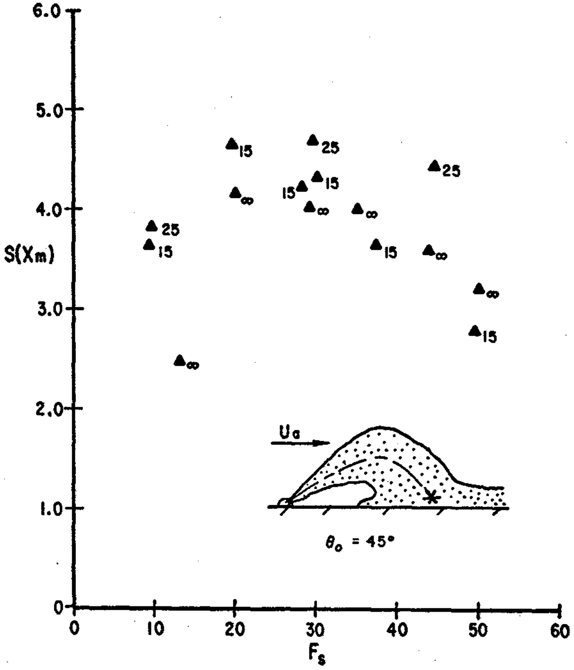

Fig. 12. Centerline Dilution at X_m for Flowing Environment, $\theta_o = 45°$

current generally results in smaller dilutions at Y_m than in the stagnant case as shown in Figure 11 and larger dilutions at X_m due to elongation of the trajectory before bottom impingement, as shown in Figure 12. However, the data for $S(X_m)$ suggest that the higher current case ($K = 15$) may result in dilutions at X_m about equal to those for the stagnant case due to joint effects of entraining horizontal momentum from the current and of the reentrainment of dense fluid due to the reattachment eddy. Subsequently, counter to the experience with round jets in a current, where the current acted to extend the plume and to enhace mixing before bottom impingement, the slot jet in a current is greatly influenced by the reattachment eddy and enhanced downstream dilution may not be realized.

The normalized maximum height of rise and the normalized maximum horizontal displacement for the $\theta_o = 90°$ case, a vertical discharge, are shown as functions of F_s and K in Figures 13 and 14, respectively. The stagnant case does not provide the greatest Y_m/B values (Figure 13) because, in that instance, the effluent falls back on itself and, in the flowing case, the bending of the jet causes it to impinge downstream of the slot as shown in Figure 14. That difference between the stagnant and flowing cases is shown again in the dilution data at Y_m in Figure 15, where dilutions for the flowing cases are larger than for the stagnant case. Similar results are shown in Figure 16 for $S(X_m)$. One would expect that for values of K smaller than examined in these experiments (larger relative currents), the discharge would be bent over so much that dilutions at X_m would decrease below those for the stagnant case as occurred in the results for the $\theta_o = 45°$ discharge (Figure 12)

Applicability to Discharges Which Are Not Two-Dimensional

The results of this study apply strictly to the case of a slot discharge located at the bottom of the receiving water and extending completely across the receiving water (strict two-dimensionality). It has been shown that the reattachment eddy downstream of the discharge has a pronounced effect on the jet behavior. Because the existence and strength of that eddy depend on the physical situation studied, caution is necessary in the application of the results found to discharge situations which are physically different.

A slot discharge on the bottom of and extending across a river or stream clearly resembles the case studied. However, a slot discharge elevated some distance above the bottom would permit entrainment flow to the lower side of the slot and weaken or possibly eliminate the reattachment eddy. In such a case, it is expected that the integral-similarity model predictions would more readily reflect the jet behavior than the experimental results reported here. Likewise, a slot discharge of finite length, not extending entirely across the receiving water body, could induce flow inward along the axis of the slot for entrainment and diminish the strength of the eddy. The merging of adjacent round jets of a multiport diffuser to form a slot jet may also constitute a different physical situation. Although an "equivalent" slot for a multiport diffuser discharge is easily calculated, the flow between the initial round jets would probably result in a weaker reattachment eddy than would be formed had the initial discharge geometry been a slot.

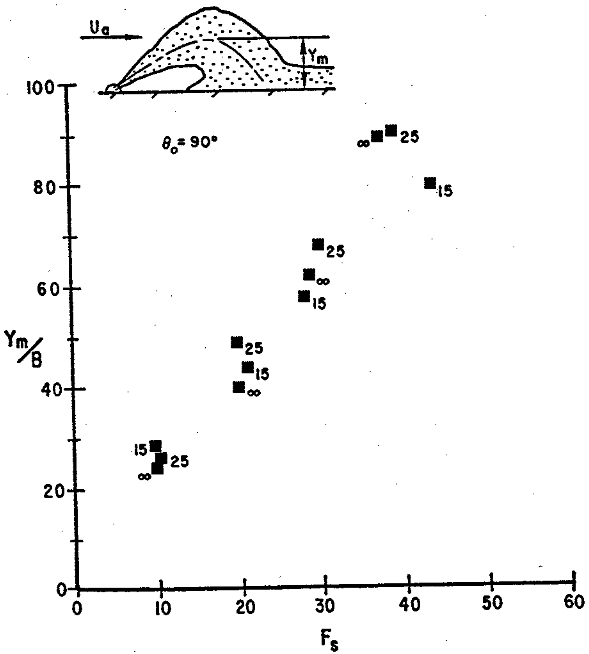

Fig. 13. Normalized Maximum Height of Rise for Flowing Environment, $\theta_o = 90°$

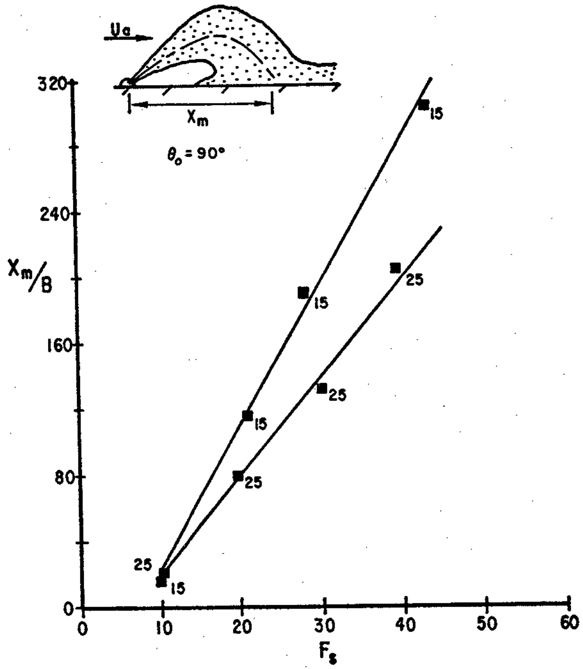

Fig. 14. Normalized Maximum Horizontal Displacement for Flowing Environment, $\theta_o = 90°$

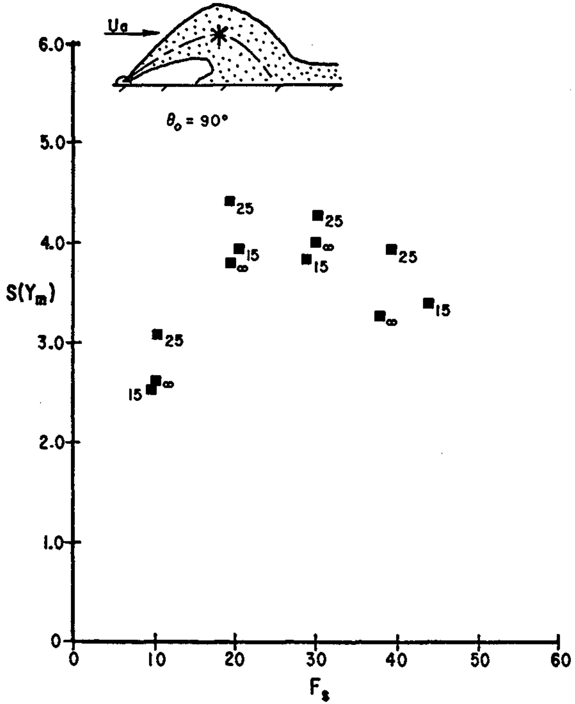

Fig. 15. Centerline Dilution at Y_m for Flowing Environment, $\theta_o = 90°$

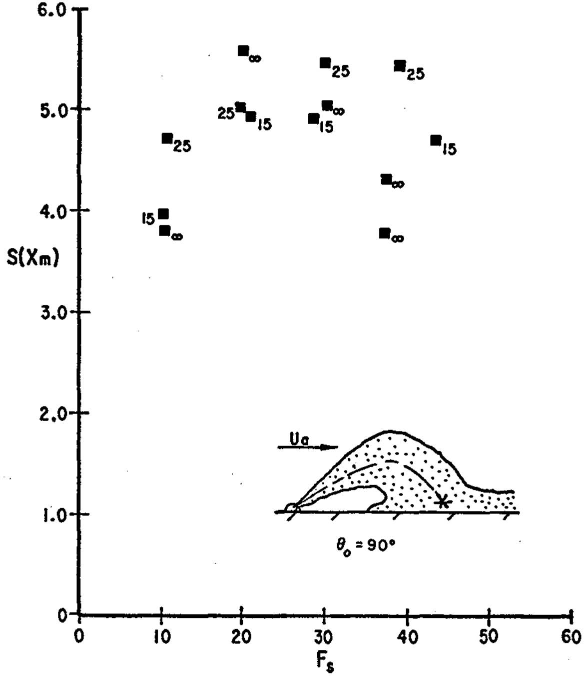

Fig. 16. Centerline Dilution at X_m for Flowing Environment, $\theta_o = 90°$

All of the physical situations mentioned above result in the possibility that the reattachment eddy may be weakened and hence that the dilution may be increased over that predicted by the experimental data reported for this study. The sensitivity of the results to these factors and the degree to which dilution predictions can be increased remains to be studied. The results for the two-dimensional case reported here do, however, provide the lower-bound or "worst case" on the mixing to be realized from a slot or slot-like discharge of a negatively buoyant effluent. The upper bound would be that behavior predicted by the model for receiving-water environment with no boundary interference.

Conclusions

The behavior of submerged negatively buoyant slot jets discharged near the bottom of the receiving water is influenced by the reattachment eddy created between the jet and the bottom. The trajectories of such jets in stagnant and flowing environments are lower and flatter than would be predicted in the absence of the bottom boundary. Centerline dilutions at the maximum height of rise and the maximum horizontal displacement are reduced considerably relative to predictions for the unbounded case. Contrary to experience with negatively buoyant round jets discharged near the bottom, the dilution at the maximum height of rise for a given discharge angle does not increase monotonically with increases in the discharge densimetric Froude number. The existence of the reattachment eddy in the slot discharge case results in the dilution at that point decreasing for Froude numbers greater than about 30. The strictly two-dimensional case studied provides a lower bound for dilution and trajectory predictions for cases of discharges from finite-length slots and discharges from multiport diffusers which merge to form slots.

References

1. Turner, J. S., "Jets and Plumes with Negative or Reversing Buoyancy," J. Fluid Mechanics, 26, 779-792, 1966.

2. Zeitoun, M. A., et al., "Conceptual Designs of Outfall Systems for Desalting Plants," Research and Development Progress Report No. 550, Office of Saline Water, U.S. Dept. of Interior, Washington, D.C. May, 1970.

3. Morton, B. R., "Forced Plumes," J. Fluid Mechanics, 5, 151-163, 1954.

4. Abraham, G., "Jets with Negative Buoyancy in Homogeneous Fluid," J. Hydraulic Research, 5, 235-248, 1967.

5. Holly, F. M. and Grace, J. L., "Model Studies of Dense Jets in Flowing Fluid," J. Hydraulics Div., Proc. Amer. Soc. Civil Engr., 98, 1921-1933, 1972.

6. Pincince, A. B. and List, E. J., "Disposal of Brine into an Estuary," J. Water Pollution Control Fed., 45, 2335-2344, 1973.

7. Anderson, J. L., et al., "Negatively Buoyant Jets in a Cross Flow," Environmental Protection Technology Series, EPA-660/2-73-012, U.S. Environmental Protection Agency, 1973.

8. Shahrabani, D. M, and Ditmars, J. D., "Negatively Buoyant Slot Jets in Stagnant and Flowing Environments," Ocean Engineering Report No. 8, Dept. of Civil Engineering, University of Delaware, Newark, Delaware, June 1976.

9. Fan, L.-N. and Brooks, N. H., "Numerical Solution of Turbulent Buoyant Jet Problems," W. M. Keck Lab. for Hydraulics and Water Resources, Report No. KH-R-18, Calif. Institute of Tech., Pasadena, January 1969.