CHAPTER 42

SET-UP AND RUN-UP IN SHOALING BREAKERS

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ABSTRACT

This paper reports the results of a series of laboratory experiments with periodic waves breaking on uniformly sloping impermeable beaches with the object of distinguishing set-up from dynamic shoreline motions due to partial reflection, the combination of which is normally referred to as 'run-up'. The principal findings are:

1. The mean set-up across the breaker zone was observed to be linear with mean slope proportional to the square of the bottom slope. The mean slope was independent of frequency over slopes of 0.04 or less, and increased with wave period over steeper slopes.

2. The dynamic run-up range was found to be proportional to the square of wave period times beach slope, in agreement with the equation of motion for a nearly frictionless block sliding on corresponding slopes under gravity.

3. The total run-up was poorly correlated with Hunt's empirical formula, nor could any reasonable deterministic justification of this formula be deduced from the present results.

4. Transient run-up was observed to considerably exceed steady-state values in most cases, suggesting that time-dependent momentum flux should be considered in the run-up of variable (natural) waves.

INTRODUCTION

Although commonly lumped together under the ubiquitous headings of <u>run-up</u> and <u>run-down</u>, it is now generally recognized that the shoreline fluctuations in water level observable in the presence of steady, periodic waves breaking on a sloping beach comprise an equilibrium that is the result of at least two different physical effects: (1) periodic oscillations at wave frequency which are the result of partial reflection from the shore slope of that portion of the initial energy of each incident wave not expended through breaking; and, (2) a super-elevation of the mean level against the shore caused by the gradient of momentum flux associated with wave decay in the surf zone.

The transition from a lowering of mean level under the breaker line (set-down) to a positive maximum (set-up) somewhere within the range of periodic shoreline excursions was predicted in principle by Longuet-Higgins and Stewart (1964) as a particular application of their more general theory for momentum flux in non-breaking waves. It was examined in some detail during a series of experiments by Bowen, et al (1968), in which waves of several frequencies and amplitudes were breaking on a uniform beach slope ($S = dD/dx = 0.082$). It was found that the mean surface gradient $-d\bar{\eta}/dx$ was nearly linear, varied inversely with frequency, and was nearly independent of breaker height. Assuming, as roughly observed, a constant empirical proportionality factor γ between breaker height H and total water depth $(\bar{\eta} + D)$ within the surf zone, they were able to integrate the momentum flux equation of Longuet-Higgins and Stewart to obtain a linear ratio

$$- (d\bar{\eta}/dx)/S = (1 + 8/3\gamma^2)^{-1} \quad (1)$$

The authors note that, although the shoreline set-up $\bar{\eta}_s$ comprised the majority of the observed total run-up R, the latter was still in reasonable agreement with the empirical formula of Hunt (1969)

$$R = HS(H/L_0)^{-\frac{1}{2}} \quad (2)$$

where H is a wave height ordinarily taken as the corresponding deep water wave height H_0, but, in fact, is most commonly obtained by linear transformation from heights measured in a uniform channel ahead of the slope. Lastly, Bowen, et al suggest that the ratio of beach-to deep water wave-steepness $S/(H_0/L_0)^{\frac{1}{2}}$ may have particular relevance to conditions in the surf zone, since it also appears in the breaking criterion of Iribarren and Nogales (1949). In an attempt to better estimate the quantitative predictions of their slope theory, they present a plot of $\bar{\gamma}$, the average of many determinations of γ across the surf zone, versus $S/(H_0/L_0)^{\frac{1}{2}}$, which includes data for two other beach slopes reported by Putnam (1945), but without drawing any further conclusions.

A more deterministic attempt to predict surf zone set-up is that of Hwang and Divoky (1970), who numerically integrated the momentum flux equation, assuming that (spilling) breakers can be represented by cnoidal waves, whose energy is dissipated at a rate equivalent to a constant fraction of that for a bore of equal local height. Although the many assumptions of this theory are difficult to defend, it provides a means of calculating the set-up and wave height decay simultaneously, and the latter, at least, appears to be in much better agreement with observations than the linear decay assumed by Bowen et al (1968). However, their computed set-up profiles are convex upwards, while the bulk of experimental results suggest that they are linear, or even concave upwards. Even so, the differences are small, and the method deserves further study.

Meanwhile, Battjes (1974a, b) has attempted by dimensional arguments to show that the steepness ratio

$$\xi \equiv S/(H/L_0)^{\frac{1}{2}} \quad (3)$$

is a similarity parameter that, depending upon the interpretation of H, can be used to categorize a wide variety of surf zone phenomena; such as, breaking, breaker type, breaker depth, energy reflectance, and the number of waves simultaneously present between the breaker point and the shoreline. Additionally, Battjes and Roos (1976, in press) present some results of experiments with water velocities and elevations within the swash zone, and try to fit them into the same mold. But it is difficult to judge the accuracy of these results or conclusions for lack of adequate descriptions of equipment, methodology, or measurement error.

The present paper presents a portion of the results of a two year series of laboratory studies of the dynamics of periodic waves breaking on uniform impermeable beach slopes, in which the time histories of surface elevation, crest velocity, and horizontal fluid velocity, were measured at a plurality of observation stations before and after breaking, as function of slope and wave period. Particular attention was paid to the mean, and periodic, water level excursions across the breaker zone in an attempt to distinguish set-up from the dynamic shoreline excursions caused by reflections. Those experiments dealing with breaking kinematics are described elsewhere (Van Dorn, 1977), and this paper is restricted to shoreline effects, except as regards some pertinent new results from the above paper.

INSTRUMENTATION AND PROCEDURES

All experiments were conducted in a 0.5 m wide by 24 m long glass-sides wave channel with a constant still water depth of 36 cm. Twelve combinations of three plate-glass beach slopes ($S = 0.022$, 0.040, and 0.083) and four wave periods ($T = 1.65$, 2.37, 3.43, and 4.80 sec) covered the ordinary range of prototype wave conditions for length and time scales of 16:1 and 4:1, respectively. Waves were generated by a planar paddle, hinged 55 cm beneath the channel floor, and driven sinusoidally at the top by a crank shaper. All data runs commenced after at least 5 minutes of generator operation, found by experiment to be about the minimum for low-frequency start-up transients to decay and stable equilibrium to be achieved. Measurements involved the following types of instrumentation.

Surface elevation η versus time, referred to still water, was monitored simultaneously by a plurality of digital (shorting-contact) wave staffs and analog subsurface pressure gages. They were supplemented at specific locations by stroboscopic and flash photography through the channel walls. Comparison of data from these three sources soon established that elevations computed hydrostatically from subsurface pressure were substantially lower than the staff and photo elevations, which generally agreed within 1 cm. The discrepancy was greatest under the wave crests, where it sometimes amounted to 50 percent, diminishing to zero at the shoreline; trough elevations by all three methods agreed within 0.2 cm (Figure 1). This result may substantially affect the assumption of mean hydrostatic equilibrium basic to the theory of time-averaged momentum flux. It may also affect the interpretation of mean elevation given below. Overall elevation accuracy was poorest in vicinity of the plunge and rebound regions of intense breaking, especially on the 0.083 slope, where, in fact, the admixture of air and water made any definition of free surface elevation questionable.

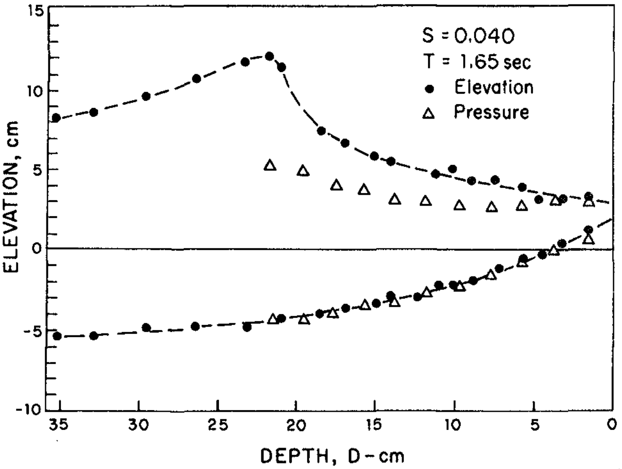

Figure 1. Hodographs of maximum and minimum waves elevations as an individual wave moves toward shore. Pressure and staff elevations agree under troughs, but peak pressure is clearly a poor indication of crest elevation.

Mean elevation $\bar{\eta}$ was observed at fifteen stations evenly distributed between the respective slope toes and the still water shoreline. The sensors comprised 1.0 mm diameter glass tubes taped to channel wall, with their lower ends 1.6 mm above the glass slope. These tubes were connected by plastic tubing via a glass capillary restriction to 2.5 cm diameter, open-ended thistle tubes mounted vertically outside the channel. The time constant of these 'hydraulic hi-pass filters' was adjusted to one minute by varying the capillary lengths. The free surface elevations within the thistle tubes could be rapidly determined by an electric-contact prick probe mounted on a micrometer depth gage. It was found possible to replicate elevations within 0.002 cm after the channel had stabilized, and that annulment of channel seiche fluctuations within these tubes provided the best evidence for stability. While the glass beaches were sealed to the channel walls with rubber gasketing, the slope toes were unsealed, since it was found that sealing them resulted in slow mean elevation drift due to minute seepage between the ground, but unsealed, abutting ends of the glass sections.

Shoreline elevation was monitored versus time by a digital comb-type, shorting contact, wave staff suspended over each beach so that the comb tips cleared the glass surface by about 1 mm, found experimentally to be the minimum elevation required to reduce spurious signals produced by meniscus puddling of water between contacts. Contact spacing was 2 cm on the 0.083 and 0.040 slopes, and 4 cm on the 0.022 slope. The corresponding relative vertical (run-up) resolution of these staffs was always less than 0.2 mm, although the absolute dynamic run-up range may have been slightly larger than reported, owing to elevation of the contacts above the beach. A more-serious problem is the possibility of underestimating the run-up range by virtue of spurious contact shorting in the thin retreating film from a previous wave as a new bore advanced over the slope. The only ready answer to this question seems to be the nearness of the observations to the limiting possible range, equivalent to the sliding of a frictionless block under gravity only (see Figure 8 and relevant discussion).

Data acquisition of all electric signals was accomplished on a Statos Mark III digital strip chart recorder. This instrument has a 2 khz response, 100 data increments per channel, and prints at a rate of 100 samples per second.

BREAKER CLASSIFICATION

All waves in the present experiments were breaking, and it is appropriate to place them in proper context before discussing results. All relevant breaking point variables are listed in Table 1. Here, phase velocity C_b was obtained from the slope of arrival time curves (accurate to 2%), but later found (Van Dorn, 1977) to be closely approximated by

$$C_b = (2g\eta_b)^{\frac{1}{2}} = (1.54\, gH_b)^{\frac{1}{2}} \tag{4}$$

which is substantially slower than the commonly accepted shallow water velocity: $\{g(\eta_b + D_b)\}^{\frac{1}{2}}$. Breaking wavelength is taken as $L_b = C_bT$.

With this preamble, Figure 2 compares breaking steepness H_b/L_b versus beach slope S for the present data on a field of similar data subjectively classified by Galvin (1968) according to breaker type*. The two solid lines $\xi_b = 0.4$ and $\xi_b = 2.0$ represent Battjes (1974a) reinterpretation of Galvin's 'inshore' criteria separating spilling, plunging, and surging or collapsing (non-breaking) waves, where $(\xi_b \equiv S/(H_b/L_o)^{\frac{1}{2}} = 0.32S/(H_b/L_b)$; but the dashed line $\xi_b = 0.6$ appears to be more consistent with the stroboscopic sequences taken during the present study. By either criterion, about half the present waves were spilling and half plunging; but most students of this complicated subject would agree that there is a continuous apparent increase of breaking intensity as either slope or period increases to ward the point where no breaking occurs.

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* This representation omits Galvin's "plunging altered by reflection", since it has no statistical correlation with any other breaker type.

SLOPE S	PERIOD T, sec	BREAKER HEIGHT H_b, cm	BREAKER DEPTH D_b, cm	PHASE SPEED C_b, cm/s	BREAKER SET-DOWN $\bar{\eta}_b$, cm		SET-UP GRADIENT $m = dD/dx$	MAXIMUM SET-UP $\bar{\eta}_s$, cm		RUN-UP RANGE ΔY, cm	RELATIVE RUN-UP R/H_o		TRANS. RUN-UP Y_t, cm
					OBS.	EQ (5)		OBS.	EQ (8)		OBS.	EQ (11)	
0.022	1.65	16.6	20.8	160	-0.53	-0.71	0.0019	1.6	0.91	0.17	1.5	1.6	4.0
	2.37	15.8	18.9	157	-0.44	-0.69	0.0021	1.3	0.78	0.26	1.3	1.9	4.3
	3.43	13.0	13.8	141	-0.25	-0.62	0.0018	0.8	0.44	0.52	1.4	2.0	2.8
	4.80	13.0	15.4	141	-0.23	-0.65	0.0017	0.7	0.54	0.69	1.3	2.5	3.3
0.040	1.65	16.4	21.7	165	-0.69	-0.55	0.0053	2.7	2.8	0.48	2.9	3.0	7.2
	2.37	14.4	16.9	149	-0.45	-0.68	0.0055	1.7	1.9	0.96	2.7	3.5	5.6
	3.43	11.8	11.1	131	-0.21	-0.56	0.0053	1.5	1.1	1.92	2.9	3.7	5.4
	4.80	11.9	11.1	137	-0.21	-0.54	0.0055	1.2	1.1	3.24	3.5	4.7	6.3
0.083	1.65	15.6	18.3	147	-0.64	-0.64	0.018	2.9	6.3	1.84	8.0	6.4	13
	2.37	12.7	10.8	142	-0.81	-0.71	0.0260	3.4	3.3	3.34	5.1	6.8	9
	3.43	14.8	9.3	145	-0.97	-1.26	0.047	4.0	1.9	8.85	11.1	9.3	10
	4.80	11.2	8.2	169	-1.00	-0.85	?	?	2.0	14.4	12.0	10.3	11

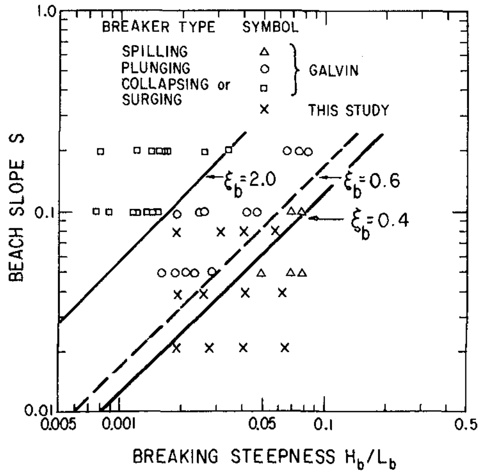

Figure 2. Classification of present breaker types according to Battjes' reinterpretation of Galvin's observations.

RESULTS

As a basic perspective of the results obtained in these experiments, Figures 3, 4, and 5 show, for the three respective beach slopes, comparisons of surf zone variables, ordered downward with increasing wave period. From left to right in each of the twelve period-slope combinations, mean surface elevation (black dots) is increasingly depressed below still water level as the break point (arrows) is approached, reaches a minimum in the vicinity of breaking, and then rises linearly toward the shoreline, although the data do not extend beyond the still water intersection with the beach. The open bars along the beach face give the dynamical range ΔY of steady state water motion, measured vertically from still water level, and whose upper limit represents the engineer's interpretation of run-up. The tic marks on the beach face labeled Y_t give the maximum recorded transient water level excursion associated with the initial surge at the start of each data run. Relevant numerical values of all data are included in Table 1.

Pre-Breaking Set-Down

The depression of mean elevation (set-down) in shoaling water predicted by Longuet-Higgins and Stewart (1964) has been shown by Bowen, et al (1968) to be in good accord with observation where wave growth is governed by linear conservation of energy flux, and to become too small as wave steepness increases beyond the linear range. Near breaking, this trend was reversed: predicted set-down continues to increase, whereas the experimental values were found to reach a flat maximum depression in the vicinity of the break point.

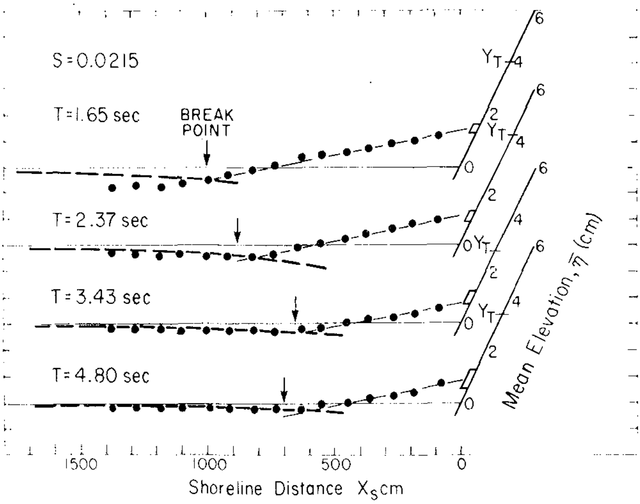

Figure 3. Set-down, set-up, and run-up range for the 0.022 slope.

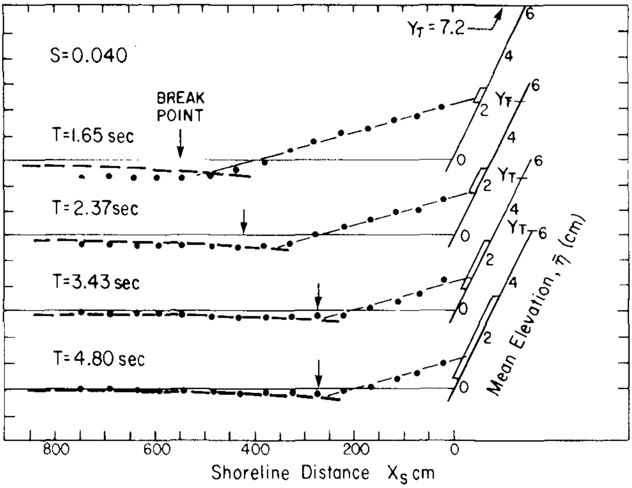

Figure 4. Set-down, set-up, and run-up range for the 0.040 slope.

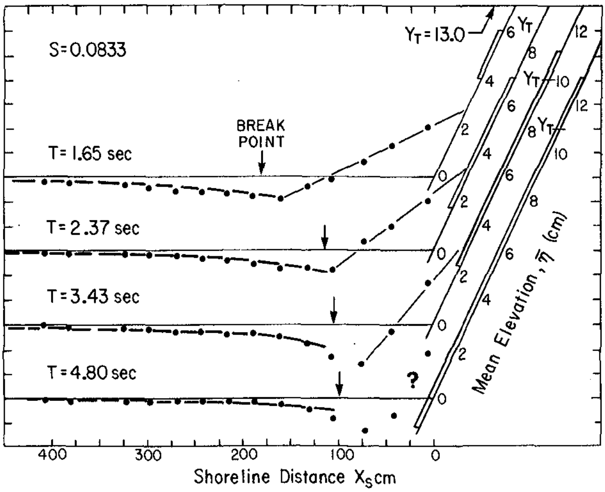

Figure 5. Set-down, set-up, and run-up range for the 0.083 slope.

Much the same results were obtained over the three present slopes, the steepest of which (S = 0.083) was the same as that used by Bowen, et al whose experiments were also conducted in the same wave channel. However, in an attempt to find a better approximation to the set-down under steep waves, the dashed curves shown in Figures 3-5 were calculated from the expression

$$\bar{\eta} = - \frac{H^2 k}{8 \sinh 2kD} \tag{5}$$

using observed values of H and D, and taking $k = 2\pi/CT$, where C was observed phase speed. It was hoped that the intersections of these curves with corresponding set-up lines within the surf zone might lead to a means of estimating maximum shoreline set-up, provided that these intersections could somehow be linked to breaker characteristics, and hence to deep water properties (Van Dorn, 1977). As can be seen the computed curves fit the data quite well over most of the growth range, except for those where T = 1.65 sec on the 0.022 and 0.040 slopes, where the observed set-down is unexpectedly large in deep water. The reason for this is not known: the experiments were repeat three times with always the same result.

Set-Up

In common with Bowen, et al (1968), set-up within the surf zone was found to be approximately linear, and, on the 0.083 slope, the mean surface slope also appears to increase with wave period, although the definition is doubtful for T = 3.43 sec, and completely uncertain for T = 4.80 sec, for both of which photographs indicate extremely violent turbulence extending clea to the bottom over most of the very-limited "surf zone". Seemingly, no simple meaning can be attached to 'mean elevation' under such conditions, where there is only one wave or less present simultaneously.

By contrast, the slopes of the set-up lines over the 0.022 and 0.040 slopes appear to be constant and independent of frequency within experimental error. Figure 6 is a plot of set-up slope $m = d\bar{\eta}/dx$ versus beach slope S which includes all present data together with others reported by Bowen, et al and Putnam (1945), where any possible variation due to frequency is represented by error bars. If the line

$$m = 3.4S^2 \qquad (6)$$

can be assumed a reasonable fit to the distribution shown, then (1) requires that

$$3\gamma^2/8 = 3.4S/(1 - 3.4S) \qquad (7)$$

From the geometry of Figures 3-5, it is evident that, to the extent that the set-down curves and set-up lines intersect at the breaking point (where the wave properties are reasonably predictable), then (1) can be integrated from this intersection ($\bar{\eta} = \bar{\eta}_b$) to obtain the maximum shoreline set-up $\bar{\eta}_s$:

$$\bar{\eta}_s = \bar{\eta}_b + \{3.4S/(1 - 3.4S)\}\,(\bar{\eta}_b + D_b) \qquad (8)$$

Values of $\bar{\eta}_s$ computed from (8) by inserting breaking data into (5) to obtain $\bar{\eta}_b$ are given in Table 1. Comparing these with corresponding values obtained by extrapolating the observed mean slopes to intersection with the beaches shows too great a disparity for (8) to be regarded as a satisfactory prediction method. The principal sources of error are the neglect of the influence of frequency on the 0.083 slope, and the fact that the breaking points do not necessarily coincide with the intersections of the set-down and set-up intersections. Even so, (8) gives set-up values much closer to observations than the shallow water approximation, $\bar{\eta}_s = 0.3H_b$ suggested by Battjes (1974, p 58).

One of the assumptions basic to the set-up theory of Bowen, et al (1968) is the proportionality between breaker height and total water depth across the surf zone. Figure 7 shows plots of observed wave height versus total water depth and horizontal distance, all normalized to their respective breaking values. While the data for the 0.083 slope are scattered, they conceivably could be represented by straight lines, but over both smaller slopes wave height drops rather abruptly from the breaking point, and tends smoothly to some finite value at the still water shoreline. There appears to be relatively little systematic correlation with frequency. This behavior is qualitatively very similar to that predicted by the numerical set-up model of Hwang and Divoky (1970), although their appropriate bottom slopes are numerically too small. This difference might be resolved by a better energy loss coefficient.

Run-Up

Evidence of periodic shoreline oscillations at wave frequency is the most sensitive test that energy is being reflected from a sloping beach. Le Mehaute, et al (1966) and Putnam (1945) observed no perturbation of the mean level over a slope of 1:100, but the run-up range ΔY in these experiments was readily measurable on all three slopes, and increased regularly with slope and wave period, as shown by the open bars along the slopes in Figures 3-5.

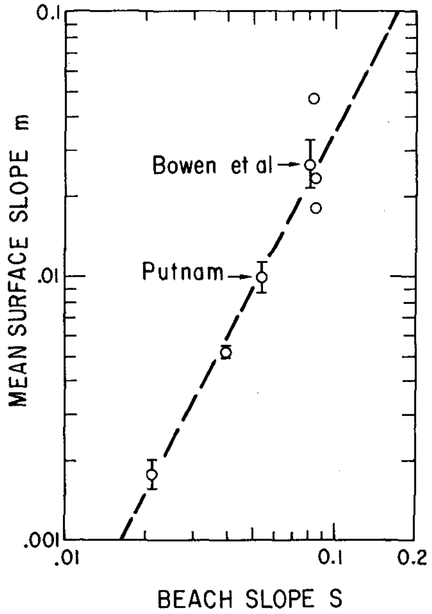

Figure 6. Observed variation of linear mean set-up slope versus beach slope.

In analyzing the motion of bores on slopes, Shen and Meyer (1963) remark that the majority of the motion in x-t space above the still water level should be expected to obey the parabolic law

$$x = u_o t - \frac{1}{2} gSt^2 \qquad (9)$$

where u_0 is the characteristic bore speed at $t = 0$. Equation (9), of course, also describes the motion of a frictionless block projected with velocity u_0 up a slope S. For the periodic shoreline oscillations considered here, the intuitively reasonable picture is that each breaker runs up or the backwash from its predecessor at intervals of a wave period; the whole motion being somehow superimposed on the mean shoreline set-up. Differentiating (9), and noting that $dx/dt = C = 0$ and $x = x_m = \Delta Y/S$ (say) when $t = T/2$, it is easily shown that

$$\Delta Y = gT^2S^2/8 \qquad (10)$$

Thus, to the extent that the waves can be considered frictionless bores, and that $S = \tan\alpha = \sin\alpha$, where α is the beach slope angle to the horizontal, (10) should be descriptive of the run-up range. The test of this estimate i Figure 8, where ΔY is plotted versus $L_0S^2 = gT^2S^2/2\pi$ for all present dat as well as for that reported by Bowen, et al (1968) and Battjes and Roos (1977, in press)*.

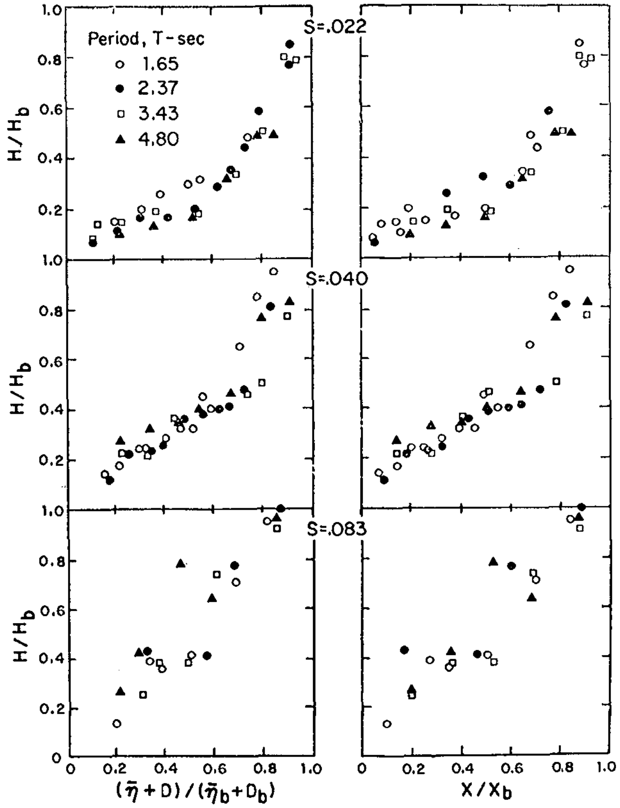

Figure 7. Wave height versus total water depth and surf zone width, all normalized to breaking values.

The straight line labeled frictionless reflection represents (10) in these coordinates, and parallels the data very well. The present experiments, being conducted over plate glass slopes gives results much closer to the frictionless limit that those of Bowen, et al over a plywood slope, and that of Battjes is not described. The conclusion is that run-up range is independent of breaker height, and depends significantly on bottom friction. Indeed, such experiments might provide a sensitive test of frictional losses.

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* For Bowen, et al, ΔY was taken as twice run-up, minus mean set-up; for Battjes and Roos, it was taken as $\Delta X \csc x$, where ΔX was the observed water excursion along the slope.

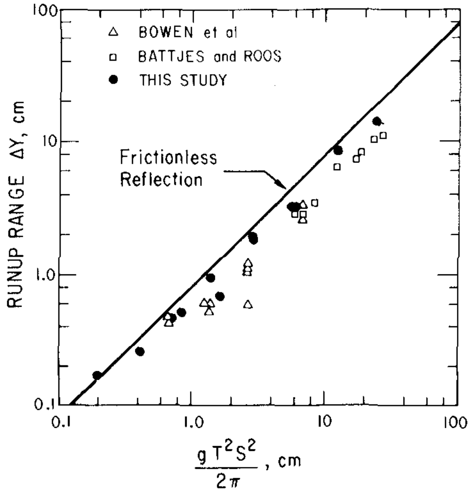

Figure 8. Run-up range versus beach slope and wave period.

Total Run-Up

Nothing said so far provides a clue as to how set-up and run-up range combine to produce the observed maximum run-up, and the above results shed little light on this interesting problem. While no reliable means for predicting set-up has been discovered, Figures 3-5 show clearly that it always amounts to a substantial fraction of the run-up range, since the run-down seldom (if ever) goes below still water level. Second, run-up range, normalized to deep water wave height, is nearly the square of Hunt's (1969) relative run-up:

$$R/H_0 = (L_0/H_0)^{\frac{1}{2}}S \qquad (11)$$

which is generally quoted as a reliable run-up index. At least, the comparisons given in Table 1 show that (11) is not a good approximation for small slopes. The values of H_0 used in the calculations were obtained from Van Dorn (1977), and are believed accurate to 10 percent.

Transient Run-Up

Figures 3-5 and the last column in Table 1 give observed values of transient run-up Y_t resulting from the initial surge associated with generato start up. The transient is manifested on the run-up records as a single intumescence, having a duration equal to 3-5 wave periods, that reflect back-and-forth between the wave paddle and beach slope, slowing decaying after several minutes. These transient surges are of interest because their amplitudes substantially exceed the steady-state run-up in many cases, and because they can be expected to occur whenever the momentum flux is varied rapidly, as for example, when waves come in groups. Because their characteristics clearly depend upon the shape of the advancing energy front, no attempt was made here to relate them to other fixed parameters.

ACKNOWLEDGMENTS

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REFERENCES

Battjes, J. A., 1974a, "Computation of set-up, longshore currents, run-up and overtopping due to wind-generated waves", *Delft University of Tech.*, Report No 74-2, 244 pp.

Battjes, J. A., 1974b, "Surf similarity", *Proc. 14th Coastal Eng. Conf.*, Amer. Soc. Civil Eng., Vol 1, Chapter 26, p 466-77.

Battjes, J. A. and A. Roos, 1976 in press, "Characteristics of flow in periodic wave run-up", *Jour. Harbors and Waterways*, Amer. Soc. Civil Eng.

Bowen, A. J., D. L. Inman and V. P. Simmons, 1968, "Wave 'Set-Down' and Set-Up", *Jour. Geophys. Res.*, vol 73, no 8, p 2569-77.

Galvin, C. J. Jr., 1968, "Breaker type classification on three laboratory beaches", *Jour. Geophys. Res.*, vol 73, no 12, p 3651-59.

Hunt, I. A., 1959, "Design of seawalls and breakwaters", *U. S. Corps of Eng.*, Lake Survey, Detroit, 49 pp.

Hwang, Li-San, and D. Divoky, 1970, "Breaking wave set-up and decay on gentle slopes", *Proc. 12th Conf. on Coastal Eng.*, Amer. Soc. Civil Eng., Vol 1, Chapter 23, p 377-89.

Iribarren, C. R. and C. Nogales, 1949, "Protection des ports, Section II", *Comm. 4, XVIIth Int. Nav. Congress, Lisbon*, p 31-80.

Longuet-Higgins, M. S. and R. W. Stewart, 1964, "Radiation stress in water waves, a physical discussion with application", *Deep-Sea Res.*, vol 11, p 529.

Le Mehuate, B. J., G. F. Snow and L. M. Webb, 1966, "Gravity waves on bottom slopes", *National Engineering Science Company*, Pasadena, California, Final Report, Volume 1, 79 pp.

Putnam, J. A., 1945, "Preliminary report of model studies on the transition of waves in shallow water", *Univ. Calif. Berkeley*, Report HE-116-106.

Shen, M. C. and R. E. Meyer, 1963, "Climb of a bore on a beach, Part 3. Run-Up", *Jour. Fluid Mech.*, vol 16, part 1, p 113-125.

Van Dorn, W. G., 1977, "Breaking invariants in shoaling waves", *Jour. Geophys. Res.*