CHAPTER 134

MATHEMATICAL MODEL OF SALINITY INTRUSION IN THE DELTA OF THE PO RIVER

by

Renzo Dazzi - Mario Tomasino

ABSTRACT

A general numerical model capable of reproducing long internal waves in stratified fluids has been constructed with the aim of investigating the salt wedge penetration in the Delta of the Po river, where the installation of a 2640 MW thermo-electric plant is foreseen.

The working hypothesis of the model, in accordance with the actual phenomenology of the river, is the one-dimensional homogeneous motion of two fluid layers of different density.

The main original aspects of the numerical computation are:

1) - The use of two different space steps (1 km for the fresh water layer; 200 m for salt water) simultaneously allowing a good description of internal waves (the velocity of which is much smaller than that of the external ones) and making it possible to work with economic (100 sec) time steps.
2) - The straightforward description of the wedge head, obtained by making it always freely correspond to one of the grid points.

The model, which has been tested on actual events, reproduces reality with a very good approximation; it also gives evidence of the small relevance of the interfacial stress coefficient in unsteady tidal generated motion of the salt wedge.

Renzo Dazzi - C.N.R. - Laboratorio per lo Studio della Dinamica delle Grandi Masse - 1364 S.Polo - Venice - Italy

Mario Tomasino - ENEL/C.R.I.S. -Servizio Idrologico- Corso del Popolo 245 - Venice - Italy

1) INTRODUCTION

Two years ago the Italian National Authority for Electric Energy (ENEL) decided to construct a thermo-electric plant with a large productive capacity (2640 MW) in the zone of the Delta of the river Po (fig. 1). This zone was chosen for both technical and social reasons, with the scope of giving a contribution to industrialisation and raising the standard of living of the inhabitants of the underdeveloped Delta region. The technical reasons of the choise are obvious, considering that this estuarine site, which collects 24% of the Italian water flow, completely satisfies the cooling water necessities of such a plant.

The installation is foreseen on the main branch of the Delta, (5 km from the sea - fig. 2) with two intakes and two outfalls on the river and on an adjacent lagoon. The cooling of the station will necessitate a variable withdrawal from 80 m^3/sec to 320 m^3/sec when the plant is fully operative.

The mean year discharge of the river Po at Pontelagoscuro (the main observation station along the river) (fig. 1) is 1500 m^3/sec; the Pila branch, where the intakes will be located, has only about 50% of this discharge. The natural discharge of this branch is therefore comparable with the foreseen withdrawals, consequently the probability of a modification of the fluvial régime due to the installation is not so remote.

In order to investigate and predict the possible changes we developed an ordinary numerical hydrodynamical one-dimensional homogenous model, which simulated the network of the Delta from Pontelagoscuro to the sea (a length of 92 km of the main course of the river, as well as 160 km of the lateral branches).

This model gave very good results /1, 2/ in the hindcasting of events with normal and high (up to 6000 m^3/sec) Pila discharge, but failed for Pila mean discharge lower than 500 m^3/sec, particularly during spring tides, when amplitude in the open sea facing the Delta is about 80 cm. .In fact during spring tides which occur at low discharge, the model gave a strong overevaluation of the upstream (negative) discharge (fig. 3).

Field measurements made in these conditions show that return currents are present and a salt wedge is formed; salt and fresh water flows are well identified and the stratification is very strong as the sharp interruption in the measured vertical profiles of salinity shows (fig. 4); it was revealed that the wedge under the fresh water penetrated up to 10 km from the river mouth (fig.2). These were the reasons why the homogenous model did not work correctly, its construction hypothesis being far away from the actual motion.

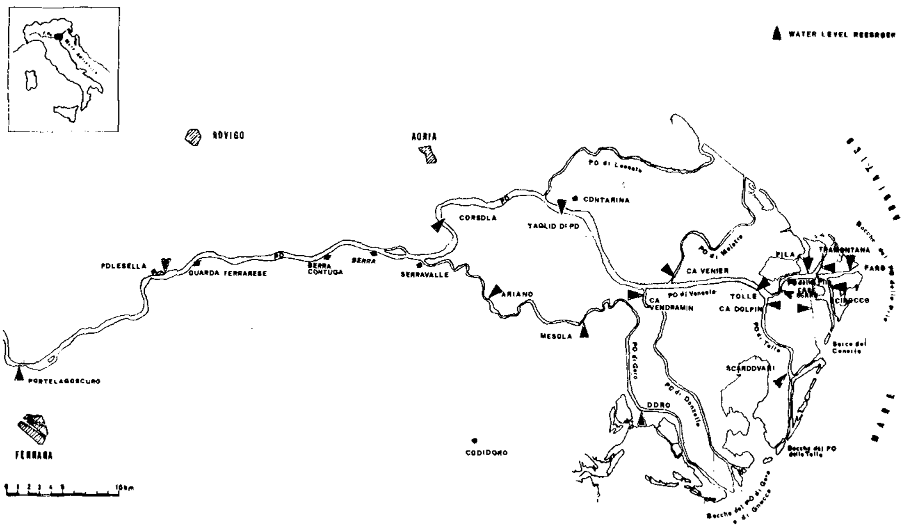


Fig. 1 - Delta of Po River

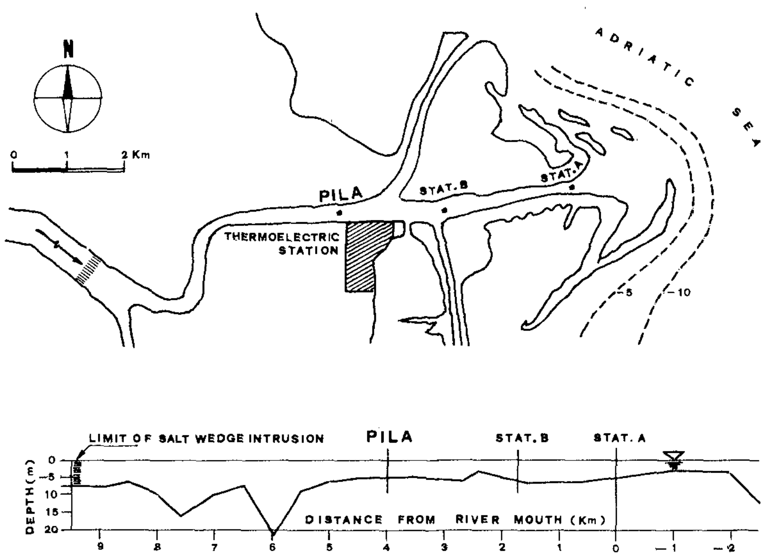


Fig. 2 - Thermoelectric plant location and longitudinal profile of the river bed

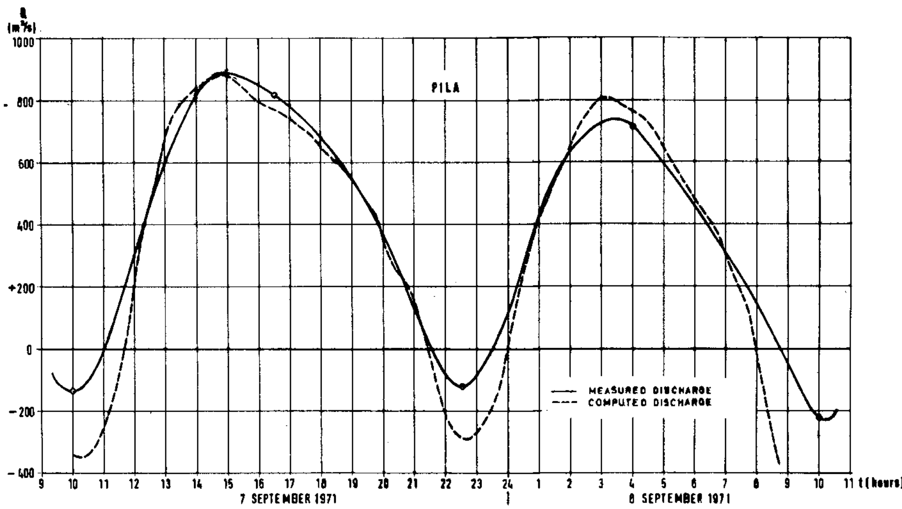


Fig. 3 - Simulation with density return currents

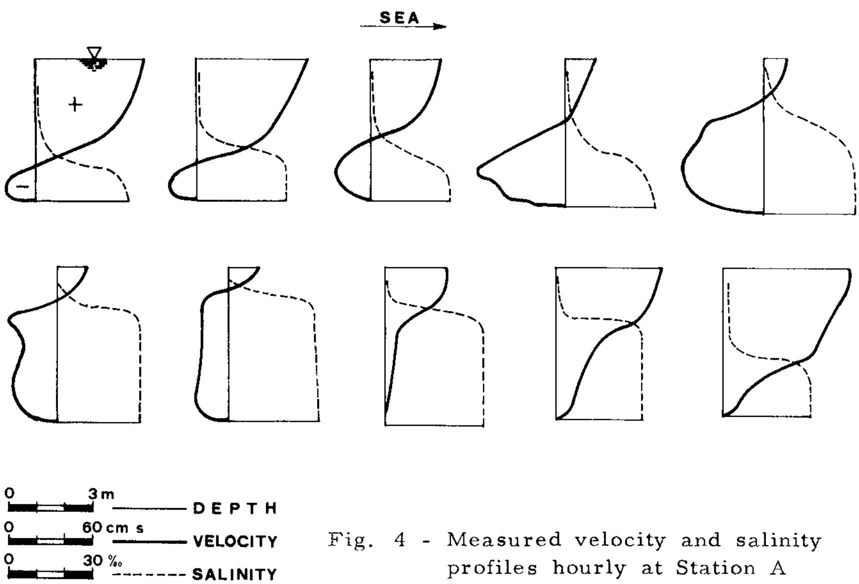


Fig. 4 - Measured velocity and salinity profiles hourly at Station A

It was therefore decided to build a model capable of reproducing the physical flow patterns and the sea water penetration in the river, with the aim of answering the main water quality questions (salinity and partially temperature) involved in the probable modification of the existing salt wedge behaviour.

It would be worthwhile briefly summarizing the possible consequences of this change:

	Effect	Damage
Environment modifications	1) Marine biota modification	Fish and plancton destruction
	2) Salt penetration in underground aquifers	Agricultural damage
	3) Sedimentation (flocculation) rate increase	Dredging
Technical effects	4) Recirculation of cooling water	Increase in water temperature

2) PHYSICAL FOUNDATIONS

The evidence of strong stratification enables the fluvial motion to be treated with the schematization of two homogenous layers, each with a constant density, considering the salt or mass transfer from one layer to the other as negligible: the two layers only interact by momentum exchange along the interface and reciprocal pressure effects.

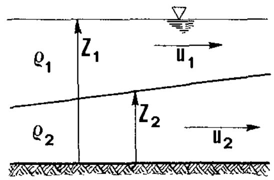
With these hypotheses* which are fully justified by the measurements, the hydrodynamical equations for both layers are the same as for the homogenous one-layered flow, plus:

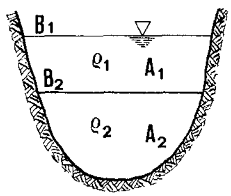
1) - one term in the continuity equation of the upper layer wich describes the discharge contribution of the lower layer
2) - one term in the momentum equation of the lower layer which takes into account the pressure exerted by the upper layer on the lower one
3) - a couple of terms in both the momentum equations which represent the interfacial stresses

*In these assumptions also the hydrostatic hypothesis is made for both layers, pratically generating /3/ a vertical integrated model of flow, which is only valid for long waves (this is precisely our field of interest)

$$\left.\begin{aligned}
&\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}+g\frac{\partial Z_1}{\partial x}+\frac{\lambda}{8R_1}\left(u_1-u_2\right)\left|u_1-u_2\right|+\frac{g\,u_1\left|u_1\right|}{C^2R'_1}=0\\
&\frac{\partial u_2}{\partial t}+u_2\frac{\partial u_2}{\partial x}+g\frac{\partial Z_2}{\partial x}+g\frac{\varrho_1}{\varrho_2}\frac{\partial Z_1}{\partial x}-\frac{\lambda}{8R_2}\left(u_1-u_2\right)\left|u_1-u_2\right|+\frac{g\,u_2\left|u_2\right|}{C^2R'_2}=0
\end{aligned}\right\}\text{momentum equations}$$

$$\left.\begin{aligned}
&\frac{\partial Z_1}{\partial t}B_1-\frac{\partial Z_2}{\partial t}B_2+\frac{\partial u_1}{\partial x}A_1=0\\
&\frac{\partial Z_2}{\partial t}B_2+\frac{\partial u_2}{\partial x}A_2=0
\end{aligned}\right\}\text{continuity equations}$$





where:

t = time
x = distance coordinate
g = gravity acceleration
ϱ_1, ϱ_2 = densities of the layers
u_1, u_2 = current mean velocities
Z_1, Z_2 = levels of the free surface and of the interface
B_1, B_2 = width of the free surface and of the interface
A_1, A_2 = cross section areas
R_1, R_2, R'_1, R'_2 = hydraulic radius given by the ratio between the cross section areas and the length of the friction contour
$C = \left[m^{1/2}/sec\right]$ Chézy coefficient
λ = dimensionless Darcy-Weisbach coefficient

(Quantities with index 1 refer to upper layer and with index 2 to the lower layer)

The head losses in the momentum equations are expressed:

1) - the bottom stress by the Chézy formula:

$$v = C\sqrt{R\,i}$$

2) - the interfacial stress by the Darcy-Weisbach formula

$$v = \sqrt{\frac{8\, g\, R\, i}{\lambda}}$$

where i represents the surface slope.

The set of four equations, which were firstly investigated for infinitely wide currents by Schijf and Schönfeld in 1953 /4/, defines two kinds of propagation velocity for small disturbances: one for the external waves i.e. surface waves, the other for internal waves i.e. the interface waves. These velocities can be easily determined /5/ by applying the Cauchy-Kowalewsky theorem to the set, putting:

$$A_x - c \cdot A_t = 0$$

where:
c = wave propagation speed
A_x = matrix of the space derivative coefficients
A_t = matrix of the time derivative coefficients

This equation, still in the case of infinitely wide flows, produces a fourth degree equation:

$$(c - V_1)^2 \cdot (c - V_2)^2 - g\, h_2\, (c - V_1)^2 - g\, h_1\, (c - V_2)^2 + \varepsilon\, g^2\, h_1\, h_2 = 0$$

where:
h_1, h_2 = depth of the two layers

$$\varepsilon = \frac{\varrho_1 - \varrho_2}{\varrho_2}$$

whose four roots are the above mentioned propagation velocities of small perturbations in the fluid /6/.

Assuming the flow velocities as negligible, c_e (velocity of the external waves) and c_i (velocity of the internal waves) become:

$$c_e = \pm \sqrt{\frac{g\,(h_1 + h_2)}{2} + \frac{g}{2}\sqrt{(h_1 + h_2)^2 - 4\,\varepsilon\, h_1\, h_2}}$$

$$c_i = \pm \sqrt{\frac{g\,(h_1 + h_2)}{2} - \frac{g}{2}\sqrt{(h_1 + h_2)^2 - 4\,\varepsilon\, h_1\, h_2}}$$

If ε is small (as in the case of fresh water over salt water) the two expressions can be approximated by:

$$c_e \simeq \pm \sqrt{g \left(h_1 + h_2\right)}$$

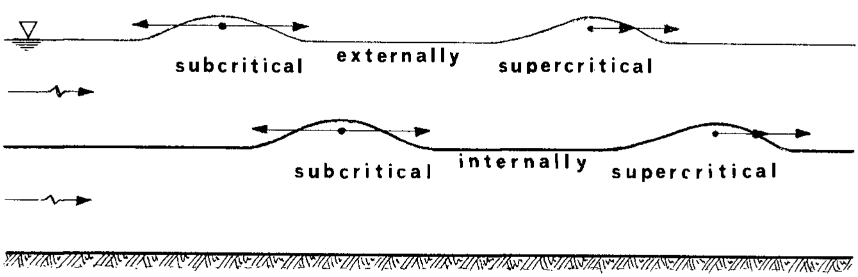
$$c_i \simeq \pm \sqrt{\varepsilon\, g \frac{h_1\, h_2}{\left(h_1 + h_2\right)}}$$

It immediately ensues that external wave velocity is very close to $\sqrt{g\,h}$, i.e. the propagation velocity in the one-layer homogeneous flow; and in ratio, it results larger in magnitude than the internal wave velocity, being:

$$\frac{c_e}{c_i} \gtrsim \frac{2}{\sqrt{\varepsilon}}$$

The definition of c_i, c_e and the ratio $\frac{c_e}{c_i}$, whose physical meanings are clearly understood, is the most important aspect for what concerns the definition of the boundary conditions and the integration space step of the finite difference scheme used for solving the set of differential equations.

Another important feature which immediately descends from the definition of the propagation velocities, is that the two layered flow can be internally or externally sub or supercritical, depending upon whether it is possible for an internal or external wave to move both up and downstream.



This behaviour is connected with the densimetric Froude number definition (in analogy with the one-layer flow) which individuates the internal sub or supercritical status of the flow /4/. Even though the densimetric Froude number is important in characterising the flow, it will not explicitely be used in the construction of the model.

3) THE FINITE DIFFERENCE SCHEME

The set of partial differential equations was integrated using a finite difference scheme, staggered both in space and time and expressing all the derivatives in the central form. All the terms present in the four equations are strictly written in this way, only neglecting the convective acceleration terms in the momentum equations. This is usual practice in hydrodynamic subcritical numerical computations and is permitted by the small relevance of these terms in the global balance of the momentum equations.

Defining the difference operator in such a straightforward way we assumed also a linear variation of all the variables, (both in time and space) expressing, when needed, the value in an intermediate point or instant as the mean between the precedent and the following values.

The geometry of the river bed is introduced into the model by defining the values of B_*, A_*, R_*, R'_* (*) as functions of the levels Z_* in every computation section and using a particular subprogram operating on the contour of the cross sections given by points. Hereafter $F(Z_*)$ will indicate the general level depending functions B_*, A_*, R_*, R'_*.

Defining:
Z1, Z2, U1, U2, B1, B2, A1, A2, R1, R2, R1', R2' as the same variables defined as $Z_1, Z_2, u_1, u_2, B_1, B_2, A_1, A_2, R_1, R_2, R_1', R_2'$ in the set of differential equations,
T = integration time step
X = integration space step
m = index defining the computation instant, so $t = m \cdot T$
n = index defining the computation section, so $x = n \cdot X$
and considering that Z1, Z2 are evaluated corresponding to integer values of m and n whilst U1, U2 are evaluated in correspondence to:

$$t = \left(m + \frac{1}{2}\right) \cdot T \qquad\qquad X = \left(n + \frac{1}{2}\right) \cdot X$$

the continuity equations are discretized as follows:

$$Z1_{m,n} = Z1_{m-1,n} + \left(\frac{B2}{B1}\right)_{m-\frac{1}{2},n} \cdot \left(Z2_{m,n} - Z2_{m-1,n}\right) +$$

$$+ \frac{T}{X} \cdot \frac{1}{B1_{m-\frac{1}{2},n}} \cdot \left[\left(A1 \cdot U1\right)_{n-\frac{1}{2}} - \left(A1 \cdot U1\right)_{n+\frac{1}{2}}\right]_{m-\frac{1}{2}}$$

(*) The asterisk represents 1 or 2.

$$Z2_{m,n} = Z2_{m-1,n} + \frac{T}{X} \cdot \frac{1}{B2_{m-\frac{1}{2},n}} \cdot \left[\left(A2 \cdot U2\right)_{n-\frac{1}{2}} - \left(A2 \cdot U2\right)_{n+\frac{1}{2}}\right]_{m-\frac{1}{2}}$$

where:

$$A2_{m-\frac{1}{2},n-\frac{1}{2}} = F\left(\frac{Z2_{m,n} + Z2_{m,n-1} + Z2_{m-1,n} + Z2_{m-1,n-1}}{4}\right)$$

$$A2_{m-\frac{1}{2},n+\frac{1}{2}} = F\left(\frac{Z2_{m,n+1} + Z2_{m,n} + Z2_{m-1,n+1} + Z2_{m-1,n}}{4}\right)$$

$$A1_{m-\frac{1}{2},n-\frac{1}{2}} = F\left(\frac{Z1_{m,n} + Z1_{m,n-1} + Z1_{m-1,n} + Z1_{m-1,n-1}}{4}, A2_{m-\frac{1}{2},n-\frac{1}{2}}\right)$$

$$A1_{m-\frac{1}{2},n+\frac{1}{2}} = F\left(\frac{Z1_{m,n+1} + Z1_{m,n} + Z1_{m-1,n+1} + Z1_{m-1,n}}{4}, A2_{m-\frac{1}{2},n+\frac{1}{2}}\right)$$

$$B*_{m-\frac{1}{2},n} = F\left(\frac{Z*_{m,n} + Z*_{m-1,n}}{2}\right)$$

The finite difference scheme is partially implicit because in the continuity equations the unknown Z1 or Z2, in one point, depends on the unknown values of the same variable Z1 or Z2 in the surrounding points in the same instant. This requires a simultaneous solution of the set of continuity equations for each layer. In solving them, the lower layer set is tackled first in order to furnish known values of the lower layer discharge contribution to the upper layer set. Both the sets of nonlinear algebraic equations obtained by discretizing the continuity equations are solved by iterative procedures.

The momentum equations are discretized as follows:

$$U1_{m+\frac{1}{2},n+\frac{1}{2}} = U1_{m-\frac{1}{2},n+\frac{1}{2}} - \frac{g \cdot T}{X}\left(Z1_{n+1} - Z_{n}\right)_{m} - \frac{\lambda \cdot T}{8 \cdot R1_{m,n+\frac{1}{2}}} \cdot \frac{1}{4} \cdot$$

$$\cdot \left(U1_{m+\frac{1}{2}} + U1_{m-\frac{1}{2}} - U2_{m+\frac{1}{2}} - U2_{m-\frac{1}{2}}\right)_{n+\frac{1}{2}} \cdot \left|U1_{m+\frac{1}{2}} + U1_{m-\frac{1}{2}} - U2_{m+\frac{1}{2}} - U2_{m-\frac{1}{2}}\right|_{n+\frac{1}{2}}$$

$$- \frac{g \cdot T}{C^2 \cdot R1'_{m,n+\frac{1}{2}}} \cdot \frac{1}{4}\left(U1_{m+\frac{1}{2}} + U1_{m-\frac{1}{2}}\right)_{n+\frac{1}{2}} \cdot \left|U1_{m+\frac{1}{2}} + U1_{m-\frac{1}{2}}\right|_{n+\frac{1}{2}}$$

$$U2_{m+\frac{1}{2},n+\frac{1}{2}} = U2_{m-\frac{1}{2},n+\frac{1}{2}} - \frac{g \cdot T}{X} \cdot \left(Z2_{n+1} - Z2_{n} \right)_m - \frac{g \cdot T}{X} \cdot \frac{\varrho_1}{\varrho_2} \cdot \left(Z1_{n+1} - Z1_{n} \right)_m +$$

$$+ \frac{\lambda \cdot T}{8 \cdot R2_{m,n+\frac{1}{2}}} \cdot \frac{1}{4} \left(U1_{m+\frac{1}{2}} + U1_{m-\frac{1}{2}} - U2_{m+\frac{1}{2}} - U2_{m-\frac{1}{2}} \right)_{n+\frac{1}{2}} \cdot \left| U1_{m+\frac{1}{2}} +\right.$$

$$\left. + U1_{m-\frac{1}{2}} - U2_{m+\frac{1}{2}} - U2_{m-\frac{1}{2}} \right|_{n+\frac{1}{2}} - \frac{g \cdot T}{C^2 \cdot R2'_{m,n+\frac{1}{2}}} \cdot \frac{1}{4} \cdot$$

$$\cdot \left(U2_{m+\frac{1}{2}} + U2_{m-\frac{1}{2}} \right)_{n+\frac{1}{2}} \cdot \left| U2_{m+\frac{1}{2}} + U2_{m-\frac{1}{2}} \right|_{n+\frac{1}{2}}$$

where:

$$R2, R2' = F \left(\frac{Z2_{m,n} + Z2_{m,n+1}}{2} \right)$$

$$R1, R1' = F \left(\frac{Z2_{m,n} + Z2_{m,n+1}}{2} , \frac{Z1_{m,n} + Z1_{m,n+1}}{2} \right)$$

These equations link together the values of each U1 and U2 which are relative to the same point, due to the fact that the interfacial stress is expressed as a function of them both; then, for each grid point, the values of U1 and U2 are simultaneously determined by iteratively solving the two momentum equations of the upper and lower layer.

The difference scheme did not prove unconditionally stable in the practical simulations and slightly exceeded (in a version which also took into account the frictional terms), the theoretical values of the time steps allowed by the usual criterion of stability:

$$\frac{X}{T} \geq c$$

In this case we consider c_e as c, as the external wave velocity is more restrictive in respect to the criterion.

4) MODEL CONSTRUCTION

Steps

The practical realisation of the model began with the choice of the space step, i.e. by the definition of the density of level and velocity computation points. The choice was conditioned by the ratio

$\frac{c_e}{c_i}$, which in our case, being $\varrho_1 \simeq 1 g/cm^3$ and $\varrho_2 \simeq 1.025 \; g/cm^3$, gives $c_e \gtrsim \frac{2}{\sqrt{\varepsilon}} \cdot c_i \simeq 12 \cdot c_i$

The internal wave velocity is less than $\frac{1}{12}$ of that of the external wave; the ratio between wave lengths of internal and external waves of the same period can, in first approximation, be considered the same.

The most relevant tidal component in the Delta is M2, with a period of 12h 25', and an approximative 280 km wave length, as evaluated on the basis of the mean Po Delta depth of 4 m (*). The overtide generation (i.e. higher harmonics) originating from the shallow water propagation does not attain (probably being overcome by dissipative phenomena) valuable effects for harmonics higher than the third one. Difference schemes of the same type as ours need about 30 points per wave length to give an almost perfect reproduction of a long wave propagation both in phase and amplitude /7/.

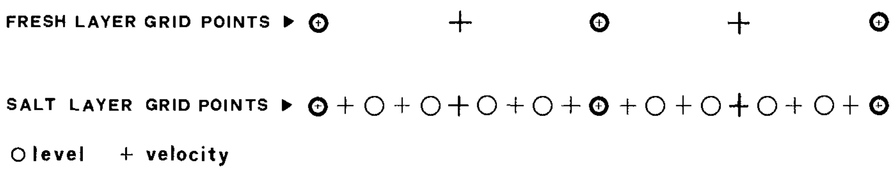
The step used in our previous one-layer model was 1 km, from which an accurate representation until the M2 fourth harmonic was obtained; as shown this is more than needed. A description of the internal wave with the same accuracy of the external one would require a space step of at least 1/12 of the previous one; considering that the external wave 1 km step was even too small, we adopted a 200 m step for this model, retaining it adequate to describe the tide induced penetration of the wedge in the river.

The first simulations made with this space step were revealed as uneconomical from the machine-time point of view, as they gave unstable results for time steps higher than 50 sec. This is explained considering that the mathematical velocity, i.e. $\frac{X}{T}$, in this case is lower than the physical one of the external wave; in fact:

$$\frac{X}{T} = \frac{200}{50} = 4 < 6.26 = \sqrt{9.81 \cdot 4} = \sqrt{g\,h} \simeq c_e$$

The Courant-Friedrichs-Lewy criterion for the stability condition of a finite difference scheme ($\frac{X}{T} \geq \sqrt{g\,h}$) is strictly valid for the linear case /7/, but it empirically proved valid for this one too.

(*) To evaluate the wave propagation speed it is necessary to take into account the depth, averaged on the whole of each cross section of the river.



As the instabilities took origin in the surface waves, we decided to adopt a wider step of 1 km again, for the upper layer only, contemporarily obtaining the same degree of accuracy for the upper and lower flow representation and having the possibility of working with larger (more economical) time steps.

Convergence

At this point the quality of the lower flow representation, obtained by using different steps for the two layers, was examined. This was achieved by comparing the results of two simulation made with the same time step (50 sec only in order to satisfy stability requirements) and different space steps (200 m and 100 m). The simulations referred to a 2 km extension of a typical section of the river bed and used actual hydrological data as input.

The Godunov-Ryabenki theorem /8/, which affirms the convergence of a finite difference scheme once its stability is ensured, gives a tool with which to decide when a step is small enough to well represent a certain perturbation. This is the case for the 100 m and 200 m steps: in fact both simulations provide similar results as regards shape and velocity propagation of the wedge (fig. 5); the head velocity is almost the same and the more irregular longitudinal profile of the wedge reveals the presence of higher frequencies in the 100 m simulation. This is logical if considered in relation with the sharp input used(abrupt raising of the salt water level at sea) because it already contains high frequencies which are only reproducible in a denser grid. Fig. 5 only represents the wedge penetration; retraction occurs with a very slow decrease of salt level at sea, and in this case the two simulations produce closely resembling profiles. We then judged the 200 m step as sufficient for the simulation of the wedge in the river.

Hence, we finally adopted space steps of 1 km for the fresh water layer and 200 m for salt water; it was then easy to work with time steps of 100 sec, which are reasonable for computer time.

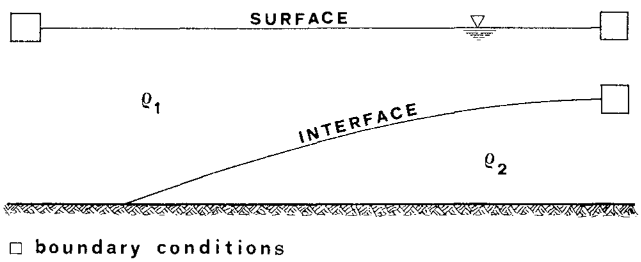
This scheme is not valid if the last seaward section of the model is completely occupied with salt water (in this case there would be a one-layer motion, the computation of which would require a 1 km space step for a time step of 100 sec); this has never happened during field measurements.

Boundaries

The boundary conditions which are mathematically required by the two-layer flow set of differential equations are defined, as previously mentioned, by internal and external wave velocity at upstream and downstream boundaries of the model. The number of conditions to be imposed is given by the number of perturbations entering the model through each boundary, as the theory of characteristics /5/, /11/ illustrates. For a subcritical flow there are two conditions (only one internal and one external wave can enter the model) at each boundary.

In order to allow the model to reproduce the wedge penetration, the upstream boundary has been placed far enough away from the sea to ensure that it cannot be reached by the wedge head.

Therefore only one upstream boundary condition (one-layer subcritical flow) is needed: the recorded fresh water level was used in the simulations. At the downstream boundary we presumed the motion as both internally and externally subcritical. This implies the use of two boundary conditions, which were the fresh and salt water level at the river mouth. However, when the salt wedge retracts and disappears, the fresh water level is sufficient to enable the model to work. Consequently two different kinds of motion can be reproduced: one layer for the normal flow of the river or two layers in presence of the wedge.



Using the internal critical condition of the motion in correspondence with the widening of the river cross section /9/ it is possible to avoid defining two boundary conditions at the mouth, because it univocally links the salt water level to that of the fresh water. For the sake of simplicity it has been used by several authors /10/. It has not been used in this model for three reasons:

1) - to have the possibility of testing the model simulations must deal with actual events; the lowest recording gauge in the river is placed before the widening of the river cross section; this is the only available downstream input.
2) - In our opinion the individuation of the critical section from the river morphology is very difficult. Moreover, as personally evidenced (presence of well marked surface fronts), in unsteady motion, the passage to critical conditions does not occur in a fixed point, instead the critical section migrates along the river following the tidal evolution.
3) - Furthermore, the main aim of the model is to predict the effects of future interventions on the river; therefore it is necessary to place the boundary conditions far enough away to render them independent from inner model events. So the critical conditions should be introduced into the model, allowing them to feel the consequences of fluvial régime modifications.

In conclusion the model boundary conditions allowed us to construct a physically respondent scheme, which overcame the comparison with reality.

As regards prediction, the seaward boundary conditions require another solution which will consist of introducing a small bi-dimensional model of the sea in front of the mouth, in order to have an input that is only dependent on tide level.

Wedge head

The motion of the wedge head has been modelled in the simplest way: its position, which is in fact a movable inner boundary in the model, always corresponds to one of the lower grid points, where it vanishes. Two situations are then possible:

1) - if the upper flow is oriented downwards, the head can only advance if its slope (the pressure of the upper layer is less important) prevails on the interfacial friction. Viceversa if the slope is too weak, the head diminishes in volume and when the interface reaches the bottom it retracts by one space step and so on;
2) - if the upper flow is oriented upwards, the head advances one space step for each time step, fastly decreasing in amplitude, then rapidly becoming negligible.

It is interesting to point out that this procedure is exactly the same as for normal one-layer homogeneous models. In fact, if we consider a linearised model with constant depth and use a time step which is too large to verify the identity between physical and mathematical velocity, it is well known that the numerical wave (which propagates one space step for each time step) travels faster than the real wave, but its amplitude rapidly decreases.

This form of modelling only identifies the position of the head relative to the space step dimension i.e. with an approximation of 200 m. It is not important to get a higher precision, as made with more complicated schematisations /5/, because the head position is defined in correspondence with the small thickness of the lower layer, where the physical processes /12/ are not exactly the same as those formulated by the equations. For these reasons the lower flow velocity is left free on the head region to overcome the critical value (internal critical velocity tends to zero as the thickness of the layer decreases). However the head velocity in this scheme, as said before, is a purely "numerical" velocity and therefore it is logically free from physical bonds.

Interfacial stress

The friction between the layers, which is expressed by the Darcy-Weisbach formula, depends on the values of the coefficient λ. This coefficient is usually given as a function of the Reynolds and Froude numbers; many formulae and experimental laboratory results furnish a wide series of values, ranging from 10^{-3} to 10^{-1} /13, 14, 15/.

As a first simple hypothesis we made several computations assuming λ as a constant, and examining its effect on the wedge behaviour. As order of magnitude of λ, we assumed the values which were found valid in other wedge computations /10, 16/. Simulations were made for an arrested wedge, i.e. in a steady condition, imposing a slope of 5 cm in 3 km to the free surface. The results of fig. 6 show that the length of penetration of the arrested wedge is very sensitive to λ variations from 0.003 to 0.006, changing the penetration length from 2800 to 2000 m. Thus a precise definition of the coefficient would appear very important.

In the actual régime of the Delta an arrested wedge has never been recognised by field measurements. The possibility of its occurrance is really doubtful: in fact a steady condition in the tidal zone could only last for a few hours. The small propagation velocity of the internal wave causes the wedge to feel for a long time the influx of the initial unsteady condition. We then verified the effects of different values of λ simulating an unsteady motion on the true river geometry and using actual data as input. Fig. 7, showing the maximum penetration extent of the wedge, illustrates how the different values of λ scarcely affect the wedge.

The small relevance of the friction coefficient in these simulations is not only explained by the particular shape of the longitudinal profile of the river, which presents a relatively steep

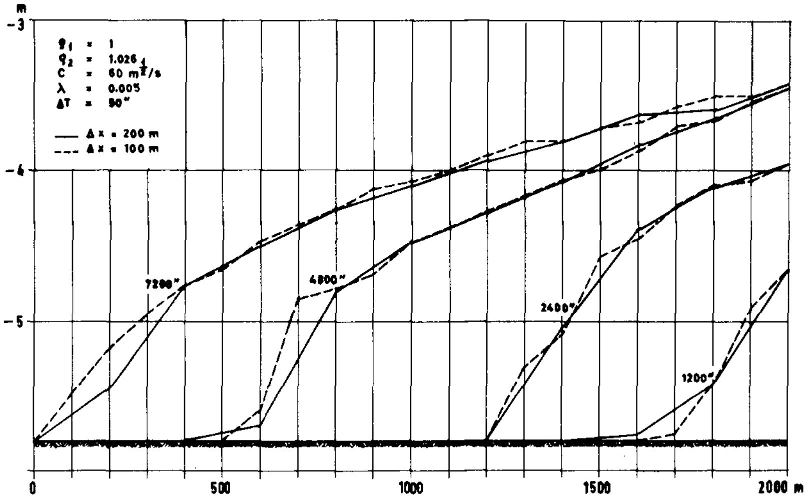


Fig. 5 - Effects of different space steps on the computed salt wedge profile

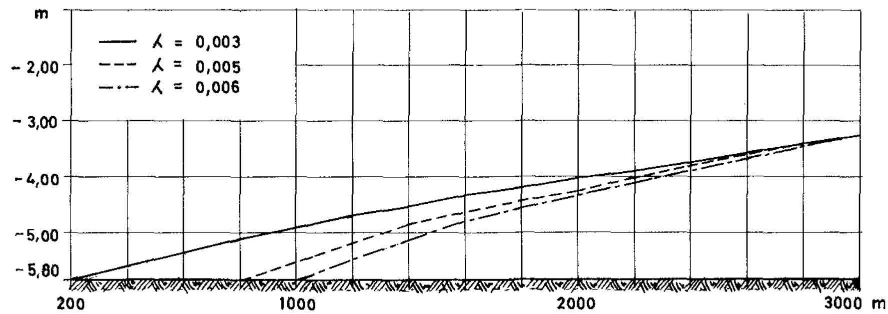


Fig. 6 - Arrested wedge

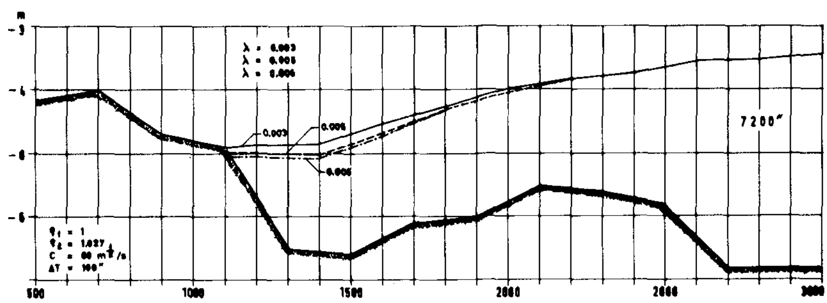


Fig. 7 - Variations of Darcy-Weisbach coefficient effects on the maximum extent of the salt wedge

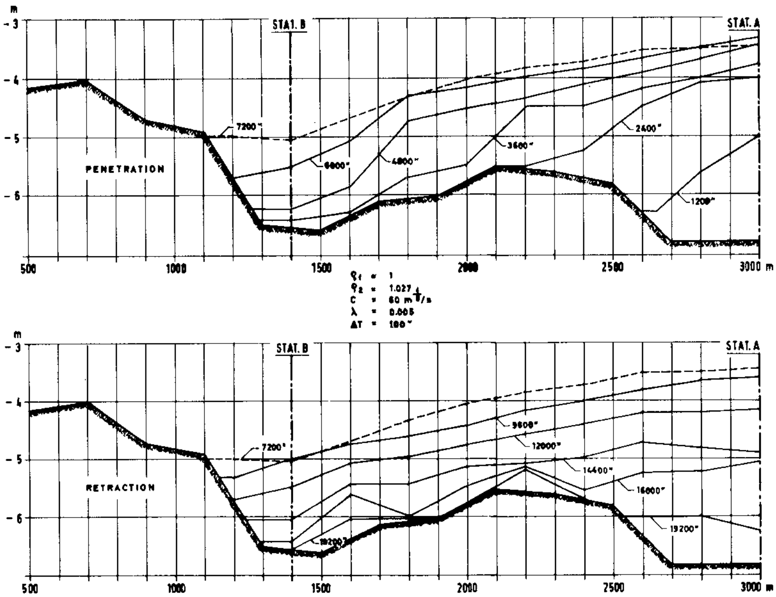


Fig. 8 - Simulation of the event of 22nd January 1974 - Computed longitudinal profile of the salinity intrusion

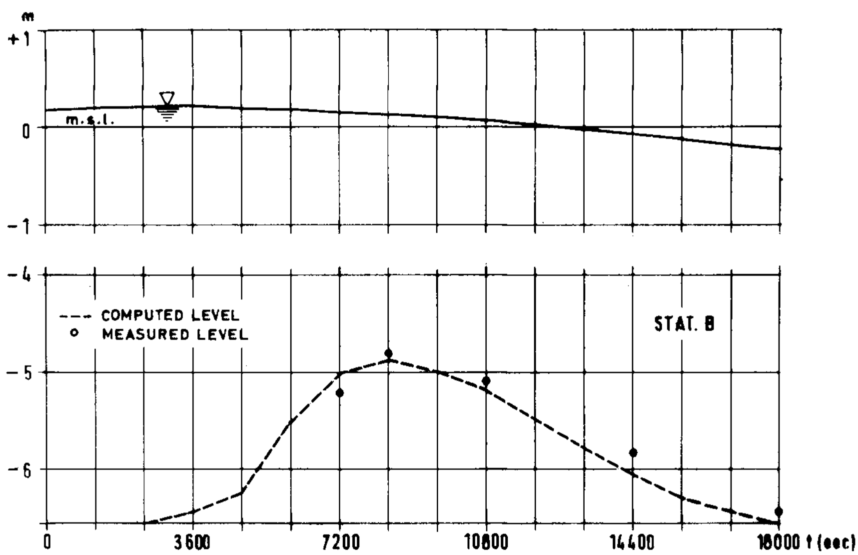


Fig. 9 - Simulation of the event of 22nd January 1974 - Computed and measured salt water level

bottom slope in correspondence with the extreme positions of the wedge head. In fact in the dynamic process of wedge penetration the friction between the layers plays a smaller role than the lower layer slope, which constitutes the main propulsive force for the wedge motion.

On the other hand, during penetration, both the upper and lower flow are oriented upwards, therefore diminishing the frictional effect which is in full action when the upper layer inverts its direction and blocks the salt water. For these reasons the interfacial stress coefficient does not need a sophisticated evaluation and therefore it has been considered a constant.

5) RESULTS AND CONCLUSIONS

On the basis of the previous considerations, the model has been applied to actual events for which sufficient information was available. The bottom friction coefficient (Chézy formula) was evaluated by a set of simulations in absence of salt water. Its value /1, 2/ is well defined (60 $m^{1/2}$/sec) and its individuation was immediate.

Thereafter the interfacial friction coefficient was individuated by attempts as the one which best fitted the measurements. Figs. 8 and 9 show the results of a simulation of an event during which the wedge only penetrated for about 3 km upstream from station A (fig. 2). The simulation well reproduces (fig. 9) the sudden raising and the milder lowering of the salt water level, which is one of the main characteristics of the wedge. These results clearly demonstrate the reliability of the model in its present structure, in reproducing the two-layer motion.

Two main problems still remain open, both connected with the physical properties of the two-layer motion. First the interfacial processes: as seen, we solved them considering λ as a constant and the interface as impermeable. Actually the interface, whose phenomenology determine the value of λ is not impermeable. In fact, as the relative velocity between the two layers increases the small perturbations of the interface grow in the form of billows which for higher velocity degenerate in eddies penetrating both layers /17/. in form of an interlayer. The time scale of the wedge phenomenology (6 hours) does not allow the interlayer to extend to the whole depth; therefore the two-layer schematization remains valid, even if it generates a more complex flow pattern. The second point is more important to enable the model to predict. As said before, the downstream passage to critical condition has to be introduced into the model. In doing so it must be noted that the critical condition activates not-yet completely known mixing processes between salt and fresh water. These processes can not be described by the two-layer approximation only. Then the mere introduction in a model of internal supercritical flows /18/ is physically insufficient to represent the actual estuarine hydrodynamics.

REFERENCES

/1/ - Dazzi R. and Tomasino M. (1973), "Modello matematico per lo studio di un estuario o delta", L'Energia Elettrica No 8, p. 477-486

/2/ - Tomasino M. and Dazzi R., "Il modello matematico del delta del Po", L'Energia Elettrica (In print)

/3/ - Vreugdenhil C.B. (1970), "Computation of gravity currents in estuaries", Delft Hydraulics Laboratory, publication No 86

/4/ - Schijf J.B. and Schönfeld J.C. (1953), "Theoretical consideration on the motion of salt and fresh water", Proc. Minnesota International Hydraulics Convention Joined Meeting I.A.H.R. and Hydraulics Division A.S.C.E., p. 321-333

/5/ - Boulot F., Braconnot P. and Marvaud Ph. (1967), "Détermination numérique des mouvements d'un coin salé", La Houille Blanche No 8, p. 871-877

/6/ - Poggi B. (1959), "Sul moto delle correnti stratificate", L'Energia Elettrica No 3, p. 197-208

/7/ - Leendertse J. (1967), "Aspects of a computational model for long period water wave propagation", The RAND Corporation, Santa Monica - California, memorandum RM 5294-PR

/8/ - Godunov S.K. and Ryabenki V.S. (1964), "The theory of difference schemes", North Holland Publishing Company - Amsterdam

/9/ - Stommel H. and Farmer H.G. (1952), "Abrupt change in width in two-layer open channel flow", Journal of Marine Research 11-2, p.205-214

/10/- Boulot F. and Daubert A. (1969), "Modèle mathématique de la remontée de la salinité sous une forme stratifiée en régime non permanent", XIII Congrès de l'A.I.R.H., Kioto

/11/- Landau L.D. and Lifshitz F.M. (1959), "Fluid mechanics ", Vol. 6 of Course of Theoretical Physics, Pergamon Press, Oxford

/12/- Abraham G. and Vreugdenhil C.B. (1971), "Discontinuities in stratified flows", Journal of Hydraulic Research No 3, o. 293-308

/13/- Abraham G. and Eysink W.D. (1971), "Magnitude of interfacial shear in exchange flow", Journal of Hydraulic Research No 2, p. 125-151

/14/- Pedersen B. (1972), "The friction factor for a two-layer stratified flow, immiscible and miscible fluids", Rep. 27 , p. 3-13, Tech. Univ. Denmark

/15/- Shi-Igai and Sawamoto M. (1969), "Experimental and theoretical modeling of saline wedge", I.A.H.R., Kioto p. 29-35

/16/- Vreugdenhil C.B. (1969), "Nunerical computation of fully stratified flow", I.A.H.R., Kioto

/17/- Keulegan G.H. (1966), "The mechanism of an arrested saline wedge", Estuary and Coastline Hydrodynamics, Mc Graw and Hill p. 546-574, Ippen editor

/18/- Grubert J.P. and Abbott M.B. (1972), "Numerical computation of stratified nearly horizontal flows ", Journal of the Hydraulics Division,ASCE, oct. 1972