CHAPTER 65

NUMERICAL MODELLING OF SUSPENDED SEDIMENT

by

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ABSTRACT

The present paper describes the application of a two-dimensional numerical suspended sediment model to problems having analytical solutions, as well as to laboratory and field situations.

The model is based upon an implicit finite-difference solution to a two-dimensional (longitudinal and vertical) diffusion-advection equation for suspended sediment transport. Horizontal eddy diffusion is neglected in comparison with vertical diffusion and vertical water motion is assumed negligible in comparison with the sediment fall velocity.

The various applications indicate that the greatest errors in the model are due to large spatial concentration gradients and that errors can be controlled by a suitable choice of space and time step. In addition, it is considered that the model has great flexibility and seems to have an acceptable level of accuracy, at least in the field situations tested, provided the physical parameters of the model can also be determined accurately.

INTRODUCTION

The production of larger and larger ships has resulted in the development of new ports and harbours as well as in the re-development of existing ones. The deep draughts of modern vessels requires the provision and maintenance of safe, deep, port approach channels. Dredging and/or training works may be necessary in some situations and can so interfere with the free movement of sediment on the sea bed as to produce a chain reaction of events culminating in the appearance of dangerous shoals in unwanted positions.

The consequence of engineering works can be studied with the help of in-situ field observations and/or small scale hydraulic model tests. However, the advent of the high speed computer has led, in recent years, to the development of mathematical models i.e. the analytical or numerical solution of equations which directly or indirectly describe the physical processes at work in a particular situation. The size and complexity of present-day engineering problems often requires that a digital or analogue computer is used to solve the equations.

The present paper describes a simple mathematical model which endeavours to describe the settling and dispersal of suspended sediment in two-dimensional flow situations. The authors are concerned, in particular with the accuracy and usefulness

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of the model in describing real engineering situations. Attention has been confined to quasi-steady flow situations and thus the effects of vertical flows and accelerations are not included in the model.

THE MODEL

The mathematical model is based upon an implicit finite-difference solution to the two-dimensional sediment diffusion-advection equation:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial C}{\partial y} + w_f C\right) - \frac{u \partial C}{\partial x} \qquad \ldots \ldots \ldots \ldots \quad (1)$$

where C is the sediment concentration.

x, y are horizontal and vertical co-ordinates respectively
w_f is the sediment fall velocity in still water

ε_y is a vertical sediment diffusion coefficient
u is the horizontal sediment velocity and is taken equal to the velocity of the surrounding fluid

Horizontal eddy diffusion is neglected in comparison with vertical diffusion, and vertical flows are assumed negligible in comparison with w_f.

The formulation of the model has been described elsewhere [1,2,3] and consequently only a brief outline is considered here. The three-dimensional space-time plane (x,y,t) is divided into blocks of size $\Delta x/2$, Δy, $\Delta t/2$ as indicated in Fig. 1. The differential terms in eq. 1 are then written in difference form either at the centre of each vertical block (e.g. $i\Delta x$, $(j+\frac{1}{2})\Delta y$, $(n+\frac{1}{2})\Delta t$) or at mesh points ($i\Delta x$, $j\Delta y$, $(n+\frac{1}{2})\Delta t$). For example, the following difference forms may be used at mesh points.

$$\frac{\partial C}{\partial t} = 2(C^{n+\frac{1}{2}}_{i,j} - C^{n}_{i,j})/\Delta t + O(\Delta t) \qquad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (2a)$$

$$w_f \frac{\partial C}{\partial y} = (C^{n+\frac{1}{2}}_{i,j+1} - C^{n+\frac{1}{2}}_{i,j-1})/2\Delta y + O(\Delta y^2) \qquad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (2b)$$

$$\varepsilon_y \frac{\partial^2 C}{\partial y^2} = \varepsilon^{n+\frac{1}{2}}_{i,j}(C^{n+\frac{1}{2}}_{i,j+1} - 2C^{n+\frac{1}{2}}_{i,j} + C^{n+\frac{1}{2}}_{i,j-1})/\Delta y^2 + O(\Delta y^2) \qquad \ldots \ldots \ldots \ldots \quad (2c)$$

$$u\frac{\partial C}{\partial x} = 2u^{n+\frac{1}{2}}_{i,j}(C^{n}_{i,j} - C^{n}_{i-\frac{1}{2},j})/\Delta x + O(\Delta x) \qquad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (2d)$$

where indicies $(n+\frac{1}{2})$, i, j indicate values of the parameters at time $(n+\frac{1}{2})\Delta t$, and position $i\Delta x$, $j\Delta y$. O( ) indicates that the difference equations contain small order terms which result from a Taylor Series expansion of C about the point being considered in the space-time plane. The small order terms are assumed to be negligibly small.

More accurate difference equations can be used by including some of the higher order terms. For example,

$$\frac{\partial C}{\partial y} = (C^{n+\frac{1}{2}}_{i,j-2} - 8C^{n+\frac{1}{2}}_{i,j-1} + 8C^{n+\frac{1}{2}}_{i,j+1} - C^{n+\frac{1}{2}}_{i,j+2})/12\Delta y + O(\Delta y^4) \qquad \ldots \ldots \ldots \ldots \quad (3a)$$

$$\frac{\partial^2 C}{\partial y^2} = (-C^{n+\frac{1}{2}}_{i,j+2} + 16C^{n+\frac{1}{2}}_{i,j+1} - 30C^{n+\frac{1}{2}}_{i,j} + 16C^{n+\frac{1}{2}}_{i,j-1} - C^{n+\frac{1}{2}}_{i,j-2})/12\Delta y^2 + O(\Delta y^4) \qquad \ldots \ldots \quad (3b)$$

These are particularly useful for the vertical direction where large concentration gradients may exist.

Repeated application of eq. 1 and 2 or 3 to points between the sea bed and surface produces a set of linear simultaneous equations relating the concentration at time $(n + \frac{1}{2})\Delta t$ to those occuring at time $n\Delta t$. The use of known boundary conditions at the sea bed and surface and at upstream and downstream boundaries enables any unknown concentrations outside the boundaries to be eliminated. Once initial $(n\Delta t)$ concentrations are specified the simultaneous equations can be solved by matrix methods in order to find the unknown concentrations over the vertical at position $i\Delta x$ and time $(n + \frac{1}{2})\Delta t$. This solution process may be repeated at $(i + \frac{1}{2})\Delta x$, $(i + 1)\Delta x$ etc. until the downstream boundary is reached. The time step is then advanced $(\Delta t/2)$ and the process repeated.

The replacement of differential terms by difference equations is an approximation and introduces errors into the model. Other errors are introduced by computer round-off and in the present scheme by writing the convective transport term $(u\partial C/\partial x)$ at time level $(n\Delta t)$ instead of at $(n + \frac{1}{2})\Delta t$. The magnitude of the total error present in the model can be established only if analytical solutions to a problem are known. Unfortunately this is impossible and the best that can be done is to test the model against simplified analytical solutions or the results of controlled laboratory tests. This is the approach adopted by the present authors. The model is then applied to existing field data on a variety of problems in order to examine the usefulness of the model in real situations.

NUMERICAL TESTS

One-dimensional situations

Initially the one-dimensional form of eq. 1 was used for numerical tests. This eliminates errors due to the convective transport term and enables comparisons to be made with known analytical solutions (Dobbins[4], O'Connor[15]). The model was operated with various space and time steps and for various values of the model parameters (Zein[2]). A particular sediment problem was considered in which sediment was assumed to be suddenly eroded from the sediment bed at a constant rate. The flow depth and vertical diffusion coefficient were kept constant w.r.t. space and time and the initial concentration was taken equal to zero. In addition, various difference schemes were examined (e.g. Crank Nicolson, Stone and Brian, Fully Implicit). The results may be summarised as follows:

(1) The computational scheme was stable and there were no exponential growths of errors. This is to be expected for implicit schemes. It is suggested that the best results are obtained for the particular problem studied above if $\alpha = (w_f \Delta y / \varepsilon_y) << 1$

(2) Losses or grains of mass were acceptable for reasonable space and time steps.

(3) Numerical diffusion (dissipation error) was present in one-sided schemes (e.g. Fully Implicit) but the effect was probably negligihle for reasonable space and time steps. This generally means dividing the depth into perhaps forty intervals, with the model time step determined from the equation

$$\Delta t = \Delta y/(2 \gamma w_f) \qquad \ldots \ldots \ldots \ldots \quad (4)$$

with $\gamma \geqslant 1$.

(4) The largest errors in the model were dispersion errors and arose from the nature of the bed boundary condition. This required the model to distribute a block of sediment over the flow depth at each time interval and consequently errors are propagated from the area of large concentration gradient at the bed boundary. Errors from this source could be reduced by the use of appropriate space and time intervals and are negligible as the numerical solution approaches the one-dimensional steady state solution i.e. the analytical solution to eq. 1 with ($\partial C/\partial t = u\partial C/\partial x = 0$).

(5) Difficulty is experienced in modelling the near-bed zone when $\varepsilon_y \rightarrow 0$ since $\partial C/\partial y \rightarrow \infty$. The problem can be overcome by introducing a "false bed" boundary condition or reducing the near-bed zone. (O'Connor[1], Cardona[3]).

Two-dimensional situations

The effect of errors from the longitudinal transport term are less easy to check since two-dimensional analytical solutions only exist for very simple problems. However, a series of model tests was carried out for a particular sediment problem for which an analytical solution existed (Mei[5]). The problem was one of describing the steady state (eq. 1 with $\partial C/\partial t = 0$) sediment distribution for the case of steady uniform flow down a two-dimensional channel in which one section of the bed was non-erodable (Fig. 2). The flow depth, velocity of flow and vertical diffusion coefficient were all kept constant over space and time. The sediment concentration at the bed boundary was maintained at a constant value of unity at all times. The computer solution was operated from an initial concentration of zero until a steady state solution was reached. The results for various flow and sediment parameters were considered to be good (see for example Fig. 3). Maximum errors were confined to surface levels and are associated with small absolute concentration values as well as areas with steep concentration gradients. For reasonable space and time steps errors were generally less than 10% and improved accuracy could always be obtained by a suitable reduction in space and/or time step.

A further series of numerical tests was performed with the computer model in order to check on longitudinal sediment dispersal. If eq. 1 is averaged over the flow depth, a one-dimensional diffusion-advection equation can be produced which incorporates an effective longitudinal dispersion coefficient. The magnitude of the dispersion coefficient can be calculated from a knowledge of the sediment and flow parameters (Elder[6], Sümmer[7]). It may also be calculated by operating the two-dimensional model for long enough to achieve an effective one-dimensional situation. The rate of change of the standard deviation (σ_x) of the depth averaged longitudinal distribution curves is then proportional to the effective longitudinal dispersion coefficient ($D_x = \partial \sigma_x^2/2\partial t$).

Model results are shown in Fig. 4 for a particular sediment suspension exponent ($Z = w_f/(\beta K u_*)$; β is the ratio of the vertical sediment diffusion coefficient to the vertical momentum transfer coefficient; K is Von Karman's Constant; u_* is the shear velocity equal to $\sqrt{\tau_b/\rho}$; τ_b is the applied bed shear stress; ρ is the fluid density) and for two particular velocity profiles with flow depth. The dispersion of sediment is seen to be greater than that for fluid particles alone and agrees reasonably well with theory. It should be noted, however, that the theories assumed a totally reflecting bed boundary condition while the computer solutions were operated for an absorbing bed boundary condition (Zein[2]).

The numerical tests indicated above suggest that the numerical accuracy of the model is reasonable at large diffusion times for the particular problems studied. They provide no means of testing the usefulness of the model at small diffusion times. Consequently, laboratory tests were used for this purpose.

LABORATORY TESTS

Because the computer model is applicable to two-dimensional flows, it was decided to build a special flume with flexible walls which could be moved at different flow speeds. The reduction of wall shear stress and the partial suppression of secondary motions means that a narrow flume behaves effectively as a much wider one. It was also constructed much more cheaply than a wide glass or perspex channel. The outer part of the flume was constructed of marine plywood so that an externally braced channel some 50 ft. long and 21" wide was formed. The moving walls were made of butyl rubber and were supported by a system of vertical rollers spaced at intervals of 4ft. along the flume. A 5/16" diameter rubber cord was fixed to the top of the belt to prevent it riding down the rollers. Both belts were driven simultaneously through a bevel gear and belt drive system from a variable speed electric motor located at the downstream end of the flume. A false floor was also fitted between the rollers, and roughness elements, consisting of 3/16" high, $\frac{1}{2}$" wide wooden batterns, were fixed to it at 2" centres. Thin vertical brass strips (3/8" wide) were also fixed to the sides of the false floor at 4ft. centres to prevent lateral deflection of the walls towards the centre of the flume. Water was supplied from a constant head tank in the laboratory roof and flowed into the channel through a vaned entry section. The water level was controlled by an overflow weir fitted inside an existing rectangular flume; the water being guided towards the weir by fixed vertical walls (Fig. 5).

Considerable experimental work was undertaken so as to improve entry conditions to the flume as well as to produce the best configuration of bed roughness and speed of movement of the walls. Only the downstream half of the flume was used for the sediment tests and lateral flow uniformity was achieved over some 70% of the flume width with the walls moving at some 70% of the mean flow speed. Longitudinal dispersion due to lateral inhomogeneity was further reduced by injecting sediment over the full width of the flume.

Slug injection tests were performed in the flume with polystyrene particles (s.g. 1.04) with a size range between 250-300 μ and with a measured fall velocity of 7.45×10^{-3} ft./sec. Sediment concentrations were measured by a battery of syphons with an estimated accuracy of 5-10 ppm. Water velocity profiles were measured by a miniature (1 cm diameter) current meter and water surface slopes were measured with differential manometer. Sediment concentrations were measured at three elevations and at 20, 26 and 32 seconds after sediment injection for various positions down-stream of the injection point. The experimental results indicated that maximum concentrations changed by 100% between 20 and 32 seconds. Sampling times were, of course, limited by the length of the flume.

The computer model was then operated with measured values for w_f, u and h, and with the measured sediment distribution at 20 seconds as initial values. The vertical diffusion coefficient was kept constant over space and time and was determined from the equation

$$\varepsilon_y = (w_f h)/(6Z) \qquad \ldots \ldots \ldots \ldots \ldots \ldots \quad (5)$$

while Z was determined using $\beta = 1$ and $K = 0.4$. Two values of u_* were used (viz. Table 1) based on estimates of u_* made by considering the gradient of the velocity-depth curve as well as the longitudinal slope of the water surface.

The boundary conditions used in the model were

(1) At the bed : $y = 0 : t \geqslant 0$

$$\varepsilon_y \frac{\partial C}{\partial y} = 0 \qquad \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots \quad (6)$$

(2) At the surface : $y = h : t \geqslant 0$

$$\varepsilon_y \frac{\partial C}{\partial y} = - w_f C \qquad \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots \quad (7)$$

(3) Upstream boundary : $x = 0 : t \geqslant 0 : 0 \leqslant y \leqslant h$

$$C = 0 \qquad \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots \quad (8)$$

(4) Downstream boundary.

Determined by upstream concentrations, initially set to zero.

The results for a typical set of observations are shown in Fig. 6, while the test parameters are shown in Table 1. Agreement between observations and model is considered to be good. Numerical errors, based on the one-dimensional work are considered to be negligible and may be less than ½%. Convective transport errors were minimised by using very small Δx steps. Uncertainties in the determination of w_f and u_* were tested by operating the model with these quantities changed by 20%. The effect was found to be small (Fig. 6). The largest source of error is, in fact, considered to be due to the sampling method. The syphon samplers could not determine simultaneous concentrations within the expanding sediment cloud.

TABLE 1. Model parameters used for laboratory tests

w_f ft/sec.x10^3		h ft.x10	u_{mean} ft/sec.x 10	u_{wall} ft/sec.x10	u_* ft/sec.x10^2		$_y$ ft^2/sec.x10^3		Δy ft.x10^2	Δx ft.10^2	Δt secs.
7.45	8.94	9.1	3.71	2.69	4.0	4.8	2.4	4	9.1	3.35	6.1
w_f (measured) : 7.45 x 10^{-3} ft/sec. ; u_* (slopes) : 3–5 x 10^{-2} ft/sec. ; γ =1 ;											

The technique adopted was to sample three sections in the leading part of the concentration cloud at a fixed time and then to repeat the test for the centre and trailing part of the cloud keeping one sampling station as a common control. The concentration at a point is thus the average of at least five separate tests. The scatter on experimental results is thus large since nearly instantaneous concentrations are being recorded : the computer produces the turbulent average concentration.

It should also be noted that good results were obtained in the present test with a spatially constant value of ε_y. This is due to the nature of the present problem which eliminates large concentration gradients at the lower boundary (eq.6). If entrainment was allowed from the bed boundary, ε_y would probably have to vary with elevation above the bed.

FIELD RESULTS

Comprehensive field observations are extremely scarse and those available are generally incomplete in one or more details. Unfortunately the authors were not able, due to limitations of finance, to perform field tests themselves. Three existing sets of results have, therefore, been used.

1. Sewage sludge dumping in the Irish Sea (U.K.)(Ref. 8)

The model was applied to results collected as part of a sewage sludge disposal study in the Irish Sea (Fig. 7). The vertical distribution of sludge was measured with photo-electric monitoring probes, at 6, 43 and 53 minutes after the release of a ship load of sludge containing 10.9% dry solids. The vessel's discharge speed was about 6 kts. and the initial length of sludge patch was about 1 mile. The width of the patch was observed to increase over the monitoring period from some 66 - 164 to 246 ft. The weather on the test day (5th May 1970) was good with a calm sea and light north-westerly winds (force 1-2). The water was some 100 ft. deep and the release was about mid-ebb tide with surface velocities of 3 ft/sec. Vertical stratification was small with surface and bed salinities differing by some 0.2% . Water velocity and salinity/temperature profiles were measured over the flow depth from an anchored vessel during the course of the experiment. The test area is subject to semi-diurnal tides with a range on the test day of some 19 ft.

The model was operated with observed average values of flow depth and water flow. It was assumed that the sludge settled with an effective fall velocity equal to the median value which was obtained from laboratory settling tests at the appropriate field concentration. If the particles did not flocculate, and were unaffected by turbulence etc., the model could be applied to a series of grain sizes (or fall velocities). The vertical diffusion coefficient was estimated from eq. 5 with $\beta = 1.0$, $K = 0.40$ and u_* estimated from the water velocity profile. The same boundary conditions employed in the laboratory tests were used (eq. 6 - 8) while the concentration distribution at 6 minutes was used as initial values. A good estimate of initial values can also be obtained from the initial dimensions of the sludge patch and the solid's content of the sludge cargo.

Initial calculations with the model soon demonstrated that lateral spreading was important. Model values computed without lateral spreading were in error by some 100%. The model was then modified so that at the end of each time step the sediment concentrations were spread over the estimated plume width which was also a function of depth. The results are shown in Figs. 8a and 8b and the model parameters used are shown in Table 2. Agreement between model and field results is considered good. Numerical errors are difficult to estimate but the one-dimensional work suggests they will be less than 5%. Convective transport errors are negligible since the results are for the maximum concentration area in the centre of the sludge patch. This was confirmed by reducing Δx by a factor of two. Again, the largest difference between model and observation could well be attributed to observational inaccuracy since the photo-electric probes have a reading accuracy of some 10-20 ppm.

TABLE 2. Model parameters for the Irish Sea Tests

w_f ft/sec. x 10^3	h ft.	u_* ft/sec. x 10^2	ε_y ft^2/sec. x 10	Δy ft.	Δx ft.	Δt mins.
1.39	100	8.1	5.4	1	360	6

2. Sediment spoil disposal on the Potomac Estuary (USA)(Van Der Kreeke[9])

The results from a field and mathematical model investigation into the longitudinal and vertical distribution of sediment in a dredging plant outfall plume was used to test the model. The discharged sediment had a mean size of 45 and was discharged at a rate of approximately 10 lb/sec. through a pipe-line on to a shallow plateau in the homogeneous tidal zone (Fig. 9). The discharge point was located near the estuary bed and was some 10 ft. below the water surface. Tides in the Potomac are semi-diurnal with a mean range of 2.1 ft. Field observations indicated that the maximum length of outfall plume was 3 miles and that its width remained constant except possibly at times of slack water.

The computer model was used to determine the vertical and longitudinal sediment distribution throughout a flood or ebb tide. Many of the parameters used in the model were those adopted by Van Der Kreeke who proposed a simple one-dimensional analytical model for the sediment distribution in the plume. He considered the change in vertical sediment concentration within a column of water equal to the flow depth as it drifted downstream with the depth-mean tidal velocity. Sediment entrainment and particle settling were allowed at the bottom of the block. This simple model thus neglects the longitudinal spreading action due to velocity gradients over the flow depth.

The model was operated initially with a constant spatial and temporal diffusion coefficient and with a constant water depth (10 ft.). A further test was conducted with a depth constant diffusion coefficient which varied sinusoidally with time according to the equation.

$$\varepsilon_y = \varepsilon_{y\ min} + (\varepsilon_{y\ max} - \varepsilon_{y\ min}) \sin wt$$

where $w = 2\pi/T$ and T is the tidal period.

The velocity field was synthesized by applying a sinusoidal time variation, similar to eq. 9, to the measured depth distribution of velocity at maximum flow rates.

The model boundary conditions were:

(1) At the surface : - eq. 7 was used.
(2) At the bed :

$$\varepsilon_y \frac{\partial C}{\partial y} = - w_f C^{\infty} \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \qquad (10)$$

where C^{∞} is the steady-state one-dimensional bed concentration. The value used for C^{∞} was estimated by an examination of field sediment profiles remote from the outfall plume.

(3) Upstream boundary : eq. 8 was used.
(4) Downstream boundary : one-dimensional conditions were used.
(5) Injection point : This was located five mesh intervals downstream of $x = 0$ and the concentration was held constant over the depth at 1250 ppm (Van Der Kreeke[9]). However, any space or time varying quantity could have been used).

The initial ($t = 0$) concentration was taken as zero.

The longitudinal mesh spacing (Δx) used in the present model was varied to allow for the very small velocities occuring at the start of the tide. Interpolation errors arising from the convective term were thus kept to a minimum. The mesh spacing was increased five fold once the bed velocities exceeded 0.1 ft/sec.

The model results are shown in Figs. 10 and 11, while the model test parameters are given in Table 3. Comparison with field results and with Van Der Kreeke's model is also shown.

TABLE 3. Model parameters for Potomac Tests

w_f ft/sec.x 10^3	h ft.	u_* ft/sec. x 10^2		ε_y ft/sec. x 10^2		C ppm		Δy ft.	Δx ft.	Δt min.
8.0	10	7.5	15* / 7.5	5	10* / 5	272	272* / 0	1	1.25 / 6.25	1.04

Half tidal period : 6 hours : $\beta = 1.0$: K = 0.40 : $\gamma = 1$:
* indicates max./min. values for sinusoidal variations

The computer results show how the sediment is stretched out along the estuary by the tidal currents as well as the dominating effect of the sediment fall velocity once maximum velocities have been exceeded (after + 3 hours). The effect of allowing a sinusoidal time variation in sediment entrainment (C^{∞}) and vertical diffusion coefficient is also seen. Much smaller concentrations are achieved towards slack-water since C^{∞} is zero at slack water (+ 6 hours). Comparison of Van Der Kreeke's results with the slack water results shows that the neglect of longitudinal dispersion produces higher concentrations in the middle reaches of the sediment field (100 - 1000 ft.) . Both models will show the same final values at the outfall and at distant downstream points. Figs. 10 and 11 show some differences at downstream points since a slightly different C^{∞} value was used in the two models (c.f. 197 and 272 ppm.).

The numerical model has great flexibility and problem parameters can be changed as desired with each space and time interval. For example, sediment settling on to the bed can be re-entrained at will. The simpler analytical models cannot do this and changing problem parameters is a lengthy process. However, the Potomac tests do demonstrate one difficulty of modelling shallow water systems. It is desirable, on accuracy grounds, to use a reasonable number of vertical intervals. This then means that the model time step will be small unless the sediment has an extremely small fall velocity (eq. 4). Small time steps imply, in turn, that small horizontal space steps (Δx) will be required, particularly if flow velocities are relatively small. Consequently a large amount of computer store and run-time may be required to model shallow systems with a high degree of accuracy.

The numerical accuracy of the Potomac results is difficult to establish but seems, from Figs. 10 and 11, to be adequate for engineering purposes since the two-dimensional results are at least as good as the simpler one-dimensional model which was considered adequate by Van Der Kreeke.

3. Sediment intrusion into a tidal lock - Mersey Estuary (U.K.)(O'Dell[10])

Gladstone Lock is situated at the entrance to the Mersey Estuary (U.K.) and connects part of the impounded dock system of Liverpool with the tidal Mersey Estuary (Fig. 7). The impounded water level in the dock system falls slowly over a period of time due to locking operations, gate leakage etc. The level is restored by allowing spring tides to flow freely into the lock and docks once the flood tidal level exceeds the dock water level. The difference in density of the dock and estuary water generates a combined tidal and density exchange flow (Halliwell and O'Dell[11]). The levelling process thus allows sediment to enter the dock system.

Various sediment measurements had been made in the lock as part of a general siltation study (Halliwell and O'Dell[11]). In particular, the variation of sediment concentration with time was measured continuously over a five year period at a fixed position near the estuary bed at the lock entrance. Also, vertical profiles of sediment concentration and horizontal water velocity were available throughout a level period at a station in the middle of the lock and inside the dock itself (Fig. 7). The model was, therefore, used to predict the longitudinal and vertical distribution of sediment within the lock and, in particular, illustrate the variation in vertical sediment profiles with time at the central position during a levelling period of some 110 minutes.

The model parameters are particularly difficult to define in this complex non-homogeneous flow situation. The sediment is a mixture of flocculated silt/clay and organic matter with a dispersed mean size between 5 - 10μ. Its effective size during the levelling process is, however, much higher due to the effects of flocculation, turbulence and differential particle settlements. An effective particle fall velocity was estimated by fitting analytical one-dimensional steady-state sediment profiles to other field observations in the tidal part of the estuary. In addition, field observations of similar sediment settling in the Thames Estuary (U.K.)(Owen[12]) were also used to help in deciding a value for w_f (Zein[2]).

The magnitude of the vertical diffusion coefficient was originally (Zein[2]) determined from eq. 5 but some difficulty was experienced in obtaining a good model fit, particularly in the lower half of the flow profile. Further tests were then made with ε_y determined by the equation:

$$\varepsilon_y = \frac{w_f}{Z} y(1 - y/h) \qquad \dots \dots \dots \dots \dots \dots \quad (11)$$

The model then gave good results (Cardona[3]) over the measured part of the flow depth (90%) but predicted near-bed concentrations were considered to be unreasonably high ($>$ 400,000 ppm). Finally, eq. 11 was used over the major part of the flow depth with constant values of ε_y for four vertical mesh spacing near the bed and surface.

The suspension exponent (Z) was modified to include the reduction of vertical mixing by vertical density gradients i.e.

$$Z = w_f/(\beta' K u_*) \qquad \dots \dots \dots \dots \dots \dots \quad (12a)$$

$$\beta' = (\beta K')/K \qquad \dots \dots \dots \dots \dots \dots \quad (12b)$$

where K' is a reduced Von Karman constant and is probably a function of flow Richardson number (Ri). For example, K' might be given by the equation

$$K' = K(1 + a\,\mathrm{Ri})^b$$

where the constants a, b depend on whether momentum ($a = 10$; $b = \frac{1}{2}$) or salt ($a = 3.33$; $b = -3/2$) transfer is considered (Bowden and Gilligan[13]). The coefficients appropriate to sediment transfer in non-homogeneous flow may well be equal to those for salt transfer.

The value of β' used in the present tests was 0.61 and is the value found from calculations on vertical sediment profiles in the tidal estuary almost opposite to Gladstone Lock (O'Connor[14]). Also, this value of β' corresponds to a Ri value of 0.114 which is the right order of magnitude for the vertical density stratification in the Mersey Narrows/Gladstone Lock area.

The water depth changed by some 10% over the levelling period and consequently the average value was used in the model. The measured flow field at the central observation point was used for all longitudinal positions. Continuity principles suggest that errors due to both approximations are probably less than 10%.

u_* is an exceedingly difficult parameter to estimate in this complex flow situation. Estimates of a mean value have been made by matching analytical velocity profiles with field observations (O'Connor[15]). A temporal variation in u_* was obtained initially by considering the temporal change in near bed velocity i.e.

$$u_* = K_1 u_\eta \qquad \dots \dots \dots \dots \dots \quad (14)$$

where K_1 is a constant with a possible value of 0.5 - 0.6 and u_η is the velocity at level $\eta = y/h$ below the surface: a value of η appropriate to 1m above the bed is usually used.

The model boundary conditions were:

(1) At the surface : equation 7 was used.

(2) At the bed :

$$\varepsilon_y \frac{\partial C}{\partial y} = - M\left[\left(\frac{u_*}{u_{*c}}\right)^2 - 1\right] - w_f \delta_o C$$

where $0 \leqslant \delta_o \leqslant 1$ and is dependent upon the value of u_* compared with u_{*c}. For example, $\delta_o = 1$ if $u_* \geqslant u_{*c}$ and $\delta_o = 0$ if $u_* < u_{*c}$. M is an erosion constant with a possible value of 1.7 - 2.0 gm/m^2/sec based on laboratory tests for silt/clay sediment (Cormault[16]).

(3) Lock entrance (x = 0) :

Concentration specified from field observations with an assumed depth variation based on various measurements in the area over a period of years.

(4) Lock exit (x = 1000 ft.):

The dock concentration was maintained at the same value as the final channel exit section.

Initial sediment values were obtained by interpolation from the measured field observations at the lock entrance, mid-point and exit.

The model was operated with fixed values of w_f and h and with various values of u_*, u_{*c} and M until a reasonable fit was obtained with the field observations (Fig.13). The values of the final model parameters are shown in Table 4, while the time variation of u_* is shown in Fig. 14. The effect of suppressing sediment re-entrainment (δ_o now equals zero) when $t \geqslant 90$ mins is also shown in Fig. 13.

TABLE 4. Model parameters for Gladstone Lock Tests

w_f ft/sec. x 10^3	h ft.	u_* max ft/sec. x 10^2	u_{*c} ft/sec. x 10^2	M gm/m^2/sec.	Δy ft. x 10	Δx ft.	Δt mins.
3.67	43	3.62	1.75	1.875	7.2	9.8	1.63
Level period : 110 mins ; $\beta = 1$; $\beta' = 0.61$; K' = 0.274 ; $\gamma = 1$;							

Agreement between field and model results is considered reasonable. An improved fit could be obtained by allowing M to vary spatially, w_f to vary with time and by making small adjustments to the magnitude and temporal distribution of u_*. However, the field programme was not extensive enough to allow such refinements to be made. It is also interesting to note that the values of the final parameters are those to be expected from the physical situation. Even u_*, which at first sight seems low, is probably correct since a note with the field observations indicated that a layer of fluid mud existed on the lock bed during the test period.

Various response tests were also tried with the model and the most sensitive parameters were found to be u_* or u_{*c}. A 20% change in u_{*c} produced a 14% change in the maximum concentration value at 90% depth. The response for M was about a half of that for u_{*c}. By contrast, numerical errors, based on the one-dimensional tests, were thought to be less than 2% at the maximum concentration point. Consequently, it is considered that larger errors can arise by an incorrect choice of physical parameter than from schematization errors.

CONCLUSIONS

The simple computer model is applicable to both laboratory and field situations and seems to produce acceptable results provided the physical parameters can be defined to a sufficient level of accuracy.

Numerical errors within the model are produced mainly from large spatial gradients but the errors can be controlled by a suitable choice of space and time interval.

The model is considered to be extremely flexible since model parameters can be readily varied over space and time. However, it is less useful in shallow water situations involving small flow velocities due to possible computer storage and run-time limitations.

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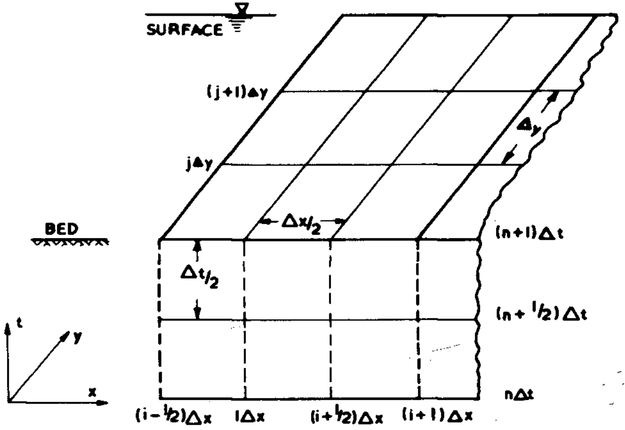


FIG. I. SPACE-TIME PLANE DISCRETIZATION

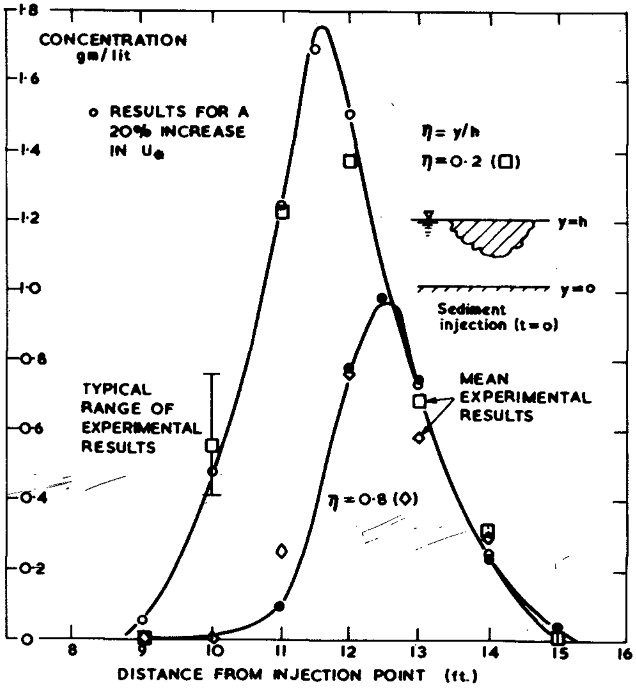


FIG. 6 CONCENTRATION DISTRIBUTION AFTER 32 SECONDS FROM INJECTION

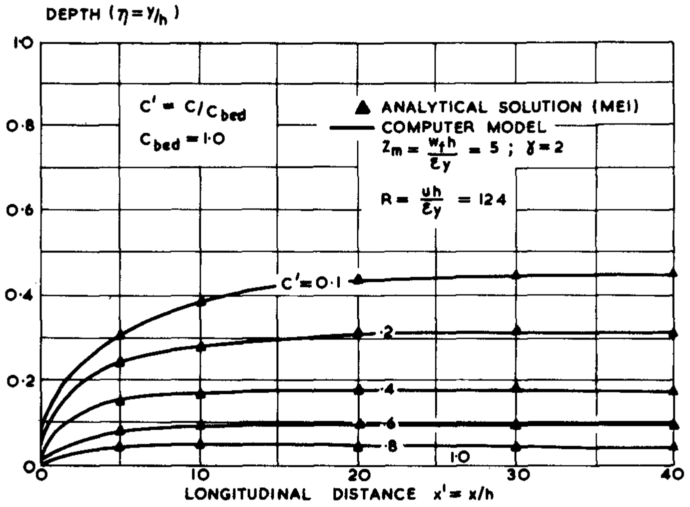


FIG. 3. DIMENSIONLESS STEADY-STATE SEDIMENT CONCENTRATIONS

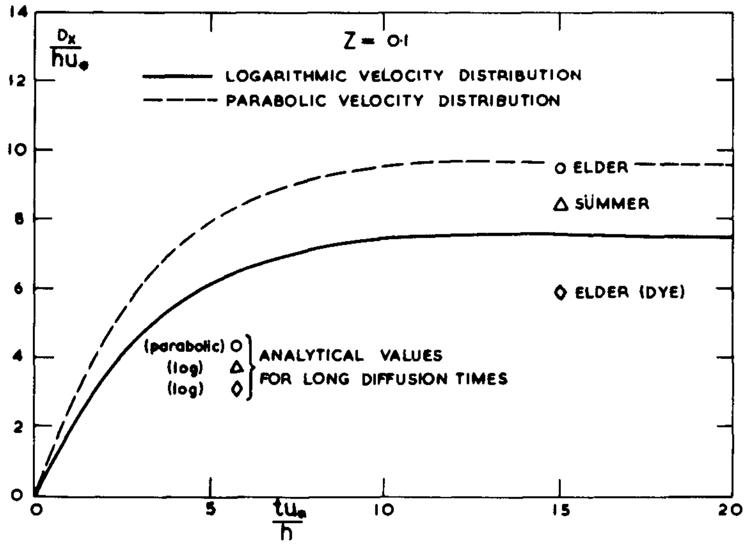


FIG. 4. COMPARISON OF ANALYTICAL AND NUMERICAL LONGITUDINAL DISPERSION COEFFICIENTS

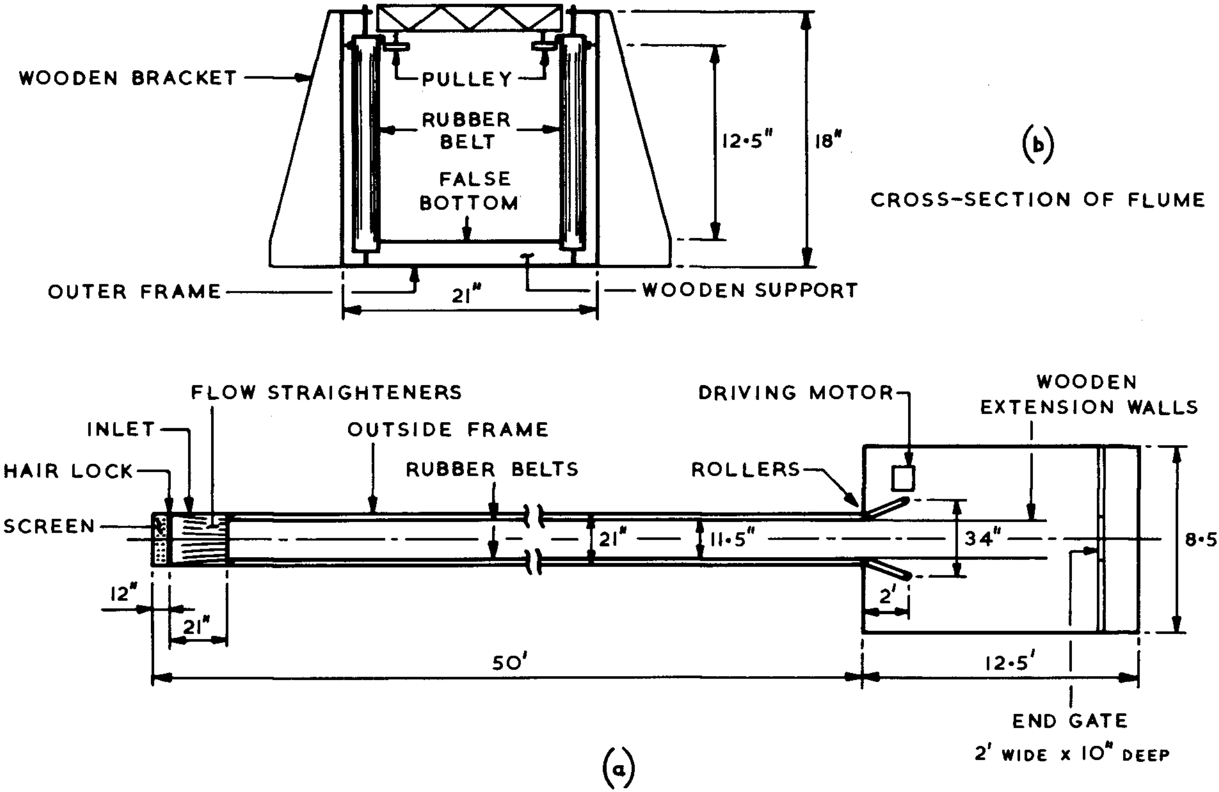


FIG.5 SCHEMATIC SKETCH OF THE APPARATUS

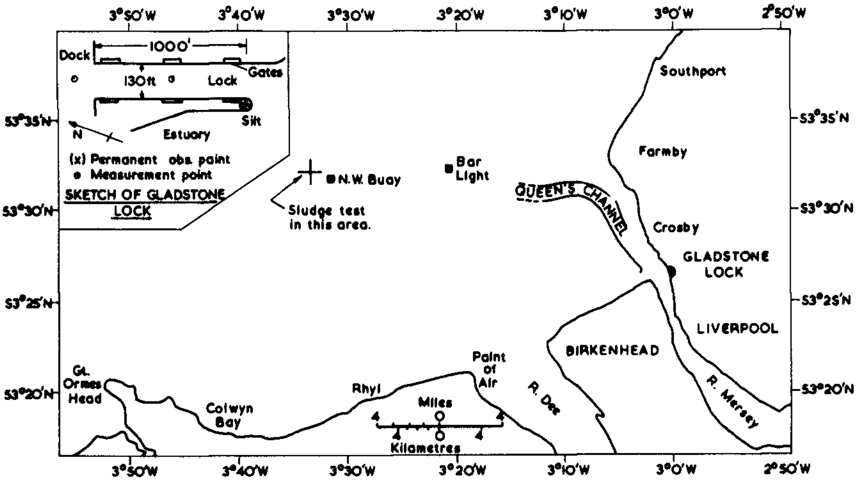


FIG. 7. IRISH SEA AND MERSEY ESTUARY LOCATION PLAN.

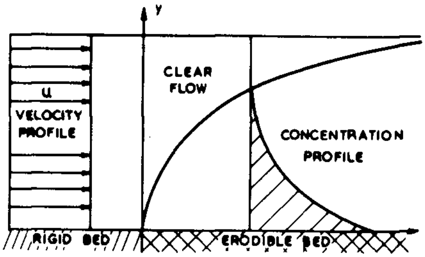


FIG. 2. TWO-DIMENSIONAL STEADY FLOW CASE

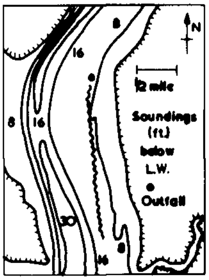
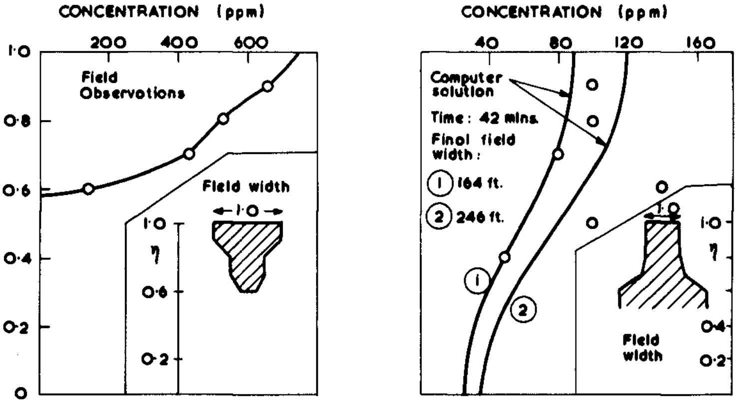


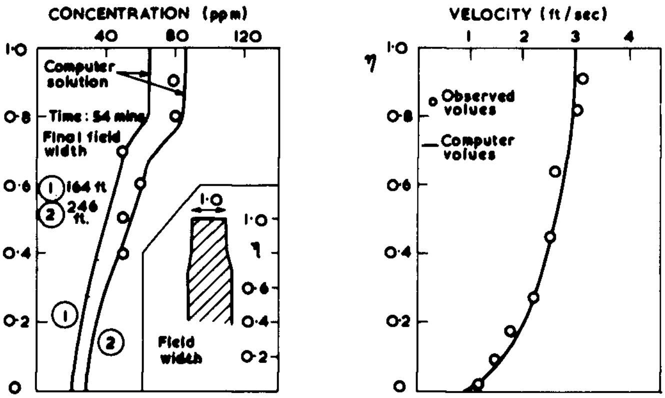
FIG. 5. POTOMAC ESTUARY SEDIMENT DISCHARGE POINT.



TIME AFTER INJECTION: 6 MINS.

TIME AFTER INJECTION: 43 MINS

FIG. 8a. COMPARISON OF MODEL AND FIELD RESULTS FOR THE IRISH SEA.



TIME AFTER INJECTION: 53 mins.

VELOCITY PROFILE

FIG. 8b. MODEL AND FIELD RESULTS FOR THE IRISH SEA TESTS.

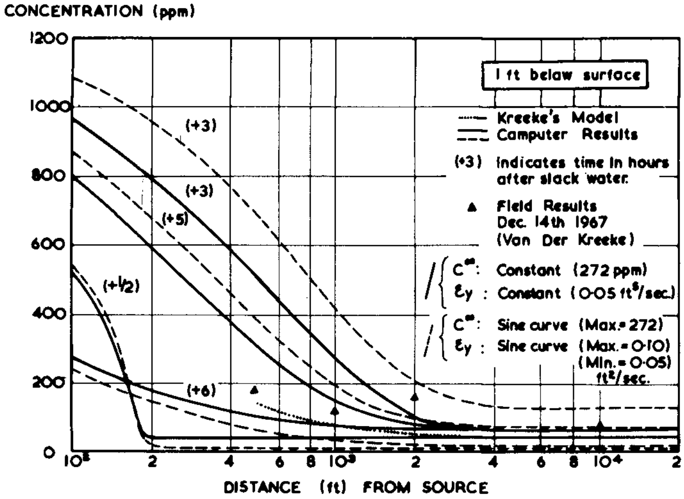


FIG. 10. COMPARISON OF FIELD AND MODEL RESULTS FOR POTOMAC ESTUARY : 1 ft. BELOW SURFACE.

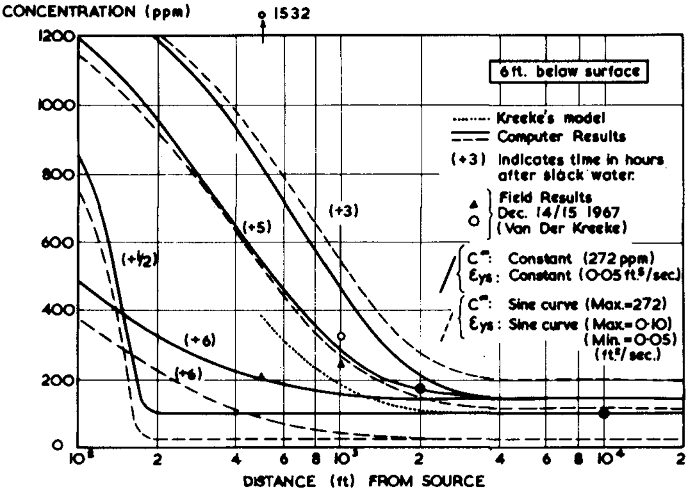


FIG. 11 COMPARISON OF FIELD AND MODEL RESULTS FOR POTOMAC ESTUARY : 6 ft BELOW SURFACE.

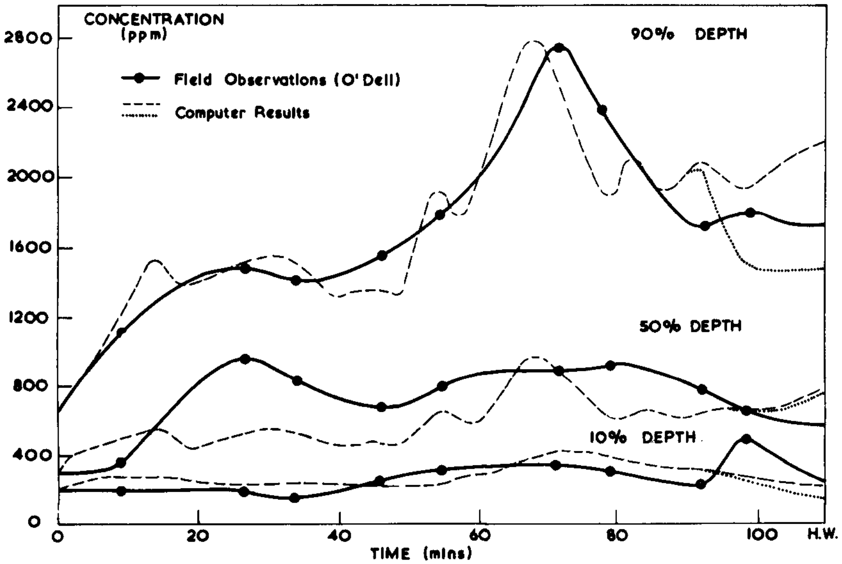


FIG. 12. COMPARISON OF FIELD AND MODEL RESULTS – GLADSTONE LOCK.

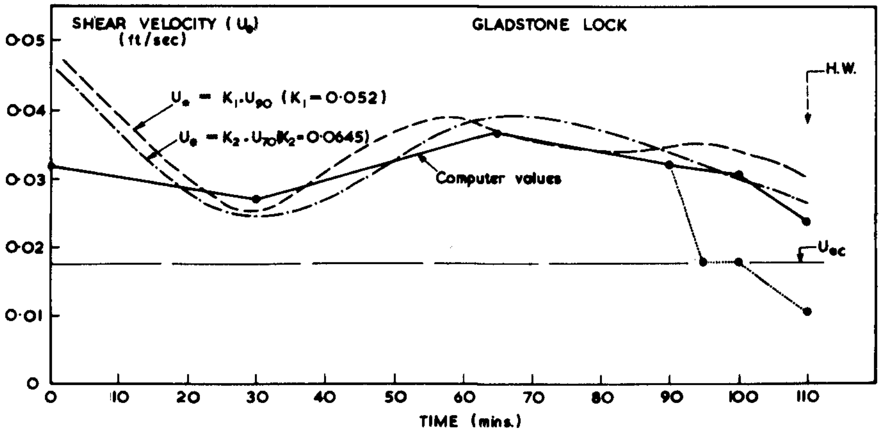


FIG. 13. COMPARISON OF U_* VALUES USED IN THE COMPUTER MODEL WITH EQUATION 14.