CHAPTER 18

A PROGRAMMABLE IRREGULAR WAVE GENERATOR

by

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ABSTRACT

An account is given of the various design considerations in the development of a low-cost programmable wave generator for a laboratory channel. The ability of this equipment to reproduce prototype spectra is critically discussed.

1. INTRODUCTION

With advances in wave recording equipment, reliable data are now becoming available on the spectra of waves likely to be encountered during storm conditions at particular sites where there are coastal engineering problems. There is therefore a need to reproduce these wave mechanically in the laboratory by methods that are sufficiently versatile to cover a wide range of conditions whilst being reasonably economical in cost.

The paper describes the development and testing of such a generator. All of the equipment, with the exception of those standard components which were obtainable commercially, was designed, fabricated, and assembled in the laboratory.

2. THE WAVE CHANNEL

The channel is of rectangular section 14m long, 0.45m wide and 0.45m deep, and has glass panels on both sides throughout its length. It is provided with an end beach of slope 1:6, consisting of non-ferrous shavings and a mattress-like material (hairlok), which is almost fully absorbent.

3. MECHANICAL SYSTEM

3.1 Choice of system. A servo-controlled electro-hydraulic system was the obvious and conventional choice for the type of programmed operation envisaged. Such a system, shown in diagrammatic form in Fig. 1 is capable of providing fast response and close control whilst satisfying the requirements of reliability and reasonable economy.

The principal components are the wave paddle, actuator (or ram), servo-valve, positional feed-back transducer and oil hydraulics motor.

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A piston-type paddle was selected because of its simplicity and because the facility would be used mainly in connection with shallow-water waves. Neoprene rubber was fitted to the periphery and this was found to be very effective in minimising leakage.

3.2 Criteria for Design. The criteria for design were the velocity and force to be applied. It was desired to produce a range of frequencies between 0.2 Hz and 3 Hz, and wave heights of up to 0.15m. For velocity, the minimum water depth (say 0.12m) was the criterion, whilst for force, the maximum water depth (say 0.35m) was the criterion. From these considerations, and utilising the curves of Gilbert, Thompson and Brewer(1) for regular waves, the maximum velocity was estimated at 0.3m/s and the maximum force at 450N. For the irregular waves the final design criteria selected were:

maximum velocity - 0.45m/s

maximum force - 450N

maximum stroke - 0.45m

3.3 Design of Components. In order to meet the dynamic requirements, a 1.1kW power unit capable of circulating 4.5 l/min (1 imp. gal/min) at a pressure of $13.8 N/mm^2$ ($2000 lbf/in^2$) was purchased. The flow demand was such that the supply pressure needed to be only 6.9 to $10.3 N/mm^2$ (1000-1500 lbf/in^2), but the wastage of power was not an important consideration in the case of equipment that would only operate intermittently.

The actuator was of the balanced, double-ended type, with the advantage of symmetry in both directions. The dimensions of the rod were 0.46m long, 19.1mm diameter, operating in a cylinder of diameter 25mm.

To be compatible with these dimensions of actuator and system dynamics a servo-valve delivering 4.54 l/min at $6.89 N/mm^2$ was required. A Moog servo-valve type 76 was selected.

A closed-loop control system was required in which the output is represented by a mechanical or electrical quantity, and this is compared with the reference input. The comparison between output and reference input results in an actuating signal which is the difference between these quantities and acts in a manner to maintain the output at the desired value. For a well-designed system, control is almost independent of the load dynamics. The feed-back element selected was a Sangamo Controls linear variable differential transducer with built-in oscillator and detector, and requiring only a DC voltage for operation. The output depends upon the load impedance and is linear to an accuracy of 1%. The transducer in effect acts as a reference for the position of the paddle since, by applying a particular voltage to the control system the paddle tends to move to the point corresponding to the given voltage.

A Moog servo-controller provides the electronic control element. Essentially, this consists of two operational amplifiers and a power supply. Overall gain may be varied over a 100.1 range.

The assemblage of the actuator, servo-valve and linear variable differential transducer is shown in Fig. 3.

3.4 Stability of Control System. Having selected the components of the control system it remained to check the stability of the system. The lowest natural frequency of the system controls the stability and in practice this is the frequency of the valve-piston-load combination which was estimated to be 20 Hz. The gain setting should be the maximum available compatible with stability.

Stability was considered using a sine wave generator with frequencies of 0.1 to 3 Hz and a system gain varying between 14 and $60s^{-1}$. For a gain of $20s^{-1}$ the motion of the paddle was smooth to the eye for frequencies down to 0.3 Hz but below this judder was evident at the ends of the stroke.

4. GENERATION OF COMMAND SIGNAL

Controlling the mechanical system is a command signal which must produce the required spectrum of waves, whilst taking into account the transfer function (paddle-to-wave motion) and the response of the generator, both of which may vary due to parameters such as water depth and system gain.

The use of an analogue white noise generator and a set of filters to produce the desired effect was rejected on the grounds that the frequency of conventional white noise generators does not usually extend down as low as that required and there are problems due to drift and output variability.

Therefore it was decided to employ digital means, taking advantage of the medium-sized computer available at the University. Punched paper tape rather than magnetic tape was chosen because of the relative cheapness of the hardware required to handle it.

The tape code is ISO8 and the format depends to some extent on the programming language. ISO8 employs 7 data bits and one parity bit.

After considering the various alternatives it was finally decided to use a tape speed of 100 characters/s with a 2-digit converter, the 1% accuracy attainable being compatible with paddle accuracy for a 0.1m stroke. The punched paper tape reader is a Computer Terminals OR 10 unit and the digital-to-analog converter a Varadyne Systems DAC-29 series.

It was necessary to design additional circuitry to provide the following functions:

(1) Operation of tape reader at a pre-determined rate.

(2) Storage of information read by tape reader.

(3) Sequencing of tape reader, storage register, and up-dating of digital to analogue converter.

(4) Smoothing and scaling of analogue signal.

These operations were performed by digital and analogue circuits built on a module basis making use of plug-in circuit cards and dual-in-line packaging of integrated circuits.

A mean value control was provided because it is convenient if the paddle is kept in a central position during the time that no signal is received from the tape recorder. The system block diagram is shown in Fig. 4. Two modes are incorporated: (1) a loop mode in which a continuous tape loop may be used to generate a periodic motion or a short length of tape to produce a solitary wave or step function (2) a trigger mode for normal program tapes, heralded by £S and terminated by £F.

Two inverting amplifiers were introduced. Three ranges of scale were provided to suit the paddle motion travel - 0.1m, 0.15m and a potentiometer control to provide any desired value.

The performance of the complete punched-tape interface unit in response to various numbers on the punched tape was investigated. For high gains of the order of $60s^{-1}$, the system was unstable, any error signal causing oscillations of 10 Hz with an amplitude of 3mm. For gains of 14 or less the response was slow and large errors in position of the order of 8% were evident. Finally, a compromise setting for the gain of approximately $30s^{-1}$ was used which resulted in positional accuracy of approximately 1% whilst maintaining a smooth motion and reasonable frequency response.

5. PRODUCTION OF COMMAND TAPES

The specification for the command tape is that the waves produced shall have the required spectral density function.

Two methods may be used for the digital simulation of sea waves:

(1) Superposition of sine waves, with random values appropriately assigned.

(2) Digital filtering of random numbers.

Goda[2] has used the first method with 50 components, with some success. It is anticipated that the non-linear transformation of waves would have a smoothing effect.

The second method does not recognise any discrete frequencies and therefore should result in a continuous distribution of energy which is more realistic of an actual sea state. The procedure generally adopted is to filter a sequence of random numbers, generated such that they possess a normal probability distribution and that the magnitude may be predicted.

The filter used may be considered as a constant parameter linear system whereby

$$a_1x_1(t) + a_1x_2(t) = a_1\left[y_1(t) + y_2(t)\right] \qquad (1)$$

the LHS representing input and the RHS representing output and

$$y(t) = \int_0^\infty h(\tau)x(t-\tau)d\tau \qquad (2)$$

or the output is given by the convolution integral of a weighting function $h(\tau)$ times the input at some time τ earlier.

A time series with specific power spectral density and probability density function may be produced by exciting a specially designed filter with white noise. The characteristic of white noise is a constant spectral density function and a specific probability distribution; for physical purposes it is band-limited otherwise it would have an infinite variance.

Borgman[3], Shvetsov and Shorin[4] use the digital version of Eq.(2):

$$\eta(t_k) = \sum_{n=-K}^{n=K} h(\tau_n)x(t_k - \tau_n)d\tau_n \qquad (3)$$

where $x(t_k)$ are the series of random numbers constituting the white noise.

The Fourier transform of $h(\tau)$ is known as the frequency response function $H(f)$ given by

$$H(f) = \int_0^\infty h(\tau)\exp(-j2\pi f\tau)d\tau \qquad (4)$$

It may be shown that if the spectrum of random numbers $S_x(f)$ is equal to unity for the range of frequencies envisaged then $H(f)=\sqrt{S(f)}$, the required spectrum.

In the present case the filter has to take account of the response function of the signal-paddle movement - wave motion system as well as the desired spectrum. By making use of the convolution theorem it is possible to perform the filtering in the frequency domain instead of the time domain and by using FFT methods considerable computing time was saved.

The 'white noise' may be regarded as a sampled time series and is generated as a series of pseudo-random numbers on a computer by a standard routine such that the probability density distribution is Gaussian and the series only repeats after 2^{43} numbers.

It may thus be arranged that the time series produced will be the sampled time series representing the water surface elevation. The length of the transforms used to simulate the filter may be the same as the number of data points it is desired to be produced. However, a more efficient method is to filter segments of the record and then to add them.

As stated earlier, it was necessary to incorporate in the command signal a transfer function from paddle motion to wave motion (wave amplitude/stroke) and this was based on small amplitude theory[5] given by:

$$\frac{a}{e} = \frac{2\sinh^2 2\pi d/L}{\sinh(2\pi d/L)\cosh(2\pi d/L) + 2\pi d/L} \qquad (5)$$

Fig. 5 shows that the agreement with theory for regular waves is quite good for frequencies between 0.5 Hz and 1 Hz, and probably acceptable to beyond these limits, dictated by secondary harmonics at the low frequency end and wave breaking at the higher.

The command signal to paddle motion in terms of amplitude and phase were incorporated. The two functions were combined to form the frequency response $T(f)$ as

$$T(f) = \frac{a}{e}(f) \times \frac{e}{R}(f) \qquad (6)$$

where R represents the command signal.

6. INVESTIGATION OF PERFORMANCE

A number of records of actual sea states at sites in the English Channel were available. These were analysed for their spectral and statistical properties.

Command tapes were then prepared using the method previously described. In all, 8 prototype records were simulated, together with 2 arbitrary spectra and 2 Moskowitz spectra. Water depth in the channel was maintained at 0.35m throughout and scales were chosen to correspond with water depths where applicable.

All the tapes used in the tests consisted of 8000 numbers giving a running time of 5min 20s.

The waves in the channel were recorded by means of a capacitance probe and tape recorder, subsequently being analysed for their spectral and statistical properties. The power spectral density function was computed by FFT directly, using Hanning smoothing for the spectral window. Over several runs, repeatability was found to be within 3 per cent.

The variation of spectral form along the channel was also examined and it was found that there was very little variation, although there was a tendency for the higher frequency energy to diminish with distance from the paddle, presumably due to turbulent dissipation.

Some typical comparative spectra are shown in Fig. 6. Also, the probability distribution of water surface elevation, $p(\eta)$, and normalised wave heights, $p(H/\sigma)$, were computed in each case and compared with Gaussian and Rayleigh distributions, respectively (Fig. 7).

Spectra from the Nab Tower were, on the whole, successfully simulated, errors in variance being less than 3%. A greater resolution would probably have revealed a shape similar to the prototype.

The Eastbourne spectrum was simulated with a variance having an accuracy within 6%, and it will be noted that the peak energy has been increased by about 15% and shifted to a slightly higher frequency. Difficulties were encountered with the production of the command tapes and this probably affected the performance.

The prototype spectra for Jersey are of interest because they exhibit double peaks, indicating chop superimposed on swell - a very difficult sea state to represent by the zero crossing method of analysis. Reproduction is not particularly good especially in respect of a lack of energy in the high frequency portion. This may be due to shortcomings of the digital filter program.

The spectral shape for the Moskowitz spectrum was reasonable but variance of the command tape was too low.

The graphical representation of the probability distribution of water surface elevation showed, in general, a positive skewness which was also exhibited by the computed values of skewness and these were greater than prototype values.

The graph of frequency response and coherence function showed that there was very little non-linearity since the coherence function differed little from unity. The frequency response showed good agreement with that predicted by sinusoidal tests.

7. CONCLUSIONS

A wave machine has been developed which is operated directly by a computer generated time series representing paddle positions. The total cost of the machine, excluding labour, was less than £1000 ($2300).

The command signal is generated by digitally filtering random numbers, the filter being designed so that the waves produced represent the desired energy spectrum. The FFT method was very efficient, but errors in the resulting time series could not be predicted with confidence. Some improvement of the filtering system is needed.

A variety of spectra, including prototype and theoretical, were used to demonstrate the feasibility of simulating sea state in this way. Generally, the performance was reasonable, but the accuracy of modelling was found to depend upon the magnitude and shape of the spectrum to be reproduced. Double-peaked spectra were the least satisfactory. In all cases, running time should be considerably increased.

The linear transfer function for the paddle to wave process was found to be reasonably valid. But wind, or other means, would need to be introduced if the higher frequencies are to be simulated correctly.

The system at present suffers from some measure of over-damping. The gain should be increased, with the introduction of dither and compensatory circuitry to minimise judder. The aim should be a flat frequency response to approximately 3 Hz.

It is hoped to produce a Mk2 model which will incorporate the improvements outlined above and to replace the punched tape input by direct on-line from a mini-computer.

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NOTATION

$H(f)$ - Frequency response function

L - Wave length

S_f - Power spectral density function

S_x - Spectrum of random numbers

a - Wave amplitude

d - Water depth

e - Stroke

$h(\tau)$ - Weighting function

$x(t_k)$ - Series of random numbers

$x(t)$ - Time dependent variable

$y(t)$ - Time dependent variable

η - Water surface elevation

σ - Standard deviation

τ - Time lag

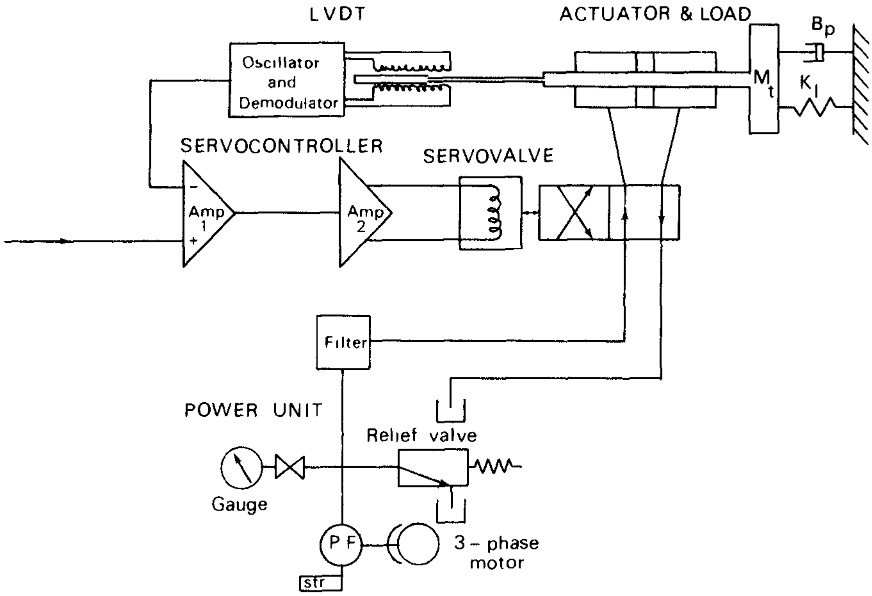

Fig 1 Hydraulic circuit

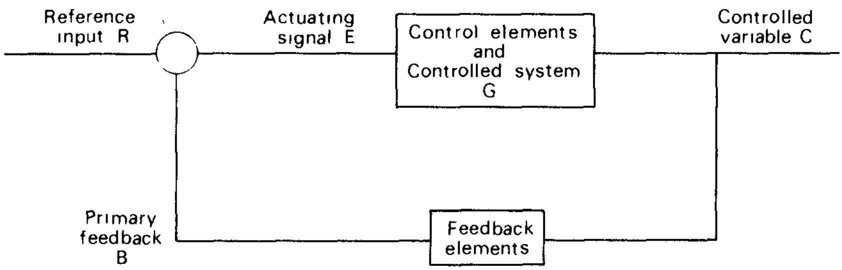

Fig 2 Closed - loop control system

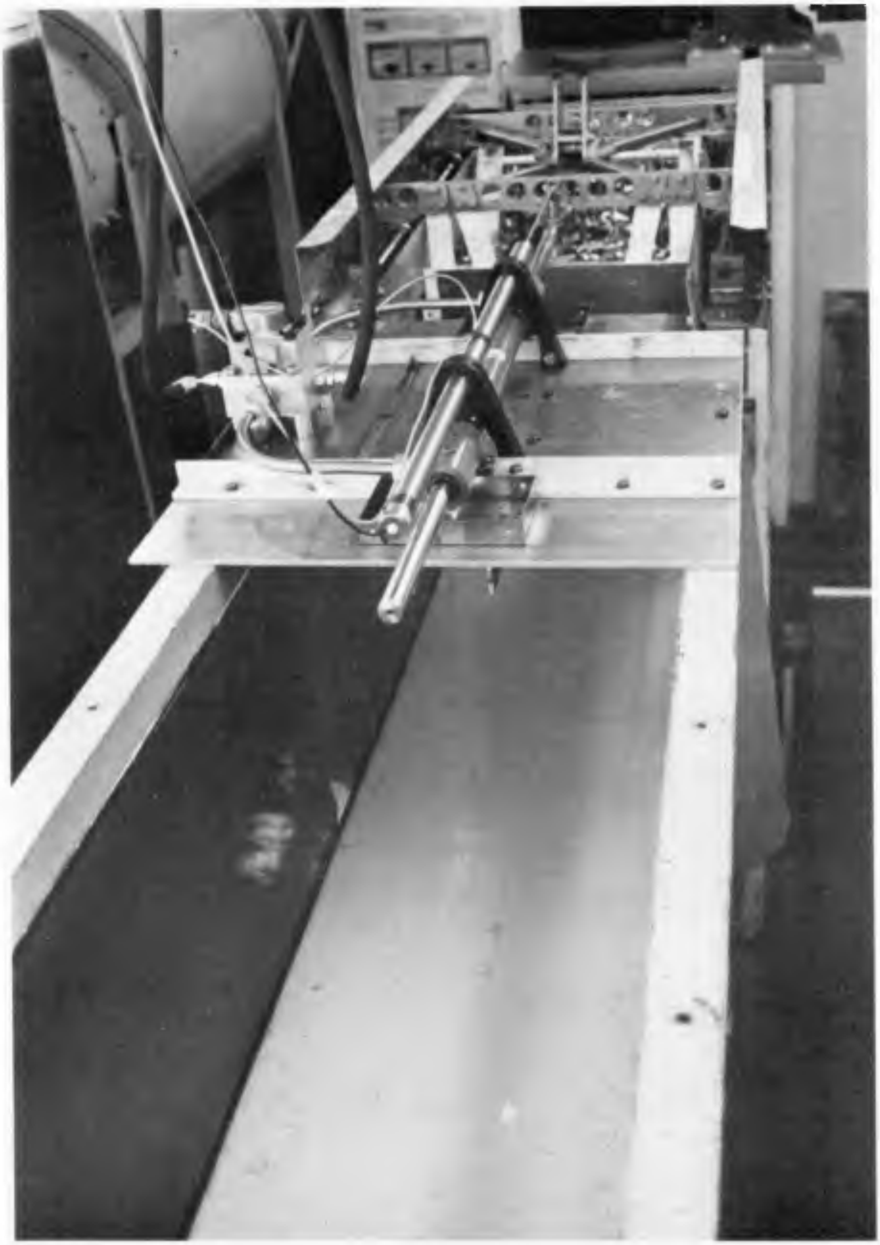

FIGURE 3 Servo-mechanism assemblage, showing actuator servo-valve and feed-back transducer

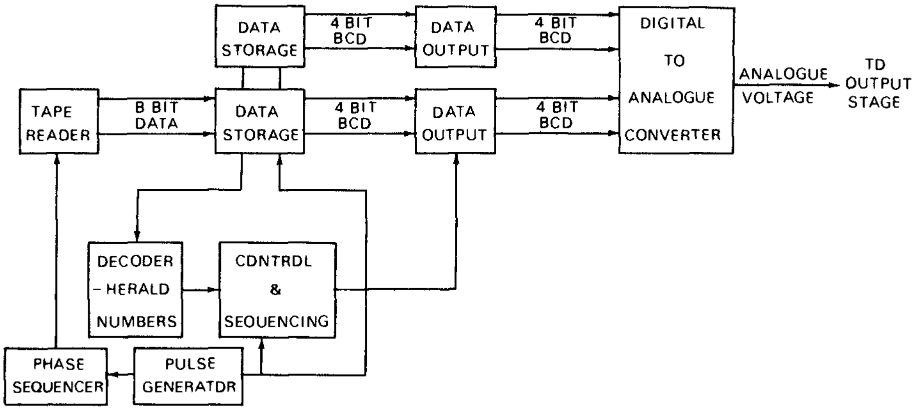

Fig 4 Block diagram of punched tape interface

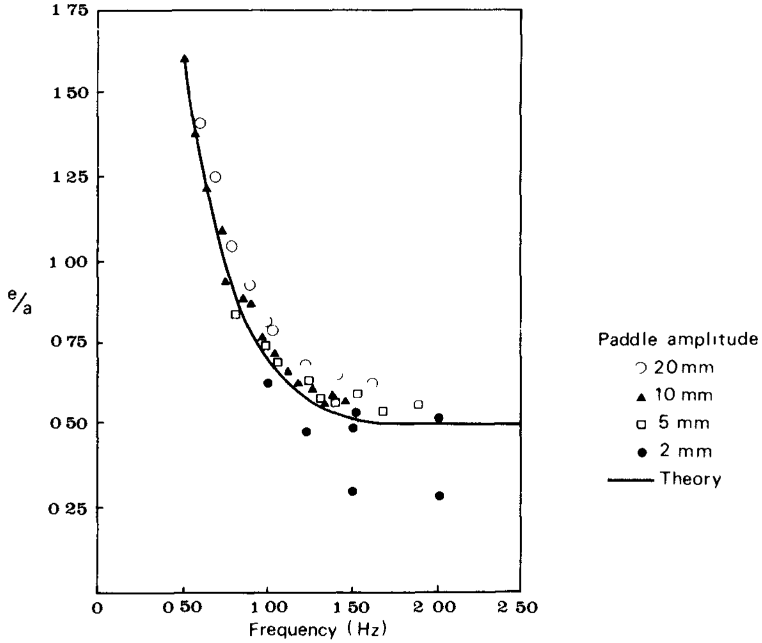

Fig 5 Comparison of experimental and theoretical values of ratio paddle amplitude / wave amplitude

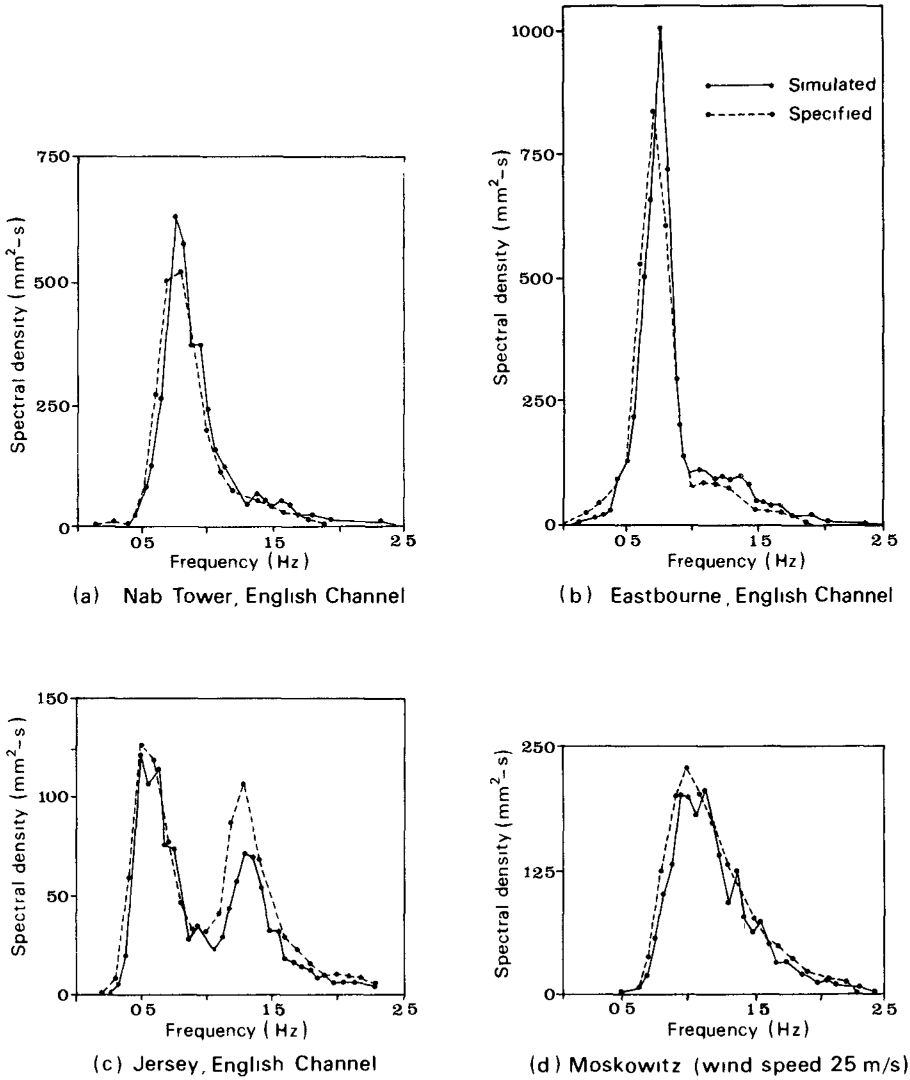

Figure 6 Simulated spectra