CHAPTER 136

The schematization for tidal computations in case of variable bottom shape.

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Synopsis.

Mathematical and physical methods can be applied for tidal studies. After general considerations on these methods, some practical aspects of tidal computations are discussed, in particular the schematization for tidal computations in case of variable bottom shape in shallow coastal waters. The relation with the coefficient of friction is dealt with. A combined one- and two-dimensional tidal computation is considered. Also an example is given of the determination of the coefficient of friction in a very shallow region; the variations, which are found in this practical case are discussed.

1. General considerations on the application of mathematical and physical methods for tidal studies.

Tidal problems may be solved by means of mathematical or physical models. Both kinds of models are approximations of the reality; in some respects in a different way.

In the analytical methods the water motion is represented in a continuous way. They can explain general physical aspects, e.g. progressive standing and Kelvin waves. These methods can only be applied in case of schematical tidal regions, and simplified assumptions e.g. the equations must be linearized. The value of the analytical method may be doubtful when it is necessary to use a computer for evaluating the solution in a particular case. An example is the evaluation of the harmonic method on a computer. This method can be applied in case one or two harmonic components must

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be considered. In case of more components a computer evaluation of an analytical solution is not recommendable.

In a physical model the practical solution is obtained by measurements at certain locations. They can represent the tide in complicated tidal regions. They show more details, e.g. eddy streets, which occur at sharp bends, and in case piers are present. Such phenomenae cannot be represented in a sufficient way in the mathematical models considered till now. The shape of the bottom can also be represented better, although it is limited in case the distortion of the scale model is considerable. On the other hand it is difficult to represent the Coriolis forces in a sufficient way in case these forces have a considerable influence on the water motion.

The numerical tidal methods have qualities between the methods, mentioned above. They can describe the tidal motion in rather complicated tidal regions, and the Coriolis force can be included in a correct way. To which extent these methods can be used is still a point of discussion and experience. This depends on the phenomenae, which must be studied, on the required data, and on the accuracy, which is demanded.

The solution by means of a numerical method is a discrete solution of linear finite difference equations. In the finite difference solution the water levels and velocities are computed at certain grid points. Usually the grid points for the water levels are different from those for the velocities. The discretization of the equations causes however difficulties, which do not occur in analytical solutions, and in physical models. Moreover it is difficult to determine the accuracy of the finite difference solution by means of mathematical formulae. It must be obtained from experience.

Two numerical evalutation techniques exist: the explicit method and the implicit method. The non-linear terms in the finite difference equations are often represented in a different way by various authors. In the explicit method the water levels, and the velocities at a future time step are immediately computed from those at previous time-steps. In an implicit method they are obtained after the simultaneous solution of a set of linear equations. Therefore the mathematical treatment of an implicit system of equations is more complicated than that of an explicit method. On the other hand in an explicit system the size of the grid net depends on a stability condition. This condition, which determines the time step, depends also on the friction, and the Coriolis coefficient. Such a condition is not necessary for an implicit system. The time step in case of the application of an implicit method, can be chosen several times greater than for an explicit scheme; it depends on the accuracy only. In particular this is of importance in coastal waters were deep gullies and shallows are found.

In the application of implicit and explicit methods instabilities of special kind, so called space-instabilities, may occur in the computation, in case the non-linear convective terms are included in the tidal equations. They may occur in particular in regions where large velocities, and variations in their directions occur, e.g. at sharp bends in an estuary. These instabilities do not occur when the convective terms are omitted. These problems are discussed in detail by Grammelveldt, 1969, and Kagan, 1970, for explicit schemes. It is shown that an explicit scheme e.g. that of Hansen's has a "computational viscosity", which depends on the grid size. In case of non-linear equations the "mathematical" wave interaction can induce a transfer of energy into the short wave range, in which it accumulates with time. Only a finite number of numerical waves can be resolved in a finite grid. This type of instability can be suppressed by introducing an artificial viscosity term, which causes a smoothing effect. However it is the question to which extent the accuracy is affected. Kagan shows that the minimum possible wave length, which is determined by the grid size, cannot be suppressed by an artificial viscosity term.

Obviously the introduction of the boundary conditions in mathematical models is much more simple than in physical models, where special apparatus must be applied.

From mathematical point of view the vertical tide, as well as the velocities should be introduced at the open boundaries, unless the convective terms in the equations of motion are small or may be neglected near the boundary. In this respect difficulties did not occur in the two-dimensional tidal computations, applied in the coastal area of the coast of the Netherlands. At the open boundaries the water levels are introduced. The velocities obtained by measurements depend often on the local conditions determined by the bottom shape. Therefore they are not used as boundary conditions. The finite difference solution of the tidal equations is correct at some distance from the boundary. The inaccuracy of the results at the coast line may be a serious objection from practical point of view.

2. Schematization of tidal regions for two-dimensional tidal computation.

2a. General considerations on the size of grids.

A one dimensional case like a river is schematized into a number of sections of equal or unequal length. The length of a section is determined such that the variation in the bottom shape is limited as possible. Considerable variations can take place at the boundaries of the sections; if necessary a separate equation of Bernoulli must be considered at these transitions. Furthermore the length depends on the locations, where the water levels and the velocities must be computed, and on the required accuracy of the results. The length of the sections in the rivers of the Netherlands is about 5 to 10 km's.

In two dimensional regions a square net is applied for numerical computations. The size of such a grid depends on the bottom shape, and moreover on the importance of the convective terms in the equations of motion.

In the sea the convective terms cause rotating currents or circulation. Circulation is also caused by the Coriolis forces. The extent of these circulations to be considered in the computations cannot be smaller than double the grid size. If smaller circulations have to be considered the grid size must be taken smaller. Therefore a finer net is required when the convective terms are important. On the other hand the number of velocity and water level points, that can be considered in the computations on the computer, determines also the size of the grid net. E.g. in the Atlantic Ocean the size must be chosen many times greater than in the North Sea, where

30 km's or smaller is to be preferred. Large grid sizes can be applied whenever variations in the bottom shape are small with respect to the depth. Such variations are much greater in the coastal zones. Also the number of tidal data in the coastal zone to be required is much greater.

The size of the grid, considered in the coastal waters of the Netherlands, is 1.6 km. This zone extends from the coast line up to 30 km in the sea, and 150 km along the coast line. For more detailed and accurate information near the coast a smaller zone is considered. The size of this zone is about 10 km perpendicular to the coast line. The grid size is 0.4 km. In this region shallows and gullies occur, and moreover the irregular shape of the coast line must be represented. The boundary conditions at the sea side of the smaller zone are obtained from the results of tidal computations in the bigger zone, Dronkers 1970. The boundary of a smaller zone must be chosen such that the tidal data are not influenced noticeable by the tidal motion in the smaller zone.

Often a square grid cannot represent the bottom shape in the mouths of estuaries, where deep gullies and shallows occur, in a correct way. One square of the grid can cover the shallows as well as the gully. In this case it should be desired to determine separately an irregular net for the gullies and the shallows. Then the length and the width of the various rectangles of the grid may become unequal, and the accuracy of the finite differences in x-direction is different from that in y-direction.

In the immediate neighbourhood of the coast line beaches occur, which are dry during a part of the tide. The slope of the bottom may be of the order of 1 m per km or more. Because of the small depths it is necessary to consider very small grid sizes. Such detailed computations are not carried out till yet.

It is necessary to check the results of tidal computations by means of vertical tide and velocity measurements. In particular it is important in coastal waters, where small differences in the vertical tide may affect the directions of the velocities considerably.

2b. Improvement on the accuracy of tidal computations by introducing modified Chézy coefficients.

The most important forces, which determine the tidal motion in the sea are the inertia forces, the Coriolis force and the gravity force. Usually the friction is of less importance, Also the convection terms can often be omitted. Friction and convection become more important in coastal waters.

The results of the computations in the coastal region of the Netherlands' Delta show that the computed velocities in the gullies are often lower, and on the shallows higher than the velocities obtained from the measurements. Accordingly the schematization must be improved by taking smaller grid sizes, or by modifying the Chézy coefficient.

Fig. 1 shows the effect of modifications in the Chézy coefficients for a part of the mouth of the Haringvliet. The Chézy coefficient has been changed in the regions within the dotted lines from 60 to 90 $m^{\frac{1}{2}}$/sec.

From tidal computations in rivers a general knowledge exists ahout the values of the friction coefficient C (Chézy) or n (Manning), as a function of depth and bottom material. In all applications it is necessary to compare the resulting velocities and water levels with those obtained from measurements. In case considerable differences occur four factors must be considered: the size of the grid net; the influence of the location of the boundary conditions; the values of the Chézy coefficients, and the influence of the convective terms, due to the variations in the velocities. In the following it will be shown how the schematization can be improved by introducing modified Chézy coefficients, which take into account the variation of the bottom shape.

The modified Chézy-coefficient of a square ($\Delta x, \Delta y$) will be determined in case of the following assumptions. The values of C are known as a function of the depth. The velocity vectors in the square are parallel, and its magnitudes do not differ considerably from the mean value. These assumptions include that the convective terms in the equations of motion can be neglected in the square. Finally it is assumed that at a certain moment the difference in head, due to the friction forces, is constant over the square.

The formula for the modified Chézy coefficient will be applied to the particular case that the velocities have directions parallel to the x-axis, and the variation in the bottom figuration only occurs in the y-direction. Let the square be subdivided in y-direction in n parts of equal depth. The total quantity of water in a cross-section of the square $\Delta x, \Delta y$ in the x-direction is,

$$a\,u\,\Delta y = a_1\,u_1\,\Delta y_1 + a_2\,u_2\,\Delta y_2 + \dots + a_n\,u_n\,\Delta y_n \qquad (1)$$

in which a_i are depths and u_i are velocity components in x-direction. Because the values of the slope of the water surface $\frac{\Delta h}{\Delta x}$ are assumed to be constant in the square in x-direction, it holds according to the formula of Chézy,

$$\frac{\Delta h}{\Delta x} = \frac{u_1^2}{C_1^2\,a_1} = \frac{u_2^2}{C_2^2\,a_2} = \dots\dots = \frac{u_n^2}{C_n^2\,a_n} = \frac{u^2}{C^2 a}\,, \qquad (2)$$

in which the velocity u, the Chézy coefficient C, and the depth a are the mean values over the square.

It follows from (1) and (2) after replacing u_m ($m = 1,2 \dots n$) by $u\,\frac{C_m\,a_m^{\frac{1}{2}}}{C\,a^{\frac{1}{2}}}$, that

$$C\,a^{3/2}\,\Delta y = C_1\,a_1^{3/2}\,\Delta y_1 + C_2\,a_2^{3/2}\,\Delta y_2 + \dots\dots + C_n\,a_n^{3/2}\,\Delta y_n.$$

In case the depth changes continuously over the cross-section Δy, the following relation is obtained for the mean value of the Chézy coefficient, C, and the mean depth a,

$$\overline{C\,a^{3/2}}\;\Delta y = \int_0^y C(y)\,a(y)^{3/2}\,dy\,. \qquad (3)$$

In the general case the velocity has components in the x- and the y-direction. Then the variable y in (3) must be replaced by the variable y', which is determined by the rotation of the axes x and y over the angle α to the axes x' and y'; α is defined by the direction of the velocities in a square. Then the factor uV, respectively vV, in which $V = (u^2 + v^2)^{\frac{1}{2}}$ must be considered in the resistance terms. Because the velocity vectors are parallel,

V can be replaced by $\frac{u}{\cos\alpha}$, respectively $\frac{v}{\sin\alpha}$.
Let a square subgrid be formed over the square (Δx, Δy) (fig.2). Then for each column in the y-direction formula (3) holds good. After addition of the results of the equation (3) for the subsequent columns, it is found,

$$\overline{C\ a^{3/2}}\ \Delta y\ \Delta x = \int_0^x \int_0^y C(x,y)\ a^{3/2}(x,y)\ dx\ dy, \tag{4}$$

in which $\overline{C\ a^{3/2}}$ is the mean value of $C\ a^{3/2}$ over the square ($\Delta x, \Delta y$). The formula (4) holds for the general case that the velocity vector is not parallel to the x-axis.

The modified value C_m of the square is obtained from $C_m\ a_m^{3/2} = \overline{C\ a^{3/2}}$, in which a_m is the mean depth in the square. Generally it holds $C_m > \bar{C}$. The analogous method can be applied to the river sections, Dronkers, 1964, Chapter XI.

3. The schematization for a combined two- and one dimensional tidal computation.

In this section the combined two-dimensional tidal computation for a part of the sea, and the one-dimensional computation for a river is demonstrated (fig.3). In case the grid size and the dimensions of the sections of the river are different, the schematization of the transition zone must be modified such, that the grid and the sections of the river fit together. In the transition section of the river this means that the width will become equal to the size of the grid: $\Delta x = \Delta y$, and that the mean velocity and the total quantity of water passing through the cross section, the discharge, does not change. This discharge is determined by the tidal conditions upriver of the transition section. Moreover the difference in head due to the resistance force must not change. Hence

$$b_1 a_1 = \Delta y\ a, \text{ and } C_1^2\ a_1 = C^2 a\ , \tag{5}$$

in which a is the mean depth of the modified section, a_1 the depth of the original section; C and C_1 are the corresponding values of the Chézy coefficient.
These equations determine the modified depth a and coefficient C.

If the width of the mouth of the river, b_1, is more than twice the value of Δy, more sections with equal width Δy, next to each other in the river mouth, must be considered.

The finite difference equations for the transition zone are mentioned below. The method for the solution of the tidal equations is dealt with in general terms.

In fig.4 the points are denoted, where the velocity components u and v, and the water levels h are computed in the transition zone from the sea to the river. An implicit scheme is applied, Leendertse, 1967. For the river the formulae of the third implicit scheme are applied, Dronkers, 1969. The water level and discharge, Q, or velocity, u, are taken at the same location, the beginning, or the end of each river section. The advantage of this method is that river sections with unequal length can be considered for the schematization of the river. An analogous method is applied in the schematization for the application of the harmonic method.

The convective terms are not considered in the formulae for the sea, however the Bernoulli term is included in the equations for the river.

Each time step consists of two parts. In the first half time step $t + \frac{1}{2}\tau$, the values of the velocity component in x-direction, u', and the water level with respect to the mean water level, h', are computed by means of implicit equations. The velocity component in y-direction in the sea, v', is determined by an explicit equation. In the second half time step $t + \frac{1}{2}\tau$, v'' and h'' are determined implicitly, and u'' is obtained from an explicit equation. No values of v have to be computed in the river, and therefore some modifications are to be made in the computational scheme.

In the following the finite difference equations are mentioned for the transition zone from the river to the sea. In fig.4 the locations of the variables with indices n and m are denoted. At the coast $m = M$, and for the river $n = N$. The most upriver section is denoted by $m = M_1$.
The squares of the grid in the sea which are on the same line as the river, have index m,N. For $M + 1 \leqslant m \leqslant M_1$, the equation of motion for the river is:

$$\frac{\delta h}{\delta x} = - \frac{1}{g} \frac{\delta u}{\delta t} - \frac{|u| u}{C^2 (a+h)} - \frac{1}{g} u \frac{\delta u}{\delta x} , \qquad (6)$$

in which h is the water level, u the velocity, a the mean depth, and b_s the mean width of the river at time t and place x. This equation is replaced by

$$h'_{m+1} - h'_m = -\frac{(\Delta x)_m}{\tau g}\left[(u'_{m+1} - u_{m+1}) - (u'_m - u_m)\right] - \tag{7}$$
$$- \frac{(\Delta x)_m}{4}\,\frac{|u_{m+1}+u_m|\,(u'_{m+1}+u'_m)}{C_m^2\,(a_m + h_m)} - \frac{1}{2g}\,(u_{m+1}+u_m)(u'_{m+1}-u'_m)\ ,$$

in which $(\Delta x)_m$ is the length of the m-th section and $\frac{1}{2}\tau$ is the half time step.

The equation of continuity,

$$b\,\frac{\delta h}{\delta t} = -\frac{\delta Au}{\delta x} \tag{8}$$

in which A is the area of a cross-section, and b the storage width, is replaced by

$$u'_{m+1} - u'_m = -\frac{(\Delta x)_m}{\tau}\,\frac{b_m}{A_m}\left[(h'_{m+1}-h_{m+1}) + (h'_m-h_m)\right] \tag{9}$$

These equations are written in the form:

$$h'_{m+1} - h'_m + \eta_m u'_{m+1} + \Theta_m u'_m = \mu_{m-1}$$
$$(m = M+1,\ldots,M_1) \tag{10}$$
$$\nu_m(h'_{m+1} + h'_m) + u'_{m+1} - u'_m = \xi_m\ ,$$

in which the coefficients depend on those of (7) and (8).

By means of the application of the sweep method in the upriver direction, the set of equations (10) can be rewritten in the form

$$u'_{m-1} = -q_{m-1}h'_m - t_m u'_m + s_{m-1} + b_{m-1}h'_{M+1}$$
$$h'_m = -p_m u'_m + r_m + a_m h'_{M+1}, \quad M + 2 \leqslant m \leqslant M_1 \tag{11}$$

Similar formulae are derived for the application of the sweep method in the downward river direction. Recurrent formulae for the computation of the coefficients q, t, etc. can he derived, Dronkers 1969.

After the successive elimination of h'_m and u'_m in the set of equations (11) in the upriver direction, and elimination in the set of equations in the downriver direction, respectively, by means of the sweep methods, the following relations are obtained between $u'_{M+1,N}$, $h'_{M+1,N}$, $u'_{M_1,N}$ and $h'_{M_1,N}$ at the beginning and at the end of the river:

$$h'_{M_1,N} + P_{M,N}\, u'_{M_1,N} + a_{M+1,N}\, h'_{M+1,N} + r_{M+1,N} = 0$$
$$P^*_{M+1,N}\, u'_{M+1,N} + h'_{M+1,N} + a^*_{M_1,N}\, h'_{M_1,N} + r^*_{M+1,N} = 0 \qquad (12)$$

The finite difference equations for the transition zone from sea to river follow next. First the equation of motion in x-direction is applied to square ABCD of fig.4 for the first half time step. This equation

$$\frac{\delta u}{\delta t} - \Omega v = -g\,\frac{\delta h}{\delta x} - \frac{gVu}{C^2 a} ,$$

in which Ω is the coefficient of Coriolis, is replaced by

$$u'_{m,n} = u_{m,n} - \frac{g\tau}{2k}\left(h'_{m+1,n} - h'_{m,n}\right) + \frac{\tau\Omega}{8}\left(v_{m,n} + v_{m,n-1}\right) -$$
$$- \frac{\tau g}{4}\left[\left\{(u_{m,N}+u_{m-1,N})^2 + (v_{m,N}+v_{m,N-1})^2\right\}^{\frac{1}{2}} + 2\,u_{m+1,N}\right] \times \qquad (13)$$
$$\times \frac{u'_{m,n}}{C^2_{m,n}\left(2a_{m,n}+h_m+h_{m+1,n}\right)} ,$$

Then the equation of continuity is applied to the square EFGH,

$$\frac{\delta h}{\delta t} + \frac{\delta au}{\delta x} + \frac{\delta av}{\delta y} = 0.$$

The finite difference equation becomes,

$$h'_{m,n} = h_{m,n} + \frac{\tau}{4k}\left(a_{m,n} + h_{m,n}\right) u'_{m,n} -$$
$$- \left(a_{m-1,n} + a_{m-1,n-1} + h_{m,n} + h_{m-1,n}\right) u'_{m-1,n} + \qquad (14)$$
$$+ \left(a_{m,n} + a_{m-1,n} + h_{m,n} + h_{m,n+1}\right) v_{m,n} +$$
$$+ \left(a_{m,n-1} + a_{m-1,n-1} + h_{m,n} + h_{m,n-1}\right) v_{m,n-1} .$$

The explicit equation for $v'_{m,n}$ follows from the finite difference equation of the motion in y-direction (see square IJKL);

$$\frac{\delta v}{\delta t} + \Omega\, u = -g\,\frac{\delta h}{\delta y} - \frac{g\, Vv}{C^2 a} ,$$

$$v'_{m,n} = v_{m,n} - \frac{\tau\Omega}{8}\,(u'_{m,n}+u'_{m-1,n}+u'_{m,n+1}) - \frac{g\tau}{2k}\,(h_{m,n+1} - h_{m,n}) -$$

$$- \tau g \frac{\left[\frac{1}{9}\,(u'_{m,n} + u'_{m-1,n} + u'_{m,n+1})^2 + v^2_{m,n}\right]^{\frac{1}{2}}}{C^2_{m,n}\,(a_{m,n} + a_{m-1,n} + h'_{m,n} + h'_{m,n+1})}\, v'_{m,n} \,. \qquad (15)$$

The unknown values in equations (13) and (14) are $u'_{m,n}$, $h'_{m,n}$, $h'_{m+1,n}$, and $u'_{m-1,n}$ etc.; $v'_{m,n}$ is the unknown value in equation (15).

In the sea these equations are applied to the grid for $1 \leqslant m \leqslant M$; at the coast $m = M$. The index n varies between $n = 1$ and $n = N+1$ and from $n = N+1$ up to $n = N_1$. A solution by means of the sweep method, determines these values (Leendertse, 1967). Equation (15) determines v in the sea.

However for $n = N$ the equations (12) for the river must be added to the system of implicit equations mentioned in (13) and (14). Furthermore a relation between $u'_{m,N}$ and $h'_{M+1,N}$ (see fig.4) must be added to the equations (12). Applying the equation of continuity to the rectangle FBCG (fig.4), it follows,

$$a_{m,N}\,(u'_{M+1,N} - u'_{m,N}) = -\frac{k}{4\tau}\left[3(h'_{M+1,N} - h_{M+1,N}) + (h'_{m,N} - h_{m,N})\right] \qquad (16)$$

The boundary condition at the coast is: $u_{m,n} = 0$ $(n \neq N)$. Also $h_{1,n}$ etc. are given at the boundary in the sea. The equations for the second time step can be set up in a similar way.

In this time step the unknown quantities are u'', h'' and v''. Then an implicit set of equations similar to (13) and (14) hold for h'' and v'', and an explicit equation for u''.

In equation (13), u' is replaced by v'', and the finite differences in x-direction are replaced by those in y direction. Furthermore the quantities which are defined for the time level t in equations (13-15) are replaced by the time level $t + \frac{1}{2}\tau$, in a way that they get the index ($'$). The equation of motion for the velocity component v is applied to square IJKL in fig.4, and the equation of continuity to the square EFGH.

Because in the equations for the river, v' and v'' do not occur, the solution of the complete set of equations of sea and river is different for the second time step. The values of v'' and h'' in the sea are determined by the set of implicit equations. The boundary value of $u_{M,N}$ at the mouth of the river is taken at time level $t + \frac{1}{2}\tau$, which is computed at the previous half time step. The equations for the river are not considered in this set of equations, because after the computation of v'' and h'', the values of u'' in the sea is determined by an explicit equation. Next values of u'' and h'' in the river are again found from a set of implicit equations for which $u''_{m,N}$ and $h''_{M_1,N}$ are the boundary condition. This set of implicit equations are solved in the same way as described above for h' and u'.

From the preceding computational scheme it appears that the dimensions of the section FB of which the length equals half of the size of the grid, must be modified such that the width is equal to the grid size k. The formulae for the modified dimensions are given in formula (5). The lengths, widths, storage widths and depths of the sections upriver of point follow from the schematization of the river.

4. The Chézy coefficient in very shallow regions.

The execution of tidal computations in very shallow regions of which the depth is small at low water or in which some parts are dry, encounter many difficulties. The convective terms can usually be neglected on the shallows because of the very small velocities. However the values and the directions of the velocities may change considerably in the transition zone from the channels to the shallow region. In this zone the values of the convective terms may be of the same order or larger in comparison with those of the other terms in the tidal equations. A very fine net must be considered for computations in such regions.

Friction forces are most important on the shallows. To get an impression about the Chézy coefficients in very shallow region, detailed tidal and velocity observations in a part of the Brouwershavense Gat of the Delta region in the Netherlands are carried out.

results are shown in relation to the depth.

It appears that considerable variations occur in the values of the Chézy coefficients for a certain depth, though in general the values decrease with decreasing depth in accordance with the expectation. Accurate computations cannot be carried out on shallows, because of the variable bottom-shape in the shallow region. This example appears too complex for schematization. Nevertheless the storage of the shallows must be taken into account in the tidal computations. Furthermore the friction term should be the only term to be considered in the equations of motion.

The determination of the Chézy coefficients in a river is dealt with by Dronkers, 1964. The measurements and computations mentioned in this section, are carried out by the Measuring Station at Zierikzee, Delta Works.

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The mean slope of the shallow in the direction of the coast is about 1:1500. The bottom material consists of fine sand (150 μ), mixed with silt.

Fig.5 shows the detailed contourlines of the depths of this region, and the locations where vertical tide and velocity measurements were taken.

The dimensions of this region are about: length 3 kms and width 2 kms. The region has been divided into 32 rectangles of which the dimensions are: length 750 m (x-direction), and width 250 m (y-direction, perpendicular to the coast). The mean depth in each rectangle has been determined with respect to the mean water level. The velocity vectors are resolved into the x-direction (u-component) and into y-direction (v-component). The tidal range was about 2.9 m. The maximum depth, where velocity measurements are taken was 3 m below mean sea level, and the minimum depth 1.4 m. The vertical velocity distribution are determined at intervals of 15 minutes. The minimum distance from the bottom was 0.2 m. The water levels are measured at intervals of 5 minutes at each gauge (see fig.5). A very accurate levelling of the gauges has been carried out for this purpose. The maximum differences in water level are of the order of 5 cm per km.

The mean velocities and their directions at any 15 minutes are represented graphically on the maps and decomposed in the x- and y-direction. The mean velocity components are determined on the sides of each rectangle (fig.5) by interpolation of the measured velocities.

After this preliminary work the equation of continuity, and the two-dimensional tidal equations of motion are applied to the water motion in each rectangle. The equation of continuity has been applied for the checking of the velocity measurements, because the quantity of water flowing to and from each rectangle at a certain instant must balance the quantity of water remaining within the rectangle at rising tide, or leaving the rectangle from high water to low water. The computations are carried out after each period of twenty minutes. The equations of motion are applied for the determination of the values of the Chézy coefficient. In fig.6 the

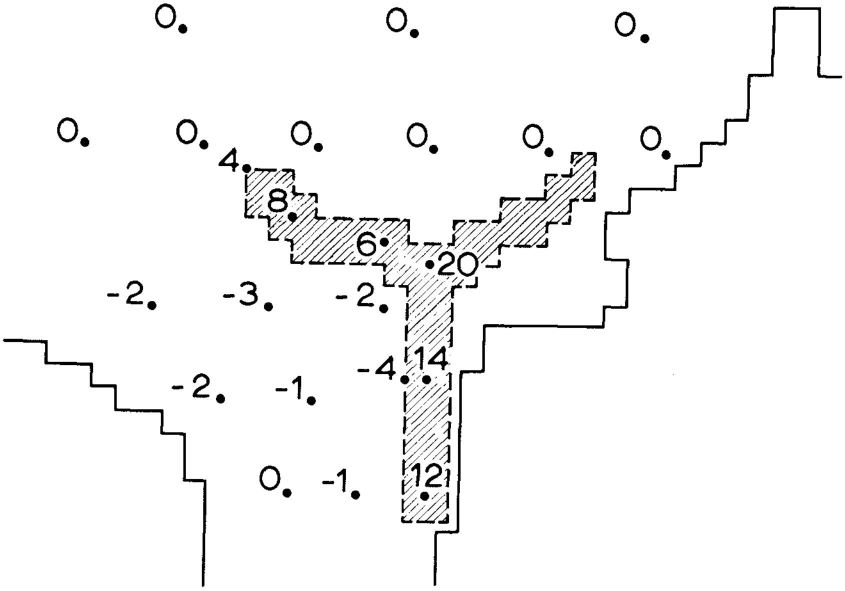

FIG. 1 INFLUENCE OF INCREASE OF CHEZY COEFFICIENT ON VELOCITIES (EXPRESSED IN %)

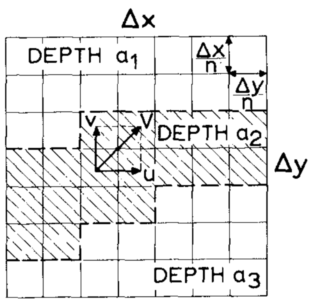

FIG.2 SCHEMATIZATION OF A SQUARE OF A GRID

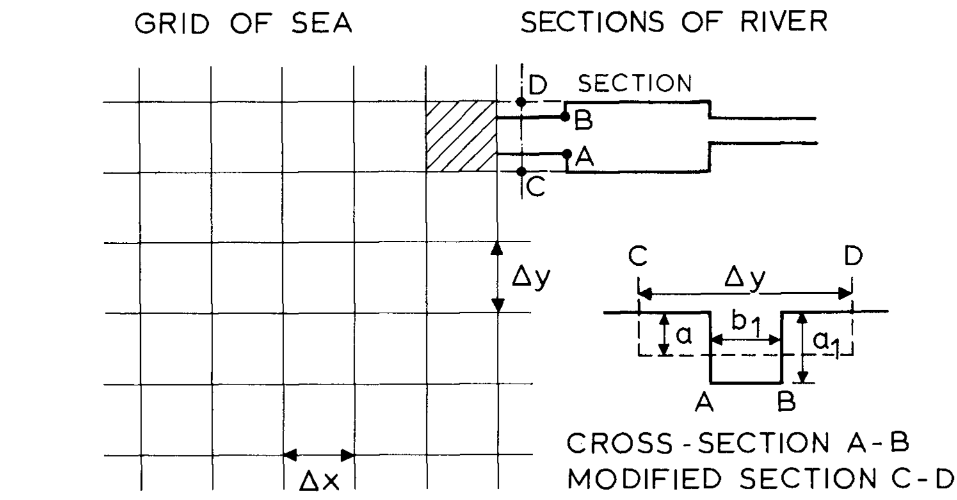

FIG. 3 COMBINED GRID OF SEA AND RIVER SECTIONS

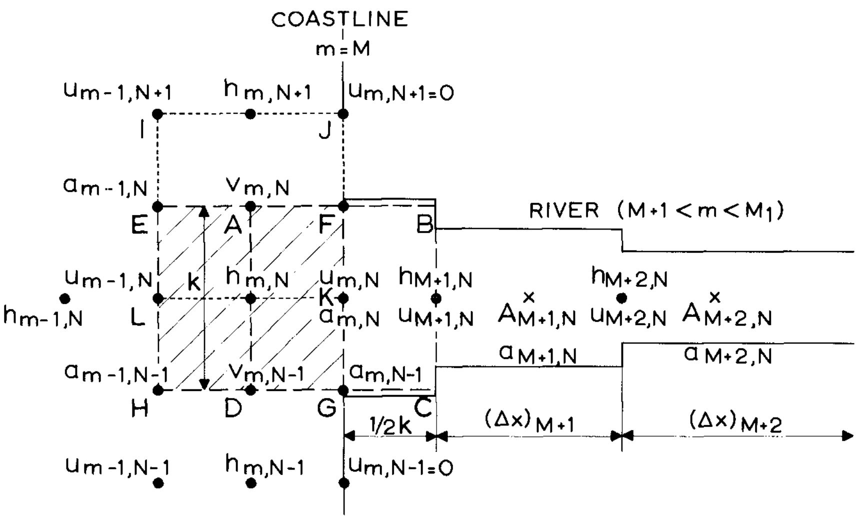

FIG.4 COMBINED GRID OF THE SEA AND RIVERSECTIONS

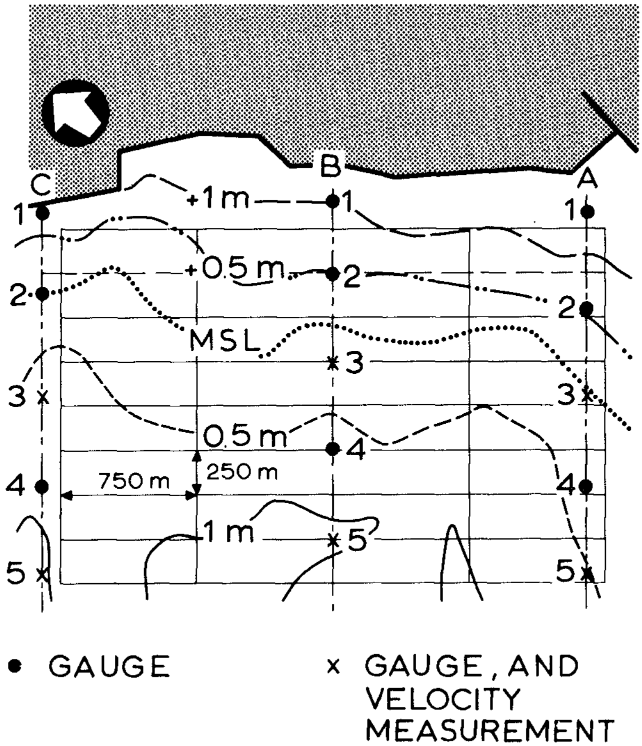

FIG. 5 CONTOUR LINES OF SHALLOW REGION, MEASURING POINTS AND GRID

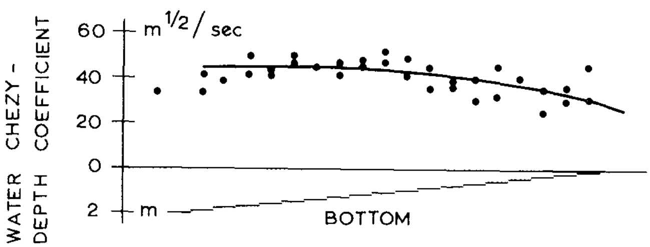

FIG.6 COEFFICIENT OF CHEZY AS FUNCTION OF DEPTH