CHAPTER 9

Equilibrium Range Spectra in Shoaling Water

by

Takeshi Ijima[1], Takahiko Matsuo[2] and Kazutami Koga[3]

Abstract

In shoaling water on sloping beach, waves break by hydraulic instability due to the finiteness of water depth, so that frequency spectra of waves in surf zone must have any limiting form similar to the equilibrium spectrum given by Phillips(1958) In this paper, authors have derived an equilibrium form of spectra for surf waves from the limiting wave condition at constant water depth by Miche(1944) and from breaking wave experiments on sloped bottom by Iversen(1952) The results are compared with surf wave spectra obtained from field observations by means of stereo-type wave meter devised by the authors(1968).

By means of this spectrum and by deep water wave spectra for various wind conditions, significant wave heights and optimum periods of limiting waves in surf zone are calculated

1 Introduction

By dimensional considerations, Phillips(1958) has shown that in the frequency spectrum of deep water wind waves the energy spectral density $\Phi(\sigma)$ of saturated high frequency component is proportional to -5 powers of frequency as shown by the following equation

$$\Phi(\sigma) = \beta g^2 \sigma^{-5} \qquad (1)$$

where g is gravity acceleration and β is non-dimensional constant, for which Phillips has given numerically 0 0117 from observed data and Pierson(1964) has obtained 0 00810 from observed spectra on the North Atlantic Ocean

Eq 1 is independent of wind conditions and is interpretted as representing a limiting spectrum for waves breaking in deep water in the state of hydraulic instability

As for waves breaking in shoaling water, the limiting wave height is determined by instability depending on water depth h and the similar limiting spectrum should exist, which, of course, is independent on wind conditions but depends only on water depth and gravity acceleration in the range of gravity waves

(1) Professor of Kyushu University, Fukuoka, Japan
(2) Lecturer, Ooita Technical College, Ooita, Japan
(3) Master Course Student, Kyushu University

Here, we assume straight shore line with constant slope and waves incident normally, so that one-dimensional waves without refraction effect

2 Derivation of equilibrium range spectrum

From observed wave record of a train of waves at a point of depth h in surf zone, apparent individual wave height $\tilde{H}$ and wave period $\tilde{T}$ of all the waves are measured in spite of breaking and non-breaking and then wave length $\tilde{L}$ of the wave at water depth h are calculated by small amplitude wave relation Then, we obtain a scatter diagram of $\tilde{H}/\tilde{L}$ related to $h/\tilde{L}$ as shown in Fig 1, which was obtained from wave record of 17 minutes long at depth of 5 7 meters on the Coast of Miyazaki (Pacific Coast of Kyushu)

The envelope curve through the upper limit of scattered points in the figure is interpreted to show the limiting wave condition for the depth h

For a train of single sinusoidal waves, the limiting condition at constant depth for h/L larger than about 1/20 is given by Miche(1944) as follows

$$H/L = \frac{1}{7}\tanh\frac{2\pi h}{L} \tag{2}$$

For deep water ($h/L \to \infty$), H/L is 1/7 and for shallow water long waves ($h/L \to 0$), eq 2 gives $H/h \approx 0\ 9$, which is somewhat larger than 0 78 by solitary wave theory For the bottom of constant slope, we have not yet theoretical relation but Iversen(1952) has shown experimental relations of H_b/H_0' and h_b/H_0' to H_0'/L_0 for bottom slopes α = 1/10, 1/20, 1/30 and 1/50, where H_0', L_0 are deep water wave height and length and H_b, h_b are wave height and water depth of breakers From these experimental relations, an empirical relation of H /L and h/L similar to eq 2 is obtained as follows

$$H/L = \frac{1}{7}\tanh\left[f(\alpha)\frac{2\pi h}{L}\right], \quad f(\alpha) = \cosh 3\ 5\alpha^{3/5} \tag{3}$$

where α is bottom slope Above relations are shown in Fig 1 for $\alpha = 0$ and 1/10 Eq 3 may give somewhat larger values for $h/L < 1/20$ for single sinusoidal waves but for actual randum waves the form of eq 3 may be assumed to represent approximate relations of limiting apparent wave height $\tilde{H}_{max}$ and length $\tilde{L}$ derived from apparent wave period $\tilde{T}$ for all $h/\tilde{L}$ Accordingly, we assume next relation for limiting apparent waves

$$\frac{\tilde{H}_{max}}{L} = C_1 \tanh\left[f(\alpha)\frac{2\pi h}{L}\right] \tag{4}$$

Where C_1 is assumed to be constant

As for actual waves with continuous spectrum, the limiting wave height $\tilde{H}_{max}$ is interpreted as the result that all the phases of component waves whose periods are within narrow range of period band between $\tilde{T} - \Delta\tilde{T}/2$ and $\tilde{T} + \Delta\tilde{T}/2$ centered at period $\tilde{T}$ happened to coincide and their spectral wave heights were summed up to attain the limiting height $\tilde{H}_{max}$ And for sufficiently narrow width of $\Delta\tilde{T}$, the component waves within the period band are considered to have nearly equal spectral wave height H_T proportional to the limiting wave height $\tilde{H}_{max}$, that is,

$$H_T \propto \tilde{H}_{max} \tag{5}$$

Therefore, from eq 4 we have

$$H_T = const\ \tilde{L}\ tanh\left[f(\alpha)\frac{2\pi h}{\tilde{L}}\right] \tag{6}$$

Assuming that the number of component waves within the period band is N and they are mutually independent, the total energy density within the period band of $\tilde{T} - \Delta\tilde{T}/2$ and $\tilde{T} + \Delta\tilde{T}/2$ is

$$N \cdot \frac{\rho g}{8}\ const\ \tilde{L}^2\ tanh^2\left[f(\alpha)\frac{2\pi h}{\tilde{L}}\right]$$

Accordingly, the total frequency spectral energy density $\Phi(\sigma)\Delta\sigma$ between $\sigma - \Delta\sigma/2$ and $\sigma + \Delta\sigma/2$ is given by the following equation.

$$\Phi(\sigma)\Delta\sigma = const\ N\ \tilde{L}^2 \cdot tanh^2\left[f(\alpha)\frac{2\pi h}{\tilde{L}}\right] \tag{7}$$

The number of component waves N in above equation is considered to become large when the period band $\Delta\tilde{T}$ becomes wide, but to become small when the period $\tilde{T}$ becomes large for limited length of wave record Accordingly, we may assume the following relation

$$const\ N = const\ \Delta\tilde{T}/\tilde{T} = const\ \Delta\sigma/\sigma,\ \left(\sigma = 2\pi/\tilde{T}\right) \tag{8}$$

Now, eq 7 is written as follows

$$\begin{aligned}\Phi(\sigma)\Delta\sigma &= const\ L^2 \cdot tanh^2\left[f(\alpha)\frac{2\pi h}{L}\right] \\ &= const\ (2\pi)^2\frac{h^{5/2}}{g^{1/2}}\frac{tanh^2\left[f(\alpha)\frac{2\pi h}{L}\right]}{\left(\frac{2\pi h}{L}\right)^2\left(\frac{\sigma^2 h}{g}\right)^{1/2}}\Delta\sigma \\ &= const\ (2\pi)^2\frac{h^{\frac{5}{2}}}{g^{1/2}}\ F\left(\alpha, \frac{\sigma^2 h}{g}\right)\Delta\sigma \end{aligned} \tag{9}$$

where

$$F\left(\alpha, \frac{\sigma^2 h}{g}\right) = \frac{\tanh^2\left[f(\alpha)\frac{2\pi h}{L}\right]}{\left(\frac{2\pi h}{L}\right)^2\left(\frac{\sigma^2 h}{g}\right)^{1/2}} \tag{10}$$

In above equations, $2\pi h/L$ is determined as a function of $\sigma^2 h/g$ by the following equation

$$\sigma^2 h/g = \frac{2\pi h}{L}\tanh\frac{2\pi h}{L} \tag{11}$$

In eq 10, when $\sigma^2 h/g$ becomes large, $2\pi h/L$ tends to $2\pi h/L_0$ and F tends to $(\sigma^2 h/g)^{-5/2}$, and when $\sigma^2 h/g$ becomes small, F tends to $f^2(\alpha)(\sigma^2 h/g)^{-1/2}$

Accordingly, from eq 9 it is seen that

$$\Phi(\sigma) = const\ (2\pi)^2 g^2 \sigma^{-5} \quad \text{for deep water waves} \tag{12}$$

$$\Phi(\sigma) = const\cdot(2\pi)^2 f^2(\alpha)\, h^2 \sigma^{-1} \quad \text{for shallow water waves} \tag{13}$$

Comparing eq 12 with eq 1,

$$const\cdot(2\pi)^2 = \beta$$

Thus, eq 9 is determined as follows

$$\Phi(\sigma) = \beta\, \frac{h^{5/2}}{g^{1/2}}\, F\left(\alpha, \frac{\sigma^2 h}{g}\right) \tag{14}$$

Above equation is considered to represent the equilibrium spectrum of surf waves

3 Comparisons with observations

The observation of surf waves is difficult because of various troubles in setting and maintaining wave meters The authors(1968) have devised a stereo-type wave meter, with which field observations at Miyazaki Coast and Nata Coast of Kyushu were carried out Fig 2 is an example of observed frequency spectra at Nata Coast. Fig.3 and 4 are non-dimensional plot of frequency spectra at Miyazaki- and Nata Coast, respectively, in which the curves are equilibrium spectra by eq 14 for $\alpha = 0$ and 1/10 with $\beta = 0\ 00810$.

In Fig.3, measured spectral densities for large $\sigma^2 h/g$ attain to the equilibrium values but for small $\sigma^2 h/g$ measured densities are lower than the latter

This seems to be due to the fact that few waves are breaking by the effect of finite depth and only high frequency waves are saturated by local wind In Fig.4 measured energy densities almost attain to the equilibrium values, which mean that almost waves are breaking by the effect of finite depth

4 Estimation of limiting significant waves in surf zone

Assuming that eq 14 is the limiting spectral energy density at water depth h, the limiting spectrum for given wind conditions at the depth may be estimated by the following considerations

In Fig 5, suppose that various spectra are drawn in linear scale and the curve MN is equilibrium spectrum at given water depth When the spectrum of off-sea wind waves is given by the curve MWQ, the spectrum to be observed at the point of depth h should be given by the curve MCQ, which is the limiting spectrum for the given wind condition. When the spectrum of offsea swell is given by the curve ASQ, the spectrum at depth h should be the curve ABCQ, which is the limiting spectrum for given swell. Fig 6 shows an example of offsea wind wave spectrum for wind speed $U = 15$ m/s, fetch length $F = 400$ km, which is proposed by the authors as shown in Appendix, and equilibrium spectra at water depth $h = 8$ meters for bottom slope $\alpha = 0$ and 1/10

Fig 7(a)(b) are significant wave height and its optimum period of limiting waves calculated by above-derived limiting spectrum at various water depth for fetch length 100 km and 400 km with wind speed 15 m/s and 30 m/s

5 Conclusions and remarks

Omitting the effect of wave refraction and directional distribution of wave spectrum, the equilibrium spectrum of surf waves at water depth h is represented by eq 14, by means of which and a proposed fetch- or duration spectrum of offsea wind waves the limiting spectrum at any depth is estimated as shown in Fig 6

Up to date, for design purposes of surf zone structures the limiting wave height by such an equation 2 or 3 is frequently used as design wave height But in some cases, it seems to be more reasonable to use such a limiting spectrum as above The effect of directional distribution of wave spectrum should be considered in future

Appendix

Derivation of Duration and Fetch Spectrum

We assume the general form of frequency spectrum as follows

$$\Phi(\sigma) = \alpha_* g^2 \sigma^{-5} \exp\left\{-\beta_*\left(\frac{g}{\sigma U_*}\right)^4\right\} \tag{A1}$$

where g is gravity acceleration, U_* is friction velocity of wind and α_*, β_* are non-dimensional constants

Above spectrum has the maximum energy density $\Phi(\sigma_0)$ at frequency σ_0 as follows

$$\Phi(\sigma_0) = \alpha_* g^2 e^{-\frac{5}{4}} \sigma_0^{-5} \tag{A2}$$

$$\frac{\sigma_0 U_*}{g} = \left(\frac{4}{5}\beta_*\right)^{1/4} \tag{A3}$$

from which we obtain

$$\beta_* = \frac{5}{4}\left(\frac{\sigma_0 U_*}{g}\right)^4 \tag{A4}$$

$$\alpha_* = \frac{e^{5/4}}{g^2} \Phi(\sigma_0)\, \sigma_0^5 \tag{A5}$$

The time developement of the maximum energy density of wind generated-wave spectrum is given by Phillips(1966) as follows

$$\Phi(\sigma_0) = A \frac{U_*^4}{g^2} \frac{\sinh(\mu\sigma_0 t)}{\mu \sigma_0}, \quad A = 240\left(\frac{\rho_a}{\rho_w}\right) \approx 3.46 \times 10^{-4} \tag{A6}$$

where ρ_a and ρ_w are densities of air and water and μ is coupling coefficient of air-sea interaction

Substituting eq A4, A5 and A6 into eq A1, we obtain as duration spectrum

$$\Phi(\sigma, t) = A e^{5/4} \frac{U_*^4}{g^2}\left(\frac{\sigma_0}{\sigma}\right)^5 \frac{\sinh(\mu\sigma_0 t)}{\mu\sigma_0} \exp\left\{-\frac{5}{4}\left(\frac{\sigma_0}{\sigma}\right)^4\right\} \tag{A7}$$

Meanwhile, Pierson(1964) has proposed the following spectrum for fully-developed wind waves

$$\Phi(\sigma) = \alpha g^2 \sigma^{-5} \exp\left\{-\beta\left(\frac{g}{\sigma U}\right)^4\right\} \tag{A8}$$

where $\alpha = 0.00810$, $\beta = 0.74$ and U is mean wind speed at 19.5 meters high above mean sea surface, which is related to U_* and U_{10} (wind speed at 10 meters above mean sea level) by the following empirical relationships

$$U = K U_* \qquad K = 1.60 + \frac{1}{\sqrt{C_{10}}} \tag{A9}$$

$$U_{10} = \frac{U_*}{\sqrt{C_{10}}} \qquad C_{10} = (0.80 + 0.114\,U_{10}) \times 10^{-3} \tag{A10}$$

For any given wind speed, the maximum energy density $\Phi(\sigma_0)$ of duration spectrum should lie always on the saturated spectrum curve of that wind speed given by eq A8, so that equating eq A6 to eq A8, we obtain next relation for σ_0 at any duration time t

$$\frac{A}{\alpha} U_*^4 \left(\frac{\sigma_0}{g}\right)^4 \frac{\sinh \mu\sigma_0 t}{\mu} = \exp\left\{-\beta\left(\frac{g}{K U_* \sigma_0}\right)^4\right\} \tag{A11}$$

Eq A7 together with eq A11 gives duration spectrum

Fetch length χ is related to duration time t by the following equation

$$\chi = \frac{1}{2} C_0 t \tag{A12}$$

where C_0 is the phase velocity of waves with frequency σ_0

Thus, fetch spectrum is obtained from duration spectrum as follows

$$\Phi(\sigma,\chi) = A e^{5/4} \frac{U_*^4}{g^2} \left(\frac{\sigma_0}{\sigma}\right)^5 \frac{\sinh(2\mu\sigma_0\chi/C_0)}{\mu\sigma_0} \exp\left\{-\frac{5}{4}\left(\frac{\sigma_0}{\sigma}\right)^4\right\} \tag{A13}$$

By the relation $\sigma_0 = g/C_0$, duration and fetch spectrum is written as follows

$$\Phi(\sigma,t) = A e^{5/4} g^2 \sigma^{-5} \frac{\sinh\left(\mu \frac{gt}{U_*} \Big/ \frac{C_0}{U_*}\right)}{\mu} \cdot \frac{1}{(C_0/U_*)^4} \exp\left\{-1.25 \frac{(g\sigma/U_*)^4}{(C_0/U_*)^4}\right\} \tag{A14}$$

$$\Phi(\sigma,\chi) = A e^{5/4} g^2 \sigma^{-5} \frac{\sinh\left(2\mu \frac{g\chi}{U_*^2} \Big/ \left(\frac{C_0}{U_*}\right)^2\right)}{\mu} \frac{1}{(C_0/U_*)^4} \exp\left\{-1.25 \frac{(g\sigma/U_*)^4}{(C_0/U_*)^4}\right\} \tag{A15}$$

In above equations, coupling coefficient μ is given by Phillips(1966) as a function of C/U_* But when fetch spectrum is given by eq A15, the significant wave height $H_{1/3}$ is shown by following equation

$$\frac{g H_{1/3}}{U_*^2} = 4.00 \sqrt{\frac{A}{5}}\, e^{5/8} \left(\frac{\sinh\left(2\mu \frac{g\chi}{U_*^2} \Big/ \left(\frac{C_0}{U_*}\right)^2\right)}{\mu}\right)^{1/2} \tag{A16}$$

And also from observation data, significant wave height is empirically related to wind speed by the following equation by Wilson(1965)

$$\frac{g H_{1/3}}{U_*^2} = \frac{K_3}{C_{10}} \left[1 - \frac{1}{\left\{\sqrt{C_{10}} K_4 \left(\frac{g\chi}{U_*^2}\right)^{1/2} + 1\right\}^2}\right], \quad K_3 = 0.30,\ K_4 = 0.004 \tag{A17}$$

Equating above two equations, coupling coefficient μ is obtained for C_o/U_* with parameter U_* as shown in Fig A1 Fig A2 is an example of fetch spectrum

References

Phillips, O M (1958) " The Equilibrium Range in the Spectrum of Wind-Generated Waves " J Fluid Mechanics, Vol 4
Miche, A (1944) " Mouvements ondulatoires de la mer en profondeur constante ou decroissante " Annales des ponts st chaussues, Vol 114
Iversen, H W (1952) " Laboratory Study of Breakers " Gravity waves Nat Bur Standards Cir No 521
Pierson, W J and L Moskowitz(1964) " A Proposed Spectral Form for Fully Developed Wind Seas based on the Similarity Theory of S A Kitaigorodskii " J Geoph Res Vol 69, No 24
Ijima, T and T. Matsuo(1968) " Observation of Surf Waves by Stereo-Wave Meter " Coastal Engineering in Japan, Vol 11
Phillips, O M (1966) " The Dynamics of the Upper Ocean " Cambridge University Press
Wilson, B W (1965) " Numerical Prediction of Ocean Waves in the North Atlantic for December, 1959 " Deutsch Hydr Z Jg 18 Ht 13

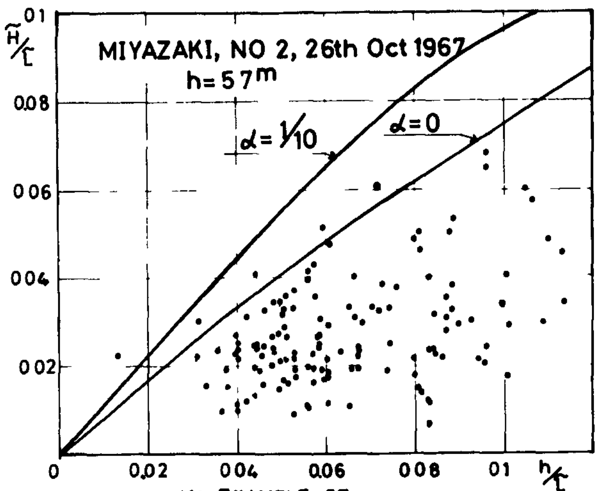

Fig -1 AN EXAMPLE OF SCATTER DIAGRAM

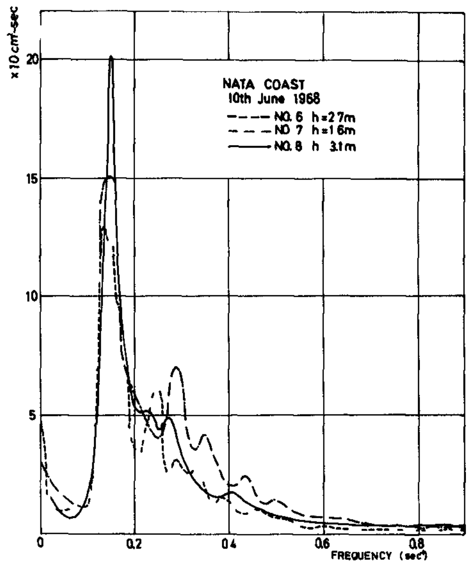

Fig 2 EXAMPLES OF FREQUENCY SPECTRA

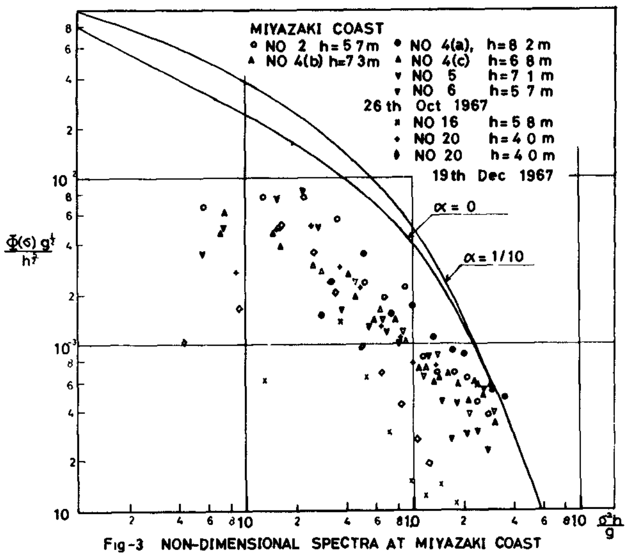

Fig-3 NON-DIMENSIONAL SPECTRA AT MIYAZAKI COAST

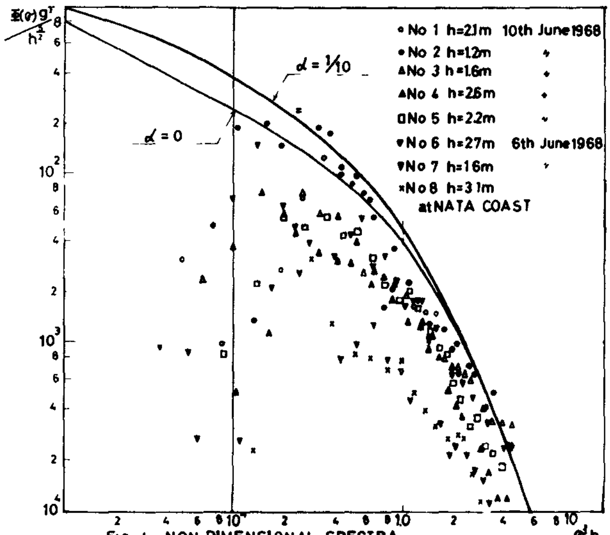

Fig-4 NON-DIMENSIONAL SPECTRA AT NATA COAST

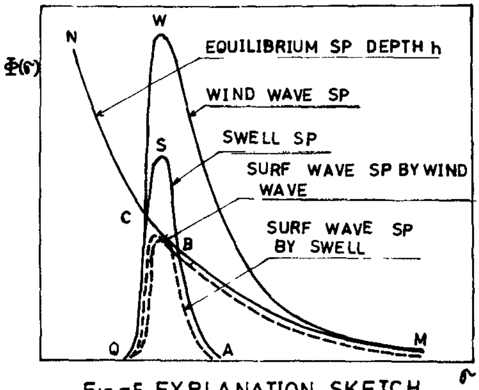

Fig -5, EXPLANATION SKETCH

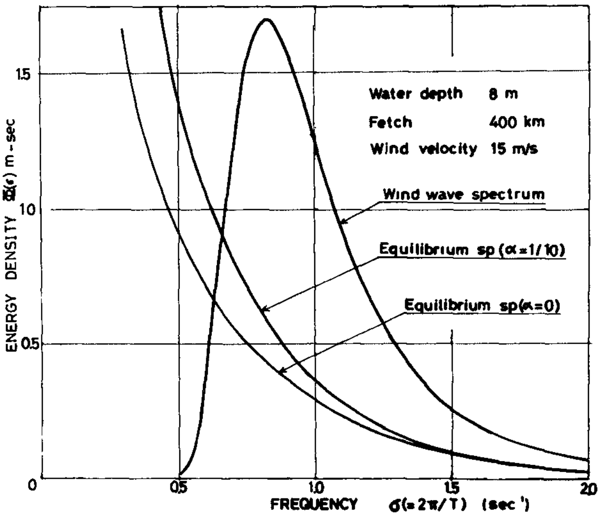

Fig -6 Limiting Spectra at Depth of 8 meters for an Offsea Wind Wave Spectrum

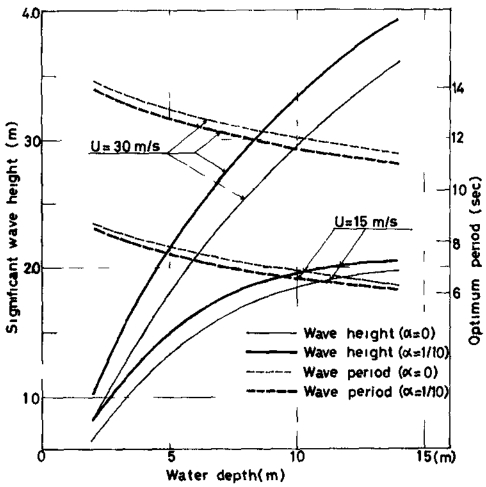

Fig-7(a) LIMITING SIGNIFICANT WAVES FOR FETCH LENGTH 100KM

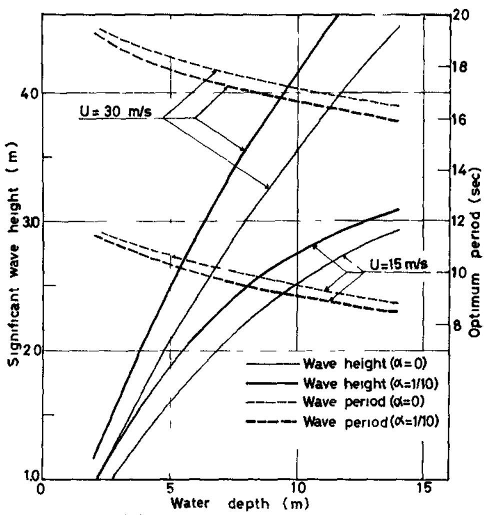

Fig -7(b) LIMITING SIGNIFICANT WAVES FOR FETCH LENGTH 400KM

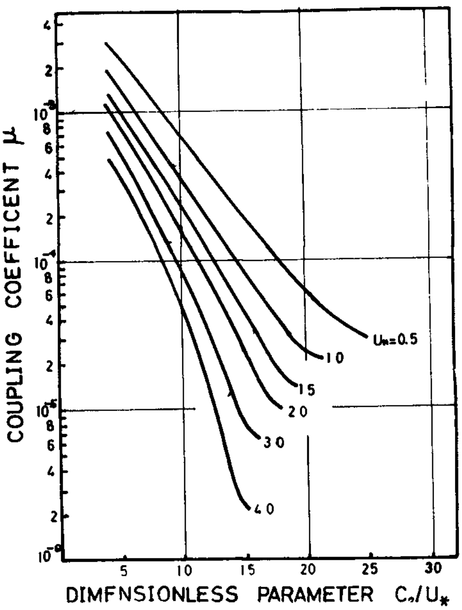

Fig-A1 RELATIONS ON μ AND C_0/U_* AND U_*

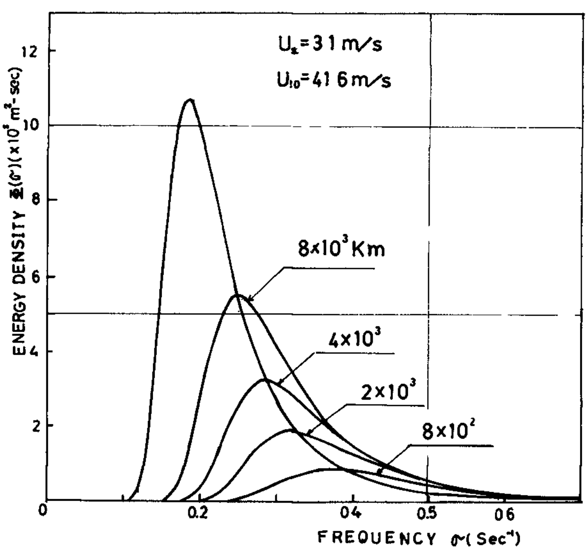

Fig-A2 FETCH SPECTRUM