

CHAPTER 91

COMPREHENSIVE TIDAL STUDY OF THE ST. LAWRENCE RIVER

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ABSTRACT

The St. Lawrence River is a major waterway connecting the Port of Montreal to the Atlantic Ocean. As is the case with so many natural rivers, extensive dredging has been necessary to provide adequate navigation conditions. About 100 years ago the Ship Channel of the St. Lawrence River was at several places only 10 ft deep and 150 ft wide; presently it has a minimum depth of 35 ft and a minimum width of 800 ft. However, further improvements are necessary, to meet present and future navigation requirements.

The Hydraulics Section of the National Research Council is studying improvement schemes by means of a hydraulic and a mathematical model of the St. Lawrence River from Montreal to Father Point, a distance of about 350 miles. The hydraulic model has been built to a horizontal scale of 1:2000 and a vertical scale of 1:120. The mathematical study uses a finite-difference approach, employing both explicit and implicit computational schemes.

This paper describes the two studies and deals briefly with the merits of operating a mathematical and hydraulic model simultaneously.

1.0 Introduction

The St. Lawrence River (Fig. 1) is the largest river in Eastern Canada. Its drainage basin above Quebec City amounts to some 400,000 square miles, which is comparable to the areas of Great Britain, France and West Germany combined. The Great Lakes on the border between Canada and the United States are part of the St. Lawrence River System. They provide large natural storage areas and, together with the extensive hydro-electric developments, tend to regulate the river flow. The mean fresh water flow at Quebec City is 320,000 cfs, the

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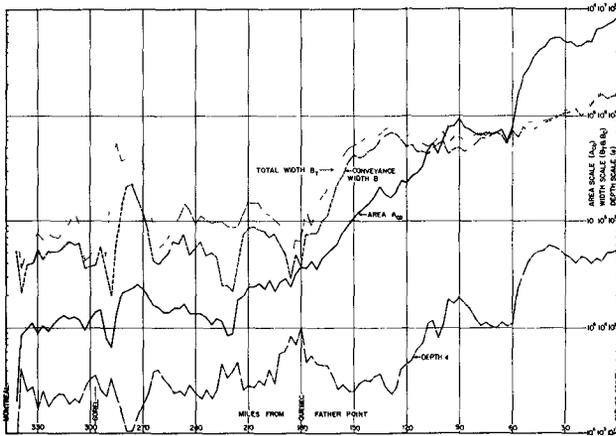


Fig. 2 Representative River Dimensions

The Department of Transport of the Canadian Government is responsible for the maintenance and improvement of the St. Lawrence Ship Channel. This must be accomplished without adversely affecting the water levels in Montreal Harbour or any other region. The Hydraulics Section of the National Research Council was asked to undertake a comprehensive study of the tidal propagation on the St. Lawrence River and to investigate the effect of proposed improvement schemes on the tide and water levels throughout the river system. Because of the importance of the problem, it was decided to conduct simultaneously, studies on a hydraulic and a mathematical model.

2.0 The Hydraulic Model

The Hydraulic Model of the St. Lawrence River at present comprises only the section from Montreal to Quebec City, a distance of 160 miles. Because of the size of the St. Lawrence River, the existing laboratory space was not sufficiently large to accommodate the entire model. While an extension to the building was under construction the

first half of the model was completed and a temporary tidal control mechanism installed at Quebec City. This allowed the calibration of the first stage of the model and also some preliminary testing.

The model has a horizontal scale of 1:2000 and a vertical scale of 1:120. In addition to the practical considerations of available floor space and required accuracy of measurements, a number of other reasons dictated the choice of these scales. For example the Lake St. Peter area is extremely shallow with low current velocities and to obtain turbulent flow in the model in this region, a high distortion is desirable. However, at other locations, the cross-sectional area and shape of the river bed changes quite abruptly; one such area is just upstream of Quebec City. With too high a distortion, it may become impossible to calibrate the model in regions of strongly varying cross sections, since the flow pattern in the model would be drastically different from that of the prototype.

The distortion of about 17, resulting from the choice of the horizontal and vertical scale of the St. Lawrence model, turned out to be close to the optimum. A smaller distortion would certainly have resulted in laminar flow conditions in Lake St. Peter, while a larger distortion would indeed have caused difficulties in the calibration of the model around Quebec City.

The first phase of the model construction was completed by the end of 1967.

The calibration of this section of the model took approximately 6 months. Two sets of boundary conditions were used. In the first case, a mean discharge was used with a spring tide, while in the second case, a nearly mean discharge was combined with a neap tide. A more complete calibration will be carried out next year, when the final section of the model from Quebec City to Father Point is completed. Calibration was achieved mainly by varying the density of the artificial roughness pattern. The roughness elements consist of thin, 3/4" wide aluminum strips. Figure 3 shows some results of the final calibration tests. At all places the water elevations in the model are within 0.10 ft of the recorded prototype water levels.

3.0 The Mathematical Model

During the course of the study several finite-difference mathematical models, both explicit and implicit, were developed to simulate the propagation of the astronomical tides on the St. Lawrence River. Of these only one will be described in this paper and for further details the reader is referred to previous publications by one of the authors (Refs. 1 and 2).

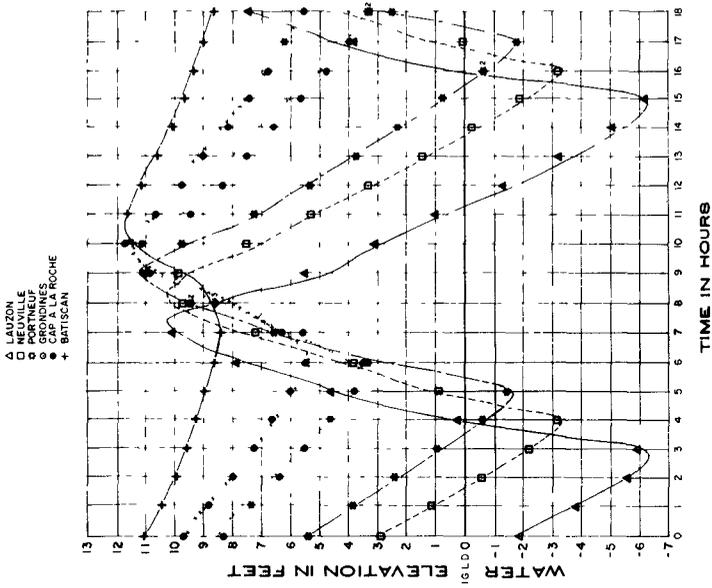
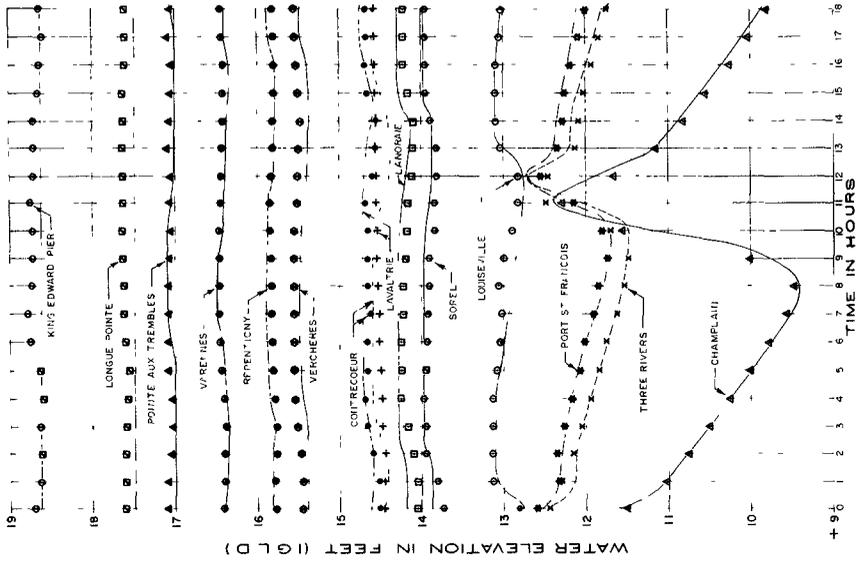


Fig. 3 Comparison of Prototype and Model Observations

The model described here is an implicit finite-difference scheme with only one island taken into account. This is a reasonable simulation since the dimensions of the other islands are small compared with the river dimensions. The model limits are Father Point and Montreal as in the hydraulic study. The river is divided into 204 longitudinal sections. Some of these sections are three miles long, while others, in regions where the river geometry varies considerably, are one mile long. The numbering scheme around Orleans Island is shown in Fig. 4; the section numbers have been circled and 52, 62 and 72 are skipped in the numbering system. The remainder of the numbering scheme is self evident. Section 1 is at Father Point, Section 207 at Montreal.

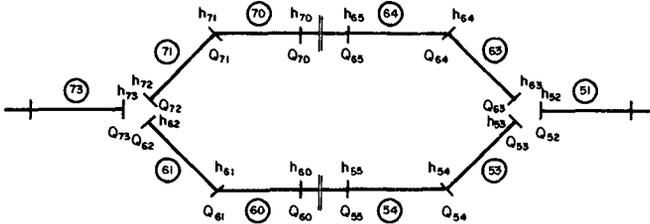


Fig. 4 Nomenclature at Orleans Island

3.1 Equations: The equations considered in this study are the equations of continuity and of motion for one-dimensional unsteady flow in open channels. These may be written respectively as:

$$\frac{\partial Q}{\partial x} + B_S \frac{\partial h}{\partial t} - \frac{Q_T}{dx} = 0 \quad (1)$$

$$\frac{\theta_1}{gA} \frac{\partial Q}{\partial t} - \left[\theta_1 \frac{\partial A}{\partial h} + \theta_2 B_S \right] \frac{Q}{gA^2} \frac{\partial h}{\partial t} - \frac{\theta_2 Q^2}{gA^3} \frac{\partial A}{\partial x}$$

$$= - \frac{\partial h}{\partial x} - \frac{Q|Q|}{C^2 A^2 d}$$
(2)

where Q is the river discharge, h the water level with respect to a certain datum (IGLD-1955), Q_T the tributary inflow, B_S the surface or storage width, θ_1 and θ_2 the momentum and energy coefficients of velocity distribution, A the conveyance area, g the acceleration due to gravity, C the Chezy friction factor and d the appropriate depth. It is evident that some simplifying assumptions have been made in the derivation of these equations.

These differential equations may be written as difference equations for each section:

$$-h'_m + \beta_m Q'_m + h'_{m+1} + \alpha_m Q'_{m+1} = \gamma_m$$
(3)

$$\delta_m h'_m - Q'_m + \delta_m h'_{m+1} + Q'_{m+1} = \epsilon_m$$
(4)

where the dash refers to the values of h and Q at the new time level $t = t + \tau$, τ being the time step chosen and where α , β , γ , δ , and ϵ are coefficients which are functions of the known tidal conditions at time $t = t$ and of the section dimensions resulting from the schematization process.

When these equations are written for each section and combined with usual conditions at the junction points and with the boundary conditions, $f_1(t)$ and $f_2(t)$, the water level variation at Father Point and the upland flow at Montreal respectively, a set of simultaneous equations resulting in a matrix as shown in Fig. 5 may be set up.

3.2 Schematization: In a finite-difference scheme as described here, the river has to be schematized into a number of sections each with its own constant dimensions. From the charts and from available dredging records it was possible to discern two distinct levels, the chart datum level and the level above which the ground is never inundated (Fig. 6). The river widths at these two levels were determined, B_{CD} is the width at chart datum, and B_T the total width. A third width, B_C , was also taken at the 6 foot depth contour, the width of the conveyance channel. These widths were obtained by planimetering the appropriate surface area for each section. In addition to these widths, the correct mileage from the reference point (Father Point), the cross-sectional area below chart datum (total river volume in the

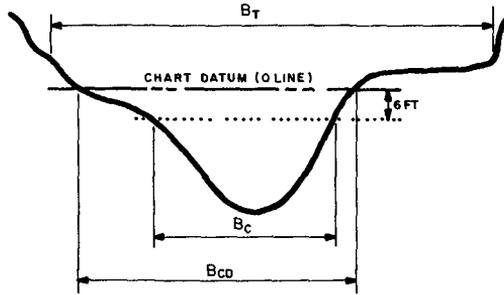


Fig. 6 Definition of Widths

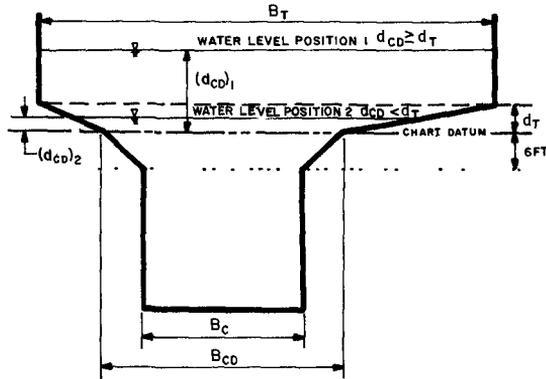


Fig. 7 Adopted Schematization

In the areas away from the island, all values in the matrix of Fig. 5 appear along the major diagonal. In this case it is possible to reduce Eqs. (3) and (4) to

$$Q'_m + q_m h'_{m+1} + t_m Q'_{m+1} = s_m \quad (5)$$

$$h'_{m+1} + p_{m+1} Q'_{m+1} = r_{m+1} \quad (6)$$

where:

$$q_m = \frac{1}{\beta_m + p_m}, \quad t_m = \frac{\alpha_m}{\beta_m + p_m}, \quad s_m = \frac{\gamma_m + r_m}{\beta_m + p_m} \quad (7)$$

$$p_{m+1} = \frac{1 + \zeta_m t_m}{\delta_m + \zeta_m q_m}, \quad r_{m+1} = \frac{\epsilon_m - \delta_m r_m + \zeta_m s_m}{\delta_m + \zeta_m q_m} \quad (8)$$

and where

$$\zeta_m = 1 + \delta_m p_m \quad (9)$$

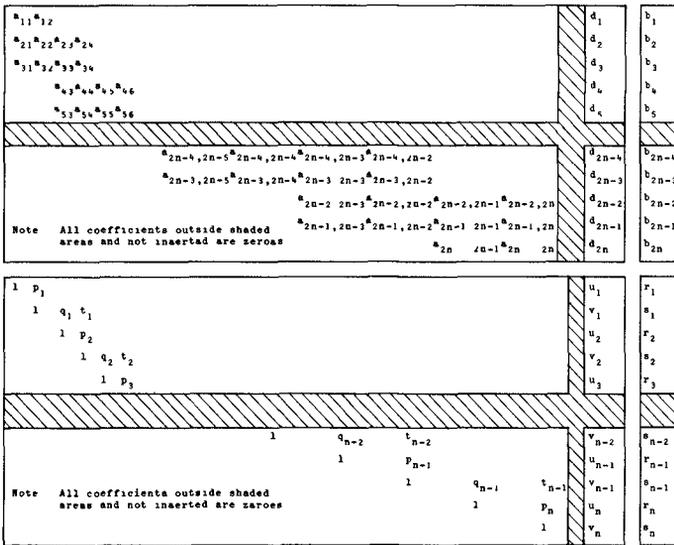


Fig. 8 Triangularization of a Matrix with an Additional Column

In the areas with junctions, the problem of triangularization is more complex since some values appear quite a distance away from the major diagonal. For the case where a value appears to the right and

above the diagonal it is possible to extend the method developed above. Referring to Fig. 8, the top matrix with an additional column denoted by d_n , may be reduced to the lower matrix using the relationships:

$$\begin{aligned}
 q_i &= \frac{a_{2i,2i+1}}{a_{2i,2i} - a_{2i,2i-1} p_1} ; t_1 = \frac{a_{2i,2i+2}}{a_{2i,2i} - a_{2i,2i-1} p_1} \\
 s_i &= \frac{b_{2i} - a_{2i,2i-1} r_1}{a_{2i,2i} - a_{2i,2i-1} p_1} , v_1 = \frac{d_{2i} - a_{2i,2i-1} u_1}{a_{2i,2i} - a_{2i,2i-1} p_1} \\
 p_{i+1} &= \frac{a_{2i+1,2i+2} + \zeta_1 t_1}{a_{2i+1,2i+1} + \zeta_1 q_1} ; r_{i+1} = \frac{b_{2i+1} - a_{2i+1,2i-1} r_1 + \zeta_1 s_1}{a_{2i+1,2i+1} + \zeta_1 q_1} \\
 u_{i+1} &= \frac{d_{2i+1} - a_{2i+1,2i-1} u_i + \zeta_1 v_1}{a_{2i+1,2i+1} + \zeta_1 q_i} \\
 \zeta_1 &= -a_{2i+1,2i} + a_{2i+1,2i-1} p_1 \\
 p_1 &= \frac{a_{12}}{a_{11}} , r_1 = \frac{b_1}{a_{11}} \text{ and } u_1 = \frac{d_1}{a_{11}}
 \end{aligned}
 \tag{10}$$

When values of an additional row appear below the band, the matrix is first triangularized and these values subsequently eliminated as in Fig. 9, using the relationships:

$$\begin{aligned}
 c &= C_{2n-1} + (-1)^n q_{n-1} L_{n-1} \\
 e &= C_{2n} + (-1)^n t_{n-1} L_{n-1} \\
 f &= (-1)^n v_{n-1} L_{n-1} + \sum_{i=1}^{n-1} (-1)^i W_1 \\
 g &= (-1)^n s_{n-1} L_{n-1} + \sum_{i=1}^{n-1} (-1)^i P_1 , \tag{11}
 \end{aligned}$$

where

$$\begin{aligned}
 L_{i+1} &= (-1)^{i+1} (C_{2i+2} - C_{2i+1} p_{i+1}) + (t_1 - q_i p_{i+1}) L_i \\
 P_{i+1} &= (-1)^{i+1} C_{2i+1} r_{i+1} + (s_1 - q_i r_{i+1}) L_i
 \end{aligned}$$

$$W_{i+1} = (-1)^{1+i} c_{2i+1} u_{i+1} + (v_1 - q_1 u_{i+1}) L_i$$

$$L_1 = c_1 p_1 - c_2$$

$$P_1 = c_1 r_1 - b_{2n+1}$$

$$W_1 = c_1 u_1 - d_{2n+1} \tag{11}$$

With these basic tools it is possible to solve the matrix in Fig. 5 and for illustration the successive steps will be given.

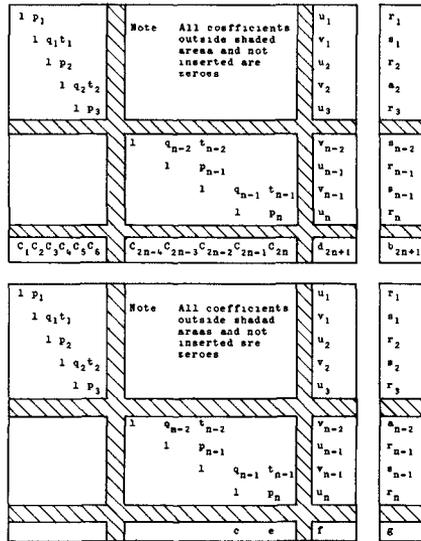
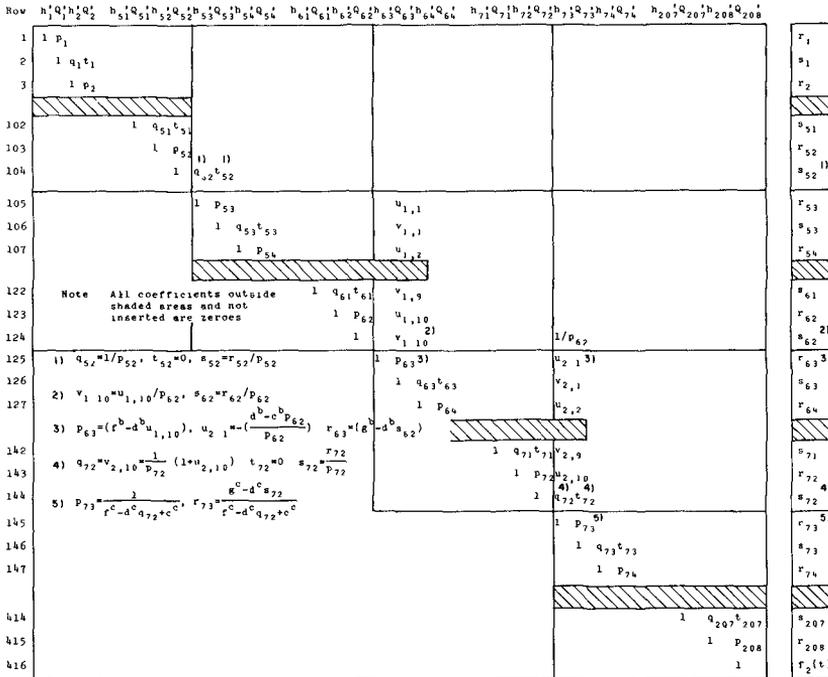


Fig. 9 Triangularization of a Matrix With an Additional Row

Triangularize to row 103 of Fig. 5 using Eqs. (5) and (6). The values slightly off the band in rows 104 and

105 may be easily eliminated. Using Eqs. (10), row 105 to 123 may be triangularized and again the h_{62} term in row 124 may be eliminated. Eliminate the h_{63} term in row 125 using (11) and then eliminate the ch_{62} and eQ_{62} terms. Similarly, using (10) rows 125 to 143 may be triangularized, and the Q_{62} term in row 145 may be eliminated using (11). The remainder of the matrix is again a single channel and may be reduced to upper triangular form using (5) and (6). The result is the matrix in Fig. 10.



this particular problem, the growth of errors was not of a magnitude as to cause this type of instability.

The time required for a typical solution was 12 minutes on an IBM 360 model 50 computer. This includes rather lengthy plotting routines.

Some results as they compare with observed prototype data are shown in Figs. 11, 12 and 13, and here it may be seen that the model results correspond quite closely to the field results, the water levels being within 0.1 ft and the time within 15 minutes. Water levels only could be compared since field current observations are at present not available.

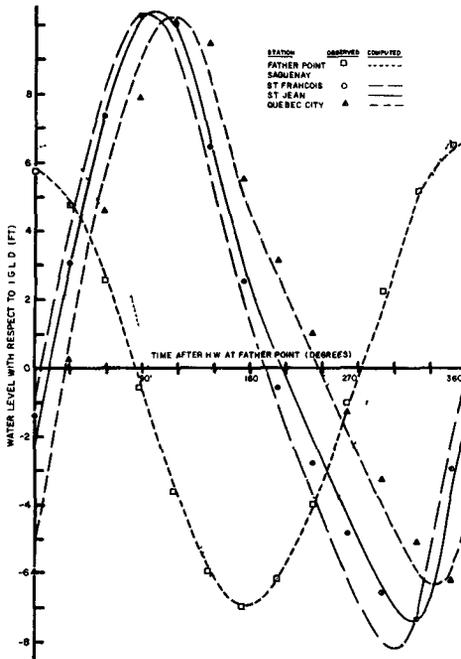


Fig.11 Comparison of Observed and Computed Water Levels

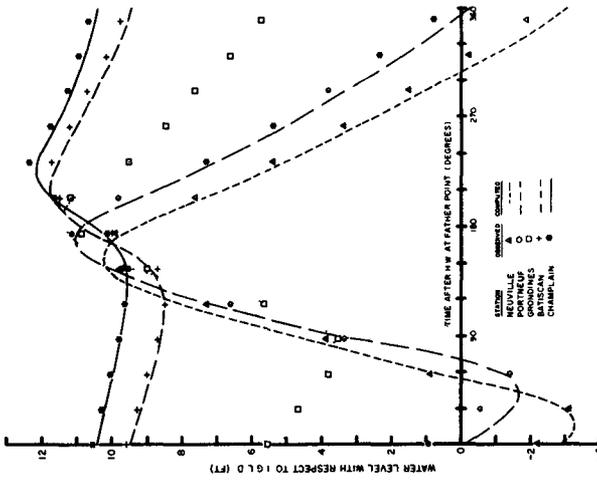


Fig. 12

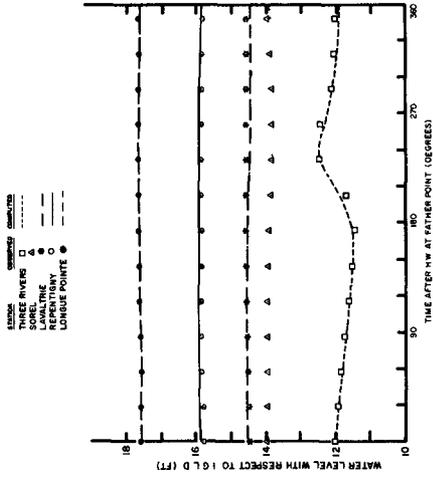


Fig. 13

Comparison of Computed and Observed Water Levels

4.0 Testing Procedures and Results

After the calibrations of the hydraulic and the mathematical model were completed, a series of tests was carried out to study the effect of some of the navigation improvement schemes. First of all, the mathematical model had shown that the change in the tidal conditions at Quebec City as a result of most of the proposed plans was nil or very small. Only the major improvement schemes, involving extensive modifications to the existing river conditions, affected the tidal propagation throughout the system. In such cases the present hydraulic model can no longer be used without changing the downstream boundary conditions. The mathematical model was used to supply the new boundary conditions for the hydraulic model.

Another demonstration of the way in which the two models complement each other involves the generation of the tide at Quebec City in the hydraulic model. The basic input to the hydraulic model is the horizontal tide at Quebec City. This is accomplished with a variable inflow, which either supplies or withdraws the tidal discharge of up to 1.5 cfs. Since no actual discharge measurements are available for the calibration condition, the mathematical model supplies this input to the control circuit of the hydraulic model. A 10 ft wide weir controlled by a feedback circuit connected to the vertical tide, acts as the final trimming device to give the exact tidal profile.

The first improvement schemes which were studied on both models, were the so-called Lake St. Peter plans. One of the plans under consideration, calls for the construction of dykes along the navigation channel through the lake. Lake St. Peter is a large shallow body of water, which now acts as an effective filter influencing the conditions upstream as well as downstream of the lake. In addition, the conditions in winter time on the St. Lawrence are very adversely affected by this shallow lake.

The first series of tests included only the construction of dykes, without changing any of the other conditions. Obviously, this is mainly of academic interest since the new cross sectional area is only 1/5 of the original area and therefore the current velocities in the new channel will be unacceptably high. However, the tests show clearly the importance of Lake St. Peter in regard to its effect on the tidal propagation in the river. Figure 14 shows the envelopes of the high and low water elevations as observed on the hydraulic model and computed on the mathematical model for a spring tide with average upland flow, with and without dykes in Lake St. Peter. The water levels

upstream of the lake have risen as much as 4 ft at Sorel and about 2.5 ft at Montreal, while just downstream of the lake the tidal range has increased considerably. The slight differences between the results of the hydraulic and the mathematical model are caused by the schematization of the delta at the upstream side of Lake St. Peter.

The second series of tests included not only the dykes along the channel, but also control structures which allow a certain amount of the flow to bypass the navigation channel over the north and south banks of the lake. The percentage of water to be diverted is governed by the maximum velocity allowed in the channel, which is in the order of 4 ft/sec. The dotted line in Fig. 14 shows the water levels resulting from a test which included this diversion of flow.

The St. Lawrence Tidal Study has shown encouraging results so far. Having the two models operating simultaneously has indeed proven to be a great advantage and will certainly continue to be of assistance when the section of the St. Lawrence River from Quebec City to Father Point is included in the hydraulic model.

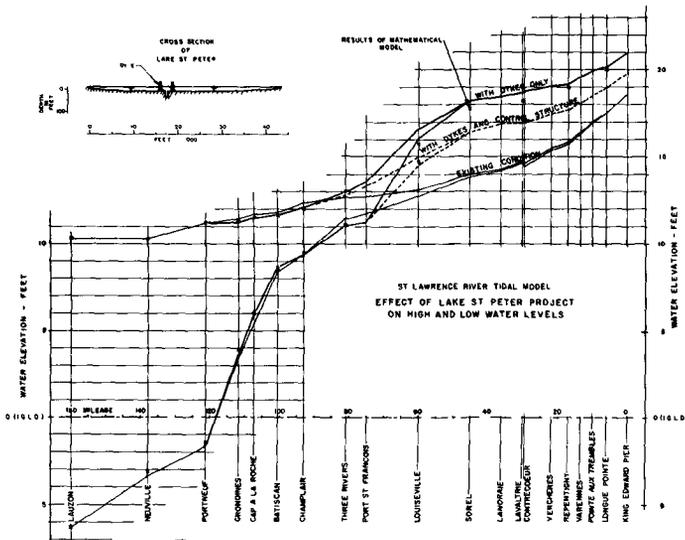


Fig. 14 Effect of Lake St. Peter Project on High and Low Water Elevations

5.0 Acknowledgments

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