CHAPTER 53

VELOCITIES IN DOWNRUSH ON RUBBLE MOUND BREAKWATERS

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ABSTRACT

This paper is a sequence to two papers, (1) and (2), previously read at Coastal Engineering Conferences. They presented a mathematical model for estimating displacements, velocities and accelerations in a downrushing wave on a rubble mound breakwater slope. Verification by photographic recording of displacements involved some uncertainties. In the present paper are reported measurements of velocities, which gave much more definite data. The correlation with velocities calculated from the model is shown to be good.

INTRODUCTION

At the VIIIth Conference on Coastal Engineering Brandtzæg (1) presented a mathematical model intended for estimating roughly the displacements, velocities and accelerations in the downrushing water on a rubble mound breakwater slope. A very few experiments indicated fair agreement between observed values and those calculated from the model.

At the Xth Conference Brandtzæg and Tørum (2) presented data from a greater number of waves of different heights and periods on slopes of 1:1,25, 1:1,5 and 1:2,0. Displacements calculated from the model were found to agree reasonably well with the observed ones, although the scatter was quite considerable. This was to be expected, since displacements had to be deduced from photographs of the wave surface profiles. As these were often rather irregular, the deduction became difficult and involved some uncertainty.

Therefore it was highly desirable to measure velocities directly, and the more so because in application of the model, velocities and accelerations are the interesting quantities. However, getting reliable readings of velocities in the rapidly accelerated downrushing water is difficult. But recently such measurements have been made, and the results are presented here. They are compared with velocities calculated from the model, partly by use of the particular values of z and β (Fig. 3) found for each wave dimension and slope, as in reference (1) (Calculation 1)and partly by use of more general values of z and β, permitting calculation without previous knowledge of specific experimental data, as in reference (2) (Calculation 2).

VELOCITY MEASUREMENTS

The tests were made in the wave channel shown in Fig. 1. The wave generator has an ordinary piston type paddle adjustable as to wave height and wave period. The wave gage was placed about 11 m in front of the model. Vertical surface displacement at this point was recorded on a Sanborn paper recorder.

The model itself was a wooden platform with a sloping front on which the cover blocks were placed on top of a layer of smaller stones, about 5 cm thick. As the stability of the armour was not at issue in these tests, the blocks were held in place with a few nails, none of which protuded above the top of the blocks.

In the tests of ref. (1) and (2) the cover blocks were arbitrarily placed with regard to orientation as well as location. In the present tests, however, it was found necessary to have a slightly smoother breakwater front (Fig. 2), so as to avoid too much air bubbles in the downrushing water, as this tended to blur the photographic recording described below.

As stated in ref. (1) the mathematical model applies to the downrushing wave only as far down as the stream is not influenced by the incoming next wave. From the wave profiles (Fig. 8) and also from the direction of velocities indicated by confetti in the water (see ref. 1, Fig. 5) it is seen that this condition holds good also somewhat below the SWL. In order to extend as far as possible the time interval during which velocity readings could be obtained, the limiting line at which velocities were measured (point A) and calculated, was shifted from the line M - N, as used in ref. (1) and (2), to the line P - Q, Fig. 3 and Fig. 8.

An Armstrong-Whitworth Miniature Current Flowmeter, Type 176/1 was placed 10 cm from the glass wall of the channel. It had a very light plastic propeller with outer diameter 1,5 cm. The range of reliable registration in a steady current covered velocities from 2 to 150 cm/sec. The instrument apparently worked satisfactorily also in unsteady currents. The propeller movements were recorded by means of a motion picture camera making some 800 - 900 exposures pro second, while a time mark was made on the film every 1/100 second.

The test programme included three slopes of the breakwater face, and with each slope three wave periods, each with two wave heights, in total 18 waves (Table 1). Due to difficulty with the camera one wave, No. 6, dropped out.

Velocities were measured during downrush of one of the first waves arriving at the model front after the generator had been started and a reasonably stable wave height had been attained. First a trial wave train was run. From the registration diagram (Example in Fig. 4a) was picked out the wave to use, and its number (n) from the front of the wave train was noted. After rest the wave generator was started again and when the nth wave arrived at the model, the fast motion camera was started. Mostly the third or fourth wave was used (Fig. 4 b).

Beside the fast camera film, giving the velocities, another motion picture film was needed to give the wave surface profiles required for determination of β and z (Fig. 3 and 8). As the two exposures could not be made similtaneously, a third wave train was run for this purpose (Fig. 4 c), and photographs taken with a camera making about 24 exposures a second. As in earlier tests, the time between exposures could be read off on the clock shown in Fig. 2. The surface profiles were drawn up, and z determined as described in ref. (1) page 456, and the values plotted against time, t, after start of downrush, in diagrams of which examples are shown in Fig. 5, 6 and 7. Smooth curves for β and z were drawn for each wave.

From the fast film strips the number of revolutions pro second of the propeller could be ascertained. From calibration curves the corresponding velocities were taken off and plotted against t, time from start of downrush. Examples are shown in Fig. 9 to 17.

Velocities measured in this way will be roughly average values. An effort was also made to register velocities by means of confetti in the water. Only a few data were obtained. They showed great variations, as was to be expected, but seemed to group themselves fairly well around the propeller-observed values.

CALCULATION 1.

Velocities calculated with β- and z-values as observed in the tests.

The definition of the mathematical model here employed and the underlying assumptions and simplifications have been set out in detail in ref. (1) and briefly summarized in ref. (2). For ease of reference the basic consept of motion is restated here (Fig. 3):

1) The body of downrushing water is considered as a triangle. That is, the surface profile is assumed to be a straight

line forming an angle β, with the breakwater front and an angle, δ, with the horizontal.

2) The triangular body is divided into individual slices, "s_u". Each slice is defined by its original distance, u, from the top, 0, of the triangle.

3) Each individual slice is taken to move integrally and independently, without regard to continuity of the fluid, but otherwise in accordance with the gravity, the pressures and the boundary resistance, frictional and inertial, acting in the fluid.

As shown in (1) and (2), the assumption that the "slices" move independently of each other should not be expected to cause significant error in the calculations, because restoring the continuity need not to any considerable degree alter the general picture of the motion.

The model leads to the following equations for the displacements, x, velocities, v, and accelerations, a, as functions of the time, t, from the start of downrush:

$$x = B^2 \ln(\mathrm{Cosh}(\frac{A}{B} t)) \qquad (1)$$

$$v = AB \cdot \mathrm{Tanh}(\frac{A}{B} t) \qquad (2)$$

$$a = \frac{A^2}{\mathrm{Cosh}^2(\frac{A}{B} t)} \qquad (3)$$

$$A^2 = \frac{g(\sin\alpha - \tan\beta \cdot \cos\alpha)}{1 + 0{,}5C_{MP}\frac{k}{z}} \qquad (4)$$

$$B^2 = (1 + 0{,}5C_{MP}\frac{k}{z}) \cdot 32 \cdot z \cdot (\log_{10}\frac{5z}{k})^2 \qquad (5)$$

Figures for C_{MP}, cover block dimension, k, and β and z (Fig. 1) are required for the calculation. Like in ref. (2) C_{MP} = 1,0 has been used here. This figure seemed reasonable on the basis of data given by Wiegel (3), Johansson (4) and others. Actually a small variation in C_{MP} does not greatly affect the calculation of velocities, as seen in Fig. 9, 12 and 15 where a value of 0,75 has been used for comparison.

In the tests of ref. (1) and (2) the characteristic linear dimension, k, of the block was defined by assuming the average volume of the blocks to be 0,5 k^3. As a slightly smoother breakwater front had to be used in the present tests, a somewhat reduced value, k = 4 cm was used in the calculations. Again, a change in k does not much affect the calculated velocity. Changing k from 4 to 3 cm causes a change of v about as great as that caused by reducing C_{MP} from 1,0 to 0,75.

For want of something better, the ordinary Prandtl equation for frictional boundary resistance was used in ref. (1), although the roughness of a rubble mound breakwater front certainly is very different from the "sand roughness" of Nikuradse. In ref. (2), however, the figure 14,8 in the last parenthesis of Eq. (5) was changed from 14,8 to 5, based on a note given in ref. (5). The same figure is used here.

The parameters β and z, important in Eq. (1) to (5), define the shape of the individual waves. How to determine appropriate figures for these parameters must depend on the objective aimed at. As the objective of Calculation 1 is to probe the possible merits of our mathematical model by comparison with specific tests, the values of β and z have been taken as nearly as possible representative of the particular waves in which velocities were measured.

Consequently, values of β and z were taken off curves like those in Fig. 5, 6 and 7 and used in CALCULATION 1. Individual values of β and z rarely deviated more than 10 per cent from the curves, corresponding to a deviation in calculated velocity of the order of 5 per cent.

With the parameter values discussed above, velocities in downrush have been calculated from Eq. (2), (4) and (5) and the resulting curves entered in the v - t - diagrams for each of the 17 waves tested (Fig. 9 to 17). On the whole the curves seem to agree fairly well with the plotted test data. Only for the three waves 10,15 and 16 do the measured velocities near the end of downrush exceed the calculated ones by some 25 to 50 per cent.

CALCULATION 2.

Velocities calculated with β- and z-values derived from general relationships.

Practical use of our mathematical model should permit estimating the order of magnitude of the velocity in downrush of a known wave on a known rubble mound breakwater slope within the range of slope steepness considered here. In such a case the appropriate β- and z-values are not known from specific tests, as in Calculation 1, but must be derived from general relationships.

The data plotted in Fig. 5 of ref. (2) indicated the following relationship between β, α and wave steepness H/L . $\beta = \alpha - \delta$, and $\delta = 6{,}56$ H/L. Similar data from the present tests are plotted in Fig. 18. They show considerably more scatter than the former. The line $\delta = 7{,}67$ H/L seems to give about the best fit. If the previous data are taken into account, an overall average value of $\delta = 7{,}0$ H/L may cover the field fairly well. However, in the present calculation $\delta = 7{,}67$ H/L has been used.

With β known, the z of any slice (Fig. 3) is given by $z = u \tan \beta$, and the z of the first slice to pass the SWL at the start of downrush is

$$z = l_{uo} \tan\beta = \frac{R}{\sin \alpha} \tan \beta$$

The uprush, R, is generally considered as being roughly proportional to the wave height, although it is surely influenced also by factors like steepness and roughness of the slope, etc. In the earlier tests, an average value of R was found to be 1,23 H. ((2) p. 980). The R - H relationship found in the present tests is shown in Fig. 19. The scatter is great. The average value, R = 1,36 H, has been used in Calculation 2, corrected for the fact that velocities were measured at the line P - Q, intersecting the slope at a point 4 cm (vertically) below the SWL. The z-value of the first slice to pass the line P - Q at the start of downrush therefore is taken to be:

$$z = \frac{R + 4 \text{ cm}}{\sin \alpha} \tan \beta$$

From these relationships velocities at the P - Q line have been calculated from Eq. (1) to (5) with otherwise the same parameters as used in Calculation 1. As explained in (2), page 981, the calculation must be done by iteration.

The resulting velocity curves have been entered in the v - t - diagrams in Fig. 9 to 17 for comparison with the test data and the curves from Calculation 1. It is seen that Calculation 2 in a number of cases gives somewhat lower velocities than both Calculation 1 and the tests, the difference being most pronounced towards the end of the downrush.

CONCLUSIONS

The comparison between observed and calculated velocities has been summarized in Fig. 20 for Calculation 1, and in Fig. 21 for Calculation 2. It is felt that the former shows about as good agreement between calculation and measurement as can reasonably be expected in this case. Calculation 2 gave velocities somewhat lower than the measured ones, in particular towards the end of the downrush. Probably some adjustment of the R- and δ- relationships would give better agreement.

In total it is concluded that the mathematical model presented in ref. (1) and (2) does provide a means by which velocities in downrush on a rubble mound breakwater slope may be roughly estimated. Discrepancies are hardly avoidable, but Calculation 1 indicates that these may be due, not as much to the mathematical model itself, as to the difficulty of predicting more accurately the values of R and δ in actual cases.

REFERENCES

(1) Anton Brandtzæg: "A Simple Mathematical Model of Wave Motion on a Rubble Mound Breakwater Slope", Proceedings, Eighth Conference on Coastal Engineering, 1963, p. 444 ff.

(2) Anton Brandtzæg and Alf Tørum: "A Simple Mathematical Model of Wave Motion on a Rubble Mound Breakwater Slope", Proceedings of the Tenth Conference on Coastal Engineering, 1966, p. 977 ff.

(3) Robert L. Wiegel. "Oceanographical Engineering" Prentice-Hall International Inc., London, 1965, pp. 269 and 270.

(4) Børje Johansson: "Vågkrafter mot en på havsbottnen liggande circulär rörledning", Institutionen för Vattenbyggnad, Kungliga Tekniska Högskolan, Stockholm, 1965.

TABLE I

Scope of tests

cot α = 1,5				cot α = 1,25				cot α = 2,0			
Wave no	H cm	T sec	H/L	Wave no	H cm	T sec	H/L	Wave no	H cm	T sec	H/L
1	15.0	1,5	0,0481	7	15,0	1,5	0,0481	13	15,0	1,5	0,0481
2	18,5	1,5	0,0593	8	18,5	1,5	0,0593	14	18,5	1,5	0,0593
3	14,0	1,9	0,0323	9	14,0	1,9	0,0323	15	14,0	1,9	0,0323
4	19,5	1,9	0,0452	9A	15,0	1,9	0,0347	15A	14,5	1,9	0,0336
5	15,0	2,3	0,0274	10	19,0	1,9	0,0440	15B	15,0	1,9	0,0345
				11	15,5	2,3	0,0283	16	18,5	1,9	0,0429
				12	19,5	2,3	0,0355	17	15,5	2,3	0,0283
				12A	19,5	2,3	0,0355	17A	15,5	2,3	0,0283
								17B	15,5	2,3	0,0283
								18	19,5	2,3	0,0355

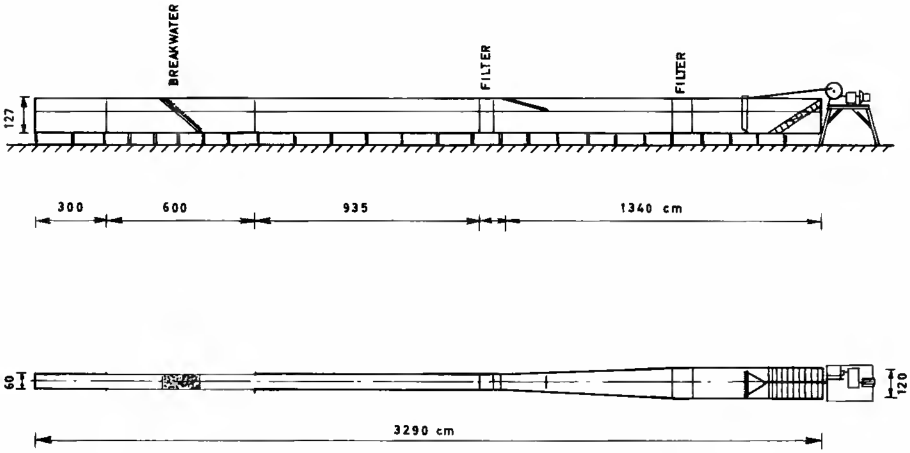

BREAKWATER
FILTER
FILTER
127
300
600
935
1340 cm
60
120
3290 cm

FIG. 1

FIG. 2

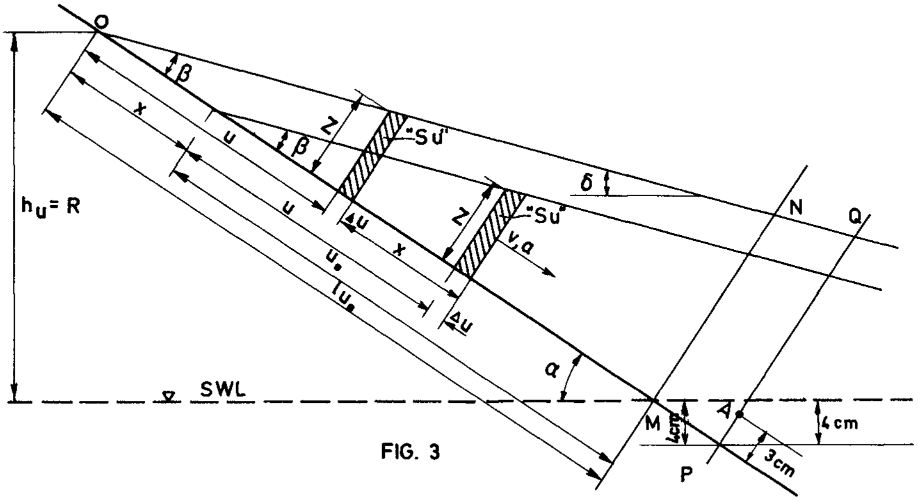
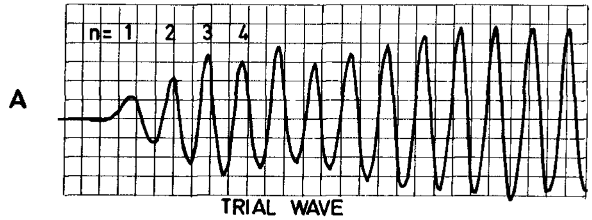
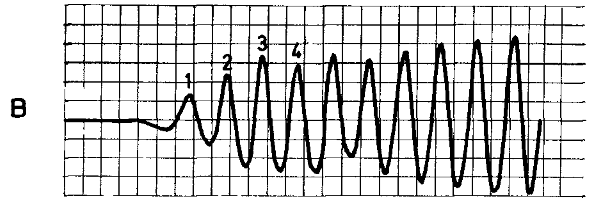


FIG. 3

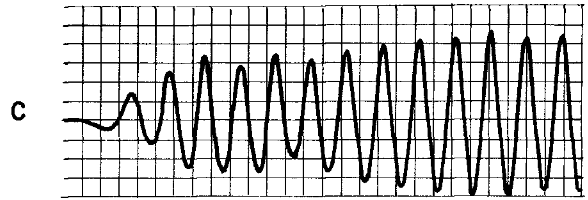
WAVE 3 H =14 cm T= 1,9 sec cot α =1,5



TRIAL WAVE



VELOCITY MEASURED AT n = 4



WAVE PROFILES TAKEN AT n=4 (see fig 8)

FIG. 4

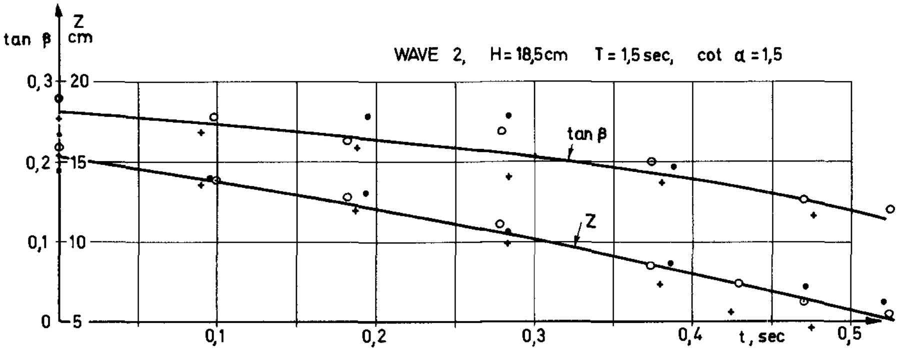

tan β
Z cm
WAVE 2, H = 18,5 cm T = 1,5 sec, cot α = 1,5
0,3
0,2
0,1
0
20
15
10
5
tan β
Z
0,1
0,2
0,3
0,4
t, sec
0,5

FIG 5

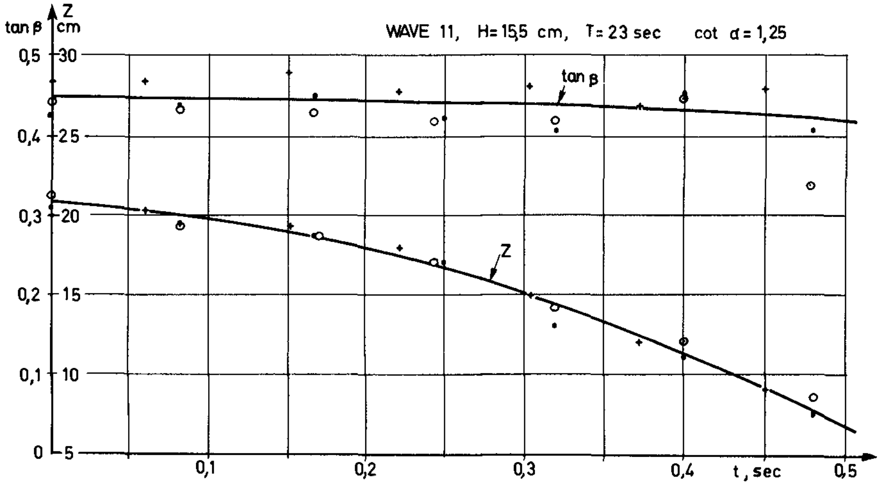

tan β
Z cm
WAVE 11, H = 15,5 cm, T = 2,3 sec cot α = 1,25
0,5
0,4
0,3
0,2
0,1
0
30
25
20
15
10
5
tan β
Z
0,1
0,2
0,3
0,4
t, sec
0,5

FIG 6

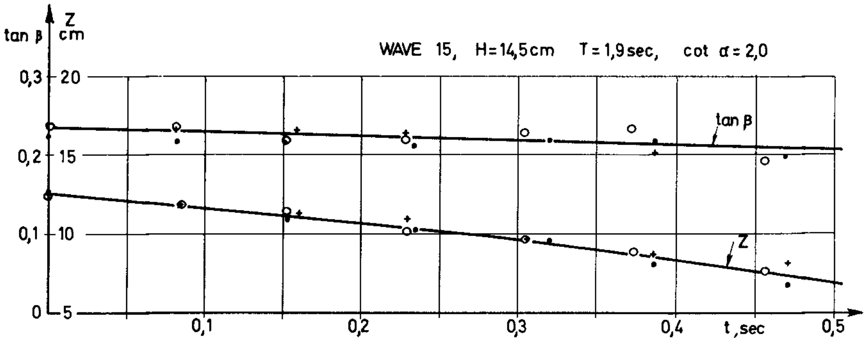

tan β
Z cm
WAVE 15, H=14,5cm T=1,9sec, cot α=2,0
0,3 20
0,2 15
0,1 10
0 5
0,1
0,2
0,3
0,4
t ,sec
0,5
tan β
Z

FIG 7

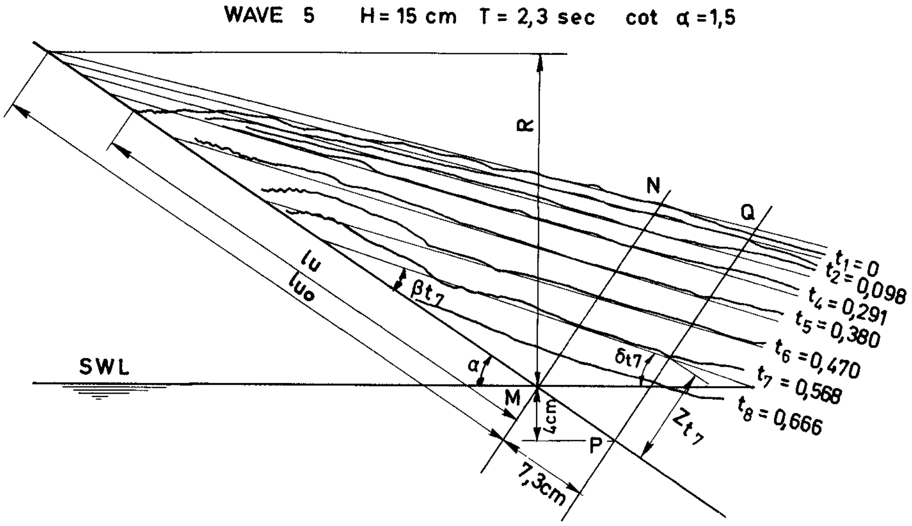

WAVE 5 H=15 cm T=2,3 sec cot α=1,5
R
N
Q
$t_1=0$
$t_2=0{,}098$
$t_4=0{,}291$
$t_5=0{,}380$
$t_6=0{,}470$
$t_7=0{,}568$
$t_8=0{,}666$
lu
lu_o
βt_7
$\delta t7$
α
SWL
M
4cm
P
Z_{t_7}
7,3cm

FIG 8

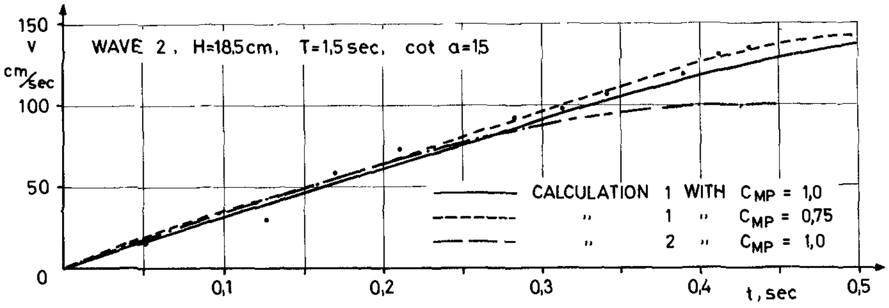


FIG 9

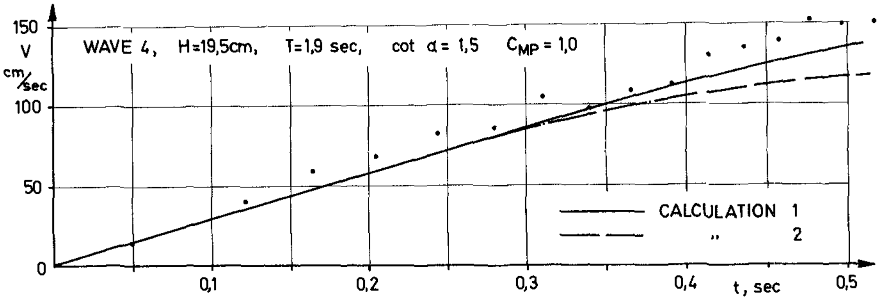


FIG 10

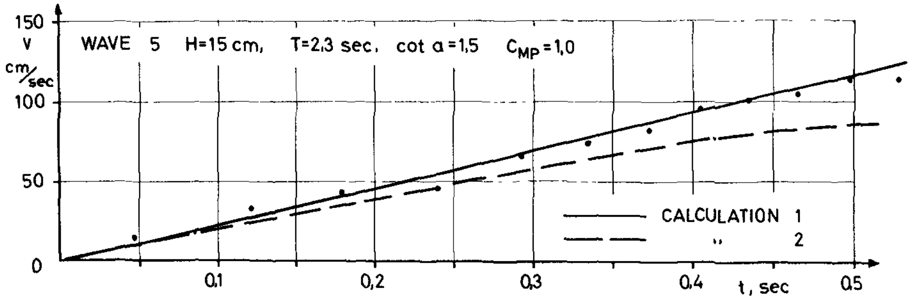


FIG 11

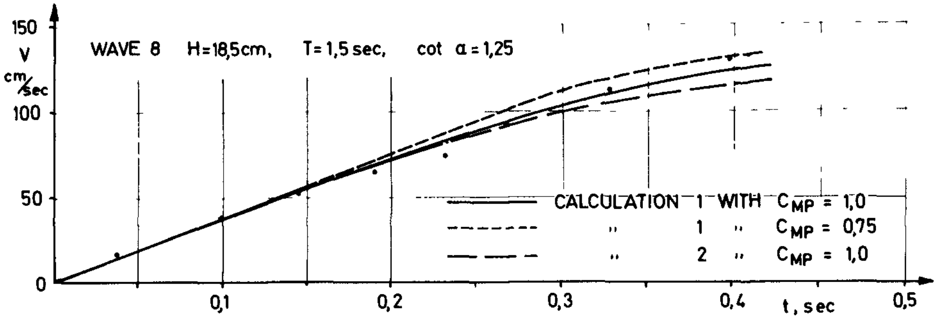


FIG 12

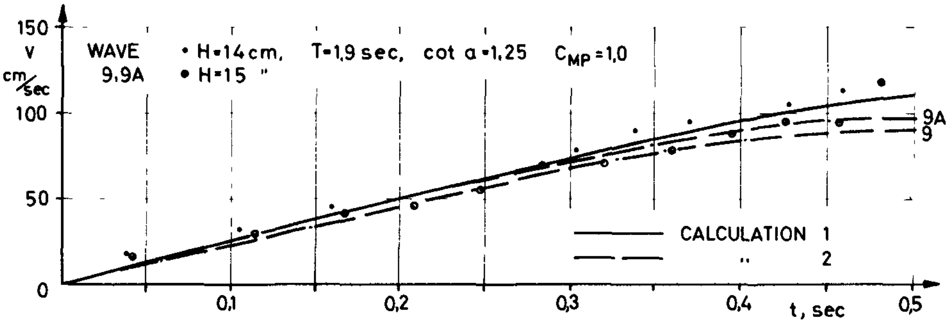


FIG. 13

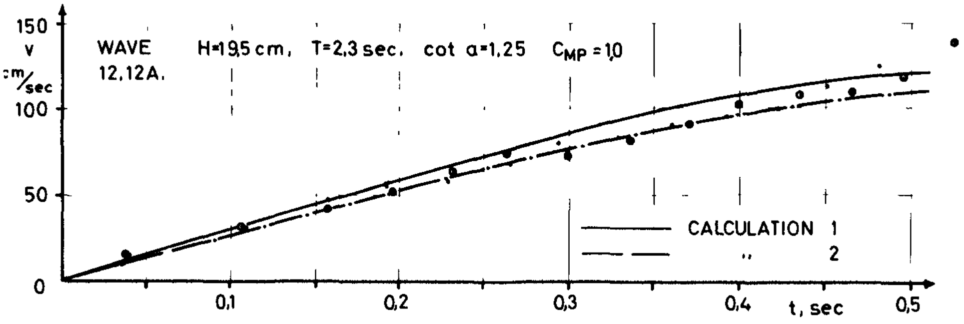


FIG 14

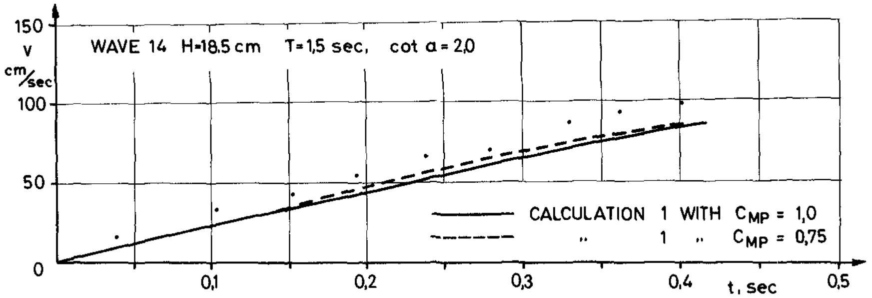


FIG 15

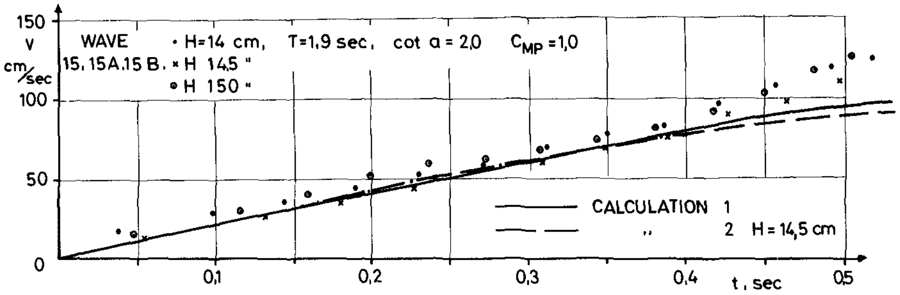


FIG 16.

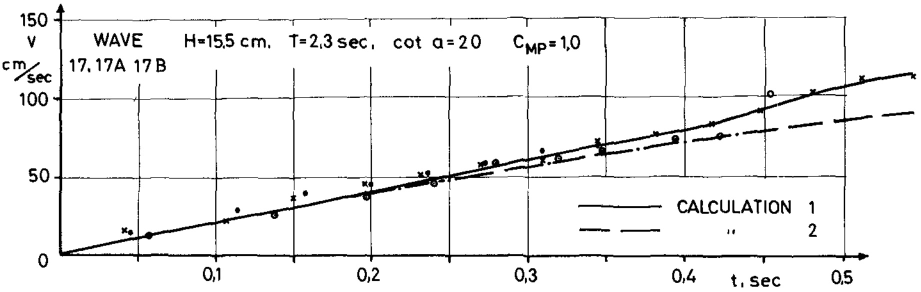


FIG 17

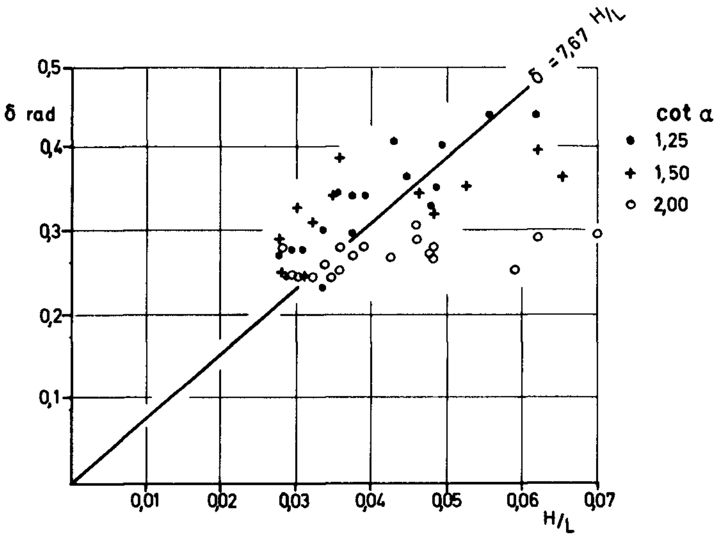


FIG. 18 ANGLE δ AS FUNCTION OF WAVE STEEPNESS

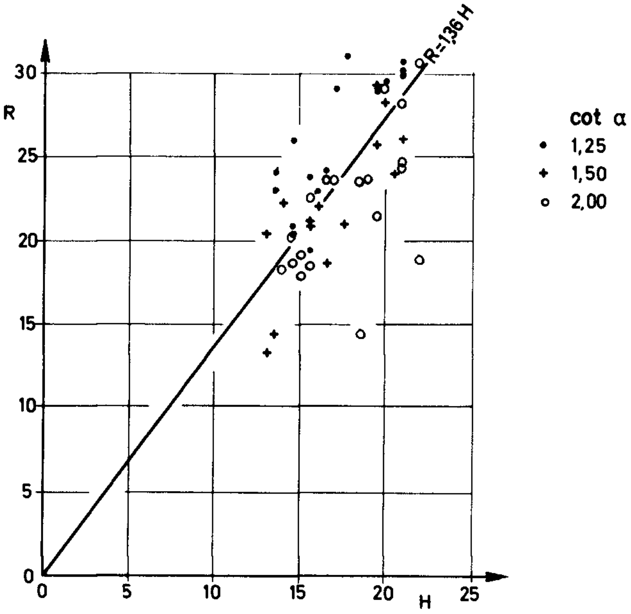


FIG 19 UPRUSH vs WAVE HEIGHT

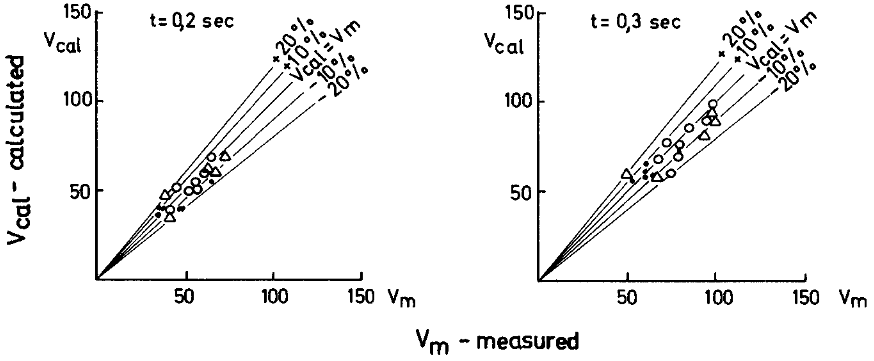


FIG 21 a

SUMMARY OF RESULTS - CALCULATION 2

$\delta = 7{,}67\ H/L,\ R' = 1{,}36\ \ H + 4{,}0\,cm\ \ C_{MP} = 1\ \ k = 4{,}0\,cm$

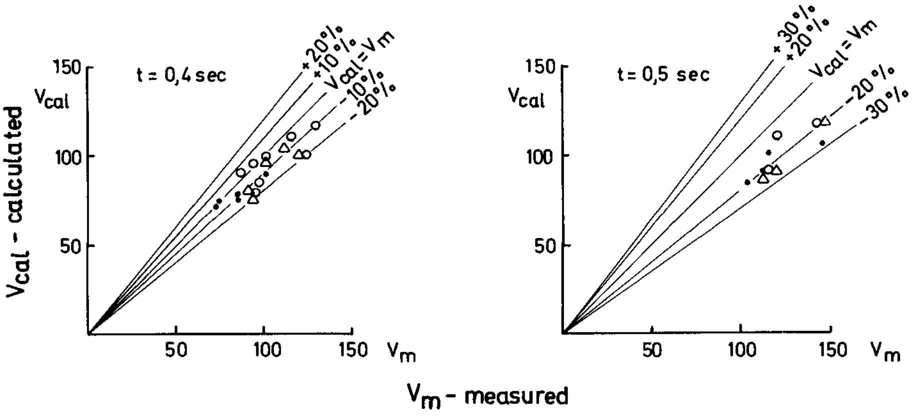


FIG 21 b

CALCULATION 2

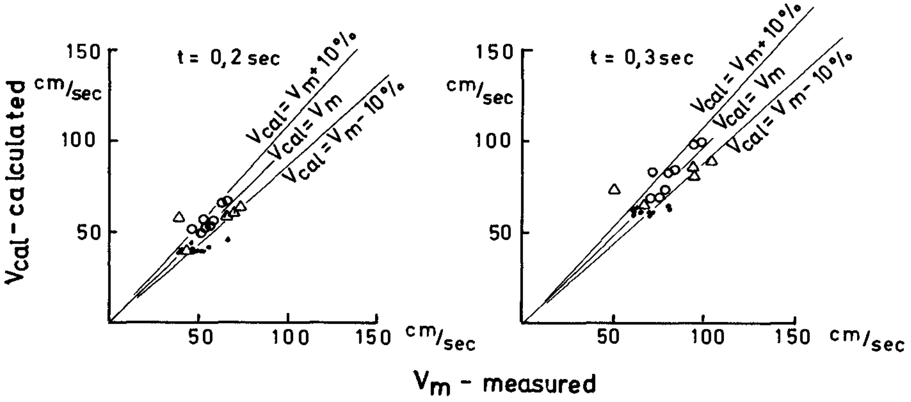


FIG 20 a
SUMMARY OF RESULTS - CALCULATION 1
β-AND Z- VALUES AS OBSERVED, $C_{MP} = 1,0$, $k = 4,0$ cm

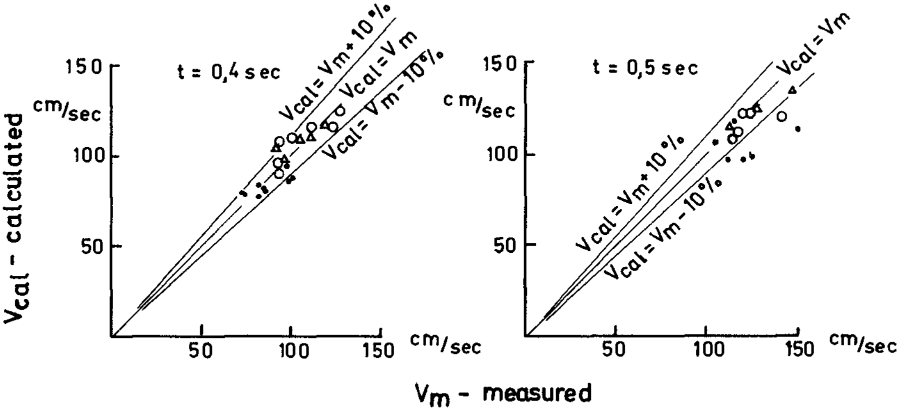


FIG. 20 b
CALCULATION 1