Chapter 34

ON OPTIMUM BREAKWATER DESIGN

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SUMMARY

A breakwater design is optimum when it results in a structure that meets the requirements at minimum total cost.

The total cost consist of cost of construction, anticipated damage and economic loss due to failure of the structure.

For any type of structure the design wave or load is governed by the condition of minimum total cost. This is worked out for some possible designs for Europoort Harbour, Rotterdam.

The data needed are often insufficient, however, for reasonable assumptions important directives can be obtained especially with respect to ranking of structures of the same type.

DEFINITION OF OPTIMUM DESIGN

Designing the cross section of a breakwater involves deciding upon:

- the type of structure;
- its dimensions.

A criterion has to be established on which the decision can be based, viz. the optimum design is that which results in a structure that meets all the requirements at minimum total cost.

As long as off-shore conditions can only be expressed in terms of statistics any breakwater will suffer damage sooner or later. So the total cost is defined as the cost of construction and the capitalized anticipated expenditure due to damage and economic loss.
For example economic loss is suffered when harbour equipment is damaged or when a harbour fails to function properly as a consequence of failure of the breakwater.

DESIGN CRITERIA

The design criteria are the requirements the structure must satisfy. They depend on:

- the functions the breakwater is expected to fulfil;
- its stability.

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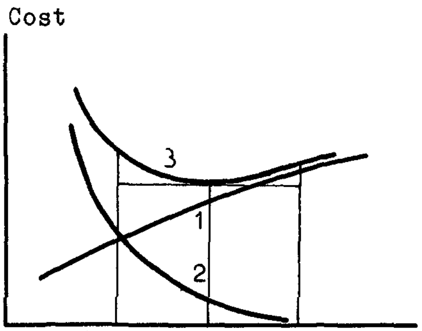
Functions:	Calling for special attention to:
- guiding of currents.	- generally no special requirements as to type of structure.
- attenuation of waves.	- crest height.
- marking of harbour entrance.	- crest height.
- providing a quay.	- crest height and width; harbour-side structure.
- retention of sand.	- inner slope.

Stability. A wave condition or wave load that will just not cause any damage can be determined for any structure. Generally, however, there is a probability that that "design wave" or "design load" as it is called will be exceeded; consequently the fact that damage will be suffered by the structure has to be accepted. If the design wave or design load is small, the cost of construction will be low but the anticipated damage will be great. As the magnitude of the design wave or load increases, the anticipated damage will decrease, due to the decreasing probability that the design conditions will be exceeded. See fig. 1.



1. cost of construction.
2. capitalized anticipated damage.
3. total cost (= 1 + 2).

Fig. 1. design wave or design load.

According to the foregoing definition of optimum design, the design wave or load must be such that the cost of construction and the capitalized anticipated damage are kept as low as possible. How to determine that minimum is called the "decision problem", which is worked out below.

The following factors are involved:

- the occurrence of various off-shore conditions;
- the relation between off-shore conditions (especially wave conditions) and the behaviour of the structure;
- the relation between the design wave or design load and the cost of construction;
- the relation between off-shore conditions, design wave or design load and anticipated damage.

OUTLINE OF THE "DECISION PROBLEM"

The way in which the above-mentioned information is worked out depends on the type of structure being considered. Two main categories will be dealt with separately:

- monolith structures (caissons, walls built of jointed blocks);

- rubble-mound structures (with cover layers of natural rock and concrete blocks).

THE OCCURRENCE OF VARIOUS OFF-SHORE CONDITIONS

In the following chapter a probability distribution curve of significant wave heights is used to describe wave attack (see specification of H_s below). The probability distribution of H_s can be derived from measurements or estimated from meteorological data by means of the well-known relations between waves, wind and fetch.
Probability distribution curves for the wave forces acting on monolith structures can be derived from the probability distribution curves of H_s combined with wave-force experiments on models.

RELATION BETWEEN OFF-SHORE CONDITIONS AND THE BEHAVIOUR OF THE STRUCTURE

Monolith structures. Wave attack is characterized by a wave force F (or a wave pressure a). The relation between wave forces and off-shore conditions must be established. If the design load F_o is exceeded, the entire cross section will be displaced (damage to part of the cross section is irrelevant). A relation can be established between displacement x, design load F_o and actual load F. A certain displacement x_o is regarded as marking the collapse of the structure. For a structure of a certain type and dimensions (i.e. a known value of F_o) this critical displacement can be related to a wave force F^* by applying the above-mentioned relation between x, F_o and F. See page 6 . Consequently, the probability of collapse is the probability of force F^* being exceeded, $p(F^*)$.
Summarizing:
$F \leqslant F_o, \quad x = o;$
$F > F_o, \quad x \neq o;$
$F = F^*, \quad x = x_o.$

Rubble-mound structures. In view of the difficulty of determining wave forces the significant wave height H_s is taken as characterizing a wave attack on the structure, provided:

- the term "significant wave height" implies that the actual distributions of wave heights and periods are applied;
- proper allowance is made for the accumulated wave attack with smaller significant wave heights (ref. 8).

Regard for those factors enables us to arrive at a design wave H_{so} corresponding to a "no damage" criterion. If the design wave H_{so} is exceeded, displacement of armour units will occur. A relation can be established between the percentage of armour units displaced, the design wave H_{so}, and actual wave height H_s. Therefore the occurrence of a certain amount of damage can be related to the occurrence of a certain H_s for a structure of a certain type and dimensions (i.e. a known value of H_{so}).

RELATION BETWEEN COST OF CONSTRUCTION AND DESIGN WAVE OR DESIGN LOAD

Monolith structures. It was assumed in the previous section that the dimensions of the structure could be determined once the design load F_o was chosen. The cost of construction can be estimated if the dimensions are known. Hence the cost of construction can be expressed as a function of F_o, $I = f(F_o)$.

Rubble-mound structures. The dimensions are related to the design wave H_{so}; consequently, the cost of construction can be expressed as:

$$I = f(H_{so}).$$

THE RELATION BETWEEN ANTICIPATED DAMAGE, OFF-SHORE CONDITIONS AND DESIGN WAVE OR DESIGN LOAD

As stated before, with every design wave or load there is a probability of damage occurring to the structure. To determine the amount of anticipated damage it is assumed that an insurance company is willing to insure it against damage. If the company insures a great number of unrelated constant risks, which need not be of the same nature (e.g. all government investment risks), and if the theoretical annual premium is s:
s = probability of damage x the cost of repairing that damage. The same premium would be charged for a single object for an infinite duration, which is (to abandon the insurance model) the average sum spent per year on repairing damage. It seems reasonable to take that premium as the anticipated damage per year.
It should be noted that under constant risk it is assumed that any damage suffered would be repaired immediately.

For *monolith* structures, partial damage to which is not considered, the anticipated damage per year is:
$s = p(F^*).W$, in which $p(F^*)$ is the probability of force F^* being exceeded in any given year, and W is the cost of repairing when a failure of the structure occurs.

For *rubble-mound* structures the anticipated damage per year is estimated as follows: For a structure of certain dimensions (i.e. a known value of H_{so}) the occurrence of a certain damage is related to the occurrence of conditions characterized by an H_s (see page 9).
If the probability of H_s being exceeded is considered, intervals ΔH_s can be chosen at which a constant amount of damage ΔW may be presumed with reasonable accuracy. Assume the probability of occurrence of waves in the interval ΔH_s in any given year is Δp. The corresponding anticipated annual damage is $\Delta p . \Delta W$. Hence the total anticipated annual damage is:

$$s = \sum \Delta p \,.\, \Delta W.$$

As already stated, the factor W may include economic loss.

The capitalized value of the sum of the "premiums" s depends on the life of the structure. If its life is 100 years or more, the capitalized anticipated damage s is:

$S = \frac{100}{\delta} s$*), in which δ is the rate of interest as % per year.

*) If interest is added continuously, the capitalized value (present value) of a sum s to be paid after t years is $e^{-\frac{\delta}{100}t}$ s. Consequently, the capitalized value of the sum of the premiums s to be paid for the lifetime T of the structure (sum of all present values) is found to be:

$$S = s\int_0^T e^{-\frac{\delta}{100}t}\,dt.$$

For T = 100 years, $S = \frac{100}{\delta} s\,(1-e^{-\delta}) \approx \frac{100}{\delta}.s.$

For T = 10 years and $\delta = 3.5\%$: $S = \frac{100}{\delta} s\,(1-e^{-\delta/10}) \approx 0.3\,\frac{100}{\delta}.s.$

THE TOTAL COST OF THE STRUCTURE

The total cost of the structure K was defined as the cost of construction I and the capitalized anticipated damage S. Hence: $K = I + S$. With the expressions already established it is then found that for:

Monolith structures: $K = f(F_0) + \frac{100}{\delta} \,.\, p(F^{*}).W.$

Rubble-mound structures: $K = f(H_{so}) + \frac{100}{\delta} \sum \Delta p \,.\, \Delta W.$

THE DESIGN WAVE OR DESIGN LOAD

The design wave or design load is governed by the condition that the cost K shall be as low as possible. It is sometimes possible to determine the minimum values of the expressions for K analytically, but it is usually easier to determine the minimum K_0 graphically. This method has the advantage that a good impression is obtained of the function K near its minimum. This is of particular importance with respect to the amount of money "wasted" if the wrong design wave or load is adopted. The latter sum is called the "regret".

The main cause of any error in the value of H_{so} or F_0 will be that the K curve itself is wrong, due to inaccuracy of the data. The respective curves K, I and S are generally similar in all cases. The cost of construction increases gradually as H_{so} or F_0 increases. See fig. 1. The capitalized damage S, however, decreases rapidly as H_{so} or F_0 increases, especially if the decrease in the probability of occurrence of H_{so} or F_0 is great. The curve K always has a rapidly decreasing portion on the left of its minimum and a slowly increasing portion on the right. Consequently the "regret" is greater for a design wave or load that is too small, than it is for a design wave that is to the same extent too great. See fig. 1. Hence one should be on the safe side when deciding upon a design wave or load.

The method of approach described in the foregoing has been culled from the determination of design levels for the Delta project in the Netherlands (ref. 1, 2 and 7).

The procedure discussed above is worked out for cases embodying optimum values for:

- the dimensions of two monolith structures;
- the dimensions (weight of stone, slope) of two rubble-mound structures.

Most of the cases were taken from studies carried out on various designs for Europoort Harbour, Rotterdam.
A plan of the harbour is given on page 17.

MONOLITH STRUCTURES

RELATION BETWEEN OFF-SHORE CONDITIONS AND THE BEHAVIOUR OF THE STRUCTURE

The wave forces acting on a monolith structure fall into two main categories:

- quasi-static forces, defined as forces that fluctuate with the same period as the incident waves (period of about 5-10 sec);
- dynamic or impact forces, defined as forces the duration of

which is short compared with the wave period (for steep barriers a corresponding period of the order of 1 sec*)).

To obtain some idea of the relation between structure displacement and wave force, an approximate and simplified calculation has been made of the positive displacements of a caisson-type breakwater with vertical weather-side front and assuming a sinussoidal load, a friction coefficient $f = 0.5$ and a non-elastic horizontal bed. The results are given in figure 2 in which the displacement x is given as a function of F/F_0.

If a displacement of the order of one metre is taken as indicating failure of the structure, it appears from figure 2 that for quasi-static loads (period $\geqslant 5$ sec) the value of F/F_0 always has to be taken as unity, because exceeding force F_0 can easily lead to a displacement of metres. Consequently, the probability of failure is $p(F_0)$. In the case of dynamic forces acting on steep barriers (period in the order of 1 sec) a displacement of about a metre will occur if $F/F_0 \simeq 2$. Adopting a value of $F/F_0 = 2$, however, will generally lead to much greater displacement, because of the accumulated displacement due to forces between F_0 and $2F_0$. So the ratio F/F_0 has to be reduced; $F/F_0 = \alpha$, with $1 < \alpha < 2$. The value of α depends on the probability distribution curve of F and the value of F_0. In this case the probability of failure is $p(\alpha F_0)$.

CAISSON WITH VERTICAL FRONT

Structure. It was concluded from the function criteria, the off-shore conditions and model investigations on wave attenuation that the minimum crest height is M.S.L. + 2 mtrs. Initial calculations showed that in view of the expense and the off-shore conditions concerned the crest height must be kept as low as possible. Hence the crest height is kept at M.S.L. + 2 mtrs. The specific gravity of the caisson with its sand fill is $\gamma = 2.1$ tons per cubic metre. The coefficient of friction is $f = 0.5$.

A diagram of the structure is given in figure 6.

The occurrence of the off-shore conditions. Both quasi-static and dynamic forces (F_{stat} and F_{dyn}) act on the exposed front of the structure. Probability distribution curves are given in figure 7. They were derived from field measurements and model investigations (see also ref. 6). The wave heights, hence the quasi-static forces are limited by the depth of the water, so for small probabilities the probability distribution curve has the same configuration as that of the water levels. Only $H_s = 5.5$ mtrs has been taken into account when determining the probability of excess of the dynamic forces. Wave pressures under the caisson are being ignored for the moment.

Relation between off-shore conditions and the behaviour of the structure. See relevant paragraph on page 5 . Assuming for the moment that quasi-static forces and dynamic forces are independent,

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*) It should be noted that the duration of overall dynamic forces is often considerably greater than the duration of local pressures.

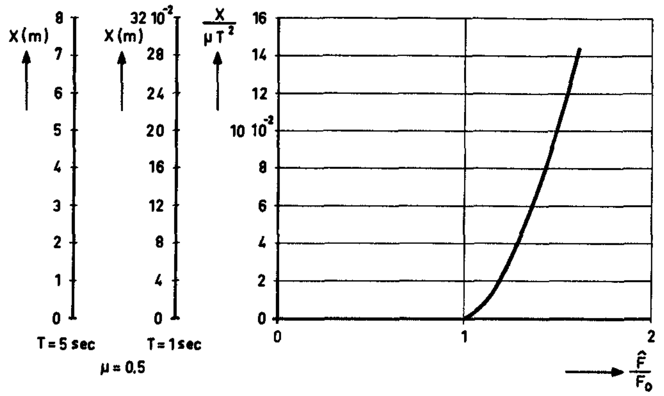


FIG. 2 DISPLACEMENT OF A CAISSON UNDER INFLUENCE OF A SINUSOIDAL FORCE

X = DISPLACEMENT IN m

$F(t)$ = WAVE FORCE = $\hat{F}\sin\omega t$

$\varepsilon F(t)$ = UPLIFT FORCE DUE TO WAVE ACTI

F_0 = DESIGN LOAD

W_{eff} = WEIGHT OF UNIT AT STILL WATER

W = WEIGHT OF UNIT IN AIR

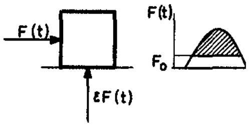
$\mu = \frac{W_{eff}}{W}$

f = BOTTOM FRICTION COEFFICIENT = $\frac{1}{2}$

T = PERIOD OF THE WAVE FORCE

ω = ANGULAR FREQUENCY = $\frac{2\pi}{T}$

g = ACCELERATION OF GRAVITY



POSITIVE DISPLACEMENT FOUND FRO

$$\hat{F}\sin\omega t - f(W_{eff} - \varepsilon\hat{F}\sin\omega t) = \frac{W_{eff}}{\mu g}\frac{d^2x}{dt^2}$$

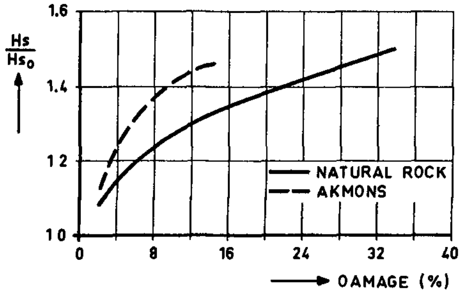
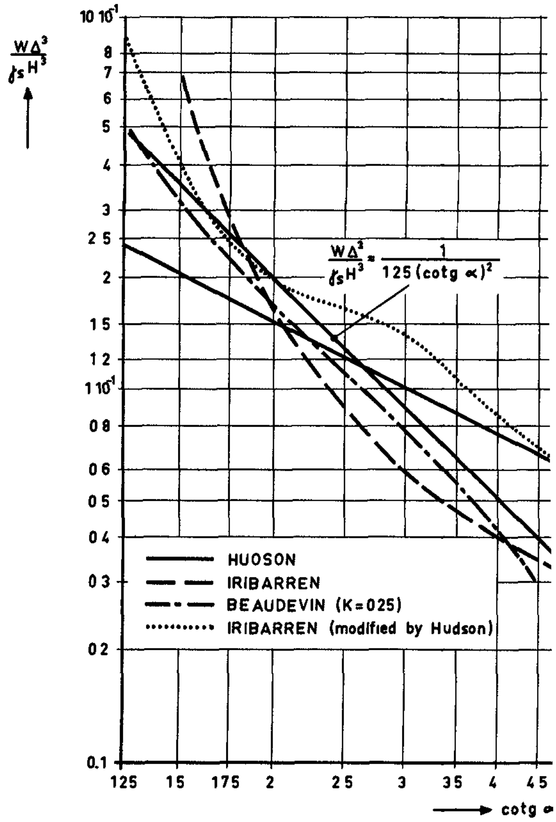


FIG. 3 DAMAGE AS A FUNCTION OF $\frac{Hs}{Hs_0}$



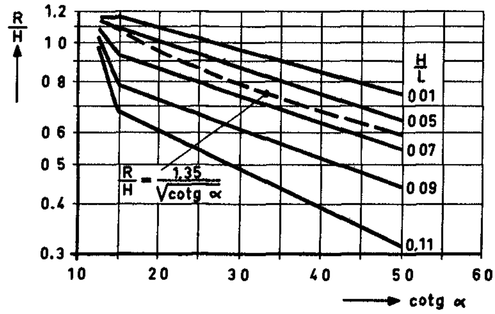


FIG. 4 RELATIVE WAVE RUN UP (AFTER HUDSON)

FIG. 5 STABILITY OF NATURAL ROC

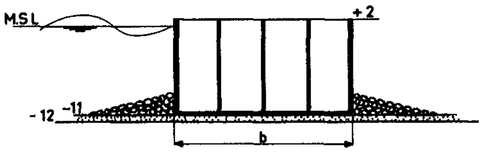
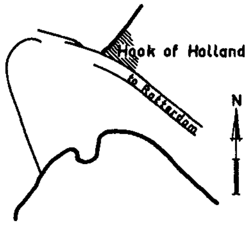


FIG. 6 CAISSON WITH VERTICAL FRONT



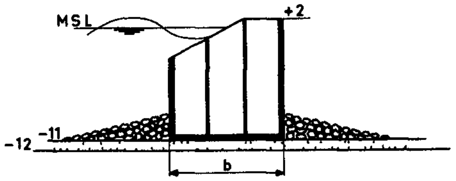


FIG. 9 CAISSON WITH COMPOSITE FRONT

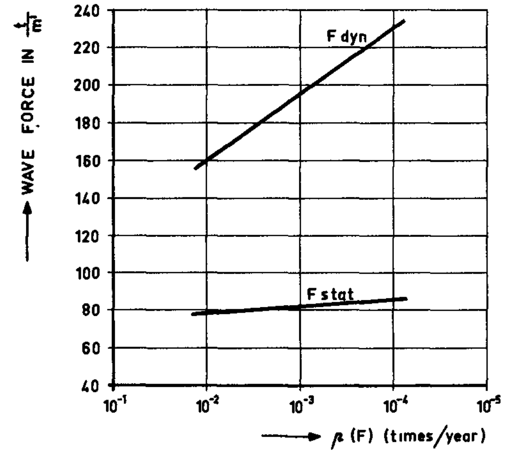


FIG. 7 PROBABILITY OF EXCESS OF F

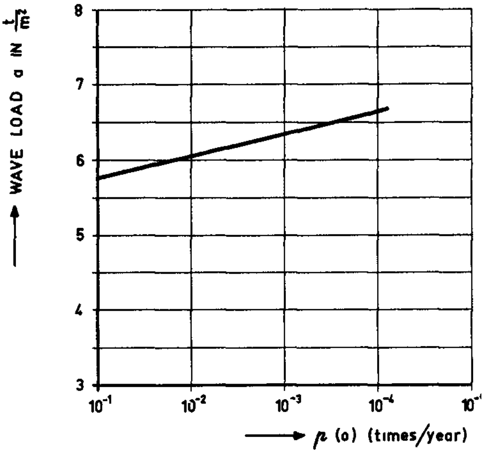


FIG. 10 PROBABILITY OF EXCESS OF

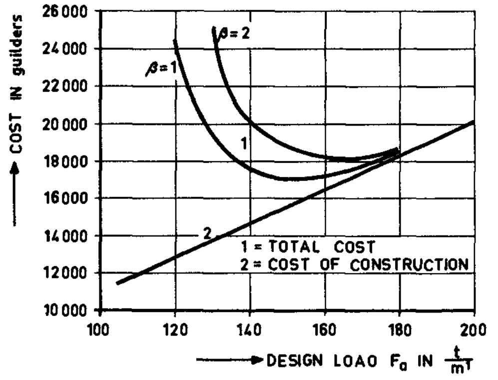


FIG. 8 COST AS A FUNCTION OF F_0

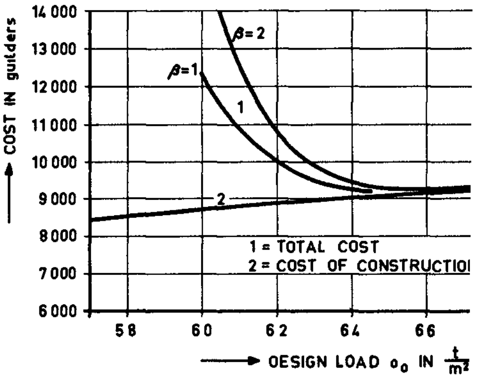


FIG. 11 COST AS A FUNCTION OF a_0

the probability of failure is:

$$p\,(\text{failure}) = p\,(F_{stat}) + p\,(F_{dyn}).$$

It may be concluded that, for the structure and off-shore conditions considered, a force $F_{dyn} = 1.2\ F_{dyn_0}$ results in a displacement which is fairly representative of the accumulated displacement due to forces $F_{dyn} > F_{dyn_0}$. It is clear from the probability distribution curves that $p(F_{stat})$ is much smaller than $p(1.2\ F_{dyn_0})$ and can be ignored. So dynamic forces only have to be considered and $p(\text{failure}) = p(1.2\ F_{dyn_0})$.

The relation between the required volume of the caisson per metre of exposed front V_{re} and design load F_{dyn_0} is

$$V_{re} = \frac{1}{f(\gamma-1)}\,F_{dyn_0} = 1.82\ F_{dyn_0}\ \text{cub.mtr.}$$

The relation between cost of construction and design load. It is assumed that the cost of construction is a linear function of V, so

$I = AV + B$ in which:

A = Dfl 50 per cub.mtr.

B = Dfl 2000 per cub.mtr. (= cost of toe protection, etc.).

$V = 1.82\ F_{dyn_0}$.

Hence: $I = 91\ F_{dyn_0} + 2000.$

The relation between anticipated damage, off-shore conditions and design load. The capitalized value of the anticipated damage is

$$S = \frac{100}{\delta}\ p(1.2\ F_{dyn_0}).W \qquad \text{(see page 4).}$$

The money per metre weather-side front involved in a failure, W, is assumed to be proportional to the cost of construction.

$$W = \beta\, I.$$

From this and the expression for I it follows that

$$S = \frac{100}{\delta}.p(1.2\ F_{dyn_0}).(91\ F_{dyn_0} + 2000).$$

The total cost of the structure, $K = I + S$.
Substitution of the expressions for I and S gives:

$$K = (91\ F_{dyn_0} + 2000).\left\{1 + \frac{100}{\delta}.\ p(1.2\ F_{dyn_0})\right\}.$$

The values of K and I have been plotted as a function of F_{dyn_0} for $\frac{100}{\delta} = 30$, $\beta = 1$ and $\beta = 2$. See figure 8.

The design load. Dimensions of structure. It is evident from figure 8 that the optimum design load F_{dyn_0} is 150 to 170 tons per metre, depending on the value for β adopted. In view of the difficulty encountered when replacing a single caisson β will generally be greater than unity. For that reason and for the reasons given on page 5. under "Design load", 170 tons per metre has been adopted for F_{dyn_0}. The probability of failure occurring in any given year is $p(1.2\ F_{dyn_0}) = 5.10^{-4}$ or once in 2000 years on an average.

V = 310 cub.mtrs. Hence the width of the caisson $b = \frac{V}{13} = 24$ mtrs.

I = Dfl 17,500.

K = Dfl 17,760 for $\beta = 1$.

K = Dfl 18,020 for $\beta = 2$.

CAISSON WITH COMPOSITE WEATHER-SIDE FRONT

Structure. The upper part of the exposed front of the caisson is inclined, so as to reduce the effect of dynamic forces on its stability. The crest height is determined in the same way as that described for a caisson with vertical front, it is M.S.L. + 2 mtrs. The specific gravity of the caisson with sand fill is $\gamma = 2.1$ tons per cub.mtr. The coefficient of friction is $f = 0.5$. The structure is shown diagrammatically in figure 9.

The occurrence of the off-shore conditions. This type of structure was tested for wave forces by the Coastal Engineering Laboratory at Copenhagen and at the Hydraulics Laboratory at Delft. Those tests and experiments on similar structures showed that:

- When the top of the vertical front is some distance below S.W.L., the dynamic forces on the vertical part are small compaired with the quasi-static forces, so they may be neglected.
- When the crest of the structure is below S.W.L. the quasi-static forces acting on the vertical part result from equally distributed wave pressures. The probability distribution curve for the magnitude of these pressures, a, is given in figure 10. It was derived from field measurements of off-shore conditions and from model tests.
- Both dynamic and quasi-static forces act on the inclined part. The angle of inclination is made equal to the angle of friction between the caisson and its foundation so as to prevent those forces from affecting the caisson's sliding stability.

The wave pressures underneath the caisson are assumed to be linear between the wave pressure at the front and the mean water pressure on the inside.

Relation between off-shore conditions and the behaviour of the structure. See relevant paragraph on page 5 . As only quasi-static forces need be considered: $p(\text{failure}) = p(a_0)$. The relation between the required volume of the caisson per metre of exposed front V_{re} and the design load a_0 is:

$$V_{re} = \frac{1}{f}\,\frac{(\text{force on vert. front})}{\gamma - 1} + \frac{\text{force underneath caisson}}{\gamma - 1}$$

or:

$$V_{re} = \frac{a_0\left(13 - \frac{b-4}{2}\right)\frac{1}{f}}{\gamma - 1} + \frac{\frac{1}{2}a_0 b}{\gamma - 1},$$

in which b is the width of the caisson. See fig. 9.
The value of b can be determined from the postulate that the required volume must equal the present volume V_{pr}. The latter is

$$V_{pr} = 13b - \tfrac{1}{4}(b-4)^2.$$

The relation between cost of construction and design load. It is assumed that the cost of construction I is a linear function of V.

$I = AV + B$ in which:

$A = \text{Dfl } 50$

B = Dfl 2000 (= cost of toe protection, etc.).
I can be expressed in terms of a_o with the expressions for V_{re} and V_{pr}. See fig. 11.

The relation between anticipated damage, off-shore conditions and design load. The capitalized value of the anticipated damage is:

$$S = \frac{100}{\delta} p(a_o).W. \quad \text{See page } 4 .$$

The money per metre weather-side front involved in a failure, W, is assumed to be proportional to the cost of construction I.

$$W = \beta I.$$

From this and the expression for I it follows that

$$S = \frac{100}{\delta} p(a_o).(50 V + 2000),$$

in which V is a function of a_o. See fig. 11.

The total cost of the structure, $K = I + S$.
Substitution of the expressions for I and S gives:

$$K = (50 V + 2000).\left\{1 + \frac{100}{\delta}.p(a_o)\right\} .$$

The values of K and I have been plotted as a function of a_o for $\frac{100}{\delta} = 30$, $\beta = 1$ and $\beta = 2$. See fig. 11.

The design load. Dimensions of structure. It is clear from figure 11 that the optimum design load $a_o = 6.6$ tons per square metre. The probability of failure occurring in any given year is $1.4\ 10^{-4}$ or once in 7,200 years on an average.

V = 143 cubic mtrs. b = 12.4 mtrs.
I = Dfl 9,150.
K = Dfl 9,190 for $\beta = 1$.
K = Dfl 9,230 for $\beta = 2$.

RUBBLE-MOUND STRUCTURES

RELATION BETWEEN OFF-SHORE CONDITIONS AND BEHAVIOUR OF THE STRUCTURE

So far but little quantitative information has become available on the damage suffered by the structure as a function of off-shore conditions. The experiments carried out mainly concerned initial damage.

With respect to the seaward cover layer, in first approximation the figure 3 for the relation between H_s/H_{so} and the occurrence of damage can be adopted. This relation was derived from tests on models carried out by the Waterways Experiment Station and the Delft Hydraulics Laboratory (ref. 4 and 8). The results should be used with caution. The damage percentages refer to the number of blocks in the area between the crest ($\cong$ S.W.L. + H_{so}) to S.W.L. - H_{so}. The damage is often found to have occurred in a more restricted area around S.W.L. For that reason and in view of the rapidly increasing damage for damage $> 30\%$ it is assumed that for 30% damage the cover layers will have become displaced locally and that the structure will have collapsed.

Information on wave attack on the crest and inner slope is even scarcer. It is known, however, that appreciable overtopping can easily cause damage. Two types of structure will be considered in this light:

- If the inner slope is faced with small category blocks, it is assumed that the structure will collapse as soon as there is appreciable overtopping. The wave run-up found by Hudson (ref. 3 and fig. 4) has been adopted as a criterion for overtopping, H being replaced by H_s.
- If the crest and upper part of the inner slope is protected by armour units such as those used for the seaward cover layer, the relation between damage and H_s/H_{so} is assumed to be the same for small crest heights $h \ll H_{so}$ as it is for the seaward cover layer, however, with the restriction that the structure is assumed to collapse at 10% damage. The latter assumption has been made in view of the fact that the damage occurs mainly along the inner crest line. It is stressed once again that these assumptions have not received adequate experimental support.

In the example of a rubble-mound structure with a cover layer of natural rock, the relation between block weight and the angle of the slope must be known. Some existing formulae are given in figure 5. For practical reasons an average curve is assumed in which the block weight is inversely proportional to $\cot g^2 \alpha$, and in which the wave height is H_s, see fig. 5. It is not the intention of the authors to propound a new formula.

RUBBLE-MOUND WITH COVER LAYER OF CONCRETE BLOCKS

Structure. From the function criteria and data on wave heights and water levels (see also ref. 9) it was concluded that the minimum crest height required was M.S.L. + 2 mtrs. This implies that mass overtopping will occur, and that consequently the crest and harbour-side slope will be subject to severe wave attack. The crest would have to be raised to at least M.S.L. + 7 mtrs. to reduce this wave attack, which appeared to be an uneconomical solution in view of the increased cross-sectional area and the relatively expensive core material. Accordingly, the crest height has been kept at M.S.L. + 2 mtrs.

As regards the slope of the structure it can be shown that for the off-shore conditions and prices concerned the steepest possible slope should be adopted. A slope of 1:1.5 was adopted for practical reasons.

The example has been worked out for a cover layer consisting of concrete akmons with a density of ρ = 2800 kilogrammes per cubic metre. A diagrammatic sketch is given in figure 12.

The occurrence of off-shore conditions. The data on wave heights were obtained from wave-recording stations in the North Sea. A probability distribution curve of H_s was derived from the data expressed as the number of storms in which a certain H_s is exceeded. See fig. 13.
For information on the distribution of individual wave heights see ref. 10.

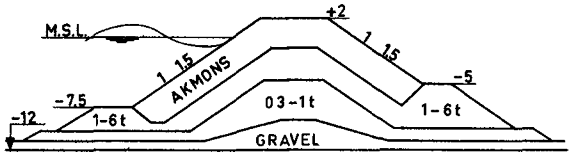


FIG. 12 STRUCTURE WITH COVER LAYER OF CONCRETE BLOCKS

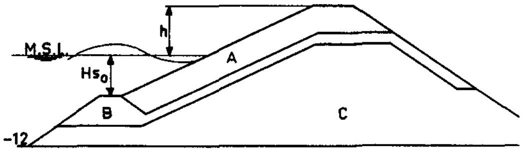


FIG. 15 STRUCTURE WITH COVER LA OF NATURAL ROCK

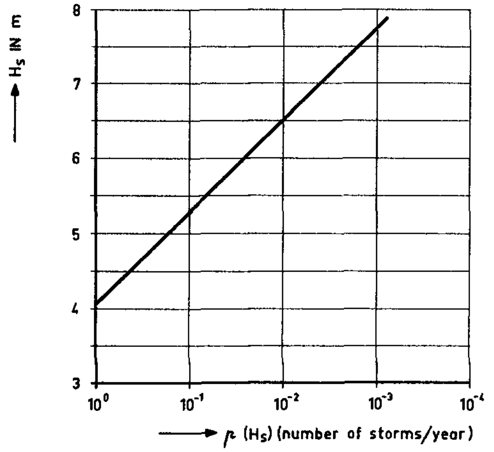


FIG. 13 PROBABILITY OF EXCESS OF Hs

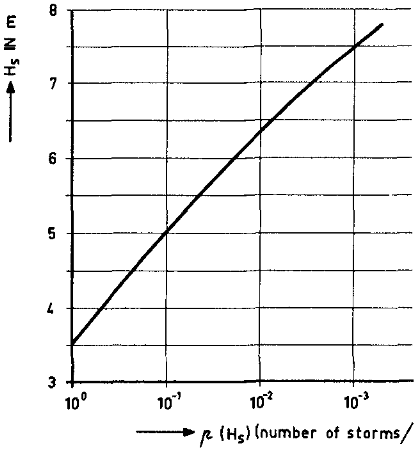


FIG 16 PROBABILITY OF EXCESS OF

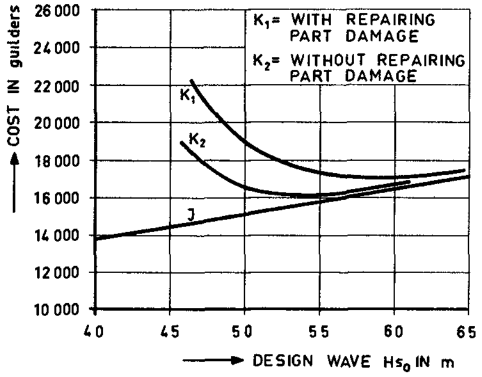


FIG. 14 COST AS A FUNCTION OF Hs_0

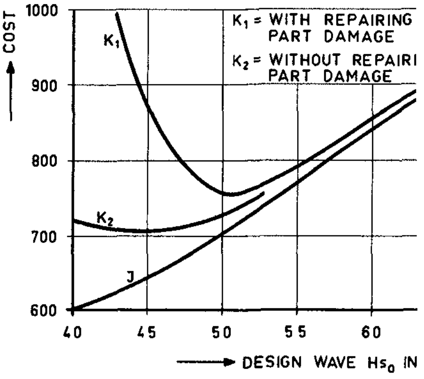


FIG. 17 COST AS A FUNCTION OF H

Relation between off-shore conditions and behaviour of the structure. See relevant paragraph on page 9 . For wave heights exceeding H_{so}, damage occurs as shown in figure 3. Here the percentage of damage refers to the whole concrete cover layer. As mentioned on page 10 it is assumed that the structure will collapse at 10% damage, hence when $H_s/H_{so} = 1.45$, see fig. 3. As regards the relation between the design wave height H_{so} and block weight (akmons), tests on models and data from ref. 8 showed that

$$W = \frac{2.8\ H_{so}{}^3}{50} \text{ ton (slope 1:1.5; } \rho = 2800 \text{ kilogrammes per cubic metre.)}$$

Relation between cost of construction and design wave. $I = f(H_{so})$. The cost of construction can be divided up into the cost of the concrete cover layers and the cost of the second layers and core. The latter two are independent of the design wave height, and are per metre weather-side front:

79 cub.mtrs.	gravel	Dfl 1580.-
84 " "	rock 0.3-1 t.	" 2270.-
100 " "	rock 1-6 t. (dumped)	" 2700.-
34 " "	" " (by crane)	" 1290.-
		Dfl 7840.-
	wastage 10%	" 780.-
		Dfl 8620.-

The cost of the cover-layers is assumed to be proportional to the total volume of concrete per metre weather-side front, Q.

$Q = CAV^{1/3}$, (ref. 8) in which:

V is the volume of a block: $V = \frac{W}{2.8} = \frac{H_{so}^3}{50}$.

C is a constant; for akmons C = 0.9.

A is the area to be covered per metre: A = 33.

Assuming a wastage of 10% of the blocks during construction, it is seen that:

$Q = 8.8\ H_{so}$.

Concrete costs Dfl 150 per cub.mtr. hence the cost of the cover-layers per metre, I_{cl}, works out at

$I_{cl} = 8.8\ .150.H_{so} = 1320\ H_{so}$.

Consequently the cost of the structure per metre:

$I = 1320\ H_{so} + 8620$.

Relation between anticipated damage, off-shore conditions and design wave. $S = \frac{100}{\delta} \sum \Delta p \ \ \Delta W$. See page 4 . Three intervals for H_s/H_{so} are considered for the occurrence of H_s. The corresponding damage percentages and the probability of occurrence follow from figures 3 and 13. The amount of damage ΔW is assumed to be: percentage of damage x cost of construction of cover-layers x 2. The latter factor 2 has been arbitrarily adopted in view of the fact that the placing of a limited number of blocks later on is more expensive. For a collapse, ΔW is assumed to be equal to the total cost of construction. For various values of H_{so}, Δp and ΔW

are given in Table 1.

As slight partial damage needs not necessarily be repaired[*], two cases will be considered:

- the amount of damage when partial damage is repaired;
- the amount of damage when partial damage is not repaired.

For $\frac{100}{\delta} = 30$ the values of s and S are given in Table 2.

The total cost of the structure. K = I + S.
The total cost of the structure for various values of H_{so} is given in table 3. The sums are also given in figure 14 in which I and K have been plotted as a function of H_{so}.

The design wave. Required block weight. In accordance with the minimum total cost criterion the design wave H_{so} is:

H_{so} = 6 mtrs. and H_{so} = 5.5 mtrs. if partial damage is not repaired.

H_{so} (m)	$p\,(H_{so})$	Probability of failure $p\,(1.45\,H_{so})$	Block weight W (tons)
6	$2.6\ 10^{-2}$	$1.8\ 10^{-4}$	12
5.5	$6.5\ 10^{-2}$	$7.5\ 10^{-4}$	9

RUBBLE MOUND WITH COVER LAYER OF NATURAL ROCK
Structure. The structure considered has a straight seaward slope in the area of severe wave attack. The cover layers are supported by a hard shoulder and the inclination of its lower slope is 1:1.5, which is assumed to be the steepest slope that is easy to construct. The material available is said to lead to the following conditions:

- the quarry provides certain quantities of various categories of blocks with a clearly defined maximum block weight (W = 7 tons ρ = 2650 kilogrammes per cubic metre).
- the harbour-side slope is protected by smaller blocks than those used for the seaward cover layer, and cannot withstand considerable overtopping (in view of the small percentage of the heaviest blocks).

The stability of the harbour-side slope faced with small blocks depends mainly on the crest height. So increasing the cross-sectional area should be effected by heightening the crest rather than by adopting a gentler harbour-side slope.
The steepest slope that is reasonably easy to construct is assumed to be $\text{cotg}\,\alpha = 1.5$.

Density of stone and water: ρ_s = 2650 and ρ_w = 1030 kilogrammes per cubic metre, respectively.

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[*] Whether or not repairs shall be carried out is also a matter of personal decision on the part of the one who is responsible for the maintenance of the structure. It is known, however, that a small percentage of damage makes no difference to the effect of successive wave attacks having greater H_s. It would be worth going to the trouble to find out whether that also holds good for appreciable damage, so as to see if repairing such damage would be of any use.

Table 1.

H_{so} (m)	I	I_{cl}	$1<H_s/H_{so}<1.3$, n=4%			$1.3<H_s/H_{so}<1.45$ n=8%			$H_s/H_{so}>1.45$; collapse		
			Δp	ΔW	$\Delta p.\Delta W$	Δp	ΔW	$\Delta p\Delta W$	Δp	ΔW	$\Delta p\Delta W$
4	13900	5280	1.01	420	430	$5.2.10^{-2}$	860	40	$3.8.10^{-2}$	13900	530
5	15220	6600	$1.6.10^{-1}$	530	80	$4.7.10^{-3}$	1060	5	$2.8.10^{-3}$	15220	40
5.5	15900	7280	$6.3.10^{-2}$	580	40	$1.6.10^{-3}$	1160	-	7.10^{-4}	15900	10
6	16540	7920	$2.5.10^{-2}$	630	15	$5.2.10^{-4}$	1260	-	$1.8.10^{-4}$	16540	3

Table 2.

H_{so} (m)	With repairing partial damage		Without repairing partial damage	
	$s=\sum\Delta p\,\Delta W$	$S=\frac{100}{\delta}s$	s	S
4	1000	30000	530	15900
5	125	3750	40	1200
5.5	50	1500	10	300
6	18	540	3	90

Table 3.

H_{so} (m)	With repairing partial damage			Without repairing partial damage	
	I	S	K	S	K
4	13900	30000	43900	15900	29800
5	15220	3750	18970	1200	16420
5.5	15900	1500	17400	300	16200
6	16540	540	17080	90	16630
6.5	17200	100	17300	20	17220

I in guilders.
n = percentage of damage.

Crest width b (in view of block size) b = 5 mtrs.
Height of hard shoulder in seaward cover layer: M.S.L. - H_{so} mtrs.
A diagrammatic sketch is given in figure 15.

The occurrence of off-shore conditions. In this example the wave conditions taken are those found along the south coast of Turkey. A probability distribution curve for H_s is given in figure 16. The water level is constant. The depth is assumed to be 12 mtrs.

Relation between off-shore conditions and behaviour of the structure. The damage to the seaward cover layer for wave heights exceeding H_{so} (see the relevant paragraph on page 9) is given in figure 3, provided that the heaviest layer is extended down to M.S.L. - H_{so} mtrs. It is assumed that the structure will collapse at 30% damage, hence when $H_s = 1.45\ H_{so}$. See fig. 3. In view of the particulars given in figure 4, it is also assumed that critical overtopping (i.e. collapse of the structure) will occur at

$H_s = \frac{h\sqrt{\text{cotg}\,\alpha}}{1.35}$, in which h is the crest height above M.S.L.
Failure will occur when the critical value for overtopping

$H'_{so} = \frac{h\sqrt{\text{cotg}\,\alpha}}{1.35}$ or the critical value for the collapse of the seaward cover layer $1.45\ H_{so}$ is exceeded. As the entire cross section is destroyed in both cases, it will always collapse at the smaller of the two values H'_{so} and $1.45\ H_{so}$. Consequently, the optimum design is obtained when $H'_{so} = 1.45\ H_{so}$.
The corresponding crest height is:

$$h = \frac{1.35 \cdot 1.45\ H_{so}}{\sqrt{\text{cotg}\,\alpha}} = \frac{1.95\ H_{so}}{\sqrt{\text{cotg}\,\alpha}}$$

The relation between the angle of inclination of the seaward cover layer and the design wave H_{so} for a given block weight W and relative density Δ is:

$$\text{cotg}\,\alpha = \sqrt[3]{\frac{\gamma_s\ H_{so}^3}{1.25\ \Delta^3 W}} . \qquad \text{See also fig. 5.}$$

Relation between cost of construction and design wave. $I = f(H_{so})$. It is assumed that the breakwater will be built with the aid of floating equipment, the cost of which is practically the same, for all the structures considered. Hence the cost of construction is determined by the quantities of the various categories of rock. The quantities for different values of H_{so} have been worked out, see table 4.

For the sake of simplicity it has been assumed that the prices of all the categories of blocks are the same and equal to the unit. So the cost of construction, I, appears in the last column in table 4.

Relation between anticipated damage, off-shore conditions and design wave. $S = \frac{100}{\delta} \sum \Delta p \,.\, \Delta W.$
The capitalized anticipated damage is determined in a manner similar to that in which the akmon cover layer is established. Five intervals

Table 4.

H_{so} (m)	cotg α	h (m)	Vol. of rock in m^3 per m'			Total
			cat. A	cat. B	cat. C	
4	1.55	6.2	77	78	446	601
4.5	1.85	6.3	92	84	470	646
5	2.2	6.4	106	91	505	702
6	2.85	6.7	140	109	594	843
7	3.6	7	182	129	647	958

Table 5.

H_{so} (m)	I	I cat.A	$1<H_s/H_{so}<1.2$, n=3%			$1.2<H_s/H_{so}<1.3$, n=9%			$1.3<H_s/H_{so}<1.4$, n=17%		
			Δp	ΔW	$\Delta p\,\Delta W$	Δp	ΔW	$\Delta p.\Delta W$	Δp	ΔW	$\Delta p\,\Delta W$
4	601	77	$3.5.10^{-1}$	4.6	1.6	8.10^{-2}	14	1.1	$3.2.10^{-2}$	26	0.8
4.5	646	92	$1.8.10^{-1}$	5.5	1	$2.9.10^{-2}$	16.6	0.5	$1.4.10^{-2}$	31	0.4
5	702	106	8.10^{-2}	6.4	0.5	$1.3.10^{-2}$	19	0.2	$4.2.10^{-3}$	36	0.2
6	843	140	$1.8.10^{-2}$	8.4	0.2	$1.3.10^{-3}$	25	-	4.10^{-4}	48	-
7	958	182	$2.8.10^{-3}$	10.9	-	-	-	-	-	-	-

Table 5.

H_{so} (m)	I	I cat.A	$1.4<H_s/H_{so}<1.45$, n=25%			$H_s/H_{so}>1.45$; collapse		
			Δp	ΔW	$\Delta p\,\Delta W$	Δp	ΔW	$\Delta p.\Delta W$
4	601	77	9.10^{-3}	39	0.4	$2.9.10^{-2}$	601	17.5
4.5	646	92	4.10^{-3}	46	0.2	8.10^{-3}	646	5.2
5	702	106	$1.2.10^{-3}$	53	0.1	$1.6.10^{-3}$	702	1.1
6	843	140	7.10^{-5}	70	-	8.10^{-5}	843	0.1
7	958	182	-	-	-	-	-	-

H_s/H_{so} are considered. The corresponding percentages of damage follow from figure 3 and refer to category A blocks, see fig. 15. The amount of damage ΔW is assumed to be: damage percentage x cost of cover layers made of category A blocks x 2. For a collapse, ΔW is assumed to be equal to the cost of construction. For various values of H_{so}, $\Delta \rho$ and ΔW are given in table 5. The damage due to the collapse of the structure is also given separately. For $\frac{100}{\delta} = 30$ the values of s and S are given in table 6.

The total cost of the structure. $K = I + S$.
The total cost of the structure for various values of H_{so} is given in table 7. The sums are also given in figure 17 in which I and K have been plotted as a function of H_{so}.

The design wave. Required slope. In accordance with the minimum total cost criterion the design wave H_{so} is:

$H_{so} = 5.1$ mtrs. and $H_{so} = 4.5$ mtrs. when partial damage is not repaired.

H_{so} (m)	$\rho(H_{so})$	Probability of failure	Angle of seaward slope
5.1	9.10^{-2}	$1.2\ 10^{-3}$	cotg $\alpha = 2.25$
4.5	$2.5.10^{-1}$	$7.5\ 10^{-3}$	cotg $\alpha = 1.85$

CONCLUSIONS

1. The decision on the type of structure and its dimensions can be based on the criterion of minimum total cost.
2. The total cost of a structure can be expressed in terms of design wave or load. To establish this relation there must be known: the off-shore conditions, the behaviour of the structure, the cost of construction and the anticipated damage.
   Although information on this is often insufficient, for reasonable assumptions important directives can be obtained with respect to minimum total cost and corresponding optimum design wave or load.
3. The method is especially of great aid when it concerns ranking of similar structures, as the errors made in costs of construction, damage etc. will in that case affect the results in the same way.
4. The scope of information needed to determine the minimum cost can be used as a bases for future investigations on breakwater design.

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Table 6.

H_{so} (m)	With repairing partial damage		Without repairing partial damage	
	$s=\sum \Delta p \cdot \Delta W$	$S = \frac{100}{\delta} s$	s	S
4	21.4	642	3.9	117
4.5	7.3	219	2.1	63
5	1.9	57	0.8	24
6	0.3	9	0.2	6
7	-	-	-	-

Table 7.

H_{so} (m)	I	With repairing partial damage		Without repairing partial damage	
		S	K	S	K
4	601	642	1243	117	718
4.5	646	219	865	63	709
5	702	57	759	24	726
6	843	9	852	6	849
7	958	-	958	-	958

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