Chapter 2

PERIODIC GRAVITY WAVES OVER A GENTLE SLOPE AT A THIRD ORDER OF APPROXIMATION

by

B. Le Méhauté and L M. Webb

National Engineering Science Company, Pasadena, California

The concepts of energy flux and group velocity for nonlinear periodic gravity waves are discussed The average energy flux, average energy per wavelength and "group velocity" are calculated for irrotational periodic gravity waves at a third order of approximation. Then the principle of conservation of transmitted energy between wave orthogonals is applied to the same order of approximation for determining the variation of wave height in decreasing depth.

The results are presented as nomographs. It is seen that the "shoaling coefficient" needs to be expressed at least at a third order of approximation to account for experimental results.

Les concepts de flux d'énergie et de vitesse de groupe sont étudiés en vue de leur application a la théorie non lineaire des ondes de gravité periodiques Le flux d'énergie moyen, l'énergie moyenne par longueur dónde et la vitesse de groupe sont calculés au troisième ordre d'approximation La variation d'amplitude d'une onde en profondeur variable est calculée au même ordre d'approximation en applicant le principe de conservation d'energie transmise entre orthogonales.

Les résultats sont presentés sous forme d'abaques, et comparés avec quelques resultats expérimentaux Il est conclus que la méthode présentée est encourageante, et reclamerait d'être travaillée à un ordre d'approximation plusélevée que le troisième ordre.

INTRODUCTION

The present study is a theoretical contribution to the problem of periodic gravity waves traveling in water of decreasing depth. It is remembered that this problem can be treated by two different methods: the analytical method and the energy method.

The analytical method consists of finding a potential function ϕ (x, y, t) for a progressive wave, as a solution of $\nabla^2 \phi = 0$ and which satisfies the usual boundary condition including that of a sloped bottom: $\phi_x - S\,\phi_y = 0$ for $y = -d(x)$ (d is the depth and oy is positive upwards, S is the bottom slope).

The general solution in form of power series can be expected to be of the following form:

$$\phi = H\phi_{10} + SH\phi_{11} + S^2 H\phi_{12} + \cdots + H^2\phi_{20} + SH^2\phi_{21} + \cdots + H^3\phi_{30} + \cdots \qquad (1)$$

The terms in $(\phi_{10}, S\phi_{11}, S^2\phi_{12} \cdots)$ are obtained from the linear theory, i. e. by taking: $\phi_{tt} - g\phi_y = 0$ as free-surface condition. (2)
Actually $H\phi_{10}$ is the classical Airy solution valid for a horizontal bottom. It has been shown (Biesel, 1951), that $H = H_1$ and

$$H_1 = \frac{H_o}{\left\{\left[1 + \dfrac{4\pi d/L}{\sinh 4\pi d/L}\right]\tanh\dfrac{2\pi d}{L}\right\}^{1/2}} \qquad (3)$$

where H_o is the deep water wave height and L the wave length.

The terms in successive powers of H $(H^2\phi_{20}, H^3\phi_{30} \cdots)$ are obtained from the Stokes Theory, i. e by taking for free-surface condition.

$$\phi_x \eta_x - \phi_y + \eta_t = 0 \quad \text{FOR } y = \eta(x,t) \qquad (4)$$

$$g\eta + \phi_t + \tfrac{1}{2}\left(\phi_x^2 + \phi_y^2\right) = F(x,t) \text{ ON } y = \eta(x,t) \qquad (5)$$

On a very gentle slope such as encountered on the continental shelf and for relatively steep waves, it is easily realized that these non-linear terms are far more important than the terms in S, (linear or nonlinear).

It is recalled that the energy method consists of assuming first that for a short distance, the wave motion on a sloped bottom is the same as on a horizontal bottom. Then when the wave motion has been so determined, it is assumed that the rate of transmission of energy is constant over a varying depth. The use of the energy method instead of the analytical method permits to take into account a number of phenomena such as bottom friction and variation of distance between wave orthogonals, which are often more important than the flow pattern deformation due to the bottom slope as given by the analytical method and represented by the terms in S. The definition of average flux

$$F_{av} = \frac{\rho}{T}\int_{t}^{t+T}\int_{-d}^{\eta} \phi_x \phi_t \, dt \, dy \tag{6}$$

is independent of the order of approximation for ϕ. Hence, writing F_{av}= constant should give the variation of wave height as function of depth at an order of approximation corresponding to the order of approximation for ϕ.

It is recalled that the application of this principle where ϕ is taken as the Airy solution gives also the relationship (3) above. It is realized that the analytical method merges with the energy method, based on the conservation of transmitted energy flux, when S tends to zero. This confirms the validity of the energy method for small values of S. Then it can be assumed that the principle of conservation of energy flux also applies to a nonlinear wave.

It is recalled also (Stoker, 1957) that the rate of propagation of energy G is given by

$$G = \frac{F_{av}}{E_{av}} \tag{7}$$

where the average energy E_{av} is:

$$E_{av} = \frac{\rho}{L}\int_{0}^{L}\left[\frac{1}{2}\int_{-d}^{\eta}\left(\phi_x^2 + \phi_y^2\right) dy + g\int_{-d}^{\eta} \eta \, d\eta\right] dx \tag{8}$$

The speed of propagation of energy happens to be also the "group velocity" G' if F_{av} and E_{av} are calculated to the first order of approximation: $G = G'$, where

$$G' = C - L\frac{dC}{dL} \tag{9}$$

This result is consistent with the fact that there is no energy passing through the nodes of a wave train and that the energy travels at the speed of the wave train, the so called group-velocity. This statement does not hold true in the case of nonlinear waves (Biesel 1952). In this case only formula (7) is valid. In fact, it is immaterial whether or not this happens to coincide with the classical group velocity formula (9) since the study of the modification of wave shoaling involves the use of the flux of energy given by formula (6) only. However, in the following, the two values G and G', given by formulas (7) and (9) respectively, have also been calculated for the sake of academic interest and to conform to tradition. It will be seen that the results given by these two procedures are different if the calculations are carried out at a third or higher order of approximation.

ENERGY FLUX, AVERAGE ENERGY AND "GROUP VELOCITY" AT A THIRD ORDER OF APPROXIMATION

In the following, these notations are used:

- H_3 - wave height at a third order of approximation
- H_1 - wave height at a first order of approximation
- H_0 - wave height in deep water
- L - wave length at a third order of approximation
- T - wave period
- C_1 - phase velocity at a first order of approximation
- C_3 - phase velocity at a third order of approximation $\} = \frac{L}{T}$
- d - water depth
- g - acceleration of gravity
- ϕ - potential function
- t - time
- x - horizontal coordinate
- η - free-surface elevation around the S. W. L.
- y - vertical coordinate from the mud line
- k - wave number $= 2\pi/L$
- σ - angular frequency $= 2\pi/T$
- ρ - fluid density
- θ - phase angle $= 2\pi\left(\frac{x}{L} - \frac{t}{T}\right)$

Subscripts

- $_0$ - refers to deep water
- $_1$ - refers to linear wave theory
- $_3$ - refers to third order wave theory

Other notations are defined as they appear in the text.

The Stokes third order wave theory as extracted from the theory at the fifth order developed by Skjelbreia and Hendrickson (1961) is defined by the following set of equations: the potential function ϕ is given by

$$\phi = \frac{L^2}{2\pi}\left[(A_{11}\lambda + A_{13}\lambda^3)\cosh ky \sin\theta + \lambda^2 A_{22}\cosh 2ky \sin 2\theta + \lambda^3 A_{33}\cosh 3ky \sin 3\theta\right] \tag{10}$$

The free-surface elevation η is given by:

$$\eta = k(\lambda\cos\theta + \lambda B_{22}\cos 2\theta + \lambda^3 B_{33}\cos 3\theta) \tag{11}$$

and the phase velocity C_3 is given by:

$$\left(\frac{L}{T}\right)^2 = C_3^2 = C_1(1 + B\lambda^2), \qquad C_1 = \frac{g}{k}\tanh kd \tag{12}$$

where λ is the real positive root of

$$\lambda^3 B_{33} + \lambda = \frac{\pi H}{L} \tag{13}$$

and with $S_* = \sinh kd$ (14)
$C_* = \cosh kd$

$$\left.\begin{aligned}
A_{11} &= \frac{1}{S_*}, \quad A_{13} = -\frac{C_1^2(5C_*^2+1)}{8 S_*^5}, \quad A_{22} = \frac{3}{8 S_*^4} \\
A_{33} &= \frac{13 - 4C_*^2}{64 S_*^7}, \quad B_{22} = \frac{C_1(2C_*^2+1)}{4 S_*^3}, \\
B_{33} &= \frac{3(8C_*^6+1)}{64 S_*^6}, \quad B = \frac{8C_*^4 - 8C_*^2 + 9}{8 S_*^4}
\end{aligned}\right\} \tag{15}$$

It is pointed out that most often, C_3^2 given in technical literature and as presented above by formula (13) is wrongly taken at a third order of approximation with $\lambda = \frac{\pi H}{L}$. In fact λ calculated from formula (13) gives a very different value for shallow water due to the fact that the term B_{33} tends to infinity when $d \to 0$. Hence in very shallow water λ should be taken as a third order of approximation as:

$$\lambda = \left[\frac{1}{B_{33}} \frac{\pi H}{L}\right]^{1/3} \tag{16}$$

In the following the full formula (13) is used, and it has been verified that due to that third order term, the correction on the wave height variation by shoaling with respect to the linear theory appears to be in the opposite direction of what it would be if λ were taken as $\frac{\pi H}{L}$. Hence, it has been judged important to emphasize this discrepancy with other papers, making use of the third order wave theory. It is easily seen that, in deep water, equation (14) becomes

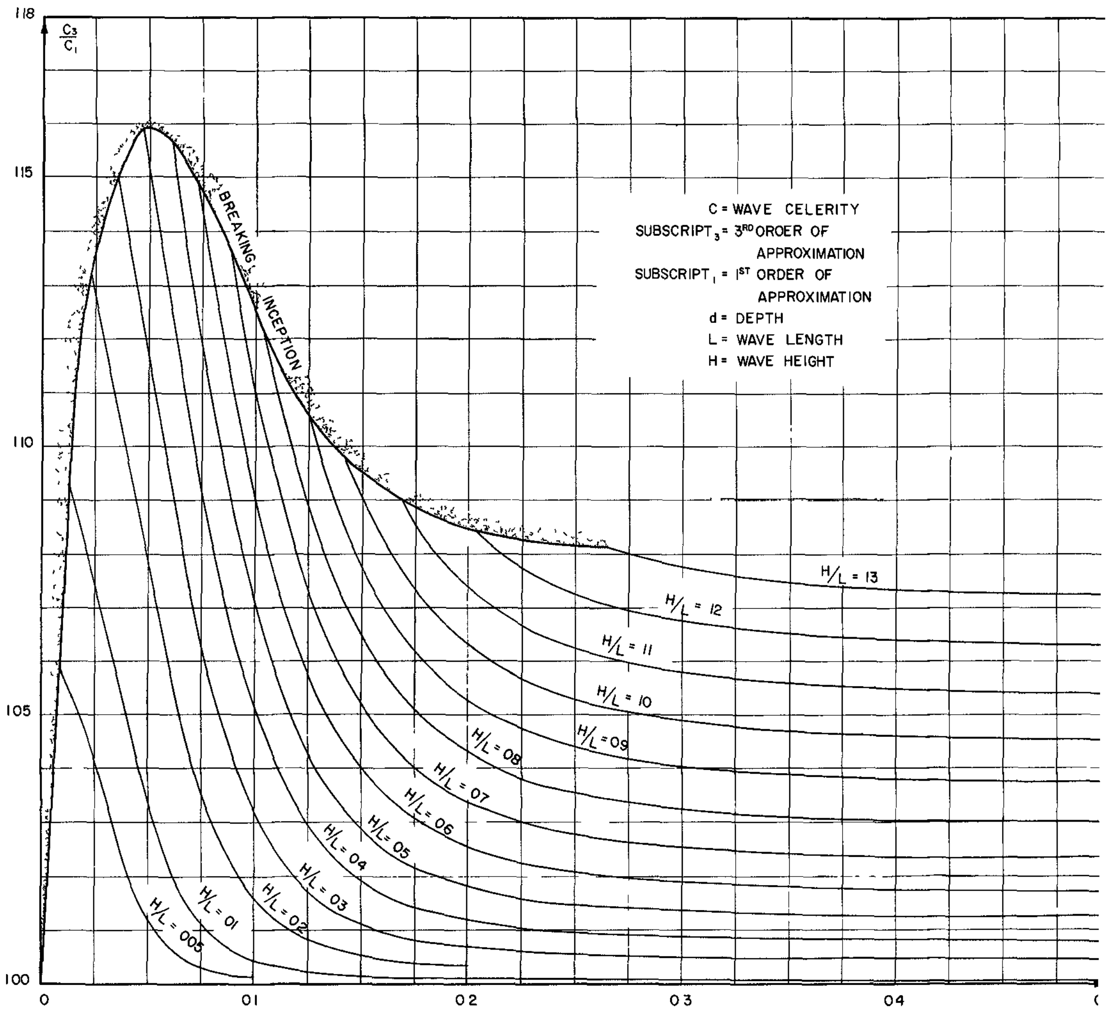
$$\frac{3}{8}\lambda_o^3 + \lambda_o = \frac{\pi H_o}{L_o} \tag{17}$$

and equation (12)

$$\left(\frac{L_o}{T}\right)^2 = \frac{g}{k}\left(1 + \lambda_o^2\right) \tag{18}$$

in which case $\lambda_o = \frac{\pi H_o}{L_o}$ is a reasonable approximation.

It is recalled that this set of equations gives λ, L and C_3, simultaneously for given values of d, T and H. The corresponding value of $\frac{C_3}{C_1}$ vs. $\frac{d}{L}$, for various values of $\frac{H}{L}$ is presented in Figure 1.



$\frac{C_3}{C_1}$
118
115
110
105
100
0
01
02
03
04
C = WAVE CELERITY
SUBSCRIPT$_3$ = 3RD ORDER OF APPROXIMATION
SUBSCRIPT$_1$ = 1ST ORDER OF APPROXIMATION
d = DEPTH
L = WAVE LENGTH
H = WAVE HEIGHT
BREAKING INCEPTION
H/L = 13
H/L = 12
H/L = 11
H/L = 10
H/L = 09
H/L = 08
H/L = 07
H/L = 06
H/L = 05
H/L = 04
H/L = 03
H/L = 02
H/L = 01
H/L = 005

FIG 1

Now, by inserting the value of φ given by (10) into (6), (8) and integrating, one obtains after lengthy but straightfoward calculations.

$$F_{av} = \frac{\pi \rho C_3^2 \lambda^2}{8 k^2 T} \cdot \frac{1}{S_*^2} \left\{ 4(S_* C_* + kd) + \lambda^2 \left[\frac{(S_* C_* + kd)}{4 S_*^6} \right. \right.$$

$$(-20\, C_*^6 + 16\, C_*^4 + 4 C_*^2 + 9) \tag{19}$$

$$\left. \left. + \frac{S_* C_*}{2 S_*^4} (16 C_*^4 + 2 C_*^2 + 9) \right] \right\} + O(\lambda^6) + \cdots$$

and

$$E_{av} = \frac{\pi \rho C_3^2 \lambda^2}{L k^2} \frac{C_*}{S_*} \left\{ 1 + \frac{\lambda^2}{32 S_*^6} (-16 C_*^6 + 56 C_*^4 - 49 C_*^2 + 27) \right\} + O(\lambda^6) + \cdots \tag{20}$$

Then the rate of energy propagation $G = \dfrac{F_{av}}{E_{av}}$ is found to be.

$$G = G_3 = \frac{C_1}{2} \left\{ 1 + \frac{2 kd}{\sinh 2kd} + \frac{\lambda^2}{32 \sinh^4 kd} \left[5(8 \cosh^4 kd - 8 \cosh^2 kd + 9) \right. \right. \tag{21}$$

$$\left. \left. - \frac{6 kd}{\sinh 2 kd} (8 \cosh^4 kd + 16 \cosh^2 kd - 3) \right] \right\}$$

which is quite different from

$$G = \frac{C_3}{2} \left[1 + 2 kd / \sinh 2 kd \right]$$

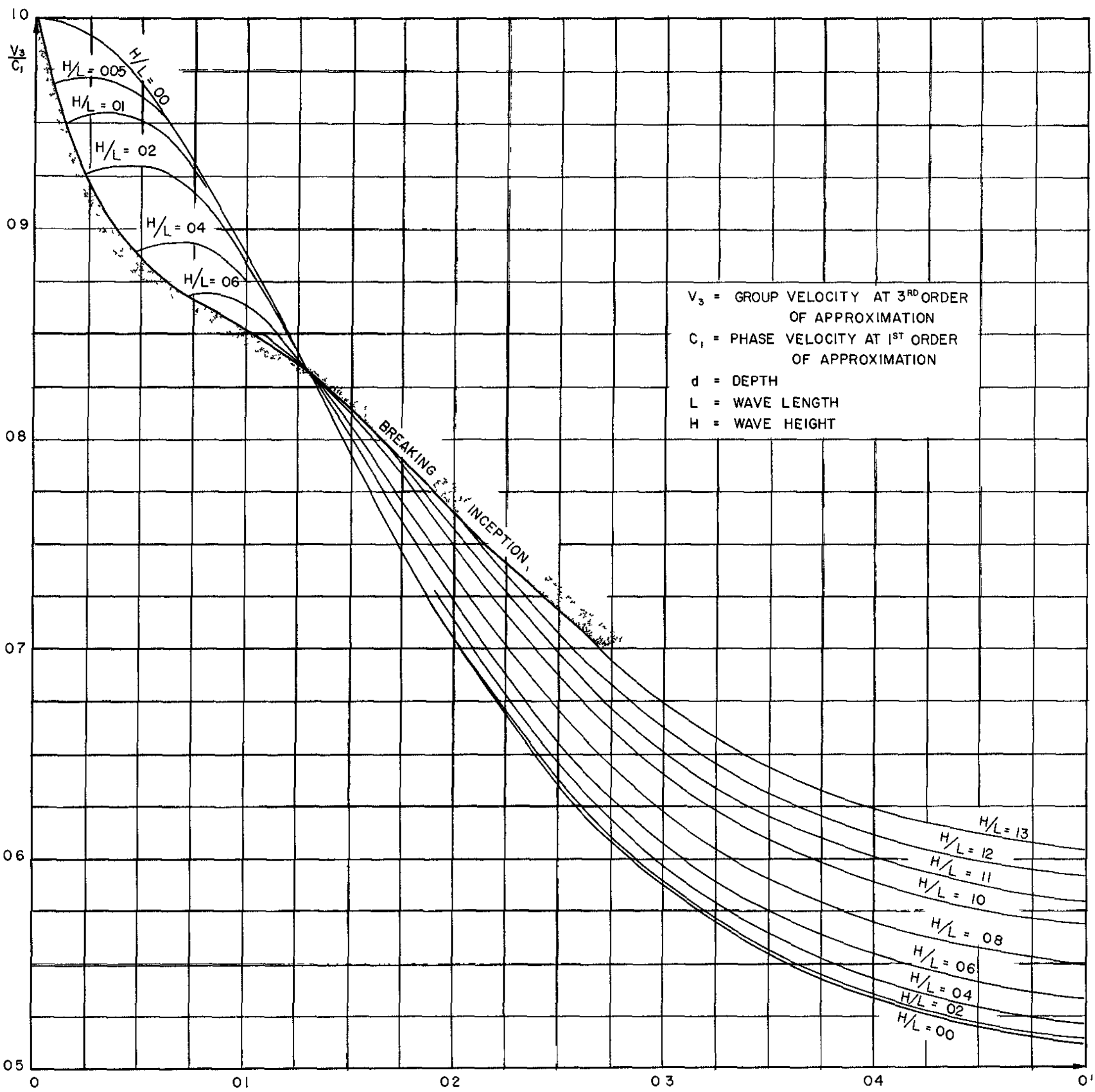
as sometimes is proposed.

If one applies the formula (9)

$$G' = G_3' = \frac{C_1}{2} \left\{ 1 + \frac{2 kd}{\sinh 2kd} + \frac{\lambda^2}{8 \sinh^4 kd} \left[-(8 \cosh^4 kd - 8 \cosh^2 kd + 9) \right. \right. \tag{22}$$

$$\left. \left. - \frac{2 kd}{\sinh 2 kd} (8 \cosh^4 kd + 28 \cosh^2 kd - 9) \right] \right\}$$

It can easily be seen that only the first terms of (21) and (22) are alike. The value of $\frac{G_3}{C_1}$, $\frac{G_3}{C_3}$ vs $\frac{d}{L}$, for various values of $\frac{H}{L}$ is presented on Figures 2 and 3 respectively. The transformation of these



1 0
0 9
0 8
0 7
0 6
0 5
$\frac{V_3}{C_1}$
0
0 1
0 2
0 3
0 4
0 1
H/L = 00
H/L = 005
H/L = 01
H/L = 02
H/L = 04
H/L = 06
H/L = 08
H/L = 10
H/L = 11
H/L = 12
H/L = 13
BREAKING INCEPTION
V_3 = GROUP VELOCITY AT 3RD ORDER OF APPROXIMATION
C_1 = PHASE VELOCITY AT 1ST ORDER OF APPROXIMATION
d = DEPTH
L = WAVE LENGTH
H = WAVE HEIGHT

FIG 2

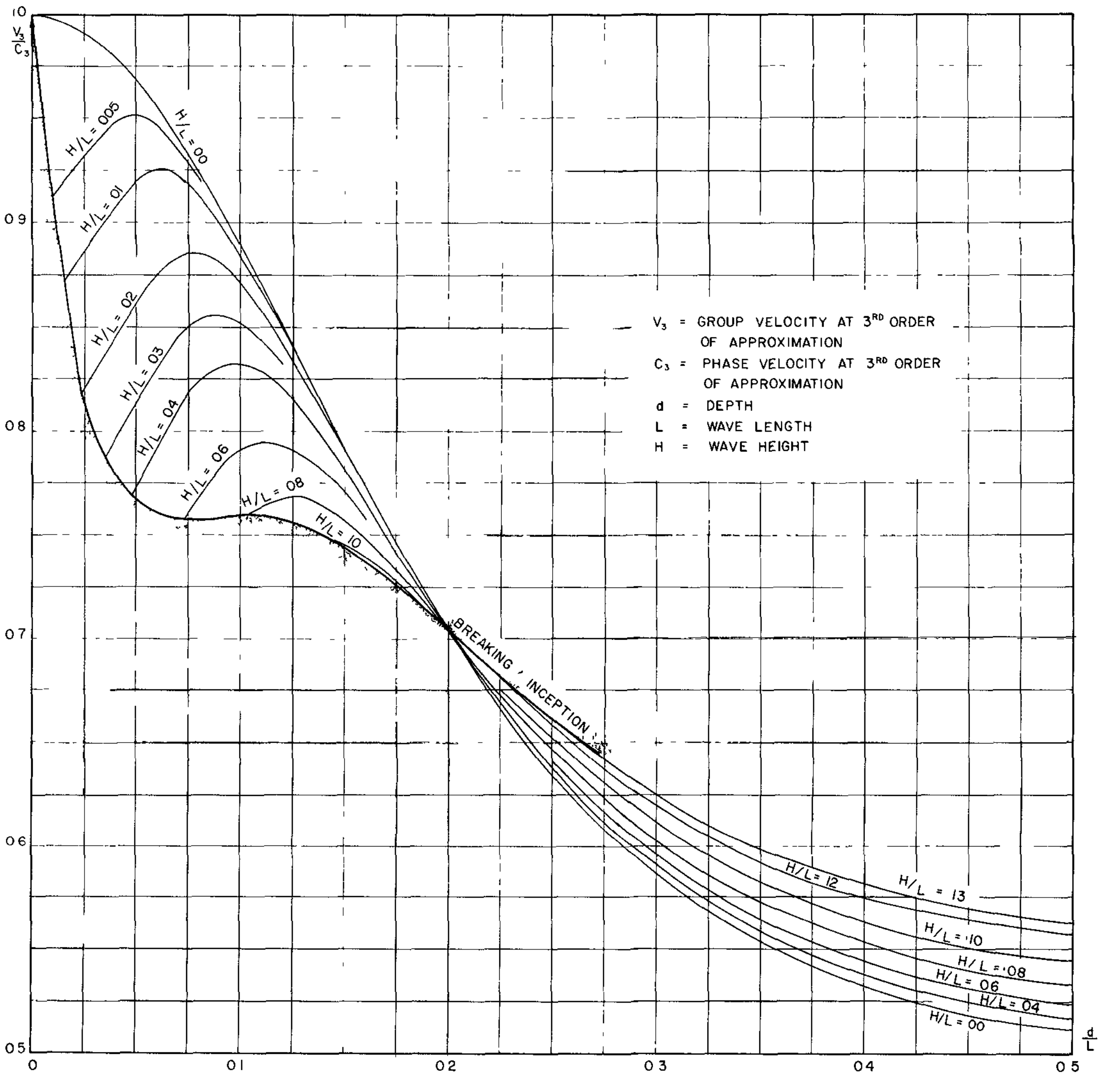

1·0
$\frac{V_3}{C_3}$
0·9
0·8
0·7
0·6
0·5
0
0·1
0·2
0·3
0·4
0·5
$\frac{d}{L}$
H/L = ·005
H/L = 00
H/L = ·01
H/L = ·02
H/L = ·03
H/L = ·04
H/L = ·06
H/L = ·08
H/L = ·10
BREAKING INCEPTION
H/L = ·12
H/L = ·13
H/L = ·10
H/L = ·08
H/L = ·06
H/L = ·04
H/L = 00
V_3 = GROUP VELOCITY AT 3RD ORDER OF APPROXIMATION
C_3 = PHASE VELOCITY AT 3RD ORDER OF APPROXIMATION
d = DEPTH
L = WAVE LENGTH
H = WAVE HEIGHT

FIG 3

results as functions of H/T^2 and d/T^2 for practical purposes has not been carried out because the primary purpose of the study is the wave shoaling

It is interesting to note that G for nonlinear waves decreases relatively as $\frac{H}{L}$ increases in shallow water, while it increases with $\frac{H}{L}$ in deep water. It can easily be verified that the results of the first order wave theory are found provided λ is taken to be equal to $\pi H/L$ in formulas (18) and (20), and in formulas (20) and (21). Then

$$F_{av} = \frac{1}{2}\rho g\left(\frac{H}{2}\right)^2 \frac{C_1}{2}\left[1 + \frac{2\kappa d}{\sinh 2\kappa d}\right] \tag{23}$$

$$E_{av} = \frac{1}{2}\rho g\left(\frac{H}{2}\right)^2 \tag{24}$$

and

$$G = G_1 = \frac{C_1}{2}\left[1 + \frac{2\kappa d}{\sinh 2\kappa d}\right] \tag{25}$$

WAVE SHOALING

Now multiplying F_{av} given in (19) by the constant value $K = \frac{32\pi}{\rho T^5}$ and taking the corresponding limit for $\frac{d}{L} \to \infty$ and writing the conservation of energy flux $K F_{av}\big|_{d=d} = K F_{av}\big|_{d \to \infty}$ gives

$$\left(\frac{L}{T^2}\right)^4 \frac{\lambda^2}{S_\kappa^2}\left\{A\left(\lambda, \frac{d}{L}\right)\right\} = \left(\frac{L_o}{T^2}\right)^4 \lambda_o^2\left[4\left(1 + \frac{3}{4}\lambda_o^2\right)\right] \tag{26}$$

Where A is the expression between bracket of formula (19) the subscript o refers to deep water. Equation (26), combined with equations (12), (13), (17) and (18) forms a system of 5 simultaneous equations which permits calculation of L_o, λ_o, L, λ and H, i e. the shoaling coefficient $\frac{H}{H_o}$ as function of $\frac{d}{T^2}$ and $\frac{H_o}{T^2}$ once H_o, T and d are given.

An outline of the method of solution is given now:

Given $\frac{d}{L}$ and $\frac{H}{L}$, λ is calculated from formula (13). Also $\frac{L}{T^2}$ is calculated from formula (12) divided by L, as function of λ, $\frac{d}{L}$ and $\frac{H}{L}$

Consequently, $\frac{d}{T^2}$ and $\frac{H}{T^2}$ are obtained. Then $\frac{32\pi F_{av}}{\rho T^5}$ is calculated from formula (26) at a depth d

By eliminating λ_o between (18) and the following equations

$$F = \left(\frac{L_o}{T^2}\right)^4 \lambda_o^2 \left[4\left(1 + \frac{3}{4}\lambda_o^2\right)\right] \tag{27}$$

and defining X as follows

$$\frac{L_o}{T^2} = \frac{g}{2\pi}(1 + X) \tag{28}$$

and

$$B = F(g/2\pi)^{-4} \tag{29}$$

gives

$$4X + 19X^2 + 36X^3 + 34X^4 + 16X^5 + 3X^6 = B \tag{30}$$

X in the correction due to the nonlinear terms Hence X remains small, as does B Consequently by inverting the polynomial we have

$$X = \frac{1}{4}B - \frac{19}{64}B^2 + \frac{578}{1024}B^3 - \frac{21,159}{16,384}B^4 + \cdots \tag{31}$$

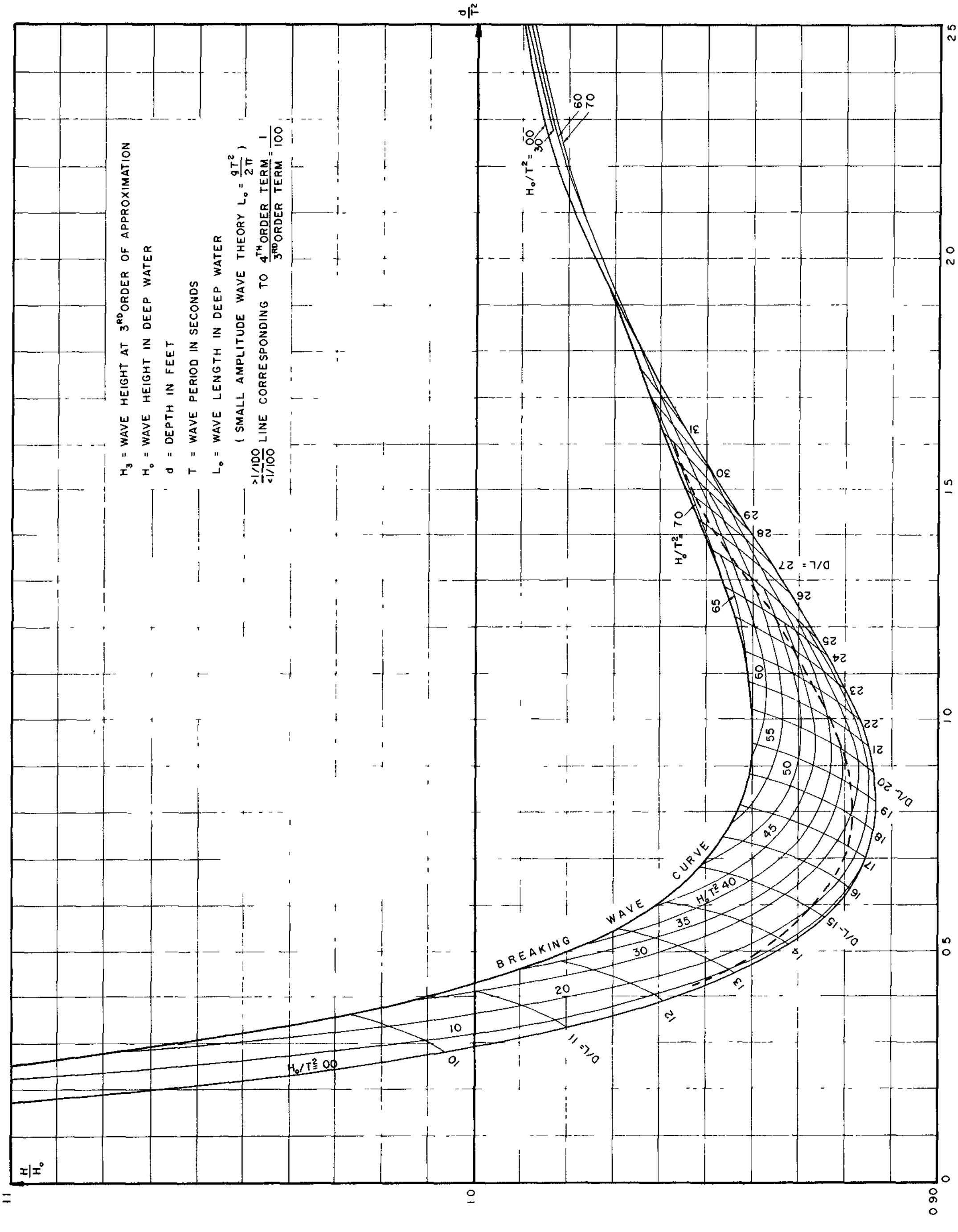
at a very good approximation Because this calculation has been done by computer, the solution (30) has been obtained by successive approximations around $X = \frac{B}{4}$ by the Newton-Raphson method. $\frac{L_o}{T^2}$ is obtained from equation (28), then λ_o from equation (18) and $\frac{H_o}{L_o}$ from equation (17) Finally $\frac{H_o}{T^2}$ and $\frac{H}{H_o}$ are obtained by simple operation. The curve $\frac{H_o}{T^2}$ = constant for various value of $\frac{d}{T^2}$ have been obtained by linear interpolation between the values found for H_o/T^2

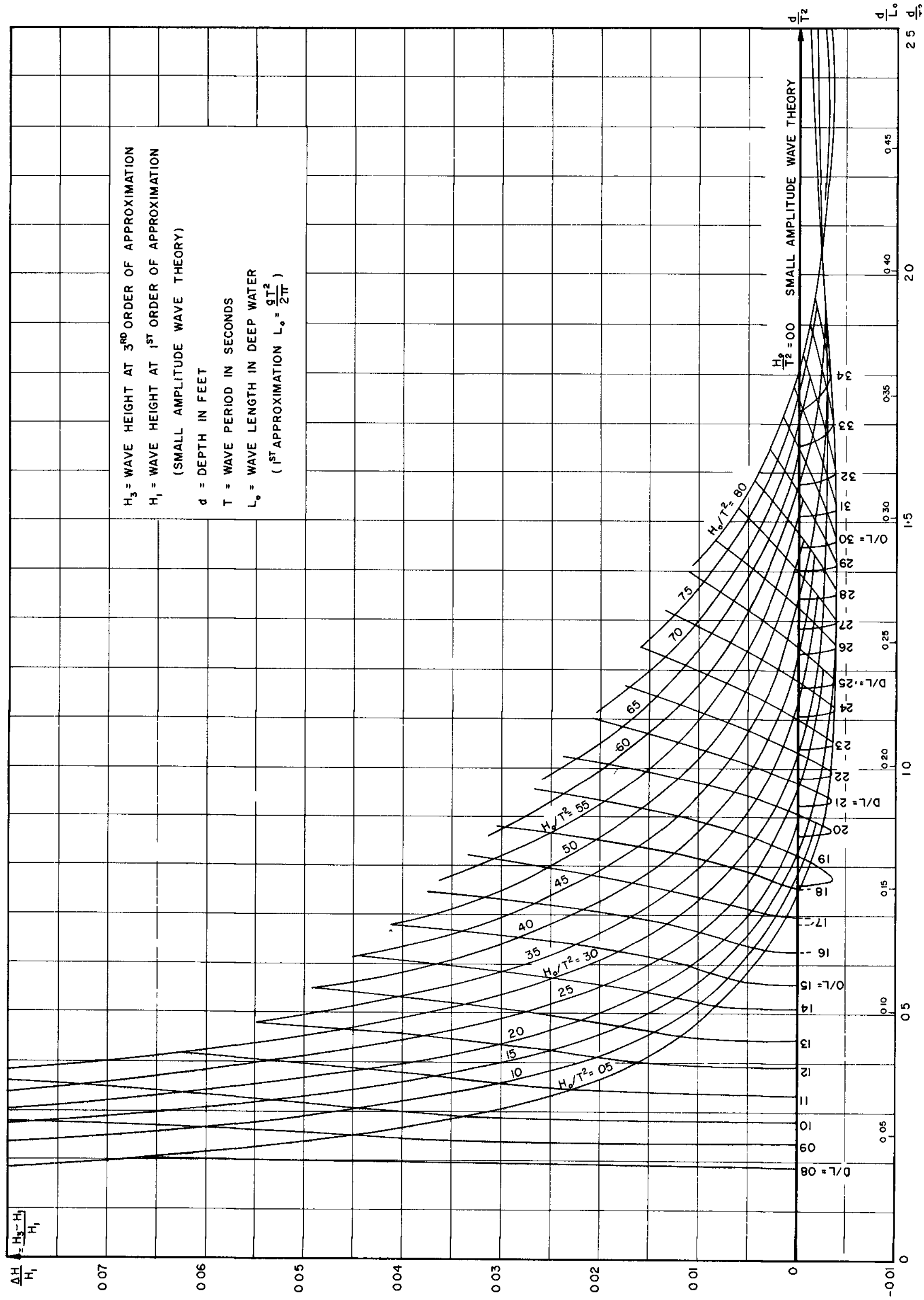
Finally the correction $\frac{\Delta H}{H_1} = \frac{H_3 - H_1}{H_1}$ due to the third order of approximation, H_1 being the value which is obtained by the linear theory (formula 3) has also been calculated as function of $\frac{d}{T^2}$ and $\frac{H_o}{T^2}$ for a given value of $\frac{d}{L}$ and $\frac{H}{L}$, by a similar process.

These results are presented on Figure (4) and (5) on which the limit for breaking criteria has been added. The complete set of tables is published in a NESCO Report SN-134.

Wilson (1964) has determined the limit after which the ratio of the fourth order term to the third order is larger than 1/100, which may be considered from an academic new point, the limit of validity of the third order wave theory It has been found that this limit is defined in shallow water by the condition ($kd < \pi/10$)

$$\frac{H}{L_o} < 0.725\left(\frac{kd}{L}\right)^2 \frac{d}{L_o} \tag{32}$$


H_3 = WAVE HEIGHT AT 3RD ORDER OF APPROXIMATION
H_o = WAVE HEIGHT IN DEEP WATER
d = DEPTH IN FEET
T = WAVE PERIOD IN SECONDS
L_o = WAVE LENGTH IN DEEP WATER
(SMALL AMPLITUDE WAVE THEORY $L_o = \frac{gT^2}{2\pi}$)
>1/100 <1/100 LINE CORRESPONDING TO $\frac{4^{TH} \text{ ORDER TERM}}{3^{RD} \text{ ORDER TERM}} = \frac{1}{100}$
BREAKING WAVE CURVE
H_o/T^2 = 00
H_o/T^2 = 40
H_o/T^2 = 70
D/L = 11
D/L = 15
D/L = 20
D/L = 27
$\frac{d}{T^2}$
$\frac{H}{H_o}$



H_3 = WAVE HEIGHT AT 3^{RD} ORDER OF APPROXIMATION
H_1 = WAVE HEIGHT AT 1^{ST} ORDER OF APPROXIMATION
(SMALL AMPLITUDE WAVE THEORY)
d = DEPTH IN FEET
T = WAVE PERIOD IN SECONDS
L_o = WAVE LENGTH IN DEEP WATER
(1^{ST} APPROXIMATION $L_o = \frac{gT^2}{2\pi}$)
SMALL AMPLITUDE WAVE THEORY
$\frac{H_o}{T^2}$ = 00
H_o/T^2 = 80
H_o/T^2 = 55
H_o/T^2 = 30
H_o/T^2 = 05
D/L = 08
D/L = 15
D/L = 21
D/L = 25
D/L = 30
$\frac{\Delta H}{H_1} = \frac{H_3 - H_1}{H_1}$
$\frac{d}{T^2}$
$\frac{d}{L_o}$

and in deep water

$$\frac{H_0}{L_0} < 0.10 \tag{33}$$

The corresponding line has also been drawn on Figure 4.

It is theoretically necessary to use a higher order of approximation or the cnoidal wave theory, if one wants an error smaller than 1/100.

It is evident that Figures (4) and (5) permit also the calculation of $H(d, X)$ after insertion of a term for bottom friction and a correcting term for taking into account the variation of width of orthogonals. In this case, the calculation has to be done step by step over an interval ΔX, by going from one line $\frac{H_0}{T^2}$ = constant to another.

THEORY VS. EXPERIMENTS

The comparison of theoretical curves $\frac{H}{H_0}$ vs. d/T^2 for various $\frac{H_0}{T^2}$ = constant with the experimental results published by Iversen (1951) where actually $L_0(= 5.12T^2)$ replaces T^2 is encouraging Figure (6). If one excepts a general tendency for the experimental points to be shifted down, a fact which can be explained by bottom friction, the following facts are found both by theory and experiments:

The steepest is the wave, the smaller is the value H/H_0 in relatively deep water and the highest is the value H/H_0 in relatively shallow water. The crossing of the curve obtained by linear theory with the third order wave theory curves on one hand and experimental curves on the other hand happens for increasing value of $\frac{d}{T^2}$ as $\frac{H_0}{T^2}$ increases. Also the scattering of isolines $\frac{H_0}{T^2}$ = constant increases as $\frac{d}{T^2}$ decreases. Due to the bottom friction, a quantitive comparison does not present a good agreement. On the other hand, the scattering of the isolines $\frac{H_0}{T^2}$ = constant is wider in the experiments of Iversen than in the theoretical curves. It can be expected that a better agreement will be obtained if the above calculations were performed at a fifth order of approximation. This task has been achieved partially only. The relationships between $\frac{L}{T^2}$, $\frac{H}{d}$ and $\frac{d}{T^2}$ are readily available from the tables developed by Skjelbreia and Hendrickso (1961). The corresponding graph is presented Figure (7). The correspondi fifth order relationship has been inserted numerically in the calculation and the corresponding shoaling effect has been calculated by hand. The results are presented in Figure (8). It should be realized that these results being a combination of third order wave theory for the energy flux with a fifth order wave theory for the wave length are not consistent. Moreover, they have been obtained by curve reading and hand calculation, i. e. with less accuracy than the previous results presented in Figures (4) and (5) obtained by computer. However, by comparing the curves of Figures (4) and (8) with the experimental results of Figure (6), it is seen that the agreement tends to increase as one uses a higher order of approximation. Consequently, it

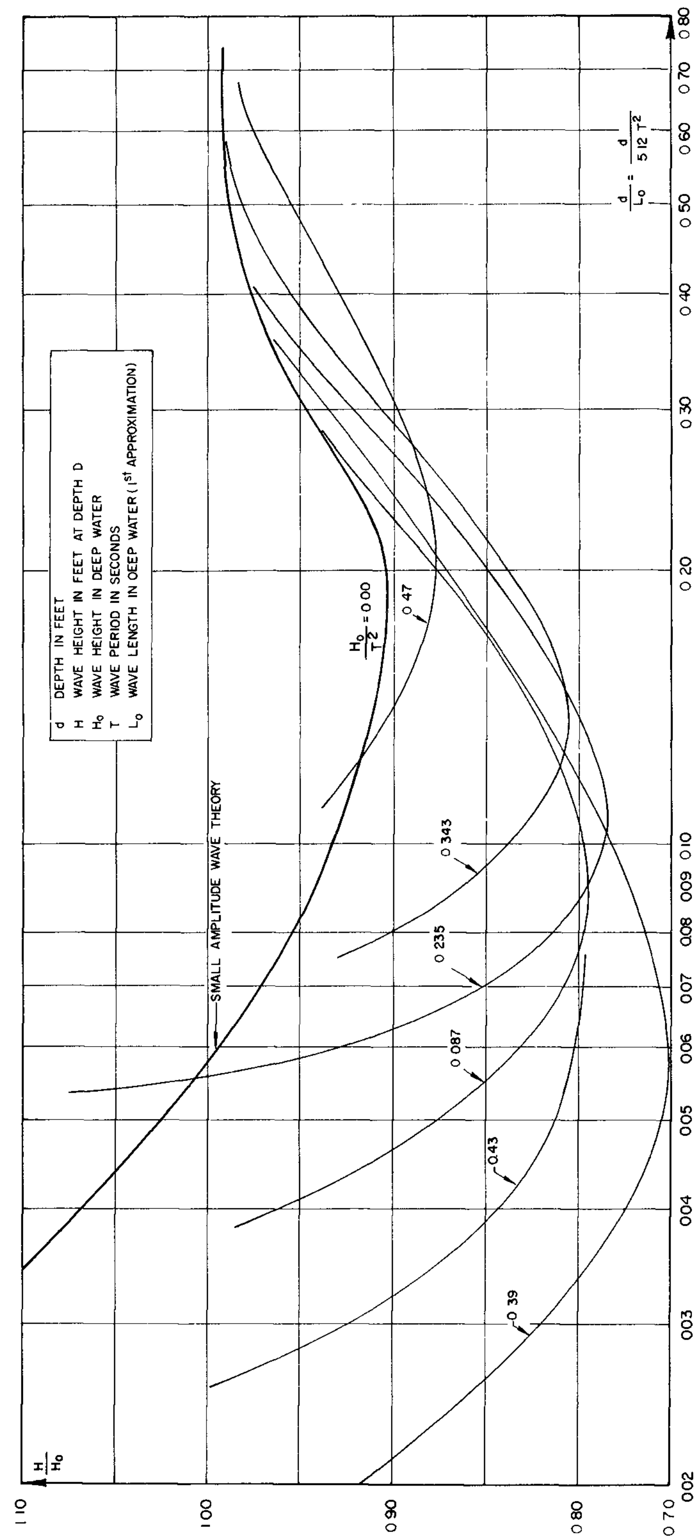

d DEPTH IN FEET
H WAVE HEIGHT IN FEET AT DEPTH D
H₀ WAVE HEIGHT IN DEEP WATER
T WAVE PERIOD IN SECONDS
L₀ WAVE LENGTH IN DEEP WATER (1st APPROXIMATION)
SMALL AMPLITUDE WAVE THEORY
H₀/T² = 0 00
0 47
0 343
0 235
0 087
0 43
0 39
H/H₀
d/L₀ = d/5 12 T²
1 10
1 00
0 90
0 80
0 70
0 02
0 03
0 04
0 05
0 06
0 07
0 08
0 09
0 10
0 20
0 30
0 40
0 50
0 60
0 70
0 80

FIG 6

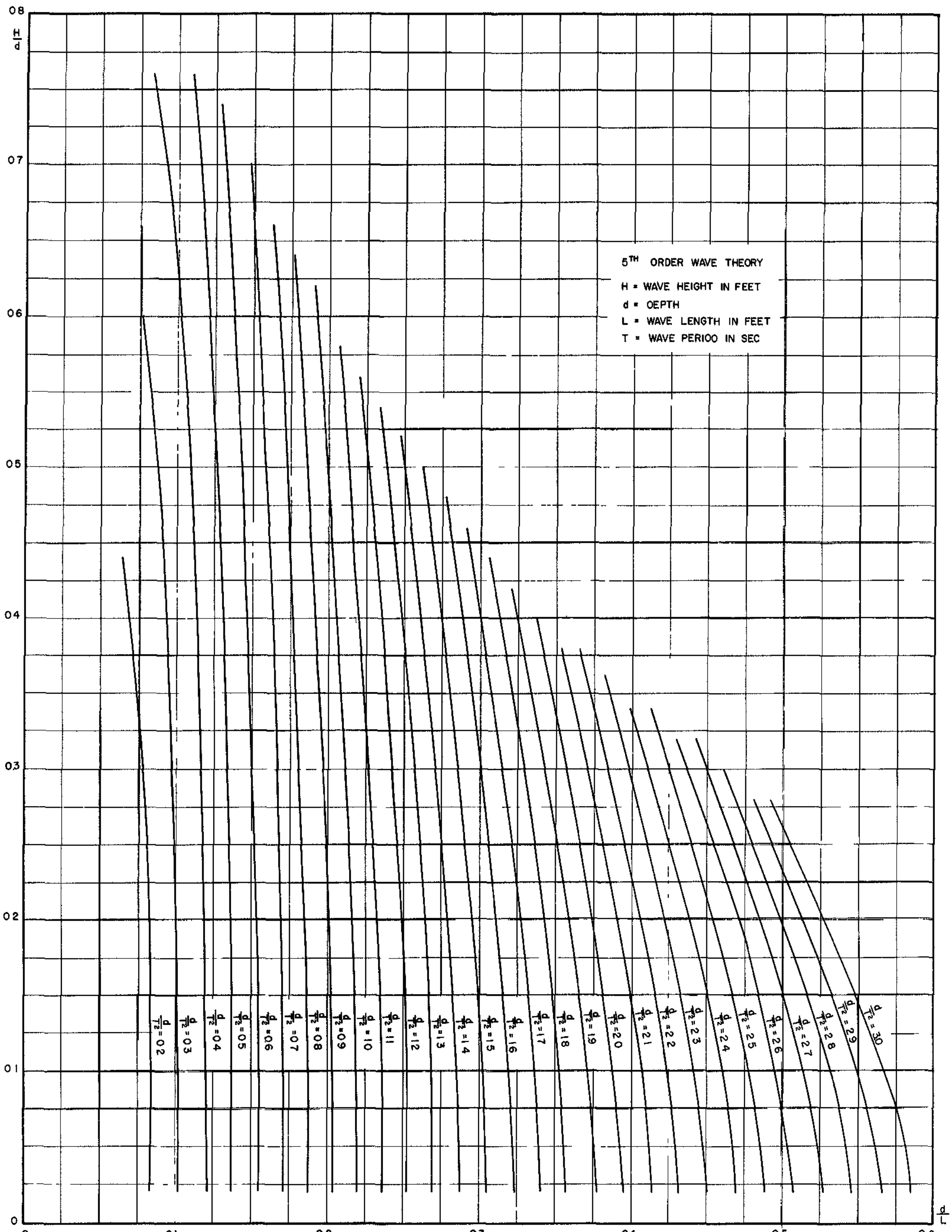

5TH ORDER WAVE THEORY
H = WAVE HEIGHT IN FEET
d = DEPTH
L = WAVE LENGTH IN FEET
T = WAVE PERIOD IN SEC
H/d
d/L
d/T² = 02
d/T² = 03
d/T² = 04
d/T² = 05
d/T² = 06
d/T² = 07
d/T² = 08
d/T² = 09
d/T² = 10
d/T² = 11
d/T² = 12
d/T² = 13
d/T² = 14
d/T² = 15
d/T² = 16
d/T² = 17
d/T² = 18
d/T² = 19
d/T² = 20
d/T² = 21
d/T² = 22
d/T² = 23
d/T² = 24
d/T² = 25
d/T² = 26
d/T² = 27
d/T² = 28
d/T² = 29
d/T² = 30

FIG 7

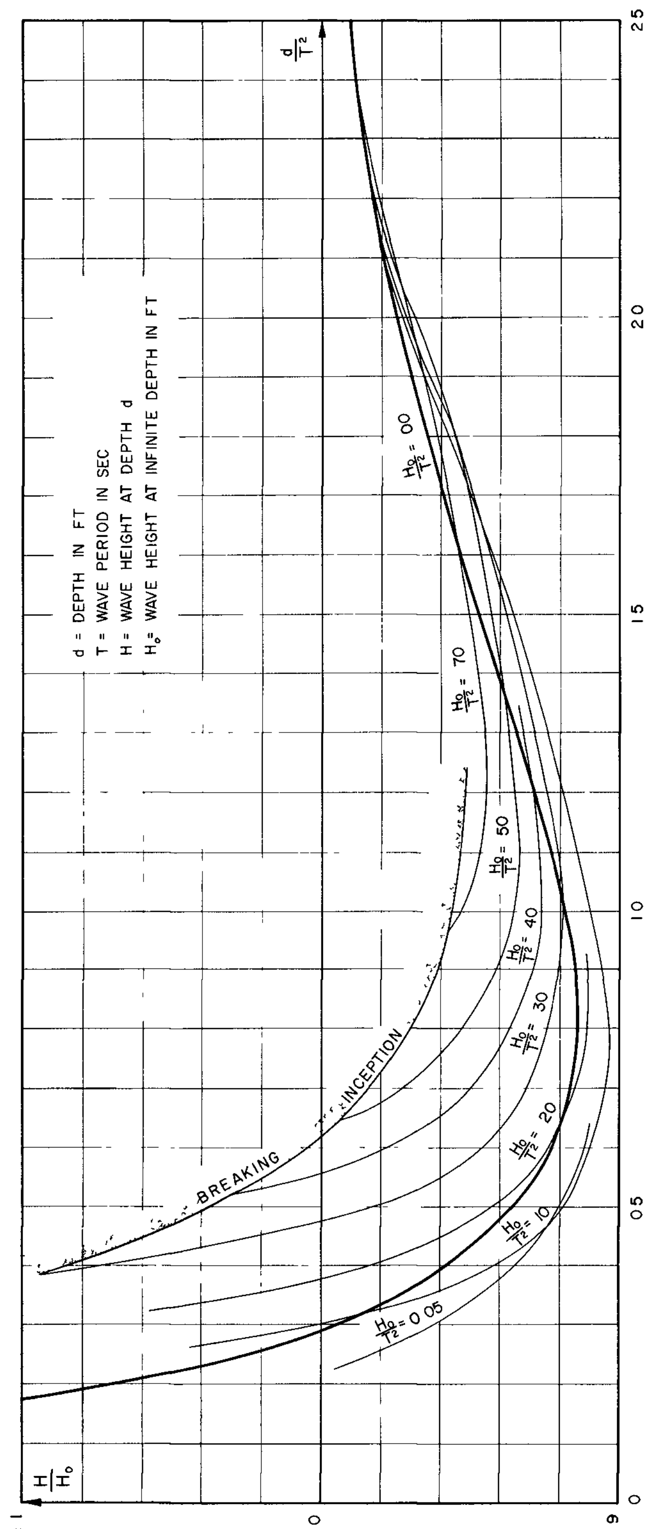

d = DEPTH IN FT
T = WAVE PERIOD IN SEC
H = WAVE HEIGHT AT DEPTH d
H_o = WAVE HEIGHT AT INFINITE DEPTH IN FT
BREAKING
INCEPTION
$\frac{H_o}{T^2}$ = 0 05
$\frac{H_o}{T^2}$ = 10
$\frac{H_o}{T^2}$ = 20
$\frac{H_o}{T^2}$ = 30
$\frac{H_o}{T^2}$ = 40
$\frac{H_o}{T^2}$ = 50
$\frac{H_o}{T^2}$ = 70
$\frac{H_o}{T^2}$ = 00
$\frac{d}{T^2}$
$\frac{H}{H_o}$
0
05
10
15
20
25
11
10
09

FIG 8

would be desirable to perform the calculation which is presently developed at a higher order of approximation. The length of the calculation is discouraging. The calculation of the shoaling coefficient by application of a similar formulation to cnoidal wave theory is probably a better method when the Ursell coefficient $\frac{H}{L}\left(\frac{L}{d}\right)^3$ reaches the value 10 or above the limit previously presented on Figure (4).

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