TAMPICO

PART 1
THEORETICAL AND OBSERVED BASIC OCEANOGRAPHIC DATA

TUXPAN

CHAPTER 1

APPROXIMATE ESTIMATIONS OF CORRELATION COEFFICIENT BETWEEN WAVE HEIGHT AND PERIOD OF SHALLOW WATER WIND WAVES

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INTRODUCTION

From the fact that the marginal frequency distributions for wave height and period of complex sea waves both follow the Rayleigh type distribution and approximately exists a linear relationship between wave height and period, Bretschneider(1959) derived wave height and energy spectra of wave period, introducing the summation function of wave height. Then he estimated the correlation coefficient r between height and period of waves as a function of non-dimensional fetch f $(= gF/U^2)$. However, his estimation seems not theoretical but empirical, being derived mainly from qulitative considerations and observed data. In this paper, the author tries to derive theoretically the equation for r as a function of f, assuming the classical energy equation for significant wave is applicable to the individual wave in complex sea. Moreover, extending the same method, he intends to estimate the coefficient for shallow water waves as a function of f and non-dimensional depth d $(= gD/U^2)$.

As the results, coefficient r for deep water waves consists fairly well with that of Bretschneider and comparing with the author's observed data, the one for shallow water waves seems to be resonable.

DERIVATION OF CORRELATION COEFFICIENT FOR DEEP WATER WAVES

After Bretschneider, the marginal frequency distributions for both wave height and period of complex sea follow the Rayleigh type distribution and in terms of correlation coefficient r between them, the energy spectra (H^2 spectra) of period T is given as follows:

$$S_{H^2}(T) = \frac{4a^2(\bar{H})^2\left[1-r+ar\left(\frac{T}{\bar{T}}\right)^2\right]^2}{1+\left(\frac{4}{\pi}-1\right)r^2}\cdot\frac{T^3}{(\bar{T})^4}\cdot e^{-\frac{\pi a^2}{4}\left(\frac{T}{\bar{T}}\right)^4} \tag{1}$$

where $a = 0.927$, H and T are height and period of individual wave, and $\bar{H}$ and $\bar{T}$ are mean height and period of successive waves in any observation period (usually in twenty minutes). Integration of (1) with T from zero to infinity gives mean square wave height proportional to the potential energy of waves.

$$\int_0^\infty S_{H^2}(T)\,dT = \overline{H^2} = \frac{4}{\pi}(\bar{H})^2 \tag{2}$$

(1) is available for both deep and shallow water waves, and significant wave height $H_{1/3}$ and period $T_{1/3}$ are related to mean height $\bar{H}$ and period $\bar{T}$, respectivelly, through r as follows:

$$H_{1/3} = 1.60\,\overline{H}, \qquad T_{1/3} = \overline{T}\sqrt{1+0.60r} \tag{3}$$

After Bretschneider's fetch graph(1958) for deep water waves, significant wave height and period are approximately expressed by the following equations in terms of non-dimensional fetch f in the range of $1 < f < 2\times10^4$.

$$\frac{gH_{1/3}}{U^2} = 0.0040\left(\frac{gF}{U^2}\right)^{0.40} \quad (4) \qquad \frac{gT_{1/3}}{2\pi U} = 0.085\left(\frac{gF}{U^2}\right)^{0.26} \quad (5)$$

where U is wind velocity, g the acceleration of gravity, F the fetch length. Therefore, when the coefficient r is given by f, wave spectra for deep water waves are fully determined by (1) in terms of f through (3) (4) and (5).

As for r, Bretschneider assumed as follows:
(i) $r = 0$ for the upper limit of f,
(ii) $r = 1.0$ for the lower limit of f,
(iii) r decreases gradually from the lower limit to the upper limit of f,
And moreover, from observed data of r for $f = 10^2 \sim 10^3$, he empirically estimated r as a function of f. His estimations are, however, not yet ultimately determined but to be revised by the future accumulation of observed data.

The author tries to estimate the coefficient r using the energy equation and the relationships (4) and (5) in the region of $1 < f < 10^4$, in which wave velocity is smaller than wind velocity.

In the case of significant wave, the transmitted wave energy P is equal to $\frac{E}{2}C$, where E is the total energy per wave length as given by $\frac{1}{8}\rho g H^2$ and C is wave velocity. In steady state, the space rate of change of P is equal to the supplied energy R_T plus R_N from wind to waves, where R_T and R_N are amount of energy supplied by tangential and normal stresses of wind, respectively. Accordingly, the energy equation becomes

$$\frac{dP}{dF} = R_T + R_N \tag{6}$$

where $P = \frac{\rho g^2}{32\pi} H^2 T$, and after Sverdrup and Munk (1947),

$$R_T = E\cdot A\cdot g\cdot\left(\frac{C}{U}\right)^{-3}\cdot\frac{1}{U} = E\,A\,g\,\frac{U^2}{C^3} \tag{7}$$

$$R_N = E\cdot A\cdot g\,\frac{U^2}{C^3}\cdot\alpha\cdot\left(1+\frac{C}{U}\right)^2 \tag{8}$$

where $E = \frac{1}{8}\rho g H^2$, $A = 2\gamma^2\frac{\rho'}{\rho}$, $\alpha = \frac{S}{2\gamma^2}$, and ρ, ρ' are densities of water and air, γ^2 the friction coefficient of wind over sea surface, and s the sheltering coefficient after Jeffreys. Sverdrup and Munk suggested $A \doteqdot 6.5\times10^{-6}$, and $\alpha = 2.5$.

(6) (7) and (8) are available to the significant wave and now we assume that they are also applicable to the individual wave. Thus from (6), taking the averages for all the successive waves in any time interval, we obtain

$$\overline{\left[\frac{dP}{dF}\right]} = \overline{R}_T + \overline{R}_N \tag{9}$$

where $\overline{\left[\frac{dP}{dF}\right]}$ is replaced by $\frac{d\overline{P}}{dF}$.

Using (1), $\overline{P}$ is written as follows:

$$\overline{P} = \frac{\rho g^2}{32\pi}\,\overline{H^2T} = \frac{\rho g^2}{32\pi}\int_0^\infty S_{H^2}(T)\cdot T\cdot dT$$

$$= \frac{\rho g^2}{32\pi}\,\frac{4\lambda^2(\overline{H})(\overline{T})}{1+(\frac{4}{\pi}-1)\gamma^2}\int_0^\infty (1-\gamma+a\gamma\tau^2)^2\tau^4 e^{-\frac{\pi a^2}{4}\tau^4}\,d\tau$$

$$= \frac{\rho g^2}{8\pi^2}\cdot\frac{1+0.2879\gamma+0.3037\gamma^2}{1+0.273\gamma^2}\cdot(\overline{H})^2\cdot\overline{T} \tag{10}$$

Now putting

$$h = \frac{g\overline{H}}{U^2}\quad,\qquad t = \frac{g\overline{T}}{2\pi U}\quad,\qquad f = \frac{gF}{U^2} \tag{11}$$

then

$$\frac{d\overline{P}}{dF} = \frac{g}{U^2}\cdot\frac{d\overline{P}}{df}\quad,\qquad (\overline{H})^2\,\overline{T} = \frac{2\pi U^5}{g^3}\cdot h^2 t \tag{12}$$

and from (3) (4) and (5),

$$h = 0.0025 f^{0.40}\quad,\qquad t = \frac{0.085}{\sqrt{1+0.60\gamma}}\, f^{0.26} \tag{13}$$

Hence

$$\frac{d\overline{P}}{dF} = 53.12\cdot\frac{d}{df}\left[(1-0.0125\gamma+0.0791\gamma^2)\cdot f^{1.06}\right]\frac{\rho U^3}{4\pi}\times10^{-8} \tag{14}$$

On the other hand, from (7) and (8),

$$\overline{R}_T = \frac{\pi^3\alpha\rho A U^2}{g}\cdot\overline{\left[\frac{H^2}{T^3}\right]} \tag{15}$$

$$\overline{R_N} = \frac{\pi^3 \alpha \rho A U^2}{g} \left\{ \left[\frac{H^2}{T^3}\right] - \frac{g}{\pi U}\left[\frac{H^2}{T^2}\right] + \frac{g^2}{4\pi^2 U^2}\left[\frac{H^2}{T}\right] \right\}$$

Similar to $\frac{d\bar{P}}{d\bar{f}}$, $\overline{R_T}$ and $\overline{R_N}$ are written in terms of f and r as follows:

$$\overline{R_T} \doteqdot 0.437 A (1 - 0.337 r - 0.707 r^2) \cdot f^{0.02} \cdot \rho U^3 \times 10^{-2} \tag{16}$$

$$\overline{R_N} \doteqdot \alpha A \Big\{ 0.437 (1 - 0.337 r) f^{0.02} \times 10^{-2} - 0.394 (1 - 0.127 r) f^{0.28} \times 10^{-3} + 0.125 (1 - 0.029 r) f^{0.54} \times 10^{-4} \Big\} \cdot \rho U^3 \tag{17}$$

Neglecting higher order terms than r^2 and substituting (9) with (14) (16) and (17), the energy equation is obtained in terms of r and f as follows:

$$\frac{d}{df}\left\{(1 - 0.012 r) f^{1.06}\right\} = 0.103 A (1 + \alpha)(1 - 0.337 r) f^{0.02} - 0.00947 A \cdot \alpha (1 - 0.127 r) f^{0.28} \times 10^{5} + 0.00031 A \alpha (1 - 0.028 r) f^{0.54} \times 10^{4} \tag{18}$$

or putting $A' = A \times 10^6$, $A'(1+\alpha) = m$ and $A'\alpha = n$, (18) becomes

$$\frac{d}{df}\left\{(1 - 0.012 r) f^{1.06}\right\} = 0.103 m (1 - 0.337 r) f^{0.02} - 0.00947 n (1 - 0.127 r) f^{0.28} + 0.00031 n (1 - 0.029 r) f^{0.54} \tag{18'}$$

where the third term in the right hand side is neglisibly smaller than the second term in the region of f considered, and also 0.012r in the left hand side is small compared with 1. Hence, to the first order approximation, (18') becomes

$$1.06 f^{0.04} = 0.103 m (1 - 0.337 r) - 0.00947 n (1 - 0.127 r) f^{0.26} \tag{19}$$

from which r is obtained as follows:

$$r = \frac{10.3 m - 0.947 f^{0.26} - 106 f^{0.04}}{3.47 m - 0.120 n f^{0.26}} \tag{20}$$

Thus for $m > 0$, $n > 0$, r is a steadily decreasing function of f.

When $A' = 6.5$, $\alpha = 2.5$, that is, $m = 22.8$, $n = 16.2$, $|r|$ becomes greater than 1.0, which is clearly unreasonable. Therefore, m and n should be selected reasonably.

According to Bretschneider's estimation, r is nearly equal to 0.9 for $f = 1$. Taking this condition into (20), m and n are related by the next equation.

$$m = 0.117n + 14.76 \tag{21}$$

And for $n = 0$, 1, and 2, r are calculated from (21) and (20), of which the results are shown in Figure 1.

From the figure, it is found that Bretschneider's estimation is the best fit for $n = 0$. When n is zero, m becomes 14.76 and $A' = 14.76$, $\alpha = 0$, which means that supply of energy from wind to waves is mainly done by tangential stress and the friction coefficient γ^2 becomes twice the one given by Munk. Such a result is somewhat different from actual phenomena, but its tendency is near to the facts that the sheltering coefficient may be much smaller than the one proposed by Jeffreys and also γ^2 may become appreciably larger than 0.0026 in some cases.

Accordingly, taking $m = 14.76$, $n = 0$, the first order approximation of r becomes

$$r = 2.97 - 2.07\, f^{0.04} \tag{22}$$

And the second order approximation is obtained from (18') as follows:

$$r = 2.97 - 2.00 f^{0.04} - 0.053\, f^{0.08} \tag{23}$$

which consists fairly well with Bretschneider's estimation in the range of $1 < f < 10^4$. Therefore, it might be not so unreasonable to assume that the energy equation for significant wave is applicable to the individual wave and in the range of f considered the energy supply by normal stress of wind may be possibly neglected for our estimation.

ESTIMATIONS OF CORRELATION COEFFICIENT FOR SHALLOW WATER WAVES

As previously described in deep water, the correlation coefficient r gradually decreases from 1.0 to 0 with increasing f, that is, with the developement of waves. While, in shallow water, Bretschneider suggested that by the effect of bottom friction the coefficient r decreases more remarkably and possibly tends to negative, that is, r variates from $+1.0$ to -1.0 with increasing f and decreasing d (non-dimensional water depth as given by gD/U^2, where D is water depth). From the assumption of Rayleigh distribution for wave height and period, however, r cannot tend to -1.0 but get to about $-0.6 \sim -0.7$ as its minimum limiting value. (Bretschneider did not show this limiting value.)

Following to the above descriptions, we assume for r in shallow water wind waves as follows:

(i) r decreases with the developement of waves,

(ii) r becomes possibly negative but cannot get to smaller than -0.7.

DEVELOPEMENT OF SHALLOW WATER WIND WAVES

Now consider the case of constant water depth D. The shallow water significant wave height H_S, period T_S, fetch length F, and water depth D are expressed non-dimensionally as follows:

$$h_S = \frac{gH_S}{U^2}, \quad t_S = \frac{gT_S}{2\pi U}, \quad f = \frac{gF}{U^2}, \quad d = \frac{gD}{U^2} \tag{24}$$

Bretschneider has shown the relations of h_S, f and d from his observations and calculations of wave height change by bottom friction, of which the result is shown in Figure 15C on Page 28d of Technical Report No.4 entitled " Shore Protection Planning and Design ", issued from Beach Erosion Board. Figure 2 in this paper is made from that Figure 15C, excepting the curves for t_S v.s. f.

It is seen from Figure 2 that for any fixed d, h_S increases with f initially along the same curve as for deep water waves and from certain point of f (say f_t) it begins to deviate and at the other point of f (say f_u) attains a steady constant state. The value of h_S at point f_t (say h_t) is expressed from the Figure 15C of Bretschneider as follows:

$$h_t = 0.0840\, d^{0.72} \tag{25}$$

On the other hand, h_t is the same for deep water waves at f_t. Thus from (4), f_t is given as follows:

$$f_t = 2020\, d^{1.8} \tag{26}$$

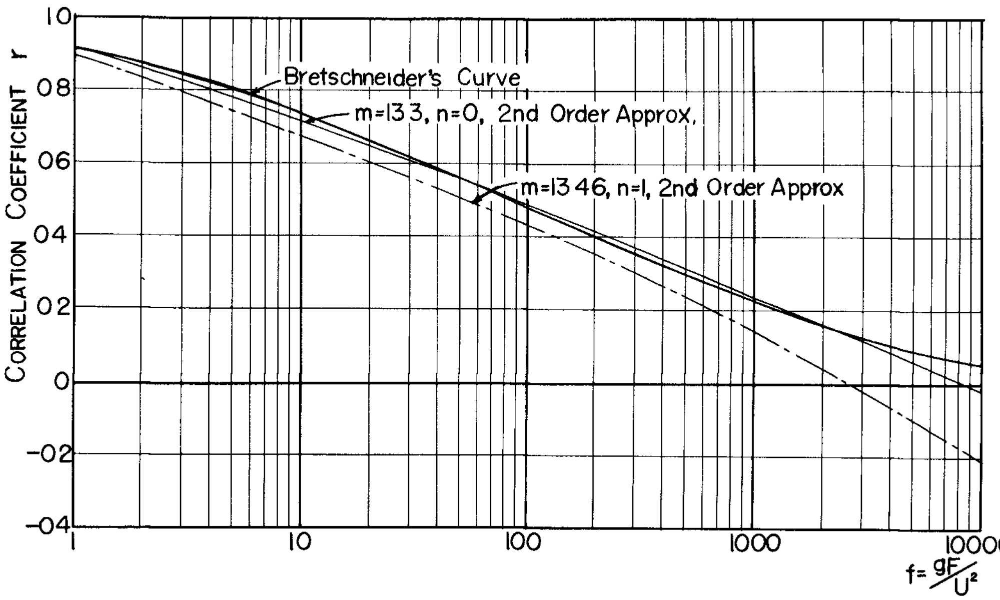

Fig. 1. Relation between correlation coefficient and non-dimensional fetch for deep water waves.

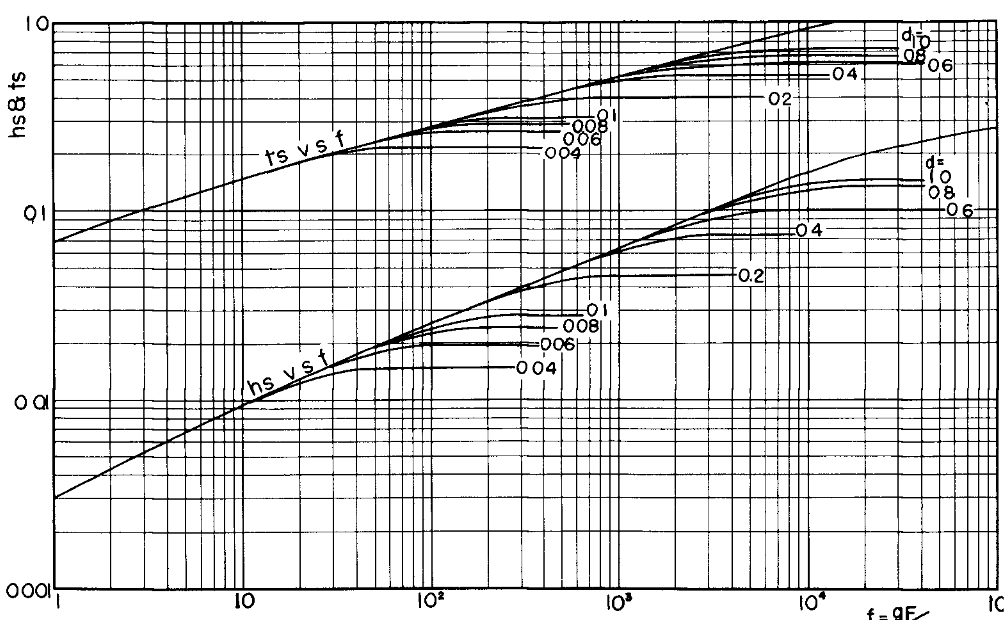

Fig. 2. Fetch graph for deep and shallow water waves.

Similarly the value of h_s at point f_u (say h_u) is obtained from that Bretschneider's Figure.

$$h_u = 0.143\, d^{0.72} \tag{27}$$

and

$$f_u = 14940\, d^{1.8} \tag{28}$$

As for wave period, Bretschneider gave no curve. Hence we derive the relation of t_s and f from the following considerations.

Similar to wave height h_s, there must be f_t , bellow which t_s is the same as that of deep water waves, and f_u, above which t_s becomes steadily constant, and also f_t and f_u are the same as those for wave height h_s. That is, both wave height and period begin to deviate from the curves of deep water waves at point f_t and get to constant at f_u.

The value of t_s at point f_t (say t_t) is obtained by substituting (5) with (26).

$$t_t = 0.615\, d^{0.468} \tag{29}$$

Theoretically, the effect of bottom friction begins to appear when wave length becomes twice the water depth. The value of t_s satisfying this condition (say $t_{t'}$) is

$$t_{t'} = \left(\frac{d}{\pi}\right)^{0.5} = 0.564\, d^{0.5} \tag{30}$$

From (5), the corresponding f (say $f_{t'}$) is

$$f_{t'} = 1445\, d^{1.92} \tag{31}$$

Hence, in the region of $0 < d < 10$, $t_{t'}$ is smaller than t_t and theoretically the effect of friction appears even when $f < f_t$.

After Bretschneider(1960), significant wave period T_s is not too critical and it is conveniently represented by wave height H_s as follows:

$$T_s = 3.86\sqrt{H_s} \tag{32}$$

where T_s is in seconds, H_s in meters. Assuming that this relationship is applicable to the region of $f > f_u$,

$$t_u = 1.924\, h_u^{0.50} \tag{33}$$

or from (27)
$$t_u = 0.728\, d^{0.36} \tag{34}$$

Thus period t_s at f_f and f_u are given by (29) and (34).

Accordingly, it is easy for us to draw smooth curves of t_s v.s. f on the fetch graph, which are tangent to those of deep water waves at f_t and asymptotically tend to the constant value at f_u. Figure 2 is thus obtained.

ENERGY EQUATION FOR SHALLOW WATER WIND WAVES

For the steady state of shallow water wind waves being affected by bottom friction, the energy equation becomes as follows corresponding to (9).

$$\frac{d\overline{P}}{dF} = \overline{R_T} + \overline{R_N} - \overline{D_f} \tag{35}$$

where $\overline{D_f}$ is the loss of energy by bottom friction, averaged for successive waves in any observed period.

The transmitted wave energy P is as follows:

$$P = \frac{\rho g^2 H^2 T}{32\pi} \cdot \tanh\frac{2\pi D}{L}\left(1 + \frac{\frac{4\pi D}{L}}{\sinh\frac{4\pi D}{L}}\right) \tag{36}$$

where D is water depth and L is wave length.

Putting

$$F_1 = \frac{1}{2}\tanh\frac{2\pi D}{L}\left(1 + \frac{\frac{4\pi D}{L}}{\sinh\frac{4\pi D}{L}}\right) \tag{37}$$

then F_1 is a function of D/L_o. (L_o is the deep water wave length of period T as given $gT^2/2\pi$.)

Putting $\overline{h} = g\overline{H}/U^2$, and $\overline{t} = g\overline{T}/2\pi U$, from (36) and (37),

$$\overline{P} = \frac{\rho g^2}{16\pi}\overline{\left[H^2 T \cdot F_1\left(\frac{2\pi D}{gT^2}\right)\right]} = \frac{\rho g^2}{16\pi}\int_0^\infty S_{H^2}(T)\cdot T\; F_1\left(\frac{2\pi D}{gT^2}\right) dT$$

$$= \frac{\rho g^2}{16\pi}\,\frac{3.434}{1+0.273r^2}(\overline{H})^2\overline{T}\int_0^\infty (1+r+0.927r\tau^2)^2\tau^4 e^{-0.675\tau^4} F_1\left(\frac{2\pi D}{g\overline{T}^2}\,\frac{1}{\tau^2}\right) d\tau$$

$$= \frac{\rho U^5}{8g}\cdot\frac{3.434}{1+0.273r^2}(\overline{h})^2\overline{t}\int_0^\infty (1-r+0.927r\tau^2)^2\tau^4 e^{-0.675\tau^4} F_1\left(\frac{d}{2\pi(\overline{t})^2}\,\frac{1}{\tau^2}\right) d\tau \tag{38}$$

Thus

$$\frac{d\overline{P}}{dF} = \frac{3.434\rho U^3}{8}\,\frac{d}{df}\left\{\frac{(\overline{h})^2\cdot\overline{t}}{1+0.273r^2}\int_0^\infty (1-r+0.927r\tau^2)^2\tau^4 e^{-0.675\tau^4} F_1\left(\frac{d}{2\pi(\overline{t})^2}\cdot\frac{1}{\tau^2}\right) d\tau\right\} \tag{39}$$

From (3), $\overline{h} = hs/1.6$, $\overline{t} = ts/\sqrt{1+0.6r}$, and (39) is written as

$$\frac{d\overline{P}}{dF} = 0.1677\frac{d}{df}\left\{(1-0.30r-0.138r^2)\,h_s^2\,t_s\,A\left(\frac{d}{2\pi(\overline{t})^2}, r\right)\right\}\rho U^3 \quad (40)$$

where

$$A\left(\frac{d}{2\pi(\overline{t})^2}, r\right) = \int_0^\infty (1-r+0.927r\tau^2)^2\tau^4 e^{-0.675\tau^4} F_1\left(\frac{d}{2\pi(\overline{t})^2}\cdot\frac{1}{\tau^2}\right)d\tau \quad (41)$$

$\overline{R}_T$ and $\overline{R}_N$ are energy supplied from wind by tangential and normal stresses, and similar to the case of deep water waves, $\overline{R}_N$ is considered to be neglisible to $\overline{R}_T$. As for R_T in shallow water waves, Kishi(1955) proposed the next equation for significant wave.

$$R_T = E\,A_s\,f\left(\frac{U^2}{C^3}\right)\frac{1}{2}\left[1+\left(\tanh\frac{2\pi D}{L}\right)^2\right]$$
$$= \frac{\pi^3 A_s}{2g}\rho U^2\frac{H^2}{T^3}\left[\coth\frac{2\pi D}{L}+\left(\coth\frac{2\pi D}{L}\right)^3\right] \quad (42)$$

where A_S corresponds to A for deep water waves, and its magnitude is considered to be in the order of $(10 \sim 20)\times 10^{-6}$.

Putting

$$F_2 = \coth\frac{2\pi D}{L} + \left(\coth\frac{2\pi D}{L}\right)^3 \quad (43)$$

F_2 is also a function of D/L_0 $(=2\pi D/gT^2)$.

From (42)

$$\overline{R}_T = \frac{\pi^3 A_s\rho U^3}{2g}\overline{\left[\frac{H^2}{T^3}\left(\coth\frac{2\pi D}{L}+\coth^3\frac{2\pi D}{L}\right)\right]}$$
$$= \frac{\pi^3 A_s\rho U^3}{2g}\,\frac{3.434}{1+0.273r^2}\cdot\frac{(\overline{H})^2}{(\overline{T})^3}\int_0^\infty(1-r+0.927r\tau^2)^2 e^{-0.675\tau^4}F_2\left(\frac{2\pi D}{g(\overline{T})^2}\cdot\frac{1}{\tau^2}\right)d\tau$$
$$= 0.0838\,A_s\,(1+0.90r-0.138r^2)\frac{h_s^2}{t_s^3}\cdot B\left(\frac{d}{2\pi(\overline{t})^2}, r\right)\rho U^3 \quad (44)$$

where

$$B\left(\frac{d}{2\pi(\overline{t})^2}, r\right) = \int_0^\infty(1-r+0.927r\tau^2)^2\cdot e^{-0.675\tau^4}F_2\left(\frac{d}{2\pi(\overline{t})^2}\,\frac{1}{\tau^2}\right)d\tau \quad (45)$$

Loss of energy by bottom friction D_f is given after Putnum and Johnson(1949).

$$D_f = \frac{4}{3}\pi^2\rho\cdot k\cdot\frac{H^3}{T^3\left(\sinh\frac{2\pi D}{L}\right)^3} \quad (46)$$

where k is the coefficient of friction, which is about 0.01 ~ 0.02 after Bretschneider. Assuming that (46) is still available to the individual wave, $\overline{D_f}$ becomes

$$\overline{D_f} = \frac{4}{3}\pi^2\rho k\overline{\left[\frac{H^3}{T^3\left(\sinh\frac{2\pi D}{L}\right)^3}\right]} = \frac{4}{3}\pi^2\rho k\int_0^\infty S_{H^3}(T)\frac{dT}{T^3\left(\sinh\frac{2\pi D}{L}\right)^3} \qquad (47)$$

where $S_{H^3}(T)$ is the H^3 spectra of T, which shall be obtained using correlation coefficient r as follows:

$$S_{H^3}(T) = \frac{6a^2(\bar{H})^3\left[1-r+ar\left(\frac{T}{\bar{T}}\right)^2\right]^3}{1+\left(\frac{12}{\pi}-3\right)r^2+\left(2-\frac{6}{\pi}\right)r^3}\frac{T^3}{(\bar{T})^4}e^{-\frac{\pi a^2}{4}\left(\frac{T}{\bar{T}}\right)^4}$$

$$= \frac{5.156(\bar{H})^3\left[1-r+0.927\left(\frac{T}{\bar{T}}\right)^2\right]^3}{1+0.820r^2+0.090r^2}\frac{T^3}{(\bar{T})^4}e^{-0.675\left(\frac{T}{\bar{T}}\right)^4} \qquad (48)$$

Hence, putting

$$F_3 = \frac{1}{\left(\sinh\frac{2\pi D}{L}\right)^3} \qquad (49)$$

F_3 is also a function of D/L_0 $(= 2\pi D/gT^2)$.

Putting

$$\alpha' = \frac{4}{3}\pi^2 k = 13.15k \qquad (50)$$

α' shall be in order of 0.1 ~ 0.2.

From (47) and (48), $\overline{D_f}$ becomes as follows:

$$\overline{D_f} = \frac{5.156\alpha'\rho}{1+0.820r+0.090r^2}\frac{(\bar{H})^3}{(\bar{T})^3}\cdot\int_0^\infty(1-r+0.927r\tau^2)^3e^{-0.675\tau^4}F_3\left(\frac{2\pi D}{g(\bar{T})^2\tau^2}\right)d\tau$$

$$= 0.005082\alpha'(1+0.90r-0.685r^2)\cdot\frac{h_s^3}{t_s^3}\cdot C\left(\frac{d}{2\pi(\bar{t})^2}, r\right)\cdot\rho U^3 \qquad (51)$$

where

$$C\left(\frac{d}{2\pi(\bar{t})^2}, r\right) = \int_0^\infty(1+r+0.927r\tau^2)^3e^{-0.675\tau^4}F_3\left(\frac{d}{2\pi(\bar{t})^2}\frac{1}{\tau^2}\right)d\tau \qquad (52)$$

Hence, substituting (35) with (40) (44) and (51), putting $\overline{R_N} = 0$, and neglecting higher order terms than r^2, energy equation becomes

$$0.1677 \frac{d}{df}\left\{(1-0.30r)\, h_s^2 \cdot t_s \cdot A\left(\frac{d}{2\pi(\bar{t})^2}, r\right)\right\}$$

$$= 0.0838 A_s(1+0.90r)\frac{h_s^2}{t_s^3} B\left(\frac{d}{2\pi(\bar{t})^2}, r\right) - 0.005082\alpha'(1+0.90r)\frac{h_s^3}{t_s^3} C\left(\frac{d}{2\pi(\bar{t})^2}, r\right) \quad (53)$$

where A, B and C are functions of r, $\bar{t}$ and d, and expressed as polynomials of r as follows:

$$A\left(\frac{d}{2\pi(\bar{t})^2}, r\right) = a_0\left(\frac{d}{2\pi(\bar{t})^2}\right) + a_1\left(\frac{d}{2\pi(\bar{t})^2}\right) r + a_2\left(\frac{d}{2\pi(\bar{t})^2}\right) r^2$$

$$B\left(\frac{d}{2\pi(\bar{t})^2}, r\right) = b_0\left(\frac{d}{2\pi(\bar{t})^2}\right) + b_1\left(\frac{d}{2\pi(\bar{t})^2}\right) r + b_2\left(\frac{d}{2\pi(\bar{t})^2}\right) r^2 \quad (54)$$

$$C\left(\frac{d}{2\pi(\bar{t})^2}, r\right) = C_0\left(\frac{d}{2\pi(\bar{t})^2}\right) + C_1\left(\frac{d}{2\pi(\bar{t})^2}\right) r + C_2\left(\frac{d}{2\pi(\bar{t})^2}\right) r^2 + C_3\left(\frac{d}{2\pi(\bar{t})^2}\right) r^3$$

where a, b, c, etc. have the following form,

$$\int_0^\infty \tau^n F_m\left(\frac{d}{2\pi(\bar{t})^2}\cdot\frac{1}{\tau^2}\right) e^{-0.675\tau^4} d\tau, \quad \begin{pmatrix} m = 1, 2, 3 \\ n = 0, 2, 4, 6, 8 \end{pmatrix}$$

which are numerically integrable. In practices, d varies from 0.02 to 1.0 and $\bar{t}$ from 0.1 to 0.8. From numerical calculations for d = 0.04, 0.06, 0.08, 0.1, 0.2, 0.4, 0.6, 0.8 and 1.0, it is known that a_0, a_1, a_2 and b_0, b_1, b_2 are all the same order of magnitude, and at any given d, their variations for the change of $\bar{t}$ are small. c_0, c_1, c_2 and c_3 are all the same order of magnitude and for any given d, they are all increasing functions of $\bar{t}$, and their rates of increase for the change of $\bar{t}$ are remarkably large.

Due to the above fact, higher order terms than r^2 can be neglected to the first approximation. For convenience of calculations, a_0, a_1, b_0, b_1 are expanded in terms of $\bar{t}$ for each d. As an example, for d = 0.06, $\bar{t}$ changes from about 0.15 to 0.25 and they are expressed in this region as follows:

$$a_0 = 0.067 + 1.28\bar{t} - 2.8(\bar{t})^2, \qquad a_1 = 0.013 + 1.074\bar{t} - 2.96(\bar{t})^2$$

$$b_0 = 2.26 - 5.2\bar{t} + 20(\bar{t})^2, \qquad b_1 = -2.275 + 0.15\bar{t} + (\bar{t})^2$$

Replacing $\bar{t}$ by $t_s/\sqrt{1+0.6r}$, and neglecting higher order terms than r^2, we obtain for d = 0.06,

$$A\left(\frac{d}{2\pi(\bar{t})^2}, r\right) = (0.067 + 1.28t_s - 2.8t_s^2) + r(0.013 + 0.690t_s - 1.28t_s^2)$$

$$B\left(\frac{d}{2\pi(\bar{t})^2}, r\right) = (2.26 - 5.2t_s + 20t_s^2) + r(-2.275 + 1.71t_s - 11.0t_s^2)$$

Similar expressions are obtained for other d.

c_o, c_1 cannot be expressed by quadratics of $\bar{t}$, c_1/c_o is expressed as quadratics of $\bar{t}$ approximately and because of $\bar{t} \fallingdotseq (1 - 0.3r)t_s$, we obtain to the first order approximation as follows:

$$C_o(\bar{t}) \fallingdotseq C_o(t_s) - 0.3rt_s C_o'(t_s) = C_o(t_s)\left(1 - 0.3rt_s \frac{C_o'}{C_o}\right)$$

where c_o'/c_o is presented approximately in quadratics of t_s. As an example, for $d = 0.06$,

$$\frac{C_1}{C_o} = 6.30 - 38.2\,t_s + 68\,t_s^2, \quad \frac{C_o'}{C_o} = 215.5 - 1505\,t_s + 2900\,t_s^2$$

Hence,

$$C\left(\frac{d}{2\pi(\bar{t})^2}, r\right) = C_o(t_s)\left(1 - 0.3rt_s\frac{C_o'}{C_o}\right)\left(1 + \frac{C_1}{C_o}r\right)$$

$$= C_o(t_s)\left\{1 + r(6.30 - 102.85\,t_s + 519.5\,t_s^2 - 870\,t_s^3)\right\}$$

Similarly, for other d, A, B, and C in (53) are presented as rational expressions in terms of r and t_s.

Thus for $d = 0.06$, (53) becomes

$$\frac{d}{df}\left[\left\{(0.01124 + 0.215t_s - 0.470t_s^2) + r(-0.0021 + 0.0513t_s - 0.0738\,t_s^2)\right\} h_s^2 t_s\right]$$
$$= A_s\left\{(0.1894 - 0.436t_s + 1.676t_s^2) + r(-0.0202 - 0.2489\,t_s + 0.587t_s^2)\right\}\frac{h_s^2}{t_s^3}$$
$$- \alpha'\left\{(0.005082 + r(0.03659 - 0.5227t_s + 2.640t_s^2 - 4.421\,t_s^3)\right\} C_o(t_s)\cdot\frac{h_s^3}{t_s^3} \qquad (55)$$

which is an example of the approximately expressed energy equation for shallow water waves.

In (55), the second term in the left hand side is neglisibly small compared with the first term for $|r| < 1$ and $t_s < 1$. Therefore, (55) becomes

$$r\left[\alpha'(0.03659 - 0.5227t_s + 2.640t_s^2 - 4.421\,t_s^3)C_o(t_s)\cdot h_s - (-0.02020 - 0.2489t_s + 0.589t_s^2)\right.$$
$$\left.\times A_s\right] = A_s(0.1894 - 0.436t_s + 1.676\,t_s^2) - 0.005082\alpha' C_o(t_s)\,h_s$$
$$- \frac{t_s^3}{h_s^2}\frac{d}{df}\left\{(0.01124 + 0.215t_s - 0.470t_s^2)\,h_s^2\,t_s\right\} \qquad (56)$$

from which r is calculated for given t_s and h_s, that is, for given f.

In the above energy equation, A_s and α' are not yet given. As for A_s, it is reasonably assumed to be nearly equal to A for deep water waves. Hence we take here $A_s = 15\times10^{-6}$. As for α', it should be in order of $0.1 \sim 0.2$ and be selected as reasonably as possible. For the purpose of it, we tried rough calculations of r for each d, putting $\alpha' =$ 0.1, 0.15, 0.20, 0.25, and 0.30.

When $\alpha' = 0.1$ and 0.15, r increases with increase of f for $d = 0.04 \sim 1.0$, which is contradictory to the asummed properties of r. When 0.20, 0.25, and 0.30, r has the tendency of decrease with increasing f for all d, which becomes more and more remarkably for larger values of α' and when $\alpha' = 0.30$, r decreases beyond -0.7, the assumed lower limit of r. Hence $\alpha' = 0.30$ is too large. In conclusions, the value of α' suitable for the assumed properties of r should be in the range of $0.20 \sim 0.25$, which means from (50) that the coefficient of friction is to be about $0.015 \sim 0.019$. Therefore we take $A_s = 15\times10^{-6}$, and $\alpha' = 0.20$.

(61) will be correct enough near $r = 0$, but apart from it, it will become erroneous, and it is not always enough to use only (61) in order to obtain r. Accordingly, we proceed as follows:

(i) When $f < f_t$, $\overline{D_f}$ becomes much smaller than $\overline{R_T}$ and r should tend to that of deep water waves. From (44) and (51),

$$\frac{\overline{D_f}}{\overline{R_T}} = 0.0606\frac{\alpha'}{A_s}\frac{C\left(\frac{d}{2\pi(\bar{t})^2}, r\right)}{B\left(\frac{d}{2\pi(\bar{t})^2}, r\right)} = 0.000808\,\frac{C_0(\bar{t}) + C_1(\bar{t})\,r}{b_0(\bar{t}) + b_1(\bar{t})\cdot r}\cdot h_s \tag{57}$$

from which the value of f satisfying the condition $\overline{D_f} < 0.02\overline{R_T}$ (say f_o) is calculated for each d, and at f_o, r should be nearly equal to that of deep water waves. After calculations, f_o is obtained approximately as a function of d as follows:

$$f_o = 1000\, d^{1.80} \tag{58}$$

which is shown in Figure 3. Compared with (26), f_o is about a half of f_t and still smaller than f_t'(31). This may be because of the existence of longer individual wave than the significant wave.

(ii) When $f_u > f > f_t$, taking $A_s = 15\times10^{-6}$, $\alpha' = 0.2$, the value of f where r becomes zero (say $f_{(r=0)}$) is obtained for each d from (56), of which the result is as follows:

$$f_{(r=0)} = 1060\, d^{1.30} \tag{59}$$

which is shown in Figure 3.

(iii) When $f > f_u$, $d\overline{P}/dF$ becomes zero and $\overline{R_T} = \overline{D_f}$. And then r should be constant. Such a constant value of r is obtained from (57), putting $\overline{D_f} = \overline{R_T}$, and is shown in Figure 4 as a function of d.

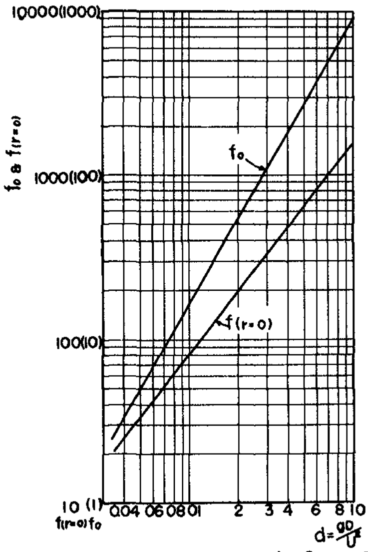

Fig. 3. f_0 and $f_{(r=o)}$ for d.

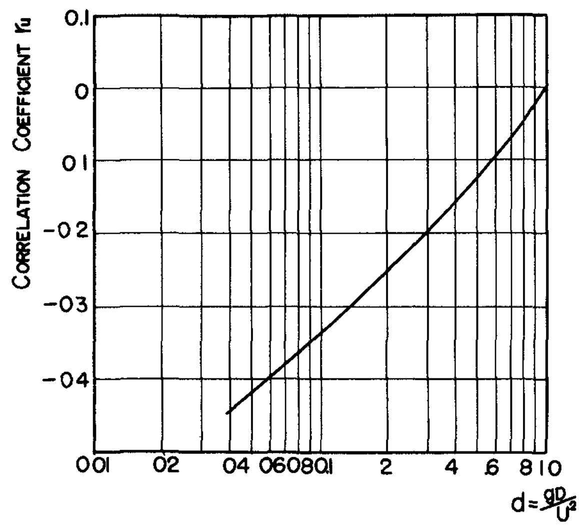

Fig. 4. Correlation coefficient for d.

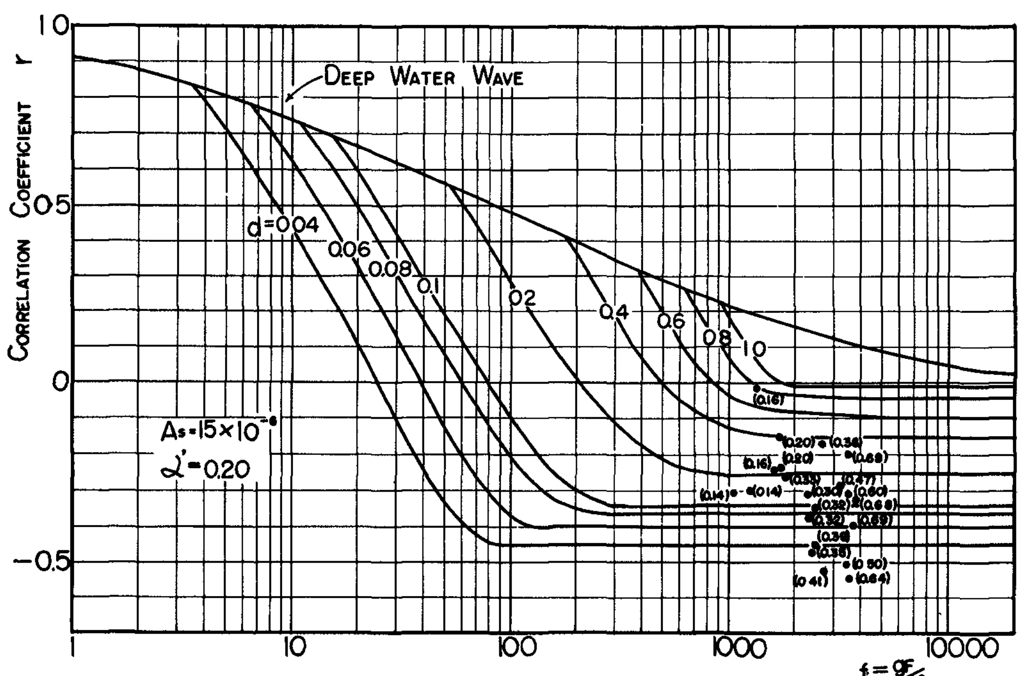

Fig. 5. Relationship between correlation coefficient and non-dimensional fetch for shallow water waves.

As mentioned aboved, at $f = f_0$, r begins to deviate from the value of r for deep water waves, and then gets to zero at $f_{(r=0)}$, and still continuously decreases down to the ultimate, constant value. Figure 5 shows the curves of r thus obtained for each d.

SOME RESULTS OF OBSERVATIONS

The above- mentioned r for shallow water waves should be verified by a lot of observation data. Wave observations are now carried on near the Port of IzumiOotsu on the east coast of Oosaka Bay by means of underwater-preesure type wave-meters, and correlation coefficients are calculated from records at the depth of -2.20 meters below L.W.L., of which some results are plotted in Figure 5. Up to date, the amount of data is not satisfactory but the above-estimated tendency of r is seen from the Figure in some degree.

CONCLUSIONS

(I) In the case of deep water waves, asumming that the energy supplied by normal stress from wind is neglisible to that by tangential stress and energy equation by significant wave is available to the individual wave, and taking $A = 2\gamma^2 \rho'/\rho = 15 \times 10^{-6}$, the correlation coefficient r for wave height and period is presented in terms of f in the region of $1 < f < 10^4$ approximately as follows:

$$r = 2.97 - 2.00 f^{0.04} - 0.053 f^{0.08}$$

(II) In the case of shallow water waves, asumming similarly to the case of deep water waves and taking the coefficient of bottom friction as $0.015 \sim 0.019$, the correlation coefficient r is given as a function of d and f, as shown in Figure 5. r decreases more rapidly than that of deep water waves and gets to a certain negative value for each d.

The above-mentioned estimation is, however, only derived by approximate calculations with simple assumptions and so it should be necessarily verified by future investigations.

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