

CHAPTER 9
STATISTICAL PROBLEMS OF SAMPLE SIZE AND SPACING ON
LAKE MICHIGAN BEACHES

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INTRODUCTION

Beaches along Lake Michigan display a wide range in texture and composition. Some are composed of fairly uniform sand or gravel, others display an irregular mixture of both. Most beach sand is predominantly quartz with variable amounts of calcite, feldspar, and heavy minerals. Some gravel beaches are composed wholly of limestone or dolomite pebbles of local origin; in others the pebbles have the varied composition of their parent glacial materials.

Lake Michigan beaches may be narrow or wide, steep or gently sloped, soft or firm, and so on, depending in part upon their orientation, the season of the year, their nearness to natural source areas, and other factors. A single beach may show a wide variety of characteristics, with notable changes in texture and composition over short distances or during short time intervals.

The problem of sampling such nonhomogeneous deposits for geological or engineering purposes has challenged numerous workers; and to the writer's knowledge no completely satisfactory sampling technique has yet been developed. How large should a single sample be? How deeply should it penetrate the beach? How far apart should samples be collected? How often should the sampling process be repeated?

Specific answers to these questions must depend on the objectives of the beach study and on the nature of particular beach areas. It is common knowledge that material transported along beaches is subject to abrasional and selective sorting processes which produce systematic changes in particle size, shape, and other attributes. The objective of many studies is to measure these systematic changes and relate them to the physical processes of the beach and nearshore zone. The information so obtained is of value in geological interpretation, engineering design, and other fields.

In the present paper a somewhat general approach is used, to bring out principles which apply to all beaches. From these, it is possible to develop specific techniques for most common situations. Sand beaches afford a suitable starting point, both because more is known about them than about gravel or mixed beaches, and because sampling problems are relatively simpler. It is known, for example, that closely spaced samples over a small area on sand beaches have similar characteristics in terms of particle size, shape, composition, and other attributes. In other words, within a local area the particle population of a beach is

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homogeneous. That is, random samples from the small area satisfy the statistical condition that they have similar properties at some selected level of significance. Moreover, average values from the samples tend to distribute themselves as a normal probability curve.

As the area of sampling is enlarged, or as the depth of sampling increases, the variability among the samples increases, and the more outlying or deeper samples begin to display average properties significantly different from those in the smaller area. Statistically this means that the larger population is no longer homogeneous at the selected significance level. On this basis a beach may be considered as having a series of homogeneous populations which grade or abut into each other as the deposits are followed along or across the beach. Each individual population has a variability due mainly to random sampling fluctuations, whereas statistically significant variations from one population to the next may be considered as being caused by variations in the "natural treatments" imposed by waves, currents, and other geological agencies.

In some instances the homogeneous areas grade imperceptibly into others, as when particle size is studied along an extensive sand beach. In other instances there are abrupt population differences, as when the texture changes abruptly from uniform sand on the backshore to stringers and patches of gravel on the foreshore. In general, changes across the beach are more pronounced than along the beach, and most beaches display a series of bands or zones parallel to the shore, each of which is homogeneous for an appreciable distance along the beach.

The several homogeneous populations that comprise a beach may be considered as a universe or set of populations which characterize the beach as a whole. The number of such populations and the relations among them, are partly a function of the significance levels selected, and partly a function of the natural gradations and abrupt changes which occur on the beach. It is also possible to consider the gradational types of population as single populations with linear or exponential gradients.

In statistical practice it is recognized that sampling procedures should be adapted to the kinds of populations being sampled. A few random samples from a homogeneous population may be sufficient to estimate the characteristics of the population, but when heterogeneity is present, the sampling plan usually involves some type of systematic or stratified sampling. Sampling procedures are designed to ensure that samples are representative of their populations, and that more than one population is not inadvertently included in any one sample.

The purpose of this paper is to show that definite answers to some problems of sampling heterogeneous beaches can be given. Sampling plans suitable for various beach conditions are discussed in the light of standard statistical procedures for sorting out and evaluating variabilities among samples.

STATISTICAL PROBLEMS OF SAMPLE SIZE AND SPACING ON LAKE MICHIGAN BEACHES BEACH POPULATIONS

The writer and his students, with collaboration of E. C. Dapples, have conducted many beach sampling experiments during the past seven years. Some of the results are reviewed here, supplemented by other data to enlarge the frame of discussion. Most of the experiments were conducted with sampling grids laid over the beaches. The grid spacing varied from less than 10 feet to as much as 100 feet between samples. Most grids involved segments of beach 200 feet or more long, commonly covering the entire width of exposed beach. In some experiments a single sample was collected at each grid point; in others multiple samples were collected around each stake.

The samples were analyzed for moisture content, particle size, particle shape, heavy mineral content, etc. In many instances a parallel set of beach firmness readings was made with a penetrometer. The analytical data were plotted on maps, and almost all of the maps show a distinct zoning or banding parallel to the shore, with each band relatively homogeneous over its length. Some beach attributes do not display this marked banding; for example, maps of particle roundness may be relatively irregular across the beach, although in some studies a marked increase in average roundness is found on the landward side in the dune sands. On the whole, however, most properties that are related to the dynamics of beach processes (such as average particle size, degree of sorting, sphericity, and density as shown by heavy mineral content) do display systematic patterns.

The concept of beach populations was suggested by the systematic map patterns and by the generally known geological processes occurring on beaches. Waves wash up to the berm but seldom cross it. Hence, the berm is a natural subdivision between a lakeward part of the beach subject to rhythmic wave action, and a backshore acted upon seldomly by waves. The backshore consists mainly of dry sand and is subject to winnowing action by the wind. Farther inland the winnowed sand may accumulate in a belt of dunes. These geological processes vary with the seasons and contribute to cross-beach variability. Similarly, currents along the shore tend to carry some particles farther than others, so that average particle size and other properties vary from one end of the beach to the other. The along-beach changes are generally more gradual than the cross-beach changes, however.

The data from the grid experiments were analyzed statistically to determine whether the observed variability of beach properties could occur within a single homogeneous population, or whether a more suitable approach would be to interpret the data as representing several populations distinct from each other. Numerous analyses showed that statistically significant differences are present for a number of properties on the conventional 5 per cent significant level. The method used is analysis of variance, a statistical technique for sorting out and evaluating variabilities. In its simplest form the method compares variances within and between groups of data. If the ratio of the between-

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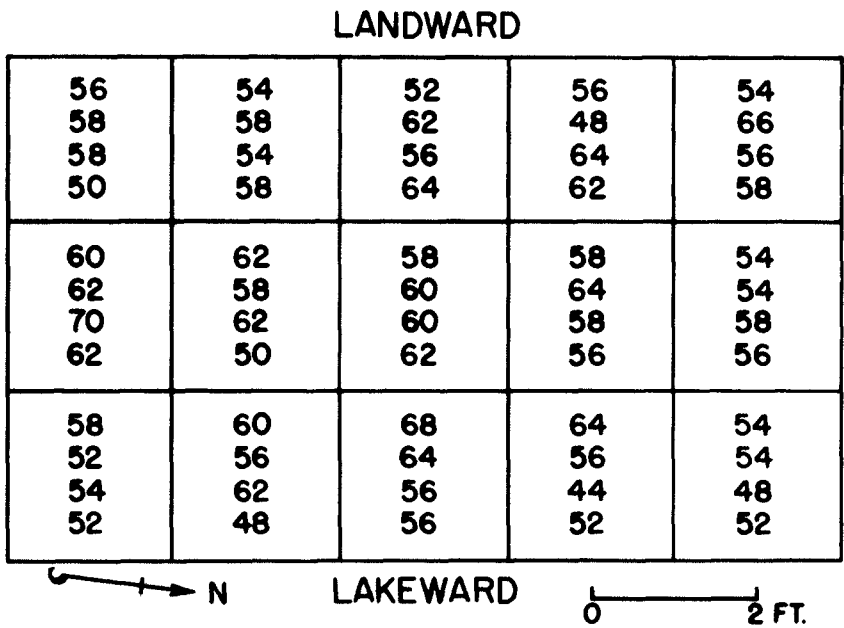


Fig. 1. Penetrometer grid on beach foreshore, Evanston, Illinois. Dial penetrometer, 6-inch penetration.

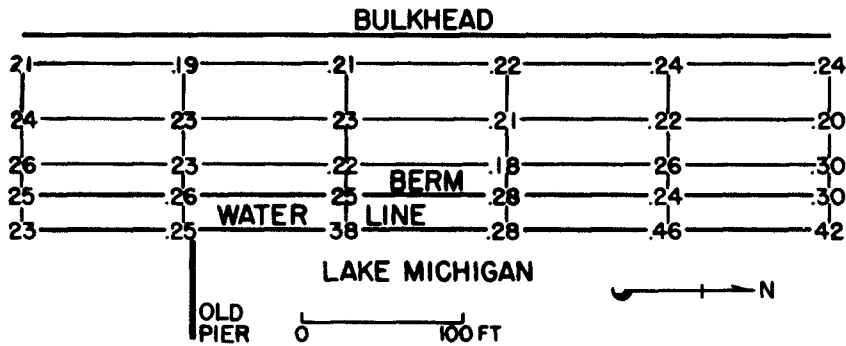


Fig. 2. Sampling grid, Lee Street Beach, Evanston, Ill. Numbers are geometric mean diameters of sand size distribution curves at sampling sites.

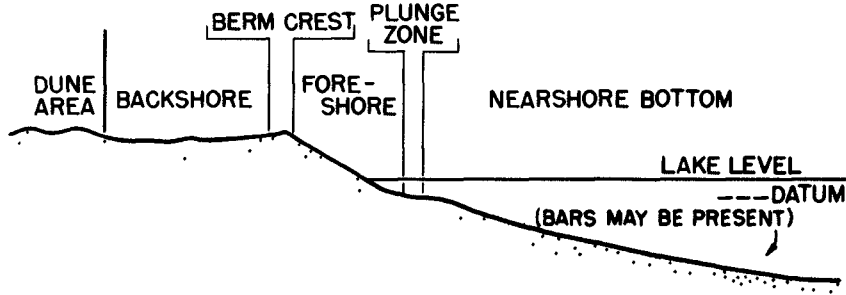


Fig. 3. Cross-section showing "population bands" on Lake Michigan beaches.

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group variance to the within-group variance exceeds a selected critical value, the difference between the groups may be considered statistically significant. In terms of the present approach, such differences may be interpreted as indicating that the populations represented by the several groups are also different.

Statisticians have developed a number of analysis of variance models during the past two decades. Several of these are directly applicable to beach problems. Full description of the methods are given in numerous recent statistics texts, such as Cochran and Cox (1950), Goulden (1952), and Kempthorne (1953). Dixon and Massey (1951) provide an excellent introduction to the subject. Five models which apply to a variety of geological problems are discussed by Krumbein and Miller (1953), and several of these are included in the following examples.

BEACH FIRMNESS

Beach firmness is a convenient property for introducing analysis of variance of beach characteristics. Penetrometers of several kinds are available, the experiments are readily performed, and the data are fairly reproducible. The experimental grid may be laid out with stakes; readings may be taken at each stake, or distributed over the rectangular areas defined by the stakes. In one study of local variability of firmness, stakes were set at 2-foot intervals over an area 6 feet wide across the beach and 10 feet wide along the beach. Each "cell" between the stakes was a 4-foot square, and four readings were made in each cell, as shown in Figure 1.

The penetrometer used in the experiment consisted of a steel rod with a gauge attached. The rod was inserted to a fixed depth in the sand, and the limiting pressure for additional penetration was read from the gauge. The larger the reading, the firmer the beach. Table 1 shows the analysis of variance of the penetration data. An intermediate step, not shown, is preparation of a "condensed table" in which the four values of each cell are combined into a single value. This analysis of variance model is a two factor form with replication, and details of computation are given in Dixon and Massey (1951, p. 134) and in Krumbein and Miller (1953). Essentially the computation consists in squaring all items in the original cells and in the condensed cells. In addition, column and row totals are squared, as is the grand total. These values, converted to sums of squares, are arranged in the analysis of variance table with appropriate degrees of freedom. The mean square or variance is then found by dividing the sums of squares by corresponding degrees of freedom. In Table 1 the total variance has been separated into four parts, representing variances due to row, column, "interaction", and residual effects.

The mean square for interaction is first evaluated against residual error at the 5 per cent confidence level, and in this instance it is non-significant. The interaction sum of squares is then added to the residual, and the total is divided by the combined degrees of freedom to give a net

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TABLE 1

ANALYSIS OF VARIANCE OF PENETROMETER
DATA FROM CAMPUS BEACH. ORIGINAL
VALUES SHOWN IN FIGURE 1

Source	Sum of Squares	Degrees of Freedom	Variance	F
Columns (along beach)	131	4	32.75	1.33 N.S.
Rows (across beach)	138	2	69.00	2.80 N.S.
Column x row interaction	229	8	28.63	1.20 N.S.
SUBTOTAL	498	14		
Residual	1077	45	23.93	
TOTAL	1575	59		

N.S. = non-significant

(Combined error = $1306/53 = 24.64$)

TABLE 2

ANALYSIS OF VARIANCE OF GEOMETRIC MEAN
DIAMETERS OF SAND SAMPLES FROM LEE STREET
BEACH, EVANSTON, APRIL, 1950

Source	Sum of Squares	Degrees of Freedom	Variance	F
Columns (along beach)	0.0173	5	0.00346	1.62 N.S.
Rows (across beach)	.0562	4	.01405	6.49 **
Residual	.0428	20	.00214	
TOTAL	0.1163	29		

N.S. = non-significant

** = significant at 1% level

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error term. When the main effects (along-beach variance and across-beach variance) are tested against the combined error, neither is significant at the 5 per cent level.

The result of this experiment is that the small grid area may be considered as having a homogeneous population of beach firmness. The variability within the rows and columns is of the same order as the variability between rows or columns, indicating that there are no significant row or column effects. In other words, there is no significant variation along or across the beach within this grid area.

PARTICLE SIZE

A second example of beach populations is furnished by Figure 2, which shows a grid 500 feet long and 100 feet wide on Lee Street Beach, Evanston. The grid extends from the backshore across the berm crest to the water line. A sample was collected at each grid point and analyzed for particle size; the figure shows the geometric mean diameters at the sampling points. Inspection of the figure shows that mean particle size increases lakeward across the grid. The berm crest was sampled in this experiment to see whether its characteristics were transitional between backshore and foreshore.

The analysis of variance is shown in Table 2. This is a two factor form with single entry (Dixon and Massey, 1951, p. 127). This simpler form does not permit evaluation of interaction effects. As the table shows, there is a highly significant difference between the rows (across beach), but no significant difference between the columns (along beach). Additional analysis indicates that the three landward rows on the backshore are homogeneous among themselves, and that the foreshore may be considered as a different population, with the berm crest representing a transition between the two. If the values in the landward three rows are averaged, the mean size is found to be 0.228 mm. The samples in the row nearest the lake average 0.337 mm. These are unbiased estimates of the two population means for particle size. The berm samples may be treated as a third average if desired.

An extension of the method of analysis also permits evaluation of the population standard deviation for sample means. Details are not given here, but the subject is discussed in Dixon and Massey (1951, p. 142). An estimate of the backshore standard deviation by these methods is 0.026 mm, and for the foreshore it is 0.096 mm. These values suggest that the backshore in this instance is finer and more homogeneous than the foreshore.

OTHER BEACH POPULATIONS

Numerous analysis of variance studies similar in form to Tables 1 and 2 have been made, on grids varying in size as previously mentioned (Krumbein and Miller, 1953; Krumbein, 1953). These studies show that each attribute of the beach, such as firmness, particle size, particle shape, moisture content, thickness of bedding, etc., has variations along and across beaches. It may be concluded, therefore, that beaches consist of a number of populations of different attributes, which together

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comprise some universe or super-population of attributes representative of beaches as a whole.

The problem of beach sampling thus appears to resolve itself to the sampling of variable populations, and the practical solution requires some method which provides representative samples of the populations or of "strata" within the super-population. In most geological or engineering studies particle size is the main attribute investigated, with mineral composition, firmness, and other characteristics important in special studies. Such attributes as moisture content, though interesting, have been studied mainly for teaching purposes.

The writer's studies on Lake Michigan sand beaches indicate that several areas of relatively homogeneous population can be discerned on most sand beaches. These population zones or bands may be made the basis for sampling the beach. Figure 3 shows the zones on a completely developed sand beach. The landward area of windblown sand, where present, grades gradually to the backshore. The backshore is relatively flat, sometimes with a gentle landward slope. The berm crest, represented as a narrow band in the figure, is a transitional area having a mixture of backshore and foreshore characteristics. This transitional zone may vary in position and width seasonally. The foreshore may be homogeneous in particle size, but usually has several moisture content populations. The foreshore extends to the average zone of breaking waves (also subject to seasonal change) where rapid and marked population changes may occur. The nearshore bottoms may be homogeneous along fairly extensive stretches of shore in moderate depths. In local instances population changes may occur near jetties and breakwaters.

The relative simple pattern of Figure 3 is subject to wide variations, especially on mixed sand and gravel beaches. Even on sand beaches the zoning may be seriously disturbed by storm waves which wash across the backshore. A major storm in Spring, 1949, scattered very coarse sand and pebbles over most of the beach at Wilmette, Illinois, in an apparently random manner. Samples collected shortly after the storm showed a wide variability in average particle size, although there were no significant row or column effects in a sampling grid 500 feet long and 150 feet wide. Apparently the storm had developed a wide but variable "foreshore" over the whole beach. In time more normal conditions redeveloped a new foreshore and berm nearer the lake, leaving a somewhat patchy but homogeneous backshore inland.

Gravel beaches present their own problems in terms of zoning. A relatively recent gravel beach at Dempster Street, Evanston, showed several steep gravel berms with distinctly different degrees of coarseness visible to the eye. Surprisingly enough, a set of samples collected on a grid which included the berms showed no significant across-beach variance in average particle size, but did show a marked size gradient along the beach.

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CYCLICAL CHANGES ON BEACHES

The foregoing discussion considers beach populations as they are at a given moment. In many studies it is desirable to record changes on the beach during storm cycles, seasons, or over longer time periods. Such changes may be noted by collecting samples on the same grids or along the same profiles at specified times. Maps of the successive beach patterns yield a visual picture of the changes, and significant differences from one pattern to the next can be evaluated by conventional geological analysis or by analysis of variance.

In 1947 the writer and his assistants, E. J. Herbaly and Carl Setzer, made a daily study of changes along a beach segment fronting Northwestern University. The segment is 225 feet long and is enclosed at both ends by pre-cast permeable groins. Maps were made each day during most of October and November. The maps showed a systematic series of sand and gravel distribution patterns during and between storm cycles.

The maps were generalized into several stages of beach development during the storm cycle. Figure 4 shows these stages. The upper map (A) shows the normal beach pattern after several days of quiet water (no waves exceeding 6 inches in height). A landward band of uniform sand gives way abruptly to a low ridge of coarse gravel marking the main berm. Lakeward of the coarse gravel berm were bands of medium and fine gravel, each marking a minor berm. No sand was exposed at the water's edge, although sand bottom was present a few feet from shore. The quiet water beach thus apparently displayed four particle populations, usually with abrupt contacts, although some gradations attributed to boundary conditions could be seen near the groins.

Several moderate storms occurred during the period of study. Some lasted for more than a day, with a gradual increase in wave height and period, followed by a tapering off to quiet water again. The first effect of a storm, observable when the waves reached a height of about 12 inches, was the stripping away of the finer gravel berm, leaving a foundation of uniform sand, as shown map B of Figure 4. As the waves increased in height to about 24 inches, the medium gravel berm was stripped off, leaving a sand zone from the water's edge to the coarse gravel berm, as shown in map C of Figure 4. In some instances the coarse gravel berm was locally breached at this stage. If the storm continued to a wave height of about 48 inches, most of the coarse gravel was stripped off, leaving a uniform sand beach to the foot of the bank. Map D of Figure 4 shows the effects of a storm during which the waves averaged about 54 inches high at the breaking point.

As the storm subsided, the coarse gravel berm was restored first, then the medium gravel berm, and finally the fine gravel, leading to a map much like the initial stage of Figure 4. This sequence of events was observed on several occasions during the study, and although no underwater samples were taken during the storms, it appeared evident that the gravel was stored offshore in one or more bars, which moved shoreward

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as the storm died down.

The important point for the present discussion is that the beach underwent a systematic series of population changes during the storm cycle. How should such a beach be characterized? During calm weather it is essentially a pebble beach; during severe storms it is entirely sand. One may perhaps argue that the beach can best be characterized by a composite sample from all four bands. Such a composite, however, would not by itself indicate the extreme variability of the beach. A preferable method would perhaps be to take samples from each band separately, weighting the averages in terms of band width and length. For detailed study it seems preferable to use principles of stratified or systematic sampling, as discussed in a later section.

SAMPLE SIZE AND DEPTH OF PENETRATION

Common practice on sand beaches is to collect samples of shallow depth and relatively small diameter. An average of present practice would perhaps be to collect a cylindrical sample from 1.5 to 3 inches in diameter and about 2 or 3 inches deep. In some instances the upper half inch of sand is first scraped off to remove stray debris. Samples of beach gravel commonly vary in volume, depending on the size of the larger pebbles present, but the writer knows of no average or standard practice for pebble beaches.

Inasmuch as a sample should be representative of the population at the point of sampling, experiments can be designed to determine the extent to which observed characteristics vary as sample size or depth are varied. Elsewhere the writer describes such an experiment on a sand beach (Krumbein, 1953), and in this paper similar data are given for medium gravel with mean size between 8 and 16 mm. The experimental design involves using a series of concentric rings which are forced to successively greater depths as sampling proceeds. The gravel in each annular space is collected and analyzed separately, and the data are combined to develop a composite cylindrical sample which grows in diameter and depth.

For the gravel sample the writer used five welded metal rings of diameters 3, 6, 9, 12, and 15 inches. The rings were forced into the beach to a depth of 2 inches, and the process was repeated four times, yielding four sets of five samples with a total cylindrical volume 15 inches in diameter and 8 inches deep. The size data for analysis of variance as shown in Table 3, which lists the geometric mean diameters in mm. of the successive analyses. The total sample weighed 27 kilo-grams, roughly 60 pounds. The lower part of Table 3 shows the analysis of variance as a two factor form with single cell entries. The between-column variance (area of sample at given depth) is not significant at the 5 per cent confidence level, but the between-row variance (depth of sample for given area) is significant at the 1 per cent level. This implies that for gravel of the size studied, the smaller rings for any given depth are not significantly different from the larger rings, but that a shallow sample of given area is significantly different from a deeper one of the same area.

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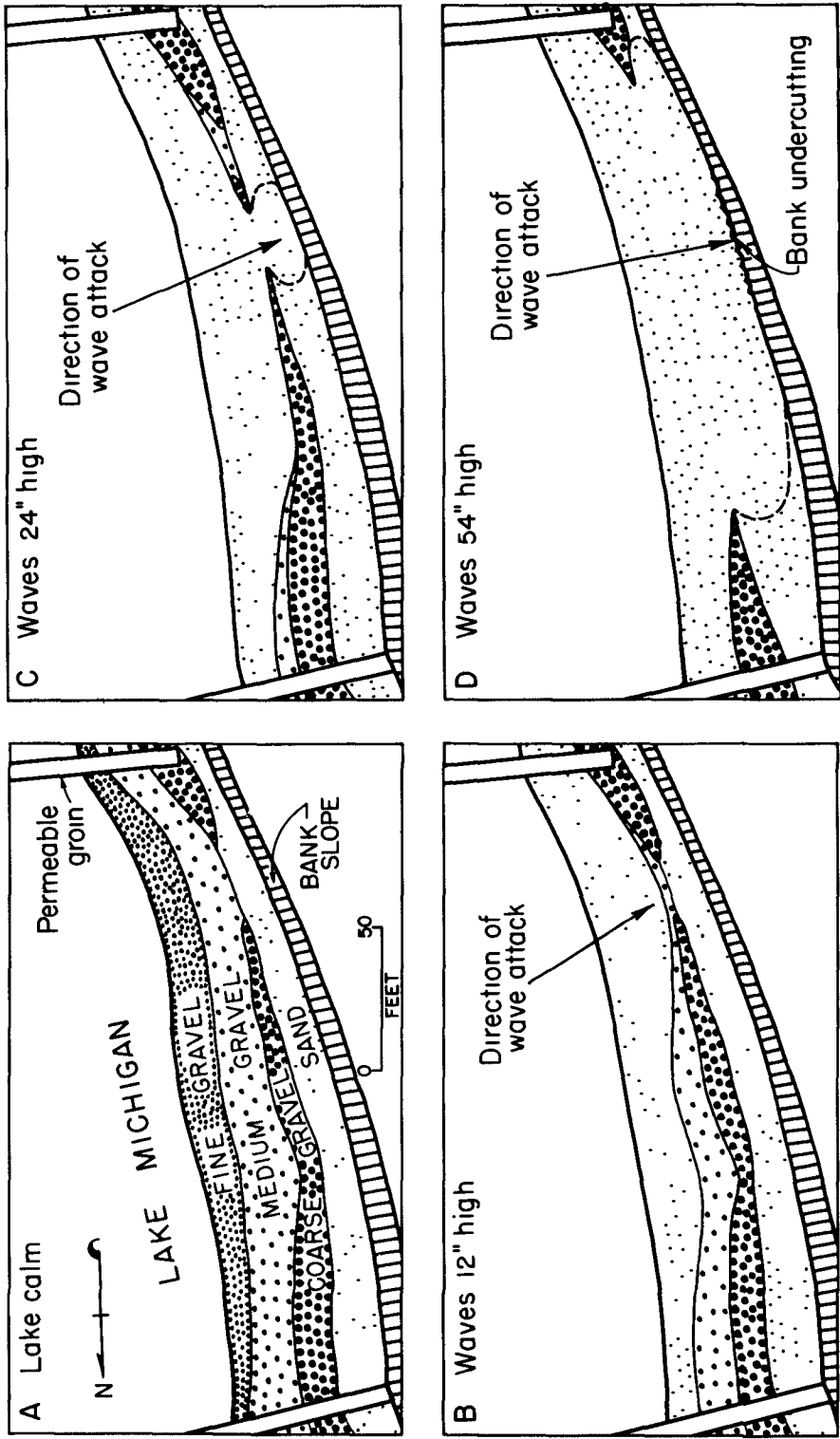


Fig. 4. Maps of small campus beach at Northwestern University, showing changes during storm cycles.

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Table 3.

ANALYSIS OF VARIANCE OF GEOMETRIC MEAN DIAMETERS
OF GRAVEL FROM CONCENTRIC RING SAMPLER, DEMPSTER
STREET BEACH, EVANSTON

		Ring Diameter				
		3"	6"	9"	12"	15"
D e p t h	2"	11.7	11.5	11.5	11.0	11.3
	4"	10.3	10.3	10.3	10.3	10.6
	6"	9.5	9.6	9.7	9.6	9.8
	8"	8.9	9.1	9.2	9.2	9.3

ANALYSIS OF VARIANCE

Source	Sum of Squares	Degrees of Freedom	Variance	F
Columns (sample area)	0.12	4	0.030	1 N.S.
Rows (sample depth)	14.43	3	4.81	150.3 **
Residual	0.38	12	0.032	
TOTAL	14.93	19		

N.S. = non-significant

** = significant at 1% level

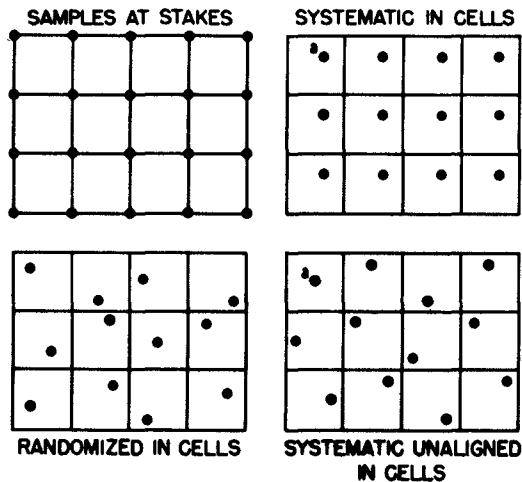


Fig. 5. Diagram showing four methods of arranging samples on a beach grid. (Adapted from Quenouille, 1949)

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The results of the gravel analysis were the same as for the earlier sand analysis, which showed that a sample 1.5 or 3 inches in diameter to a given depth was not significantly different from larger samples up to 12 inches in diameter. As with the gravel, there was a significant difference between samples of different depth. Both of these studies are only exploratory, but they suggest that a minimum and perhaps an optimum size of sample could be determined for a variety of deposits. For gravel of the size studied, the writer believes that a cylindrical sample 6 inches in diameter is amply large. The depth of penetration is more a matter of opinion, and must be decided on the basis of the purpose of the study.

The composite ring sample was collected in an area from which additional samples 6 inches in diameter and 2 inches deep were also collected. The ring sample gave essentially the same population mean as the other samples in its vicinity for samples collected in the 2-inch depth.

BEACH SAMPLING PLANS

In an earlier paper the writer suggested using each beach population or zone as a separate sampling area (Krumbein, 1953). For any one beach profile or traverse, the several zones normally present above water may be recognized by noting berm position, textural changes, relative firmness, and the like. A single sample collected at random from the central portion of each zone provides data for an estimate of the population mean. Underwater samples are collected either in terms of water depth or fixed distances from shore. Control by depth appears to be favored in practice.

The sampling procedure is improved if a minimum of four samples is collected from each population band. In this manner sufficient data are obtained to estimate population variance as well as the mean. Even two samples from each zone are preferable to single ones, if time or expense prevent collection of four. Multiple underwater samplers can be designed which provide two samples at each station.¹

The use of beach zones as a basis for sampling is a form of stratified sampling, in which the zones represent "strata" of the main population or universe, each conceivably with its own mean and variance. If the samples in each "stratum" are selected by a random process, the samples are "stratified random samples." On the other hand, if the samples in the first stratum (say the backshore) are randomized, and samples in succeeding strata are collected in the same relative positions, the samples are "systematic samples." The simplest case of systematic sampling is to collect a sample from the center of each zone or stratum.

1. The writer had the privilege of using a two-tubed sampler developed at his request by the Beach Erosion Board. The sampler consisted of two parallel pipe segments with bell-shaped mouths. When cast overboard and pulled back to the ship, two samples were collected about 16 inches apart.

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Multiple samples in each zone may be collected at random from segments of a circle drawn around the central stake. A 1-foot circle centered at the stake is divided into 45° segments numbered 1 to 8. Four numbers are drawn from a random number table (say 2, 3, 5, 8), and samples are taken 1 foot from the stake at these prenumbered positions. An alternative method is to subdivide ^{each} stratum or zone into several substrata, with a sample taken from each substratum by some random or systematic process.

If the sampling plan includes an appreciable width of beach on either side of the profile line, the problem becomes one of plane sampling. The commonest example is the square staked grid in which samples are collected at each stake. The problem may be approached from a more general viewpoint, however, as Quenouille (1949) showed. The grid may be arranged as a pattern of squares, rectangles, or other shapes. In the present discussion the square grid is used as illustration, following the essential development of Quenouille.

Figure 5 is an adaptation of a similar figure from Quenouille, showing four possible sampling plans out of a large number discussed in the original paper. Each grid is composed of square "cells" which may be considered as the sampling units. In the upper left diagram the samples are taken at the grid stakes, and in the upper right diagram they are taken with the same spacing, but at some fixed position within the cell. Both are "systematic aligned samples", with the position in cell a of the upper right diagram determined by drawing a random number for position. Once the value is selected, all subsequent samples are taken from the same cell positions.

The lower left diagram is an example of stratified sampling in which the position of the sample is separately randomized for each cell. This spreads the samples over the cells in the manner shown. The lower right diagram is an example of "unaligned systematic sampling" in which the sample in square a is randomized, succeeded by randomization of vertical co-ordinates in the remaining upper row cells, and randomization of horizontal coordinates in the remaining first column cells. Cochran (1953, p. 183) describes the process. By extending this pattern over the cells a systematic arrangement without alignment is obtained. The unaligned systematic samples have some advantages over the simpler systematic plans of the first two diagrams, in that they supply additional data on gradients in the population (Cochran, 1953, p. 184).

The writer has extended some of these plans to as many as four samples per cell. The additional samples are located by repetition of the same randomizing or systematizing process as is used for the single cell samples. For analysis of variance studies involving evaluation of interactions, multiple samples from each cell provide more information than single samples. Controlled sampling plans of this type can readily be applied to the exposed beach, but the collection of underwater samples is more complex if the ship has to be maneuvered into specific grid positions. Aligned systematic samples can be taken conveniently, however, inasmuch as the ship may follow a straight course during the sampling.

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Quenouille (1949) tested the relative efficiency of the several sampling plans for various populations, and pointed out that the efficiency varies somewhat depending on the population characteristics, and presumably on the objectives of the study. If the grids are confined to homogeneous zones on the beach, any of several may be satisfactory. If the grid straddles population zones the same cell may contain samples from more than a single population. On the other hand, if the zones are thought of as units within some "super-population" with linear or exponential gradient, then a grid which brings out the details of the gradient is presumably the best. A study of this problem for beaches by geologists or engineers in collaboration with a mathematical statistician would clarify several important details.

The use of sampling grids, especially with fairly close spacing, is perhaps more appropriate for detailed beach studies than for conventional beach surveys. In most applied problems the objective is to obtain data on beach characteristics with a minimum expenditure of time and cost. For these studies the collection of samples from beach profiles located at convenient distances along the beach appears to be suitable (Krumbein, 1953). When the areal variations on the beach are to be related more specifically to beach processes and wind winnowing, however, more detailed population studies are indicated.

The several sampling plans described above are sufficiently general to apply to most sand beach situations. The specific problem of sampling Lake Michigan beaches, with their relatively large variability of particle size, may require adjustments in grid spacing, in sample size, and in repetitions of the sampling process. Each beach presents its own special problems, but it is believed that the sampling plans described can be adapted to most situations. Beaches with mixed sand and gravel, either as patches of gravel on an otherwise sand beach, or sand beaches with scattered isolated pebbles appear to present the greatest difficulties for sampling. More study of these conditions is needed.

CONCLUDING REMARKS

The present paper is a progress report on some experimental approaches to the problem of sampling Lake Michigan beaches. It is evident that much remains to be learned about the most efficient sampling plan for any specific situation. It is believed that many of the problems can be solved by suitably designed experiments. A large background of sampling theory is available as a guide for such investigations. Cochran's book on sampling (1953) is a basic reference in this connection.

In the long run the sampling process represents an initial stage in acquiring geological and engineering information on beaches. After the samples are analyzed and the population parameters estimated, there remains the question of the relation between these parameters and the natural processes which control them. Experimental design, which is rapidly expanding in the earth sciences, provides one way of attacking the general problem. The basic importance of sampling in all subsequent interpretation of the data suggests that the sampling plan itself be an integral part of the over-all experimental design.

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This paper does not include a review of the somewhat voluminous literature on Lake Michigan beaches, inasmuch as it is directed toward the more general problem of sampling design. However, the reader is referred especially to the paper by Fisher (1954) in this symposium for an example of a comprehensive study of size and other properties along the Lake Michigan shore of Illinois. The study involves repetitive sampling along selected profiles, and includes both exposed beach and underwater samples. The maps accompanying the paper show the distribution of underwater areas of uniform and variable sand characteristics, useful in a study of population changes in the nearshore zones.

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