Chapter 14

NEW CONFIRMATION OF THE FORMULA FOR THE CALCULATION OF ROCK FILL DIKES

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In the paper entitled "Una fórmula para el cálculo de los diques de escollera" (A Formula for the Calculation of Rock Fill Dikes) published July 1938* there was presented the expression,

$$P = \frac{NA^3 d}{(\cos\alpha - \sin\alpha)^3\ (d-1)^3}$$

where

P = weight of the individual stones or blocks in kilograms
N = 15 for dikes of natural rock fill
N = 19 for dikes of artificial block fill
A = 2h = total height of the wave that breaks on the dike, measured in meters
d = specific weight of material of the stones in tons (metric) per cubic meter
α = angle of the dike's side slope with the horizontal

Before the preliminary determination of those tentative values of N, each based only on a single observed case - the natural rock fill dike of Orio and the artificial rock fill dike of San Juan de Luz, - the publication mentioned above stated that:

"It only remains now to determine the coefficient N and verify if it is sensibly constant, as seems inferred from the material submitted, or varies with the other elements of the formula.

"In the worst case, a coefficient similar to the classic and variable coefficient C of the formula of uniform flow, $V = C\sqrt{RS}$, will be considered."

In spite of the fourteen years intervening, during which the reasoning followed for the deduction of the formula has been refined, in a manner that might have been advantageously taken into account by the translators of the paper, the coefficients 15 and 19 still stand, due to the satisfactory results always obtained in numerous cases of practical application.

*Translated and published in the January 1949 Bulletin of the Beach Erosion Board, Office Chief of Engineers, Department of the Army, Wash. D.C. The coefficient K of that publication is denoted by N in this paper to avoid confusion with $K = \coth \pi \frac{H}{L}$, and similarly, the angle "a" we here use α.

The formula is actually derived for the upper slope of the dikes, and a generalization for all the depths of the structure was indicated only tentatively at the end of the paper. In this connection substantially the following was stated:

"This generalization of the formula assumes a certain margin of security, but it is not logical to apply on the sea the strict results obtained from theoretical formulas when on land it is usual to multiply them by ample safety factors.

"I should be most grateful to my colleagues who, acquainted in detail with concrete practical cases, would kindly furnish me information for refining the coefficients."

As a result of the ideas presented in the first publication of 1938 it is possible to refine somewhat the coefficients in the article entitled "Generalización de la fórmula para el cálculo de los diques de escollera y comprobación de sus coeficientes", (Generalization of the Formula for the Calculation of Rock-fill Dikes and Determination of its Coefficients) published in May 1950* in the Revista de Obras Publicas. On comparing them with results with the established experience obtained from the Argel dikes, the degree of approximation sufficient for practical application was definitely confirmed.

Besides the practical confirmation of the coefficients mentioned in the article of the Revista de Obras Publicas, the composition of the formula was verified. In this connection it was substantially stated:

As was a matter of record, in the XVII International Congress of Navigation held in Lisbon, it is satisfying that the formula deduced in the American report of Epstein and Tyrrel for reflecting rock fill dikes, starting from our expression for pressures of reflection $P = \frac{CV}{g}$**, is similar to that which was obtained in 1938 for breakwaters.

In effect, the American formula is:

$$P_t = R_t \frac{s\,H^3}{(s-1)^3\,(\mu - r)^3}$$

and ours:

$$P = \frac{NA^3 d}{(\cos\alpha - \sin\alpha)^3\,(d-1)^3}$$

*This article was translated and published in the January 1951 issue of The Bulletin of The Beach Erosion Board, Office Chief of Engineers, Depar ment of the U.S. Army, and by the Waterways Experiment Station; Translati No. 51-2.

**See the publication "Cálculo de diques verticales" (Calculation of Vertical Dikes) published 1938, translated and published in French and in English in the "Bulletin de l'Association Internationale Permanente des Congrés de Navigation", No. 23, July 1939.

in which

$P_t = P$ = weight of the individual stones
$H = A = 2h$ = wave height
$s = d$ = relative density of the material
μ = natural batter of the rock fill $\simeq 1$
$r = \tan\alpha$ = slope of the rock fill
R_t and N are coefficients.

Expressing the American formula in our notation, it becomes:

$$P = R_t \cos^3\alpha \quad \frac{A^3 d}{(\cos\alpha - \sin\alpha)^3 (d-1)^3}$$

which is ours, except only that it includes the factor $\cos^3\alpha$ in the coefficient.

It is a matter of record that on establishing our formula it was indicated that the coefficient should vary with the data of the problem. Practically the angle α, which varies most for the upper part of the dike, will not vary much, for from $\cot\alpha_1 \simeq 3$, corresponding to present rock fill dikes, it cannot get much steeper than $\cot\alpha_2 \simeq 2$, even in the reflecting dikes, because of the enormous weight of the stones this requires. Between those maximum limits the relation is:

$$\frac{\cos^3\alpha_1}{\cos^3\alpha_2} \simeq 1,2$$

which would represent only a small difference in the weight of the stones, and even less in their size, whose relation would be $\sqrt[3]{1.2} = 1.06$. Only direct observation can determine properly N or R_t, including cases of very steep slopes.

Another interesting confirmation of our formula is given now in the article entitled "Notes on Determination of Stable Underwater Breakwater Slopes" published by Kaplan in the Bulletin of the Beach Erosion Board for July of the present year.* In this article, starting from Blanchet's formula for the destruction of stone piles by a current**

$$v = K_1 \sqrt{2g \frac{w_1}{w}} \sqrt{D} \sqrt{\sin(\alpha_0 - \alpha)}$$

in which

v = velocity limit for the disintegration of the stone pile
K_1 = a dimensional coefficient which should be constant for stones of a given shape

*Kenneth Kaplan, "Notes on Determination of Stable Underwater Breakwater Slopes", Bulletin of the Beach Erosion Board, Vol. 6, No. 3, p. 20.
**Ch. Blanchet, "Formation et destruction par un courant d'eau des massifs en pierres", in "La Houille Blanche", March 1946.

D = lineal dimension of the stones
w = specific weight of the water
w_1 = specific weight of the stones
α = angle of slope with the horizontal
α_0 = angle with the horizontal of the stones' natural slope

Calling W the weight limit of the individual stones, "a" the major axis of the corresponding orbitary ellipse and T the wave period, Kaplan gets the formula:

$$\sin(\alpha - \alpha_0) = \frac{Ka}{W^{1/6}\ T}$$

where the value of the coefficient K is a function of the acceleration of gravity "g" and the specific weights of the water and of the stones.

Actually the case which is of most interest is that concerning submerged stones, or stones entirely enveloped by water. In this case it is necessary to take into account, besides the hydrodynamic thrust, the vertical hydrostatic force. It is desirable to substitute the specific weight w_1 of the stones, without deviating from Blanchet's formula, by $w_1 - w$ corresponding to these submerged stones. Therefore the formula becomes:

$$v = K_1 \sqrt{2g \frac{w_1 - w}{w}} \sqrt{D} \sqrt{\sin(\alpha_0 - \alpha)} \qquad (1)$$

Following an identical procedure of our observations of 1938 and 1950, the weight of the stones is expressed as

$$W = K_2 w_1 D^3 \qquad (2)$$

as well as the maximum orbital velocity of the water

$$v' = \frac{\pi a}{T}$$

which definitely is supposed to admit also, with a certain degree of practical approximation for such complicated problems, the application of the trochoidal theory, even in the case of steep slopes.

It is important to remember that in our first report of 1938, besides deducing the formula for the direct action of the wave breaking upon the breakwater, the same formula was also deduced, thus confirming its generalization, upon the basis of the water's descent on the slope at a reduced velocity due mainly to the roughness and permeability of the rock fill. Hence, following our own previous deduction it may be stated that

$$v = K_3\ v' = K_3 \cdot \frac{\pi a}{T} \qquad (3)$$

Thus from the equations (1), (2), and (3) the formula at once is obtained, this being duly corrected, in which

$$\sqrt{\sin (\alpha_0 - \alpha)} = \frac{K_a}{W^{1/6}\ T} \qquad (4)$$

where the coefficient K, also duly corrected, becomes:

$$K = K' \left(\frac{w}{g\ (w_1 - w)} \right)^{1/2} w_1^{1/6} \qquad (5)$$

in which K' is dimensional.

As was deduced from our publication of 1938, the maximum velocity of the wave breaking over steep slopes, being theoretically equal to its velocity, the following is obtained

$$C = v' = \sqrt{g \frac{A}{2}} = \frac{\pi a}{T} \qquad (6)^*$$

From the expressions (4) (5) and (6), in connection with our definitions $W = P$; $w_1 = d$; $w = d_1$ and the angle of the natural slope being approximately $\alpha_0 = \frac{\pi}{4}$ the following expression is readily obtained

$$P = K'' \frac{A^3}{(\cos\alpha - \sin\alpha)^3} \cdot d \left(\frac{d_1}{d - d_1} \right)^3 \qquad (7)$$

which is definitely our formula; since this expression is obtained likewise readily and without introducing previous simplifications, for any one of the processes followed in our report of 1938, in which K" is likewise dimensional.

If, in order to simplify the applications, we define the specific weights in tons per cubic meter, with which they are reduced to the relative densities, and that of the water, salt water included, being practically $d_1 = 1$ the formula is reduced to

$$P = \frac{NA^3 d}{(\cos\alpha - \sin\alpha)^3 (d-1)^3}$$

which is our plain, convenient practical formula once more confirmed in an interesting way; whose degree of approximation is more than sufficient in such complex problems, where only a slight variation in the height of the wave might give rise to greater variations than those resulting from this degree of approximation; which might also be increased by simply refining the coefficient in the manner already demonstrated in the first report of 1938.

*Actually this result does not apply to calculations in a contrary sense for the generalization of the formula for submerged slopes.