



THERMAL PERFORMANCE OF CLOSED LOOP PULSATING HEAT PIPE USING PURE AND BINARY WORKING FLUIDS

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ABSTRACT

This paper presents preliminary experimental results on thermal performance of closed loop pulsating heat pipe (CLPHP). The copper capillary tube was used having internal and external diameter 2.0 mm and 3.6 mm respectively. For all experimentation, filling ratio (FR) was 50 %, number of turns was 10 and different heat inputs of 10 to 100W were supplied to PHP. For all PHPs, Vertical bottom “heat mode” (+90°) position is maintained. The equal lengths of evaporator, adiabatic and condenser sections were 50 mm each. Working fluids are selected as Methanol, ethanol, acetone, water and different binary mixtures. The graphs are plotted, in order to study, characteristics of the thermal resistance and average evaporator temperatures at different heat input for various working fluids. Experimental study on PHP indicated that working fluid is an important factor for the performance of PHPs. The result shows that, the thermal resistance decreases more rapidly with the increase of the heating power from 20 to 60 W, whereas slowly decreases at input power above 60 W. Pure acetone gives best thermal performance in comparison with the other working fluids. No measurable difference has been recorded between the PHPs running with pure and binary mixture working fluids.

Keywords: Pulsating heat pipe, binary mixture, heat flux, natural convection

1. INTRODUCTION

Pulsating or Looped type Heat Pipes proposed and patented by Akachi (1990). This is the new member of wickless heat pipes. Their operation is based on the principle of oscillation for the working fluid and a phase change phenomena in a capillary tube. The diameter of the tube must be small enough such that liquid and vapor plugs exist. Due to its excellent features, such as high thermal performance, rapid response to high heat load, simple design and low cost, PHP has been considered as one of the promising technologies for electronic cooling, heat exchanger, cell cryopreservation, the spacecraft thermal control system, etc, Dadong and Cui (2010).

Various mathematical models have also been developed in recent years to predict the oscillating motion and heat transfer performance of the PHPs. Qu and Ma (2007) presented a mathematical model to describe the startup of a PHP. They found that the inner wall surface condition, evaporation in the hot section, superheat, bubble growth, and the amount of vapor bubble trapped in cavities affected the startup of a PHP. Shafii *et al.* (2001; 2002) concluded that the majority of the heat transfer (95%) is due to sensible heat, not due to the latent heat of vaporization. Latent heat serves only to drive the oscillating flow. They also demonstrated that the gravity force has an insignificant effect on the PHP performance.

Although extensive studies have been carried out, certain key aspects of the PHP remain poorly understood, and some analytical results obtained by different investigators are even contradictory to the experimental data. A number of researchers have conducted experimental investigations on PHPs, and the results indicated that the

heat transfer capability of PHPs mainly depends on the working fluids, evaporation/condensation lengths, inner diameters, number of turns, etc. Khandekar *et al.* (2003). Charoensawan *et al.* (2003) indicated that in vertical orientation for the 2.0 mm devices, water filled devices showed higher performance as compared to R-123 and ethanol. Whereas in 1.0 mm devices, R-123 and ethanol filled devices showed comparable performance but water showing very poor results. Khandekar *et al.* (2003) demonstrated the effect of input heat flux of the working fluid on the thermal performance of the device. Although the Eötvös number of water and ethanol was much below the prescribed maximum limit of $Eö = 4$, gravity forces were definitely seen to affect the performance. There is a smooth decrease of the thermal resistance with increasing heat power input.

Kammuang-lue *et al.* (2008) studied that, the higher latent heat of the working fluid, the higher critical heat flux. Yang *et al.* (2009) pointed out that increasing heat load clearly improves the thermal performance. Meena *et al.* (2009) concluded that as working fluids change from R123 to Ethanol and water the critical heat flux decreased. The latent heat of vaporization affects the critical heat flux. The working fluid with the lower latent heat of vaporization exhibits a higher critical heat flux. Dadong and Cui (2010) indicated that the thermal resistance decreases with the increase of the heating power at the same filling ratio. The thermal resistance decreases more slowly for the power inputs larger than 60W. For the pure working fluid PHPs, the thermal resistance is decreases in the sequence of water, ethanol, methanol and acetone. Mameli *et al.* (2011) conducted experiments on the thermal performance of a PHP working with an azeotropic mixture of water (4.5% wt.) and ethanol (95.5% wt.), in comparison to pure ethanol. No measurable difference has been recorded between the PHP

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running with the azeotropic mixture and the PHP running with pure ethanol, in terms of overall thermal resistance.

At different situations, different pure working fluids have their advantages Pachghare *et al.* (2012). But till now, mixtures used as working fluids in PHP have not been thoroughly investigated. The non-azeotropic mixtures, which have the characteristics of phase transition with temperature floating, can make heat source and working fluids match well in temperature CUI Xiaoyu *et al.* (2006). Binary fluid is quite a new developing topic in recent years. Therefore, the working fluids are used as pure and binary mixture (1:1 by volume) for the PHPs. Experimental investigation has been done to find the thermal performance of CLPHP by using different pure and binary mixture of working fluid. The various temperatures are recorded for analyzing the thermal resistance of the PHP.

2. EXPERIMENTATION

2.1. Experimental set-up

The experimental setup of CLPHP was developed and tested in the laboratory. The photograph of PHP set-up is shown in Fig. 1. All the experimentations are carried out at controlled conditions. The details of the experimental setup are shown in Fig. 2. The setup consists of a closed loop PHP, temperature recorder, power supply unit, and water tank cooling system for condenser. Both the evaporation and adiabatic sections were well thermally insulated by the proper insulation materials. The PHP, consisting of 10 turns, is made of copper capillary tube having inner diameter is 2.0 mm; the outer diameter is 3.6 mm. The pitch distance between tubes was maintained 15 mm. The PHP consists of evaporation, adiabatic and condensation sections with the height of 50 mm for each section. The heating power is provided by a carefully designed power supply unit. Heating was done by oil bath and cooling by water tank. The power meter measures the AC voltage, the current and the corresponding power simultaneously. The Filling Ratio was maintained at 50%. The heating configuration was bottom heat orientation (+90°). Ten K-type thermocouples were attached to the wall of the PHP, positions are shown in Fig. (2). Flow meter was also recorded the mass flow rate of the cooling water.

About the tube design, the most important condition of PHP is the creation of the liquid slug. According to the literature review, Akachi [15] proposed that the appearance and movement of bubbles are affected by surface tension and buoyancy in the channel. Relation of surface tension and buoyancy could be explained by the dimensionless formula shown in Eq. (1):

$$E\ddot{o} = \frac{g \cdot D^2 \cdot \rho_{liq} - \rho_{vap}}{\sigma} \quad (1)$$

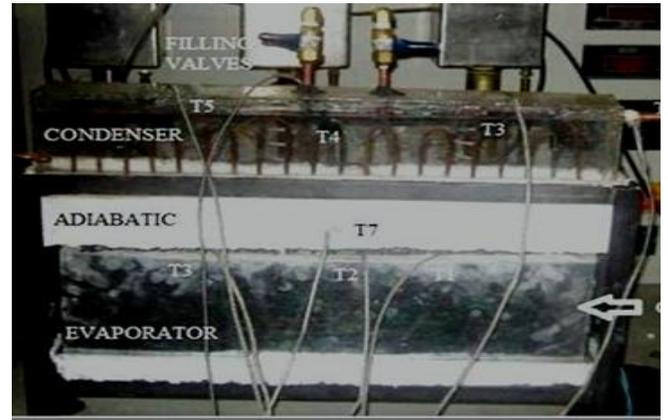


Fig. 1. Experimental setup photograph

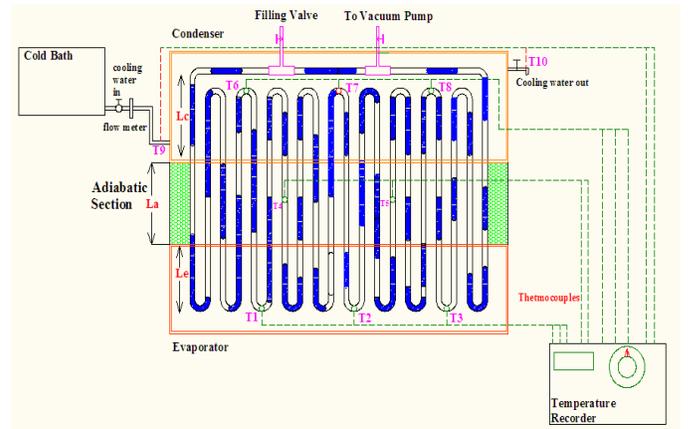


Fig. 2. Experimental setup Details.

When $E\ddot{o} \approx 4$, the bubble will get seized on both side of the wall. At this condition, the terminal velocity becomes zero and the liquid slug flow is formed. The dimension formula for the PHP diameter 'D', is given in Eq. (2).

$$D \leq 2 \cdot \sqrt{\frac{\sigma}{g \cdot \rho_{liq} - \rho_{vap}}} \quad (2)$$

PHP loop is divided into evaporator, adiabatic and condensation composing of three sections. The intact structure of PHP loop and the experimental apparatus structure are in Fig. 2.

Table 1: The thermo-physical properties of pure and binary[#] working fluid at 1 atm.

Working Fluid	T_b ($^{\circ}C$)	h_{fg} (kJ/kg)	ρ (kg/m ³)		μ (10 ⁻⁷ Ns/m ²)		k (W/m-K)		σ (10 ⁻³ N/m)	C_p (kJ/kg-K)	
			liq	vap	liq	vap	liq	vap		liq	vap
Methanol	64.7	1119.59	750.8	0.566	3291.4	109.64	0.201	0.0018	18.87	2.52	1.601
Ethanol	78.3	962.45	758.1	1.372	4452.6	102.39	0.169	0.0197	17.46	0.73	1.604
Acetone	56.2	520.56	748.5	2.123	2340.6	89.25	0.169	0.0140	19.09	2.28	1.385
Water	100.0	2251.20	958.7	0.597	2790.0	121.00	0.680	0.0248	58.91	4.22	2.034
Water-Acetone	78.1	1385.88	853.6	1.360	2565.3	105.13	0.425	0.0194	39.00	3.25	1.709
Water-Ethanol	89.2	1606.83	858.4	0.985	3621.3	111.70	0.425	0.0223	38.19	2.47	1.819
Water-Methanol	82.4	1685.40	854.8	0.582	3040.7	115.32	0.441	0.0133	38.89	3.37	1.817

[#] Calculated by: Algebraic mean value method.

2.2. Experimental procedure

Put K-type thermocouples fixed in suitable position, i.e. three thermocouples in the evaporator and three thermocouples in the condenser, two thermocouple in the adiabatic section respectively, which are connected with the temperature recorder from T_1 to T_8 as shown in Fig. 2. The heat is supplied by means of oil bath by using dielectric oil for uniform heating. Two heater of 500W each was connected in series to the power supply. It was cover with foam insulation spray type in the adiabatic section, which can prevent heat from being lost and influence in the experimental result. The vacuum is created by means of vacuum pump up to 70cm of Hg. Then, PHP is filled with the necessary working fluid. Turn on the temperature recorder and power supply, adjust the heating power initially to 10 W, and then begin to observe and note down the temperature at each point through the recorder. After the temperature curve is balanced, adjust the heating power to 20 W (plus 10 W). Again, after the temperature curve is balanced and the heating power is adjusted to 30 W. Repeat this step until 100 W. Fill in with the different working fluids at 50% filled ratio. Steps 3–6 are repeated.

3. RESULT AND DISCUSSION

The PHP performance data (for pure and binary mixture of working fluid) were obtained according to the following procedure: Heat input was stepwise increased until a quasi thermal equilibrium was established. Then, the spatial temperatures and heat input were recorded, so the thermal resistances could be determined. The thermal resistance is defined by Eq. (3).

$$R_{th} = \frac{\bar{T}_e - \bar{T}_c}{\dot{Q}} \quad (3)$$

where, \bar{T}_e and \bar{T}_c are the average evaporator and condenser surface temperatures, which is the average of three temperatures, calculated by using Eq. (4).

$$\bar{T}_i = \frac{\bar{T}_{i,1} + \bar{T}_{i,2} + \bar{T}_{i,3}}{3} \quad (4)$$

where, $i = e$ or c

\dot{Q} is heat input power to the PHP. Considering thermal losses \dot{Q} can be determined by Eq. (5) as:

$$\dot{Q} = P - \dot{Q}_{loss} \quad (5)$$

where, P is the input electrical power (measured with accuracy of $\pm 1.5\%$). \dot{Q}_{loss} is the heat loss, which was verified to be about 4% to 9%, depending on the heat load. In order to keep a good operation of working fluid in PHP, vertical bottom heat mode condition, 50% filling ratio was chosen for comparison with different working fluid as in the following test.

3.1 Effect of pure and binary mixture working fluid PHPs on the thermal resistance

From Fig. 3 and 4, it is clear that, for pure and binary working fluids of PHPs, the thermal resistances are smoothly decreases with the increasing heat input power. With increasing heat input to the device, the evaporator temperature rises resulting in a greater density gradient in the tubes. Simultaneously the liquid viscosity also drops diminishing the wall friction. Initially at low heat input upto 20W, PHP is not sufficient to sustain a stable behavior. During the starting period fluid motion is chaotic, the circulation is not initiated. The respective adjacent tubes of the PHP become and remain alternatively hot and cold

thereafter. The direction of circulation for the liquid is arbitrary; remaining fixed for a given experiment but may be changes with different experimental runs. The PHP is best suitable for all working fluid at the range of 30 to 80 W. The dry-out conditions of the water, ethanol, methanol and acetone PHPs are at 100W, 80W, 60W and 70W heat input respectively. The thermal resistances are according to the boiling temperature of the respective working fluids, shown in Table 1. The boiling point and the latent heat of vaporization of water are more as compared with other working fluid used for PHPs. Thus, water PHP can boil hardly in low power inputs.

The trends of thermal resistance over heat input for all binary mixture of PHPs are shown in Fig. 4. The PHP get dry-out for the water-methanol, water-acetone and water-ethanol is around at 85W, 80W and 90W heat input respectively, which is approximate the algebraic mean values of the boiling point of binary mixture. The temperatures are recorded for all binary mixture of working fluid PHPs. There is no quantifiable difference. It is also clear that, behavior of the thermal resistance is strongly depending on thermo-physical properties of the working fluid of PHPs. Although there are many properties of working fluid, the latent heat of vaporization is the main property that strongly affects the motion of liquid slug and vapor bubble in the tube and the heat transfer of the PHP. Therefore, the latent heat of vaporization of the working fluid will be concentrated.

In Fig. 4, water-methanol binary fluid PHP shows the better thermal performance as compared to other binary fluids up to 80W heat input. The variations of thermal resistance is according to latent heat of vaporization, see Table 1. Latent heat of vaporization for water-methanol binary fluid is high, but dynamic viscosity for water-ethanol binary fluid is more. Therefore the heat carries by water-ethanol binary fluid is low, so the thermal performance of water-ethanol is less as compared to water-acetone. Thereafter 80 W heat inputs, the behavior of all PHPs are according to boiling temperature of the binary fluids.

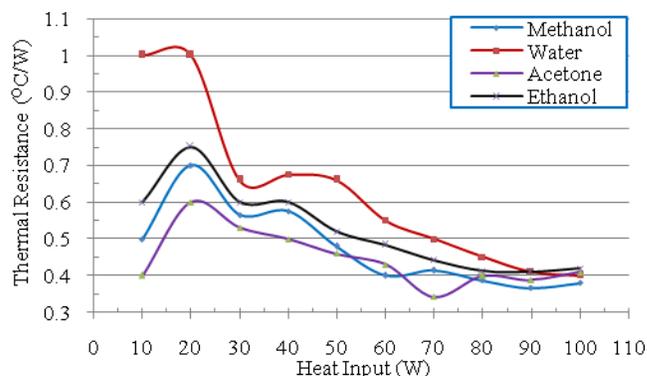


Fig. 3. Thermal resistance of pure working fluid PHP

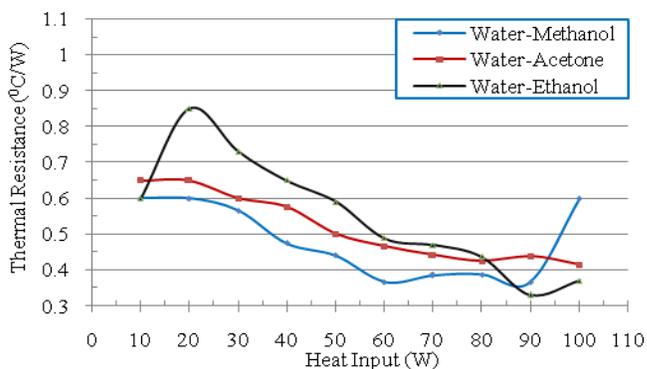


Fig. 4. Thermal resistance of binary working fluid PHP.

During starting up to 30W heat input, the thermal resistance of water PHP is more because of the more surface tension and dynamic viscosity. Therefore, the evaporation section temperature of water PHP is high and the condensation section temperature of water PHP is low, and accordingly, the thermal resistances are large. The behavior of the thermal resistance is according to the algebraic mean values of the latent heat of vaporization.

3.2 Effect of pure and binary mixture working fluids PHP on an average evaporator temperature

The behavior of the average evaporator temperature along with the heat power input in the PHP for all pure working fluids is shown in Fig. 5. In high power inputs, the temperature of the evaporation section is high enough to keep the working fluids of high boiling points can boil vehemently and smoothly flow in one direction. Therefore, all the thermal resistances reduce smoothly and the differences between the working fluids are smaller with the increasing of heating power inputs. The evaporation section temperature of methanol is low for the high $(dp/dT)_{sat}$ and the specific heat. Thus, the evaporation section temperature of methanol is low in high power inputs.

Also Fig. 6, represents the behavior of the average evaporator temperature along with the heat power input of the binary mixture working fluid in the PHP. The trends of all the binary mixture are almost same. During starting up to 20W, all the binary mixture PHPs are having same average evaporator temperature because no circulation of the fluids at low heat input. In 20 to 80 W heat input, the average evaporator temperature of all binary mixture PHPs are according to the average boiling temperature of its constituents. Thereafter 80 to 100W, mixed trend is found because of the dry-out condition at the various heat input. The trends of average evaporator temperature for Water-ethanol and water-acetone PHPs are certainly mixed. Thus, the evaporation section temperature of water-methanol is low in high power inputs.

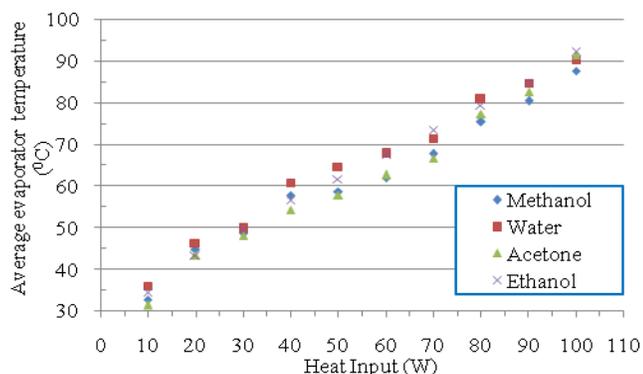


Fig. 5. Average evaporator temperature of pure working fluid PHP

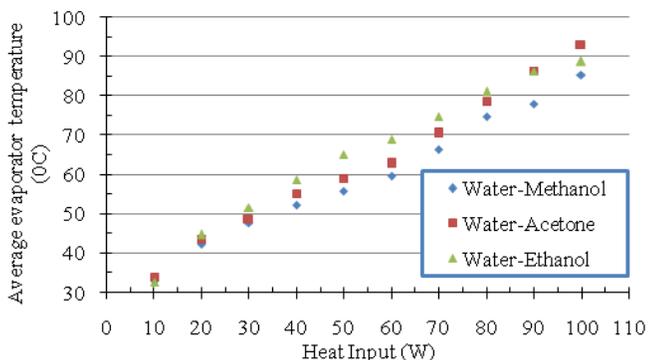


Fig. 6. Average evaporator temperature of binary working fluid PHP.

3.3 Effect of pure and its binary mixture working fluids PHP

In Fig. 7, water-methanol PHP gives the better thermal performance than pure working fluids of same constituent. It is also clear that during starting at 10 to 30W, the thermal resistance of the pure working fluids is varying drastically. But in 30 to 80W heat input, all working fluids behaves same, in 80 to 100W heating zone, water-methanol binary mixture PHP is get dry-out. Whereas in Fig. 8, water- acetone binary mixture working fluid pure acetone gives the better thermal performance than pure water and binary mixture working fluids of same constituent. Initially, thermal resistances of pure working fluid PHPs are fluctuating due to no circulation of liquid. In the high heating zone 80 to 100W, all pure and binary working fluids are stable. But in Fig. 9, water-ethanol binary mixture PHP, pure ethanol gives the better thermal performance than pure water and binary mixture of same constituent in low and moderate heat zone from 10 to 80W. As the binary fluid is in the high heat zone the PHPs gets dry-out and situation is reverse.

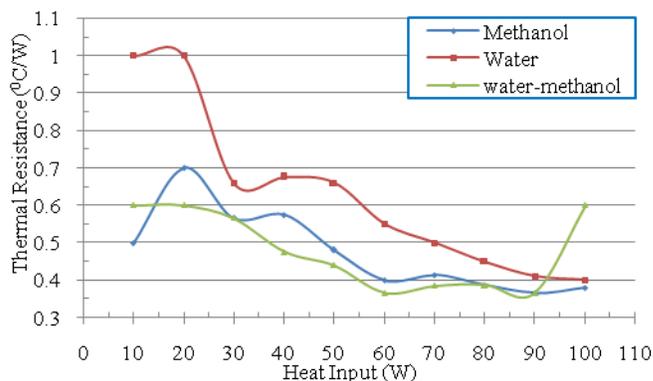


Fig. 7. Thermal resistance of water-methanol PHP

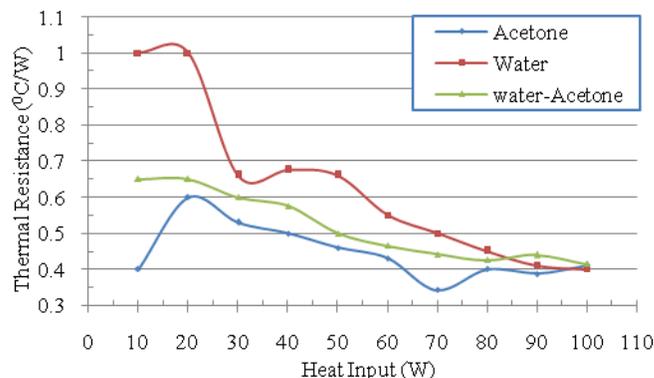


Fig. 8. Thermal resistance of water-acetone PHP

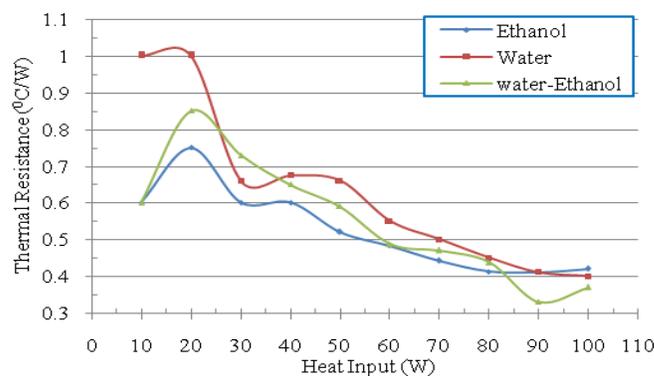


Fig. 9. Thermal resistance of water-ethanol PHP

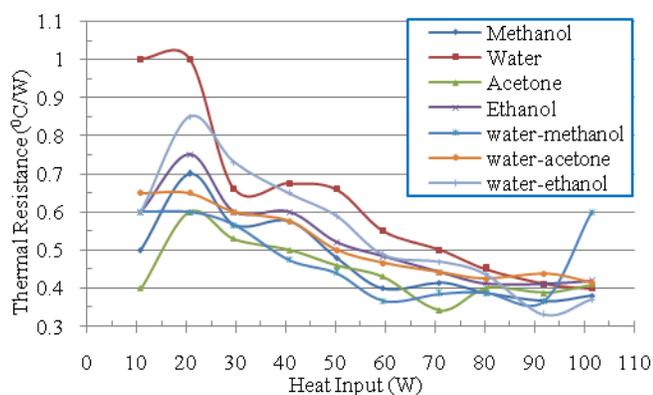


Fig. 10: Thermal resistance of all working fluid PHPs.

From Fig. 10, it is clear that trends of thermal resistance for all working fluids are approximately same in nature, as the difference in thermophysical values is minimal. In all the working fluid PHPs, pure water has more thermal resistance whereas pure acetone has lesser thermal resistance. In all PHPs, pure acetone gives best thermal performance in comparisons with the other pure and binary mixtures working fluid.

4. CONCLUSIONS

A closed loop pulsating heat pipes has been experimentally investigated to study the effects of pure and binary mixture working fluid on the thermal performance. The various working fluids (Methanol, ethanol, acetone, water and different binary mixtures) have been scrutinized. Different fluids are beneficial under different operating conditions. The binary fluid PHP shows more thermal performance as compared to the pure working fluids of same constituents. So, binary fluids are more beneficial to use in PHP for low heat input applications. The following main conclusions can be drawn from the study:

1. For pure and binary working fluids of PHP, thermal resistance is decreases with the increasing heat input. The dry-out for the water-methanol, water-acetone and water-ethanol is at 85W, 80W and 90W heat input respectively, which is approximate the algebraic mean values of the boiling point of binary mixture.
2. The evaporator temperature of methanol is low in high power inputs. A PHP of water-methanol binary mixture gives good thermal performance over other working fluids.
3. In all working fluids, pure water is having more thermal resistance whereas pure acetone is having lesser thermal resistance. So in this set-up, pure acetone gives best thermal performance in comparisons with the other pure and binary mixtures working fluid.
4. No measurable difference has been recorded between the PHP running with pure and binary mixture working fluids, in terms of overall thermal resistance. Working fluid Behavior is strongly depends on the thermo-physical properties, but latent heat of vaporization is the main property that strongly affects the thermal performance.

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NOMENCLATURE

C_p	: Specific heat	(kJ/kg·K)
D	: Diameter	(m)
$Eö$: Eötvös number = $(Bo)^2$	(----)
g	: Gravitational acceleration	(N/m ²)

P	: Electrical input power	(W)
\dot{Q}	: Heat input	(W)
R	: Resistance	(°C/W)
h	: Latent heat of phase change	(kJ/kg)
k	: Thermal conductivity	(W/m·K)
q''	: Heat flux	(W/m ²)
T	: Temperature	(°C)
\bar{T}	: Average temperature	(°C)
u	: Interfacial velocity	(m/s)

Greek Symbols

σ	: Surface tension	(N/m)
ρ	: Density	(kg/m ³)
μ	: Dynamic viscosity	(Ns/m ²)
ν	: Kinematic viscosity	(Pa.s)

Subscripts

a	: adiabatic section
b	: boiling
c	: condenser section
e	: evaporator section
fg	: fluid to gas
liq	: liquid
sat	: saturation
th	: thermal
vap	: vapor

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