Journal of Humanistic Mathematics

Teaching the Quandary of Statistical Jurisprudence: A Review-Essay on Math on Trial by Schneps and Colmez

Noah Giansiracusa
University of Georgia

Follow this and additional works at: http://scholarship.claremont.edu/jhm

Part of the Applied Statistics Commons, Evidence Commons, Jurisprudence Commons, Mathematics Commons, and the Probability Commons

Recommended Citation

Teaching the Quandary of Statistical Jurisprudence: A Review-Essay on *Math on Trial* by Schneps and Colmez

Noah Giansiracusa

Department of Mathematics, University of Georgia, Athens, Georgia, USA
noahgian@uga.edu

Synopsis

This review-essay on the mother-and-daughter collaboration *Math on Trial* stems from my recent experience using this book as the basis for a college freshman seminar on the interactions between math and law. I discuss the strengths and weaknesses of this book as an accessible introduction to this enigmatic yet deeply important topic. For those considering teaching from this text (a highly recommended endeavor) I offer some curricular suggestions.

> **Math on Trial: How Numbers Get Used and Abused in the Courtroom.** By Leila Schneps and Coralie Colmez, Basic Books, NY, 2013. (US$26.99, ISBN-10: 0465032923. 272 pages.)

1. Introduction

The phrase "guilty beyond a reasonable doubt" has developed into a pillar of many judicial systems worldwide (see [16] for an engaging history in the American context). Despite the ubiquity of these words and the cultural significance enshrining them, there is an inherent paradoxical vacuity here: *reasonable doubt* has only the meaning that we subjectively, and mercurially, impart to it. One might argue that it is indeed the inexorable flexibility of these words that renders them so powerful. Put another way, it is the lack of precise meaning that makes the concept of reasonable doubt so meaningful.

Although we cannot, and should not, confine reasonable doubt to any rigid axiomatic system, we may nonetheless assert that the expression "guilty beyond a reasonable doubt" is plausibly tantamount to an estimation that the probability of innocence, although impossible to eliminate entirely, is quite small. The devil remains in the details, as this probability is generally ill-defined, to put it mildly, and even a crude attempt at quantification leads one to wonder how small a probability of innocence suffices to convict. Thus, a statistical interpretation does not eliminate the ambiguity central to the phrase, but it does move the conversation into a new pseudo-mathematical realm.

This perspective, that the central question facing a criminal courtroom is in essence a probabilistic one, naturally leads one to wonder whether statistics could be used to uncover truth in the legal setting as it has throughout the 20th century in the scientific realm [12]. As it turns out, experimentation with this idea, and debate regarding its efficacy and ethical implications, has been playing out in our courtrooms since the late 19th century as well as in academic legal journals, most prominently in a series of *Harvard Law Review* articles from the early 1970s [6, 7, 14, 15]. Far from an obscure intellectual pursuit, the relevance of this discourse has grown rapidly over the years and is now nearly unavoidable; for instance, any legal consideration based on biological forensics, such as DNA identification, ultimately relies on subtle statistical underpinnings [1].

The book under review, *Math on Trial: How Numbers Get Used and Abused in the Courtroom* by Schneps and Colmez, provides a gentle introduction to this engrossing topic by presenting a handful of legal cases, historic and recent and some quite well-known for non-mathematical reasons, where mathematics, usually in the form of probabilistic estimates in lieu of more traditional evidence, played a prominent role. The authors do not directly broach the difficult academic debate on mathematics' broader applicability in the courtroom, but through examples and innovative elementary lessons they illustrate many pitfalls to be aware of when applying statistical reasoning to jurisprudence—and strikingly, some wrenching travesties that have befallen an unlucky few due to an overzealous use of misunderstood or misleading mathematics. In short, the book is a series of captivating criminal cases paired with accessibly presented and well-focused math lessons. Altogether, the authors proffer a compelling argument that we, as a society, can reduce the incidence of judicial error and abuse related to courtroom statis-

tics by striving to increase our collective mathematical literacy—and they deftly take an admirable step in this direction with this publication.

There are a few weaknesses of the book, however. At times the authors fall into the very mathematical traps they warn the reader about, and the legal considerations are consistently treated more superficially than the mathematical ones. In addition, there are occasions where an easily discernible bias creeps in to the narrative, most prominently in the chapter on the controversial Amanda Knox case. Nonetheless, the moral of *Math on Trial* is that we should be an informed and skeptical public, especially when it comes to quantitative courtroom reasoning, and with this in mind an astute reader can even take pleasure in locating and privately rebutting the book's minor flaws. Moreover, it is not difficult to find and consult earlier literature on the cases and mathematical issues discussed in this book in order to see other perspectives on the matter.

The very fact that a book which sets out to resolve the ambiguity surrounding murkily applied mathematics is unable to do so consistently shows how quixotic the entire venture is. Even experts will continue to disagree on the appropriateness and accuracy of mathematics as applied to the limitlessly confounding situations facing a courtroom, so in the end awareness of the general challenges is as important, if not more so, as correctness in any particular situation. In this way the book provides a wonderful pedagogical platform: students can learn much from what is presented accurately in this book, and they will learn just as much by debating and debunking what they find dubious. In short, the fundamental vagueness surrounding the phrase "guilty beyond a reasonable doubt" pervades the entire discipline of jurisprudence based on statistical reasoning, and it is therefore not surprising that any particular attempt to set the record straight mathematically, as the present book strives to do, will ultimately fall short but will nonetheless leave the reader more informed and aware of the extraordinarily important, and endlessly fascinating, issues involved.

2. What's in the book

The structure of the book is appealingly straightforward: aside from a very brief introduction and conclusion, it consists of ten chapters, each describing a particular legal case (with dates ranging from the late 19$^{\text{th}}$ century up until just a few years ago) and each beginning with a short but relevant

and easily digestible math lesson. The majority of these cases involve a statistical argument, such as a probabilistic estimate that the defendant matches some forensic evidence or an eye-witness description, and typically this statistical argument is either improperly constructed or outright baseless. The accompanying math lesson is then a simplified illustration in a hypothetical situation to clarify the problematic reasoning. In many but not all such cases, the legal decision reached using such faulty reasoning was later appealed and ultimately rejected by a higher court, often after unbelievable and unbearable hardship on the unfairly accused. The book excellently engenders sympathy for those falsely convicted and takes a laudable position of advocacy that we must increase mathematical literacy in order to prevent the reoccurrence of such judicial travesties.

The cases are, to an extent, ordered so that the corresponding mathematical issues build in complexity; sometimes the order also reflects a layering of mathematical issues. That being said, the earlier cases can in some ways be the most challenging to fully digest and reconcile. This is because when the math itself is most bare, the controversy then arises from its interpretation, and therein lie murky waters. As an example, Chapter 2 presents a case (now famous in the literature on mathematical jurisprudence, e.g., it features prominently in those aforementioned *Harvard Law Review* articles) where an eye-witness testified to a collection of physical traits that the defendant mostly matched and, although each trait was not uncommon in isolation, the combination of them all in tandem was deemed exceedingly rare. The concrete math errors highlighted here are twofold: (1) these events, such as being a beard-wearing African-American and wearing a mustache, were treated as independent when by intuition or any statistical measure they quite manifestly are not, and (2) the "estimates" given in court for the probabilities of the individual events were completely unjustified (if not altogether racist). The cleverly simple example the authors use to illustrate the first math error, which is the topic of Chapter 1 though in a far different guise, is the following: the probability that two children born are both male is $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, but if they are identical twins then their genders are completely dependent and the probability of two males rises to $\frac{1}{2}$.

This false assumption of independence results in a vast overestimate of the rarity of the joint occurrence of the physical traits which was used to argue the near certainty that the defendant was indeed the person described by the witness; this, in turn, formed the basis for a conviction. Although this false

assumption of independence and the fairly arbitrary individual probabilities asserted are central to the misuse of mathematics in this case and are worthy of careful explanation and illustration, as the authors dutifully provide, the subtler issue is how to interpret the probability of joint occurrence thus obtained. Indeed, one could imagine replacing the individual probabilities with conservative estimates, to compensate for their apparent arbitrariness, and adjusting the model to incorporate a generous degree of mutual dependence in order to obtain a probability that is perhaps still small enough to convince the jury of its rarity while at the same time less patently biased— but then how does one proceed in the judicial process with this information? Even if one accepts a statistical argument that the particular constellation of characteristics described by the witness seldom occurs even in a very large population, and that the defendant matches this description (which, as the court records show, wasn't entirely the case), the probative force of such an assertion is extremely difficult to assess.

The authors content themselves with a somewhat brief discussion of this latter point at the conclusion of their chapter, drawn largely from Tribe's article [14] where a more in-depth analysis is provided. Thus, by emphasizing the concrete issue of statistical dependence and unjustified estimates, they relegate this more complex interpretative issue to a somewhat tangential role, even though it may arguably be the deeper and more important one. In fairness, the book would likely be less readable and lose narrative flow if each case were given a detailed analysis covering the myriad complexities associated with statistical jurisprudence, rather than isolating manageable issues and building upon them through the chapters. So my comments here are less a criticism of the organization of the book than a warning to the reader that, although the math itself may seem more straightforward in the opening chapters, the interpretations and judicial applications are anything but straightforward. In general, the fundamental challenges in statistical jurisprudence come from these more open-ended issues of construal—they pose, indubitably, the problems that do not materially vanish when errors are avoided—and the cases in the early chapters are rife with these.

At the opposite extreme, the final chapter of the book focuses on the well-known Dreyfus affair. Although this case, dealing with anti-Semitism and delicate turn-of-the-century geopolitical issues, has been discussed at length in both academic and public spheres due to its historical significance [2, 11], *Math on Trial* brings attention to an important but sometimes neglected

component of the story: the reliance of the prosecution on "mathematical" arguments. Here the math, as it were, is an artifice fabricated by desperate accusers to be so impenetrably elaborate that it must be taken on faith; there is no genuine challenge of interpretation as in the early chapters, nor any simple and avoidable conceptual or computational errors, there is only a senseless witch-hunt with an outlandish cryptological argument buttressed by mathematical maneuvering to intimidate the audience and obfuscate the lack of real evidence in the case.

The mathematical aspects of this case were not a novel discovery (see [9]), but the wise inclusion of this chapter in the book accomplishes multiple worthwhile goals regardless. This chapter brings further attention to this bizarre yet important aspect of the case, it adds diversity to the types of cases presented in the book, and it lends further support to the overarching arguments of the book. Indeed, by emphasizing the mathematical aspects of this saga the authors provide an excellent lesson that math, while ultimately predicated on the search for universal truth, can in the wrong hands be a tool for precisely the opposite, namely deception and dishonesty, and thereby unscrupulous coercion. Once again the unremitting message of *Math on Trial* comes to the fore, and rightly so: the best way to disarm math as a villainous device is to better educate the citizenry in quantitative reasoning and thereby render us impervious to mathematical intimidation.

Although the math in the Dreyfus affair has a distinct flavor from that of the previous chapters, it is nonetheless an attempt at prosecutorial arguments based on statistical considerations and evidence. The chapter in the book that is the most dissimilar from the others, in that it involves no statistics whatsoever, is Chapter 8, which discusses another famous historical incident, though not specifically a legal trial, per se. Here the story is that of Charles Ponzi and his now legendary and eponymous scheme. Plenty of amusing details on Ponzi's biography and personality complement the highlighted mathematical issue, the untrammeled speed of exponential growth. The math here is quite elementary and, while nicely illustrated in a simplified model, entirely straightforward. Since this is the case, and there are really no legal issues at play, or at least none discussed in this book, the main contributions of this chapter, I feel, are a break from the pattern of improper statistical jurisprudence cases leading up to this chapter, and a well-placed levity. Many of the cases in the book are evocatively recounted and so deeply disturbing and tragic, both in terms of the crimes/accidents that occurred

and the miscarriages of justice (some of which, at least in retrospect, seem so easily avoided), that it is hard not to develop a heavy heart while reading, even if one's goal is simply to absorb the legal and mathematical issues arising. With respect to the singular Ponzi, on the other hand, at the risk of sounding callous I would say that one cannot help but feel amused and entertained by this financial debacle, especially as it is colorfully told here, despite the desperate economic times surrounding, and in part enabling, it.

The cases in the book are very well chosen to keep the reader informed and captivated. There is not a dull moment; in every chapter I found the legal situations fascinating, or the mathematical ones, and most often both in parallel. My personal favorite mathematical issue highlighted in the book comes in Chapter 5. Here the famous "birthday paradox" in basic probability theory—which is, roughly, the observation that although it is fairly uncommon to meet someone with your birthday, it is surprisingly common to find two individuals in a small group of people with the same birthday—plays out in fantastic clarity in the legal setting. A defense team, while trying to rebut forensic evidence against their client, presented literature showing that the FBI's established probability of random DNA matches could not possibly be correct because in even a small collection of samples they found a handful of matches. As Schneps and Colmez accurately explain, the confusion here is precisely the birthday paradox: the probability of finding a match between any two DNA samples in a database is orders of magnitude greater than the probability of finding a match with a fixed sample (that of the defendant, for instance), because if there are n samples then the former involves testing $\binom{n}{2} = \frac{n(n-1)}{2}$ pairs of samples. In the case at hand, it was the latter that was relevant, so this chapter is one of the rare instances where it was the defense, rather than the prosecution, that misunderstood and misapplied mathematics.

Finally, while attempting not to give away too much content—many of the cases really are page-turners!—I cannot resist mentioning that my overall favorite case in the book is that of Chapter 9. Here a large inheritance was at stake in 19$^\text{th}$ century New England, the will prescribing this inheritance was disputed, and the claimant was a remarkable woman of unbridled eccentricity—and ultimately, the richest woman in America at the time. The legal case came down almost entirely to determining the authenticity of a pair of quill pen signatures on the will (makes one nostalgic for the simplicity of such bygone eras, no?). The story itself is truly Dickensian, except for a

rather uncommon plot twist: a famous mathematician, one whose name still reverberates in the walls of the Harvard math department, was called in to testify and argue the case against the claimant.

Benjamin Peirce, working as an expert witness, developed a binomial model—at a time when statistical modeling was still in its infancy—to estimate the likelihood of signature forgery. The idea was that forged signatures would display an uncanny degree of similarity, and indeed the pair on the disputed will appeared nearly identical, so by measuring the similarity between a collection of signatures found on various documents in the late matriarch's estate, Peirce would develop a model for predicting the relative frequency of a pair of signatures matching to any specified degree. Of the 42 signatures he tested none came close to matching as flawlessly as the disputed ones did, but the data provided by these $\binom{42}{2} = 861$ sample pairs allowed him to fit the parameters of his binomial model (tedious manual labor, no doubt) which was then used to extrapolate the likelihood of extreme events. He concluded that the probability of a near perfect match as exhibited on the disputed will was astronomically small—the Victorian language used to describe such a minuscule number was far more elaborate and visceral than anything I could possibly attempt here—and could therefore be considered so unlikely a coincidence that it must be the result of duplicity. Ingenious.

Many fascinating and subtle details, well told in the book, arise concerning this model. For instance, Peirce's model empirically seemed a reasonable fit for a range of parameters but was noticeably deficient in estimating the tails of the distribution—and the conclusion he asserted rests entirely upon the fit at the tails. Moreover, the 42 sample signatures were produced over many years, with different pens and presumably in different writing positions, whereas the two will signatures were, whether legit or not, written one immediately after the other with the same pen, so again the binomial model would drastically overestimate the rarity of the event in question. Thus, as throughout the book, this use of courtroom mathematics was overzealous: very little should have been gleaned from Peirce's impressive albeit fatally flawed reasoning. Nonetheless, Peirce was an imposing intellectual figure and his model was assumed accurate based on the aura of accuracy and intimidation surrounding mathematics: the signature was deemed a forgery and the inheritance withheld. The legal deliberations of this case are not amply explored in this chapter (one wishes the authors conducted a bit more research in this direction), but presumably the court decision was indeed based

heavily on this "expert" mathematical testimony, for little else remained.

In addition to Peirce's altogether fascinating mathematical efforts—which, although ultimately short of their goal, are more thoughtful and creative than many of the shallow prosecutorial endeavors depicted in other chapters—and the phenomenal literary intrigue permeating this story, this inheritance case is particularly tantalizing for two reasons. As opposed to many of the other cases in the book, there really is little ambiguity in interpreting this probabilistic evidence. Had the statistical model been accurate, it seems fairly reasonable to base a judicial decision on the probability thus obtained; perhaps this is still an arguable point, but the interpretive nuances here are certainly less than in the earlier cases of criminal identification. Additionally, most of the cases in the book leave the reader feeling fairly confident as to the reality of what transpired (another exception is the Amanda Knox case, but more on this later). With regard to this potentially duplicitous will, however, I remain completely divided about its veracity (and the class I taught this material to was equally split) and wholly mesmerized by this peculiar case. Moreover, unique to this story I felt that we might actually be able to resolve the case correctly, after a century and a half, if we could go back to the original evidence and use more sophisticated forensic and/or mathematical methods, for instance incorporating extensive experimentally verified statistics on hand-writing reproducibility. Alas, this sentiment is likely as naive and quixotic as many of the ventures detailed in *Math on Trial*; this case will likely remain for eternity a mystery.

Much of the mathematical analysis found in this inheritance dispute chapter comes from a delightful and well-written statistical paper [10], though the Schneps and Colmez chapter rounds out the background material on this case into a more full narrative and aims the discussion at a slightly wider audience. More generally, even though many of the chapters in the book have previously been analyzed through a mathematical lens in the literature, *Math on Trial* fills an important niche by assembling these curious cases all in one place and provides contextual and mathematical background rendering them accessible to non-experts. The cumulative effect of these cases and their analysis in this book is to leave the reader feeling informed yet simultaneously mystified over the deeper implications—and both of these responses indicate, in my opinion, a very successful and worthwhile publication. Indeed, the debate over the applicability of math in the courtroom shall continue unabated, so it is prudent that the authors here have focused more on raising awareness and

resolving the concrete issues that they highlight than on attempting to settle the broader, invariably controversial and perplexing, issues surrounding statistical jurisprudence.

3. Some concerns

Having given a preview of the content of the book and a sense of its composition, I now turn to a somewhat more direct commentary. I first mention a point which, while neither positive nor negative in its own right, is important for the reader to keep in mind in order to properly contextualize the authors' arguments. This is followed by a brief mention of the one overall weakness of the book that I perceive. Finally, a list of specific, sometimes pedantic, complaints are presented. These include, unfortunately, some surprisingly elementary mathematical errors; teachers considering using this as a textbook should pay especial attention here to ensure students are not confused. It is somewhat disappointing that a professional mathematician would make such basic errors, especially in a book whose premise is that with a little thought, anyone is capable of detecting and preventing such errors. This just amplifies the point that we—whether jurors in a trial with testimony from expert witnesses, or readers of a book by expert mathematicians—should avoid the temptation to simply rely on the credentials of the experts and take the accuracy of their statements for granted; we should instead strive to take responsibility for our own understanding of the ideas presented and pro-actively think, question, and if necessary, repudiate. Since this is an attitude advocated in the book itself, we can euphemistically say that these minor mathematical mistakes in *Math on Trial* are simply opportunities to put the authors' message to work: check the math, find the errors, and don't be misled!

The general issue that I want readers to be aware of is one mentioned briefly at the conclusion of the preceding section, namely, that the book tends to focus on tangible, avoidable mathematical errors, rather than more fundamental questions of determining probative force, and practical administrative protocol, for statistical evidence. This is important to keep in mind; failure to do so can leave the reader with the false impression that by simply being smarter when applying math in the courtroom we will avoid the potential harm of statistical jurisprudence. Tribe has inveighed against this notion at length [14]; whether one agrees with him or not, one must at least admit that

this is a contentious debate that runs far deeper than mathematical literacy and the lack thereof. That being said, the limited scope of the book in this regard can, as remarked above, be viewed as a strength. Indeed, it prevents the book from sinking heedlessly into a morass of polemics. Whether or not quantitative reasoning has a place in the courtroom, it is safe to say that *bad* quantitative reasoning never does, and this book unequivocally helps gird us from a variety of instances of bad math. Thus, as long as one remains cognizant of this limited scope and does not begin to naively think problems as simple as confusing independent and dependent events are all that prevent the judicial system from embracing the quantitative revolution, one will not be led astray. In summary, one should view this book as a supplement to the existing discourse on math in the courtroom, rather than a comprehensive treatment of it.

The next general issue I wish the reader to be aware of is, as opposed to the previous one, a genuine weakness of the book—although it is vastly outweighed by the overwhelmingly many strengths and features of the book, so this criticism should not be taken too heavily. That being said, the authors' lack of legal training and background is often plainly apparent. Perhaps a collaboration between a mathematician and a legal scholar, rather than two mathematicians, would have resulted in a more well-rounded analysis, but again the shortcomings are manageable if one is willing to consult other sources on the cases under discussion. Many of these cases involve legal intricacies at least as complex, interesting, and important as the mathematical ones, yet these are almost entirely untouched in the book. Admittedly, the book is not intended as a work of legal scholarship, the emphasis really is on the math, so perhaps this is an unfair complaint. But the problem is that the two components, mathematical and judicial, are inextricably intertwined, so by treating half of the story in a perfunctory manner we often see a not entirely accurate representation of the role mathematical considerations really played in these courtroom proceedings. More pointedly, it has been remarked in another review of this book that in some of the cases discussed here the mathematical arguments, although present in the case as described, had little if any impact on the deliberations and legal ruling [4]. Thus, it is slightly misleading to suggest that correcting the math errors will in turn correct the judicial ones. Nonetheless, minimizing the number of math errors appearing in courts can only improve the situation overall, and the legal superficiality in this book does not curtail its success in this endeavor.

I now turn to some specific items and passages in the book that seem suspicious, unfairly biased, and in some cases simply erroneous. The first is a subtle and rather inoffensive one: on page 10, to convey the fragility of newborns the authors describe the infant morality rate in the 19th century as "one hundred per thousand babies." Although it appears standard to report mortality rates in units of "per thousand," in this particular case we find a curious occurrence: this figure could have been stated in the mathematically equivalent form of a reduced fraction, namely, "one in ten." Even though the numerical value is unchanged, the psychological impact of a figure like this can take different forms depending on the choice of units. This is not a criticism of the authors, it is simply an opportunity to observe that when communicating mathematics—a science of precision which aims to be an objective and universal language—there inevitably are choices to make which can influence the listener's reaction and interpretation. It is tempting to think of math as more immune to manipulation than other forms of reasoning, but in many respects it is not. This is relevant to the legal setting where one's goal is to deliberately sway the impact of evidence on the listener. Here we see manifestly a very mild example: the prevailing conventions in the field of public health, to use "per thousand" units, clashes with the convention in mathematics to present fractions in reduced form; one could thus justifiably adhere to either convention, and in doing so there is perhaps an opportunity to nudge the reader's emotional response in a particular direction based on the form of the figure reported. The authors have missed a convenient opportunity here to address this simple point, though perhaps it is sufficiently well-known that they did not feel a need to discuss it.

The next issue is the authors' treatment of the Amanda Knox case in Chapter 4. At the time this review was written, the vast majority of negative reviews of the book on Amazon.com were primarily upset at this particular chapter, and rightly so, I must admit. The authors here clearly have an agenda and fall victim to the very thing they argue most strongly against throughout the entire book: they attempt to use imposing mathematical arguments to convince us of a defendant's guilt, hoping the elegance of the math will blind us to the multitude of countervailing factors relevant to the trial. I need not explain why the details of their argument are so absurdly illogical—this is a case that has been discussed and analyzed extensively by so many others that the reader is simply suggested to read this chapter as an exercise in applying the overall philosophy of the book: think for

yourself, see through the attempted mathematical coercion, and don't trust the "experts," especially when they are clearly only applying their statistical methods to part of the story and ignoring all the evidence that invalidates their mathematical findings. Before moving on, however, I shall point out an example of the intense bias that pervades this chapter: one of the suspects, Raffaele, is described as someone who is "shy" and "favored computer science, violent manga, and knives." There is no explanation of where this bizarre and slanderous assertion came from (although Raffaele was an enthusiastic chef, and indeed chefs value knives). I hope any shy computer scientists reading this are as offended by this ridiculous passage as they should be.

Let us turn now to the actual math errors. One appears right in this same Amanda Knox chapter, in the mathematical preamble. The following situation is described: you are given one of two coins, a fair coin and a weighted coin favoring heads 70% of the time, and you wish to determine, based on the proportion of heads that occurs upon repeated tosses, the probability that the coin is the weighted one. The authors derive a probability of 58% if the experiment is a single toss resulting in a heads. However, this is incorrect: there is no well-defined answer to the question as phrased. A simple application of Bayes' Theorem reveals that *if* one assumes a flat prior distribution—that is, half the time you are given the biased coin and half the time you are given the fair coin—then indeed this is the correct posterior probability upon seeing one heads. But the assumption is absolutely crucial to even make sense of the question, so leaving it implicit is in bad taste and incorrectly suggesting that it is unnecessary is far worse. This is quite unfortunate as it demonstrates a fundamental misunderstanding of Bayesian statistics on behalf of the authors, and one of the main approaches to statistical jurisprudence suggested in the literature is predicated entirely on Bayesian statistics [6]. So, reader be warned.

The next math error is perhaps even simpler and illustrates a mere sloppiness on the part of the authors and the editor. On page 190, the authors attempt to compute the probability that a die rolled six times will land on the number six exactly three times. The correct answer is $(\frac{1}{6})^3(\frac{5}{6})^3\binom{6}{3} \cong 5\%$, corresponding to the fact that one needs to hit a particular number three times, avoid this number three times, and that this can happen in any order among the six rolls. This is about half the figure presented in the book; the author's error is that they left off the term $(\frac{5}{6})^3$, so they have in fact computed the probability of getting *at least* three sixes rather than exactly this

many. A simple error, but a surprising one from a math professor writing a book about how to avoid simple math errors.

Finally, not an error but a somewhat revealing passage: on page 105 the authors assert that Bayes' Theorem is "used quite frequently in legal situations" and they then proceed to estimate the probability of a defendant's guilt using this. Of course, anyone who has read Tribe's influential work [14] knows that Bayes' Theorem is extraordinarily problematic in the legal setting, and although there are on-going arguments about its applicability and there have been a handful, perhaps even many, legal cases where Bayes' Theorem has been employed, to assert without grounding that these instances are "frequent" among all legal cases seems a fairly unscholarly exaggeration.

4. How and why to teach this topic

The subject of mathematics in the courtroom offers a wonderful pedagogical opportunity for many reasons, and I am extremely grateful that the book under review now exists so that the barriers-to-entry for teaching it are drastically reduced. Firstly, most students who do not major in mathematics have a false impression that being able to understand mathematical arguments of any kind would require years of advanced study. On the contrary, the mathematics that has been used in the courtroom, and which is discussed in this book, is elementary; not even calculus is required. Even though notions from probability theory and statistics are used, one really needs only a minimal, conceptual understanding to follow, and refute, the arguments—and this is something I truly believe any college student can pick up during a semester while going through these cases, regardless of mathematical background and aptitude. This is empowering: students learn that rather than fearing math, they can face it, dissect it, argue about it. Conquering one's fear of math, which this experience can contribute to, goes a long way toward helping one learn math, since otherwise one faces the substantial impediment of untamed anxiety.

Secondly, often students who do exhibit math phobias trace their anxiety and inability to understand math to the fact that the concepts are so abstract and removed from real-world situations. By grounding the mathematical notions that arise in this book (statistical sampling, binomial coefficients, exponential growth, etc.) in the very real context of court cases, students can anchor their understanding to concrete situations and see the

abstract concepts in action. For instance, conditional probability and Bayes' Theorem can be a challenge for some, treated as formulas to be memorized without intuitive understanding; teaching an introductory course on statistical jurisprudence provides one with the opportunity to explain the theory of subjective probability [13] in terms of jurors' willingness to bet on legal outcomes and to view Bayes' Theorem as a systematic way of revising one's probabilistic belief in an assertion based on the addition of new evidence. We, the math educators, often attempt to motivate calculus through fairly contrived examples, but here in the legal setting we can do better. Not only are the examples anything but contrived (they are, after all, real cases where the decisions determine the fate of individuals!), but we are not simply illustrating calculus or any other particular topic, we are illustrating mathematical reasoning more generally. I believe this is a unique and magnificent educational opportunity.

Thirdly, roughly a quarter of all Americans will serve on a jury at some point in their lives [3]. Since our students are no exception, it seems judicious to prepare them for this judicial experience. Familiarity with mathematical courtroom reasoning is only a tiny component of being an informed participant in the legal process, but every bit helps—and if nothing else, this offers a way to get students from a broad range of backgrounds actively thinking about legal issues.

These are just some of the reasons that come to mind to teach the topic of *Math on Trial*; in summary, it shows that math is both accessible and relevant, and it raises the overall quantitative literacy of our students and exposes them to fascinating judicial matters which many might otherwise overlook in their education. There are surely more reasons than this, but allow me to turn now to the next question: once you decide to teach this topic—whether in a freshman seminar (as I have recently done) or as a special topics math major course or anything in between—what concrete steps should you take to use and perhaps supplement the book under review?

To begin with, the division of the book into individual cases, prefaced with simple math lessons, lends itself quite naturally to the classroom. Each case can be read on its own, and if the class is not aimed at math majors then the instructor can supplement the math lessons as appropriate. It is essentially guaranteed that a robust conversation amongst the students will be possible if so desired. More ambitious pedagogical options are available

too; for instance, the class can be split into groups, and when a particular case is discussed two groups could be assigned to it, one the defense and one the prosecution, and class time could then be used as a sort of mock trial.

The chapters sometimes reference each other, thus providing a nice unity to the book, but nonetheless it would not be difficult to teach a course using only a selection of the chapters, and to reorder them if so inclined. For some of the chapters the instructor may prefer to use an original source on the mathematical analysis, especially if a slightly higher level of discussion is sought. For instance, for the disputed will case there is [10], and for the Dreyfus affair there is [9]. An engaging assignment one could build into the course is to have students, individually or in groups, find legal cases not discussed in the book where math played a role and then analyze these cases using the ideas and methods discussed and accumulated throughout the semester. One of my students wrote a report on the famous O.J. Simpson trial, since an argument was made there concerning the conditional probability (empirically obtained) that a murder committed was by a former lover given a history of domestic abuse in the erstwhile relationship. Conveniently, there is a consortium called *Bayes and the Law* which hosts an extensive webpage listing legal cases that involved probabilistic reasoning (https://sites.google.com/site/bayeslegal/). This is a marvelous resource for students looking to find cases to write about, or for instructors looking to broaden or customize their curriculum.

With these suggestions in mind, I turn now to what I view as the most important piece of curricular advice, which is that the original Harvard Law Review debate referenced in the introduction is absolutely indispensable to a well-rounded course on statistical jurisprudence. I am undecided as to whether it is better to begin with these papers and then turn to *Math on Trial* or to do the reverse. The papers are more difficult to read, which suggests postponing them might be best, but they frame the material so beautifully that one might gain more from the book having first seen the big picture presented in the papers. Since these papers are in essence a sequence of rebuttals, it might work to start with [6], which advocates using Bayesian methods in the courtroom, then do Tribe's monumental [14] which directly confronts this suggestion and reaches quite the opposite conclusion, then proceed to *Math on Trial* to get more concrete cases under one's belt, then return to [7, 15] to see the academic debate continue. If one has time only for one paper beyond the book, I recommend it be [14]. This paper thoughtfully

lays out the entire landscape of mathematics in the courtroom and has been influential for decades since—although it certainly isn't the final word on the matter, nor should one feel obligated to agree with it. There is also the nice paper [8] written by Kaye eight years after the Harvard papers which adds an important dimension to the debate.

Throughout the course, whatever the structure one chooses and exact sequence of cases/papers one selects, I think it is important to keep the discourse lively and provide ample room for the students to debate and question everything. This should not be taught like most math courses, in a standard lecture format. There is room for plenty of helpful lectures, but like the legal profession itself, this material lives through debate and should be taught accordingly.

References

[1] Craig Adam, *Essential Mathematics and Statistics for Forensic Science*, Wiley-Blackwell, West Sussix UK, 2010.

[2] Jean-Denis Bredin, *The Affair: The Case of Alfred Dreyfus* (trans. Jeffrey Mehlman), George Braziller, New York NY, 1986.

[3] The Defense Research Institute national poll on the civil justice system (2012); available at https://www.dri.org/News/PollResults2012, accessed on January 17, 2016.

[4] Paul Edelman, "Burden of proof: A review of Math on Trial," *Notices of the American Mathematical Society*, Volume **60** Number 7 (2013), pages 910–914.

[6] Michael Finkelstein and William Fairley, "A Bayesian approach to identification evidence," *Harvard Law Review*, Volume **83** Number 3 (1970), pages 489–517.

[6] Michael Finkelstein and William Fairley, "A Bayesian approach to identification evidence," *Harvard Law Review*, Volume **83** Number 3 (1970), pages 489–517.

[7] Michael Finkelstein and William Fairley, "The continuing debate over mathematics in the law of evidence: A comment on 'Trial by mathematics'," *Harvard Law Review*, Volume **84** Number 8 (1971), pages 1801–1809.

[8] David Kaye, "The laws of probability and the law of the land," *University of Chicago Law Review*, Volume **41** Number 1 (1979), pages 34–56.

[9] David Kaye, "Revisiting 'Dreyfus': A more complete account of a trial by mathematics," *Minnesota Law Review*, Volume **91** (2007), pages 825–835.

[10] Paul Meier and Sandy Zabell, "Benjamin Peirce and the Howland will," *Journal of the American Statistical Association*, Volume **75** Number 371 (1980), 497–506.

[11] Piers Paul Read, *The Dreyfus Affair:The Story of the Most Infamous Miscarriage of Justice in French History*, Bloomsbury Publishing, London UK, 2013.

[12] David Salsburg, *The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth century*, Holt Paperbacks, New York NY, 2002.

[13] Leonard Savage, *The Foundations of Statistics*, Wiley, 1954.

[14] Lawrence Tribe, "Trial by mathematics: Precision and ritual in the legal process," *Harvard Law Review*, Volume **84** Number 6 (1971), pages 1329–1393.

[15] Lawrence Tribe, "The continuing debate over mathematics in the law of evidence: A further critique of mathematical proof," *Harvard Law Review*, Volume **84** Number 8 (1971), pages 1810–1820.

[16] James Whitman, *The Origins of Reasonable Doubt*, Yale University Press, New Haven CT, 2008.