

Article Supplement: Gilbert Gets the Flu

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Recommended Citation

Raley, J. "Article Supplement: Gilbert Gets the Flu," *Journal of Humanistic Mathematics*, Volume 3 Issue 2 (July 2013), pages 102-120.
DOI: 10.5642/jhumath.201302.08 . Available at: <http://scholarship.claremont.edu/jhm/vol3/iss2/8>

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Gilbert Gets the Flu

Janna Raley

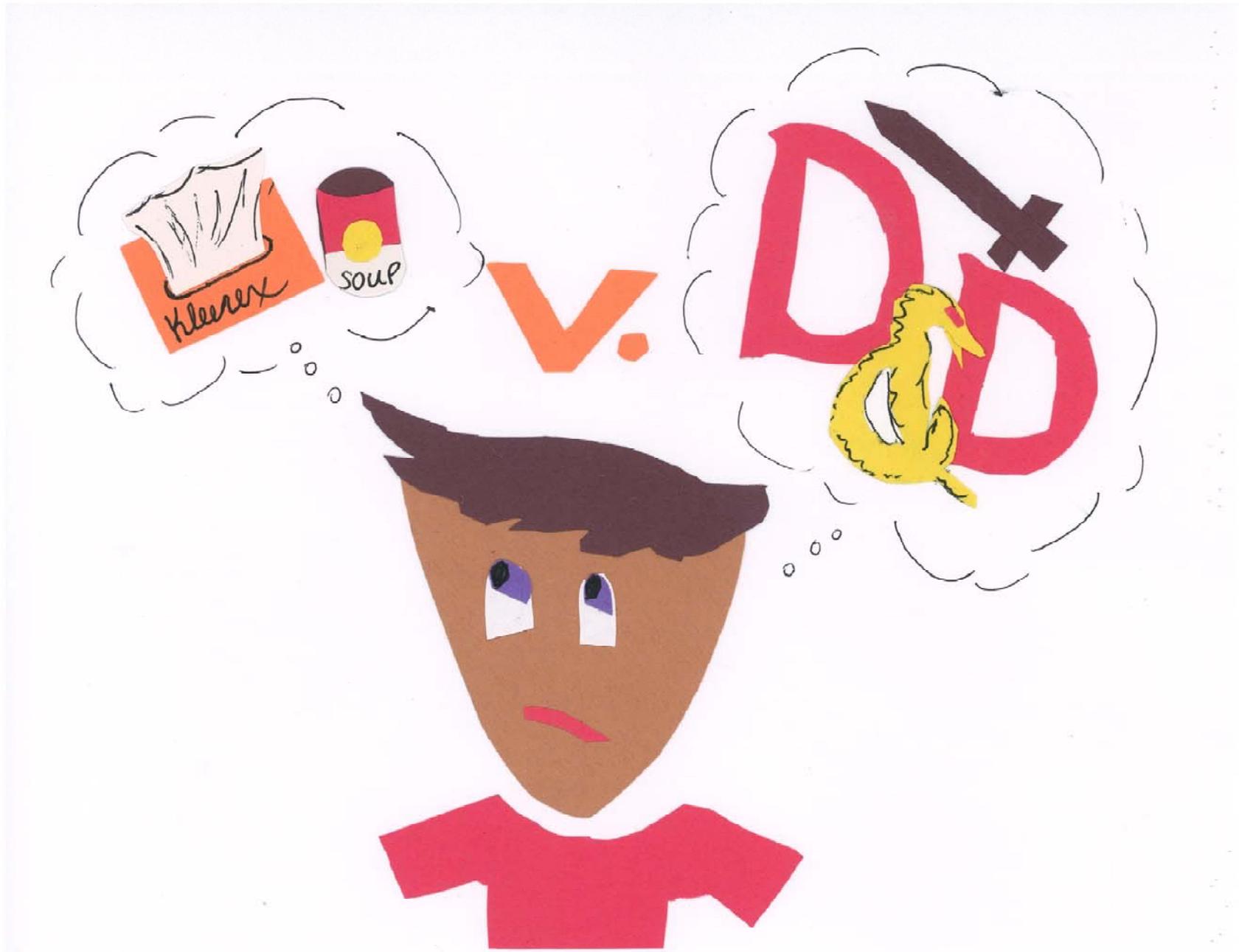


Introduction

It was a dark and stormy night.

Gilbert could feel the icy chill in the air as he walked down the boulevard. He tried to calm his nerves, but how could he? He knew there was a killer hiding in the shadows, lurking in the doorways. Fear swept over him. Every person he passed was a threat, but it was not their fault. It was what they might have inside of them. Oh, how he wished that he was running from something tangible; something he could see coming. Instead, his foe was an invisible mist. It danced on the breath of the person to his left and clung to the hand of his dearest friend. Gilbert could not see it, but he knew it well. Its name was Influenza.

Gilbert was nervous and knew he was acting out of fear rather than logic as he hurried down the emptying streets. Panicking, he ducked inside of the ominous building. Not a word was said as he simply signed his name and waited his turn. His heart was nearly beating out of his chest as he wondered how that woman behind the desk could look at him with such calmness. Her accomplice came from the back with a small package. He could see the glint of the needle that was contained in that inconspicuous plastic. Without knowing quite how it worked, he knew that small instrument had enormous power over his future. The man in the white gloves opened the package, and, for a moment, time stood still. Gilbert had this one chance to clear his head and make sure he was making the right decision. He didn't turn to the gods or to that dyed green rabbit's foot he carried in his back pocket. (He did, however, make a mental note to throw that out later because he realized that dead animal parts are usually useless and entirely creepy). No, he turned to the one thing that could provide him with the answer. He turned to game theory.



Model

Gilbert took advantage of this strange moment of frozen time in order to best model his current predicament. He knew the flu was upon his little town and he knew that he could catch it from those around him who became infected. As you probably could guess from his name, Gilbert was a happy little man who enjoyed his life and did not exactly want to get sick. Therefore, he knew he had a positive utility from avoiding the flu and a negative one should he fall ill.

His positive utility from remaining healthy was derived from things like the fact that he would not have to call in sick to his job as a computer programmer, would not miss his weekly Dungeons & Dragons meeting, and could overall just avoid the general disgustingness associated with running a fever and hacking up a lung. In the opposite scenario where he contracted the flu, he would obtain a negative utility, or, in other words, incur a cost of getting sick. These included the monetary costs such as his lost wages, his visit to the doctor, a bottle of Advil, seven boxes of Kleenexes, and four cans of soup. The cost also takes into consideration the non-monetary consequences of spending a week in bed. For example, he knows that Herbert, the Dungeon Master, will take advantage of his absence and start a campaign against his character, the monk, if he is not there to defend himself. Overall, Gilbert would just hate to miss out on his life's fulfilling activities for a whole week. Gilbert is also not the world's toughest person and just really does not want to suffer though a week of being sick. (Especially since his recent move out of his mom's basement means she would not be able to take care of him).



However, as much as he doesn't want to get sick, he also does not want to get the vaccine to prevent it. First of all, his insurance will not cover the entire cost of the shot so he will have to shell out twenty bucks. It is just plain inconvenient to get vaccinated. Secondly, did you see how big that needle is? As previously mentioned, Gilbert is not very tough, and there is a real chance that he might pass out. General wimpiness aside, there are also risks that may be associated with getting the vaccine. Bonny, who works in Accounts Payable, swears that she had negative side effects from getting the shot, and Gilbert worries he might suffer the same fate. It does not necessarily matter how realistic these risks are; Gilbert's worries make him more hesitant to get the vaccine and therefore can be included in his cost for getting the shot.

All is well and good as Gilbert can accurately identify his costs and benefits of either getting or not getting sick and getting the vaccine. However, Gilbert knows that his decision is not an isolated one and instead is a game with strategic interactions. His decision impacts that of others, and those of others can alter his decision. Since the flu is a communicable illness, he knows that fewer sick people mean his chances of getting infected decrease as well. Therefore, as others get vaccinated, the probability that he will get sick goes down even if he does not get vaccinated himself. On the other hand, if few people get vaccinated, his chances of getting sick increase. Likewise, other people will take his decision into account as they evaluate their own choices.

So while sitting there in that cold clinic chair, Gilbert assigns the following variables to these components of his model in order to conceptualize his payoff structure.

- Ω : utility of not getting the flu
- c : cost of getting sick
- p : cost of getting the vaccine
- N : number of people other than him who could potentially get sick
- H : number of people other than him who get vaccinated.

If Gilbert gets vaccinated, he will not get the flu; but he also knows he will have to incur the costs of getting the vaccine. In reality, Gilbert knows there is a small chance that he could get the flu despite getting the vaccine, but since the odds of that are small, he opts to simplify his model by assuming the shot will be one hundred percent effective. Therefore, he models his payoff of getting the vaccine as follows: payoff from vaccination = $\Omega - p$.

Gilbert also knows that if everyone else is vaccinated, there is no possible way for him to catch the flu so he will not get sick; therefore, he knows he will experience his positive utility of staying healthy with a probability equal to one. However, for every person who does not get vaccinated, his chances of catching the virus go up. He estimates that the probability of getting sick is $\beta \left(\frac{N-H}{N} \right)$, where β is a proportionality factor between 0 and 1. This means that the probability of not getting sick is $1 - \beta \left(\frac{N-H}{N} \right)$. His payoff of not getting the vaccine is thus

$$\left(1 - \beta \left(\frac{N-H}{N} \right) \right) (\Omega) + \beta \left(\frac{N-H}{N} \right) (\Omega - c) = \Omega - \beta \left(\frac{N-H}{N} \right) c.$$

Decision problem

Okay, so Gilbert has modeled his predicament, but he does not feel like he has gotten any closer to making a decision. He shifts uncomfortably in his seat knowing that he must think fast. Eventually time will unfreeze, but he has yet to decide if he really wants to become a human pincushion. Gosh golly, he wishes he had thought all this through before he got himself in this situation. He tries not to look at the needle while he continues to ponder his fate.

What he needs to evaluate are the expected values from the different outcomes. He also needs to predict what others are going to do. Now, this may come as a surprise, but Gilbert lacks certain social skills so it simply doesn't occur to him that anyone would have different utilities than he does. Therefore, in his model he simply assumes that all players are identical. He looks back at the needle and decides there is not a chance on earth (or in that fiery place below) that anyone else would opt to get vaccinated. Therefore, assuming no one else gets vaccinated, Gilbert wonders if he should follow suit. If no one else is vaccinating, then $H = 0$ and he will get a payoff of $\Omega - \beta c$ if he does not get vaccinated. He decides to compare this to the alternative of getting the vaccination: $\Omega - \beta c$ vs. $\Omega - p$.

Gilbert knows that his cost from getting sick (c) is sufficiently high and that he really could just man up and get the shot. The probability of getting sick is also sufficiently high. Therefore, in this situation he realizes that his expected value of getting the shot is actually *higher* than not getting it (meaning $\beta c > p$), assuming that no one else is getting vaccinated. Gilbert remembers from his game theory class that the solution to a game is called a "Nash Equilibrium" if no one can do better by changing their strategy given

what everyone else is doing. Therefore, Gilberts know that there cannot be a Nash Equilibrium where no one gets vaccinated since he can do better by getting vaccinated when no one else does.

He's got it all figured out now. He gets vaccinated and so does everyone, right? Wait, wait, hold the phone. Remembering that needle, Gilbert realizes that if he is not going to get sick, then he really doesn't want to go through with this form of socially accepted torture. He goes back to his model. If everyone else is vaccinated, then he can't get sick and with certainty he will get to enjoy all the benefits of his healthy, D&D filled life. Most importantly, he can do all of that without the cost of getting the vaccine.

Clearly, there cannot be a Nash Equilibrium when everyone gets vaccinated because someone can always do better by freeriding. Gilbert is really good at freeriding and a sly grin spreads across his face as he thinks about the fact that he listens to NPR every day but has not donated once. He just might be able to get away with that same concept here.

But again he realizes that he has not fully thought through this complex issue. There is not a Nash Equilibrium where no one gets vaccinated, and there is not a Nash Equilibrium where everyone gets vaccinated. How then do we know how many people would get vaccinated? Just as Gilbert begins to ponder the physics of time warp (like seriously, that man has been holding that needle out for such a long time that his arm has just *got* to be getting tired), he turns back to the matter at hand to evaluate the model.

He decided that, at any point where the expected value of getting vaccinated exceeds that of not getting vaccinated, people should opt to get the shot. However, when the expected value of getting vaccinated becomes less than that of not getting vaccinated, then people should forego the torture. Mathematically, it looks as follows:

- Vaccinate when $\Omega - \beta \left(\frac{N-H}{N} \right) c \leq \Omega - p$;
- Do not vaccinate when $\Omega - \beta \left(\frac{N-H}{N} \right) c \geq \Omega - p$.

In order to determine the precise number of people who should get vaccinated in equilibrium, it is necessary to find the value of H such that

$$\Omega - \beta \left(\frac{N - (H - 1)}{N} \right) c \leq \Omega - p < \Omega - \beta \left(\frac{N - H}{N} \right) c.$$

That is the point when enough players have vaccinated so that no one else would find it beneficial to get vaccinated.

Easy enough, some people get the vaccine and others do not. The number who gets the shot ultimately depends on the magnitudes of the utilities and costs. However, Gilbert still does not quite get it. If everyone is identical, then how come some people get the shot and others do not. This symmetric game has an asymmetric equilibrium. However, what if, instead of some arbitrary people always getting the vaccination and others opting not to, people all acted identically in the sense that they all vaccinated with the same probability? (Gilbert remembered then that this is what game theorists call a “mixed strategy.”)

He decides to look at the probability that he will vaccinate. He realizes that, given other people’s strategies, he will be willing to randomize when he is indifferent between vaccinating and not vaccinating—which happens when the payoffs of the two options are equal. However, Gilbert wants to know what specific probability he should use to randomize. He realizes that he should look at what

other people are going to do. He thinks back to his role-playing arch nemesis, Herbert, and analyzes that particular symmetric two-player game.

A Symmetric Two-Player Game Played with Herbert

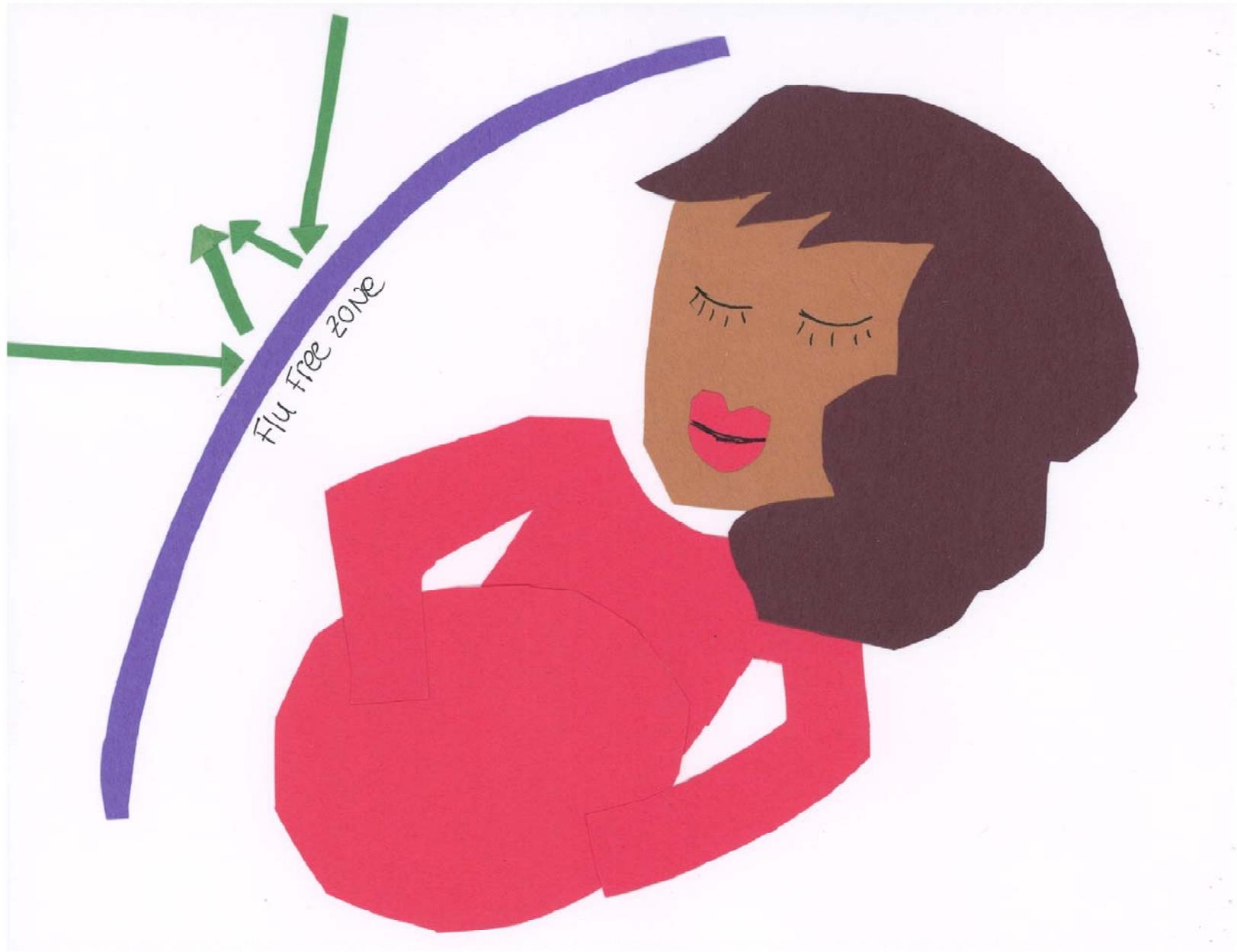
Gilbert's problem: Assuming that Herbert chooses to vaccinate with probability v , Gilbert's payoff from vaccinating is $\Omega - p$, and his payoff from not vaccinating is $v(\Omega) + (1 - v)(\Omega - \beta c)$. Gilbert would be willing to randomize when the two payoffs are equal:

$$\Omega - p = v(\Omega) + (1 - v)(\Omega - \beta c),$$

which yields $v = 1 - \frac{p}{\beta c}$.

Because this is a symmetric problem, Herbert's problem is identical to the one above, meaning that there is a Nash Equilibrium when both Gilbert and Herbert each get vaccinated with probability $v = 1 - \frac{p}{\beta c}$.

Then, in a moment of social clarity, Gilbert realizes that not everyone is like Herbert. In fact, he has noticed that most people actually are *not* like Herbert. He begins to think of his sister Bethany. Bethany is his twin sister, but they could not be any less alike. Personality differences aside, Bethany is also noticeably different in one particular regard. She is pregnant. Apparently, the flu can cause premature births and complications in pregnant women (Pregnant Women, 2012). If Gilbert is going to model his optimal strategy for vaccination against someone like her, there are new issues he realizes he needs to account for. Primarily, pregnant women have much higher costs of getting sick with the flu. This means that this is no longer a symmetric game. In order to simplify the



model for Gilbert's mental ease, he will simply assume that instead of c , Beth's cost of getting the flu is equal to γ , where $\gamma > c$.

Therefore the game is as follows.

A Non-Symmetric Two-Player Game Played with Bethany

Gilbert's problem: Assuming that Bethany chooses to vaccinate with probability v and that Gilbert vaccinates with probability q ,

Gilbert's payoff from vaccinating is $\Omega - p$, and his payoff from not vaccinating is $v(\Omega) + (1 - v)(\Omega - \beta c)$. Gilbert would be

willing to randomize when the two payoffs are equal; hence,

$$q = \begin{cases} 1 & \text{if } \Omega - p > v(\Omega) + (1 - v)(\Omega - \beta c) \\ \text{any number in } [0,1] & \text{if } \Omega - p = v(\Omega) + (1 - v)(\Omega - \beta c). \\ 0 & \text{if } \Omega - p < v(\Omega) + (1 - v)(\Omega - \beta c) \end{cases}$$

Bethany's problem (besides the fact that Gilbert is her brother): Given that Gilbert chooses to vaccinate with probability q ,

Bethany's payoff from vaccinating is $\Omega - p$, and her payoff from not vaccinating is $q(\Omega) + (1 - q)(\Omega - \beta\gamma)$. Bethany would be

willing to randomize when the two payoffs are equal; therefore,

$$v = \begin{cases} 1 & \text{if } \Omega - p > q(\Omega) + (1 - q)(\Omega - \beta\gamma) \\ \text{any number in } [0,1] & \text{if } \Omega - p = q(\Omega) + (1 - q)(\Omega - \beta\gamma). \\ 0 & \text{if } \Omega - p < q(\Omega) + (1 - q)(\Omega - \beta\gamma) \end{cases}$$

There are three Nash equilibria: one in which $(q, v) = (1,0)$; one in which $(q, v) = (0,1)$; and one in which Gilbert vaccinates with probability $1 - \frac{p}{\beta\gamma}$ and Bethany vaccinates with probability $1 - \frac{p}{\beta c}$.

Other Considerations

Gilbert is smart enough to realize that his simplifications in the model may cause it to be slightly less exact than he would like it to be. However, he decided that this model captures the essence of the problem more accurately than other potential methods. He considered approaching this as a sequential game. However, this form makes it more difficult to accurately portray the changing probability of getting infected given how many others are vaccinated. It also works best to assume that everyone is making simultaneous decisions, even though in reality there are usually different stages of vaccinations throughout the flu season. There also tends to be imperfect information in regards to who else gets vaccinated. However, Gilbert realizes he is nosey enough to be able to know what everyone around him is going to do. Plus, due to his limited social life, he is presumably dealing with a small N . This makes this particular model ideal for his decision making process.

Gilbert also knows that the flu shot is a wonderful example of a good with a positive externality. Since eventually it will be worthwhile for some to free-ride on others' vaccinations, this often results in a less than socially optimal amount of vaccination. Gilbert makes his choices based on his own risks rather than considering the fact that if he gets the flu he may pass along a deadly virus to his elderly neighbor. There could be models that take others' valuation into consideration. However, this tends to not be how individuals approach the flu shot. It would be more how public entities would want individuals to value vaccinations.

Public and private entities have tried to manipulate the payoff structures in order to incentivize individuals into vaccinating at the socially optimal level. For example, there is now a new vaccine type, the intradermal influenza vaccine, which injects the vaccination dose skin deep rather than to the muscles. There have been advertisements for this particular vaccine which have tried to portray it as being less painful and, of course, low on side effects (Intradermal Influenza (Flu) Vaccination, 2012). (Gilbert is going to be very upset when he finds out there may have been a less painful option available.) Employers have tried to lower other aspects of the costs associated with the flu by providing the vaccinations for free. It is in their best interest to do so because preventing illness minimizes missed work days. Other public campaigns have focused on spreading awareness about individuals' impacts on the general flu outbreaks. "The Flu Ends with U" is a debatably catchy phrase used by the CDC to encourage vaccinations.



Ending

SHIZHAM! Suddenly time unfreezes and the cold beads of sweat fall from Gilbert's brow and splash on the floor. The man grins down at him and Gilbert swears the malice was palpable. "Ready?!" the man asks. Gilbert chokes back his fears. He glances over at that woman from behind the desk. She smiles back at him. In that moment all of the models and all that theory just fade away.

"*ready*," he whispers.

"What?"

"**READY**," bellows Gilbert.

And as the needle pierces his arm, he sees himself begin to change. No longer is he sitting there in the clinic. Instead, he pictures himself in the Kingdoms of Kalamar. Except this time, he is no longer that hunched, lackluster monk that Hebert casts him as. Instead, he is in charge of his own destiny. He is now a...cleric...no, a mage...no. He is a knight. He brandishes his imaginary sword and his cuirass shines under the pale examination light.

Fearless, he walks to the front desk....

Works Cited

- Intradermal Influenza (Flu) Vaccination.* (2012, July 16). Retrieved November 28, 2012, from Centers for Disease Control and Prevention, National Center for Immunization and Respiratory Diseases (NCIRD):
http://www.cdc.gov/flu/protect/vaccine/qa_intradermal-vaccine.htm
- Pregnant Women.* (2012). Retrieved November 27, 2012, from Flu.gov: <http://www.flu.gov/at-risk/pregnant/index.html>