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Barry Cobb,

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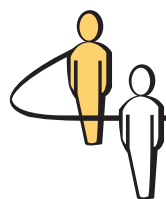
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Graphical Models for Economic Profit Maximization

Barry Cobb

Department of Economics and Business, Virginia Military Institute, Lexington, Virginia 24450,
cobbbr@vmi.edu

Decision trees and influence diagrams are utilized to analyze and solve profit maximization problems from economics. As a complement to traditional analytical methods for solving these problems, use of these graphical representations allows students to learn about the effects of uncertainty on pricing and capacity choices. Decision trees are first used to model a firm's production capacity and pricing decisions when these choices are made simultaneously under certain and uncertain demand. The decision tree approach is next extended to a situation where the firm makes its capacity and price selections under different information constraints. The use of these problems as part of a case assignment in a writing-intensive managerial economics course is discussed. Influence diagrams are also presented as an alternative modeling technique that can easily accommodate more potential values for decision variables when solving these problems. By studying the effects of uncertainty on profit maximization problems, students can also learn to appreciate that dealing with uncertainty is important in many business decisions.

Key words: decision analysis; decision tree; economic decisions; graphical model; influence diagram; managerial economics; probability; profit maximization; uncertainty

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1. Introduction

Profit maximization is introduced in economics principles textbooks under the assumption that firms face certain market demand for their products. Students learn to solve deterministic profit maximization problems using marginal analysis concepts by finding simple algebraic solutions for optimal prices and production quantities. Unfortunately, even textbooks designed for upper-level undergraduate managerial economics courses do not provide students with modeling approaches that relax the assumption of certain demand. Additionally, most of the literature from the economics and managerial accounting fields on pricing and capacity planning decisions under uncertainty (see, for example, [Banker and Hughes 1994](#), [Göx 2002](#)) employs strictly mathematical approaches using algebra and stochastic calculus to draw conclusions about the effect of demand uncertainty on these decisions.

This paper combines the concepts of marginal analysis from economics with intuitive graphical modeling approaches from decision analysis to facilitate a more robust understanding of the complexity of capacity and pricing decisions in the face of uncertain demand and economic conditions. Such examples can stimulate students to improve their economic reasoning ability and help students realize that uncertainty

affects decision making in many contexts. Additionally, this is important for students who will become managers because businesses in practice face uncertain future demand.

Decision analysis is a structured approach to decision making that allows managers to gain enhanced insight and sharper intuition about the problems they face. Quantitative representations of decision problems that incorporate uncertainty are important components of decision analysis. Two such representations are called *decision trees* and *influence diagrams*. By instituting simple models such as decision trees and influence diagrams, economists, managers, and students can leverage concepts from decision analysis to improve resource allocation and better understand the factors that affect optimal pricing and production decisions.

Decision trees—as opposed to a strictly mathematical approach to handling uncertainty—were selected as pedagogical tools in a recent writing-intensive undergraduate managerial economics course for two reasons. (1) Most of the students had prior exposure to these models from an operations management class, and (2) the instructor was comfortable teaching students to use them. In general, graphical models are widely suggested as a valuable methodology for introducing students to new concepts in

economics. For instance, Hey (2005) describes a solely graphical approach for teaching the dependence of product demand on consumer preferences. Wilkins (1992) suggests that an introduction of economic concepts through a graphical approach followed by a careful transition to a mathematical model of an economic problem provides valuable insights for students. When used to address pricing and capacity planning problems, decision trees allow students to visualize the different scenarios a firm faces and the relationship of the chance and decision variables in the problem.

The remainder of this paper applies decision trees and influence diagrams to a monopolist's profit maximization problem from economics under various assumptions about the knowledge of the actual demand function. The primary audiences for these examples are students in upper-level elective economics courses who have completed a course that included an introduction to decision trees (for example, a management science, operations management, or operations research course). The next section describes the example problem that is used to illustrate the decision tree and influence diagram techniques. In §3, these solutions are presented for decision tree models and, in §4, influence diagram models are presented. Section 4.3 illustrates an extension to traditional influence diagram models that allows continuous decision variables. Section 5 concludes the paper. The appendix contains the student version of the managerial economics assignment.

2. Example

Suppose a monopoly faces the demand function

$$Q(P, Z) = 10 - P + Z \quad (1)$$

for its single product, where Z is a random demand "shock" and P is the price set by the firm. Thus, the firm faces a linear demand function with an intercept of 10 *plus* the value of the demand shock and a slope of -1 . When price increases by \$1.00, quantity demanded for the firm's product decreases by one unit.

The company incurs \$1.00 per unit in labor costs and \$1.00 per unit in materials costs. The labor charge is a capacity cost in the sense that once the firm decides what quantity to produce, it sets its labor force accordingly. The firm must therefore set its production capacity (K) and price to maximize profits. The capacity choice of the firm limits the number of units it produces and sells, so the firm's profit is determined by the function

$$\pi(K, P, Q) = (P - 1) \cdot \text{Min}\{K, Q\} - K. \quad (2)$$

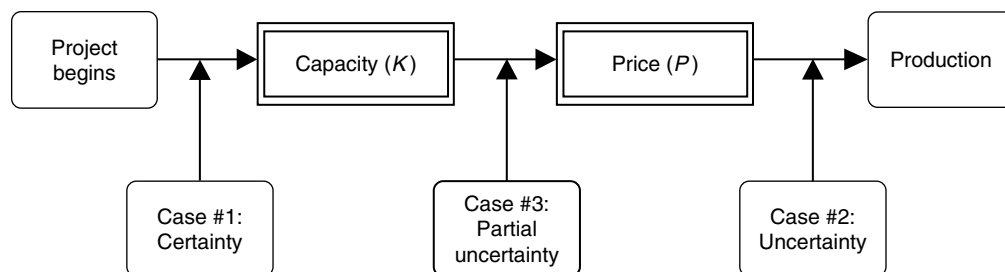
Ultimately, the number of units sold by the firm is the minimum of its production capacity and the number of units demanded by customers (calculated according to (1)).

In microeconomics courses, such profit maximization problems are introduced under the assumption that the true value of the demand shock is known at the time the price and production capacity decisions are made. This allows the firm to establish its price and capacity where marginal revenue equals marginal cost, with all capacity and materials costs behaving as variable expenses. However, uncertainty surrounding market demand complicates the pricing and capacity decisions. If either price or production capacity must be selected without complete knowledge of demand, the decision becomes more difficult.

Suppose that the random demand shock Z in (1) can be interpreted as the normally distributed error term in a linear demand function estimated from data using least squares regression and assume further that the standard deviation of Z equals one, i.e., $Z \sim N(0, 1)$. The solution to the problem depends critically on the timing of the resolution of demand uncertainty. The firm chooses a production capacity (K), then subsequently decides its price (P). The firm could potentially become aware of the true demand function at one of three possible times, shown in Figure 1 and defined by Göx (2002):

1. **Certainty:** The true value of the demand shock is *known* prior to choosing *both* capacity and price. In this case, the firm should produce where marginal revenue (based on the resulting demand function) equals marginal cost.

Figure 1 Arrival of Information on the True Value of the Demand Shock Under the Three Information Scenarios



2. **Uncertainty:** The true value of the demand shock is *unknown* at the time capacity and price are chosen.

3. **Partial Uncertainty:** The true value of the demand shock is known prior to setting price but after capacity is chosen

This paper utilizes decision analysis models to ensure that the uncertainty surrounding the market demand is correctly incorporated into the price and capacity decisions. This frames the profit maximization problem in the context of a decision-making problem under uncertainty to which decision trees and influence diagrams can be applied.

The next section uses decision trees to solve the example problem.

3. Decision Tree Solution

The *decision tree* model was derived by Raiffa and Schlaifer (1961) from the extensive-form game representation of von Neumann and Morgenstern (1947). In this model, a decision maker can be seen as playing a game against an opponent called *nature* that randomly generates outcomes for chance variables that affect the eventual payoffs. Decision trees are solved via dynamic programming by calculating expected values at chance nodes and maximizing utility at decision nodes, as demonstrated in examples later in the paper.

In this section, decision tree solutions to the profit maximization problem under the three information scenarios are presented. In the case of the certainty scenario, the decision tree solution is compared to an analytical solution obtained using marginal analysis.

Two of these scenarios were recently included as part of a case assignment in an upper-level undergraduate managerial economics elective taught as a writing-intensive course (see the appendix for specific details). Students were required to find the firm's optimal strategy in the certainty and partial uncertainty scenarios. In the former scenario, students used the marginal-revenue-equals-marginal-cost profit maximization rule whereas in the latter situation, the students developed a decision tree model. All students had completed a statistics course so they understood the concept of a probability distribution. In previous class sessions, a brief introduction to formulating and solving decision trees was provided. This was a review for most students because they were previously introduced to decision trees in an operations management course. Some assistance was provided on an individual basis to students who needed additional review.

3.1. Certainty Scenario

In the assignment mentioned previously, students were instructed to assume $P(Z = -2) = 0.16$,

$P(Z = 0) = 0.68$, and $P(Z = 2) = 0.16$. These probabilities represent a three-point discrete approximation to the standard normal probability density function (pdf), but they were simply introduced to the students as a discrete probability distribution for Z .

Consider the three possible values for the demand shock: $Z = -2$, $Z = 0$, or $Z = 2$. When these values are inserted in Equation (1), the results show that the firm will experience demand according to $Q(P) = 8 - P$, $Q(P) = 10 - P$, or $Q(P) = 12 - P$. These correspond to simplified inverse demand functions $P(Q) = 8 - Q$, $P(Q) = 10 - Q$, and $P(Q) = 12 - Q$, respectively.

Because the production quantity will be chosen with complete knowledge of demand, optimal production quantity and capacity will be equal ($Q^* = K^*$). The firm will only pay the labor charge for capacity for the units it can produce *and* sell, making its total cost function $TC(Q) = 2Q$. An example of a computational question from an economics textbook that incorporates one of these scenarios is as follows (Baye 2006, p. 291): "Suppose the inverse demand function for a monopolist's product is given by $P = 10 - Q$ and the cost function is given by $TC(Q) = 2Q$. Determine the profit-maximizing price and quantity and the maximum profits." Only the numbers in the equations from the previous citation are changed from the source.

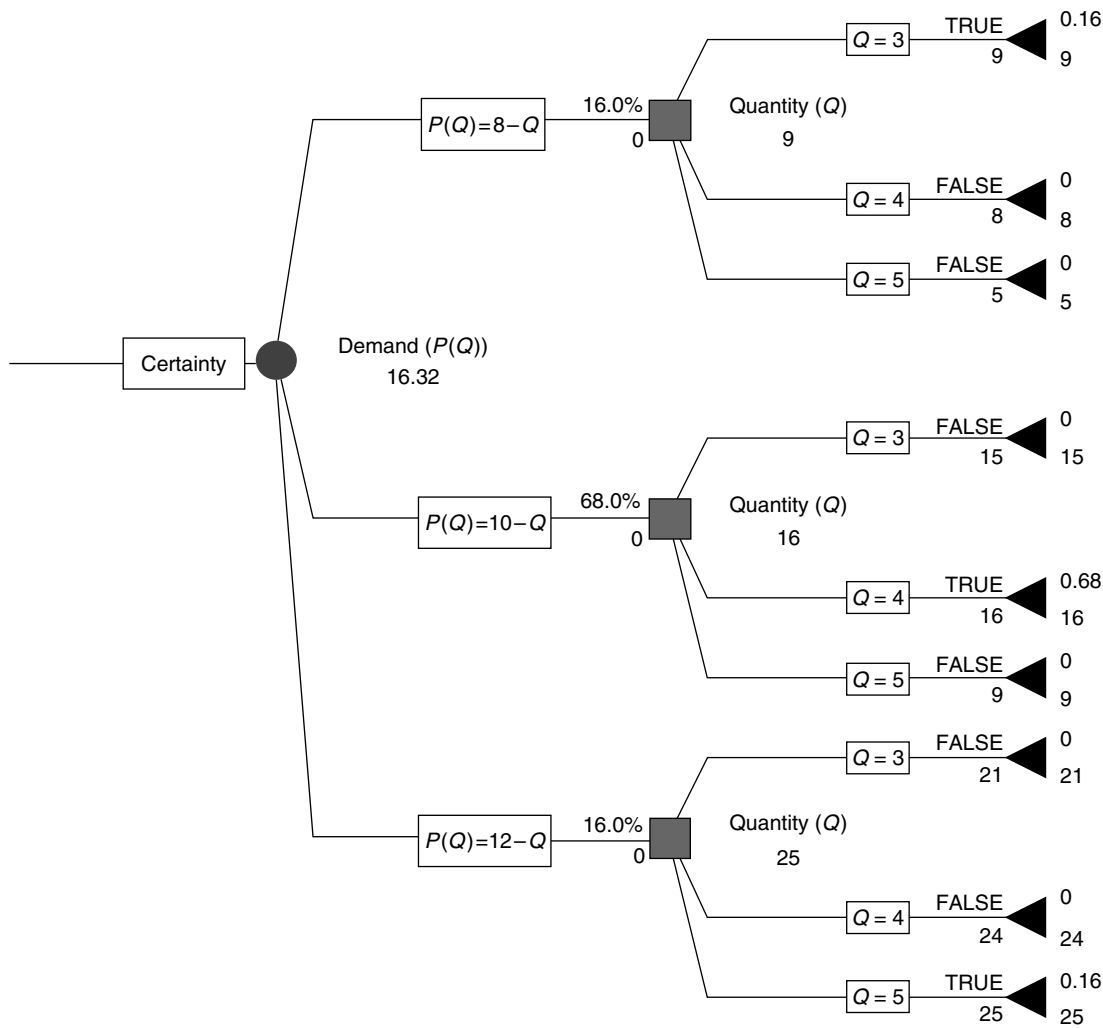
Based on a constant marginal cost $MC = \$2.00$, the firm determines optimal quantity Q^* and prices (P^*) in the certainty scenario as shown in Table 1. Table 1 goes step by step through the process of constructing a total revenue function, differentiating this function to find a marginal revenue function, then setting the result equal to the constant marginal cost and solving for the optimal value of Q . Thus, once the firm observes the demand shock, it adjusts its demand function accordingly and determines its optimal production and pricing strategies by setting marginal revenue (MR) equal to marginal cost (MC). The optimal value for P is found by substituting the optimal value for Q back into the inverse demand function.

The solutions in Table 1 can be validated using a decision tree model that considers three possible values $Q = 3$, $Q = 4$, and $Q = 5$ for the production quan-

Table 1 Optimal Solution to the Monopoly's Production Problem Under Certainty

	$Z = -2$	$Z = 0$	$Z = 2$
Inverse demand ($P(Q)$)	$8 - Q$	$10 - Q$	$12 - Q$
Total revenue ($TR(Q) = P(Q) \cdot Q$)	$8Q - Q^2$	$10Q - Q^2$	$12Q - Q^2$
$MR(Q) = dTR(Q)/dQ$	$8 - 2Q$	$10 - 2Q$	$12 - 2Q$
$MR(Q) = MC$	$8 - 2Q = 2$	$10 - 2Q = 2$	$12 - 2Q = 2$
Q^*	3	4	5
P^*	5	6	7
Profit = $TR(Q^*) - TC(Q^*)$	9	16	25

Figure 2 Decision Tree Model for the Firm's Problem Under Certainty



tity variable, as shown in Figure 2. The decision tree models in this paper were produced using the Excel add-in software Precision Tree produced by Palisade Corp. "<http://www.palisade.com>." The selection of the possible values 3, 4, and 5 for Q (and similar values for decision variables throughout this paper) is admittedly arbitrary and the selections are made with the knowledge of the solutions presented in Table 1. In general, selection of such values for a decision tree may result from trial and error where the analyst starts with some basic knowledge of reasonable values for the particular problem and adjusts these based on analysis of resulting trees in an iterative process.

The profit for each possible combination of demand and price is shown at the end of each branch. For example, when demand is given by $P(Q) = 10 - Q$ and quantity is chosen as $Q = 3$, the resulting price is $P(3) = 10 - 3 = 7$. At this end point, profit is calculated as $TR - TC = 7 \cdot 3 - 2 \cdot 3 = 15$. Price is not shown in the decision tree, because it is chosen at the same time

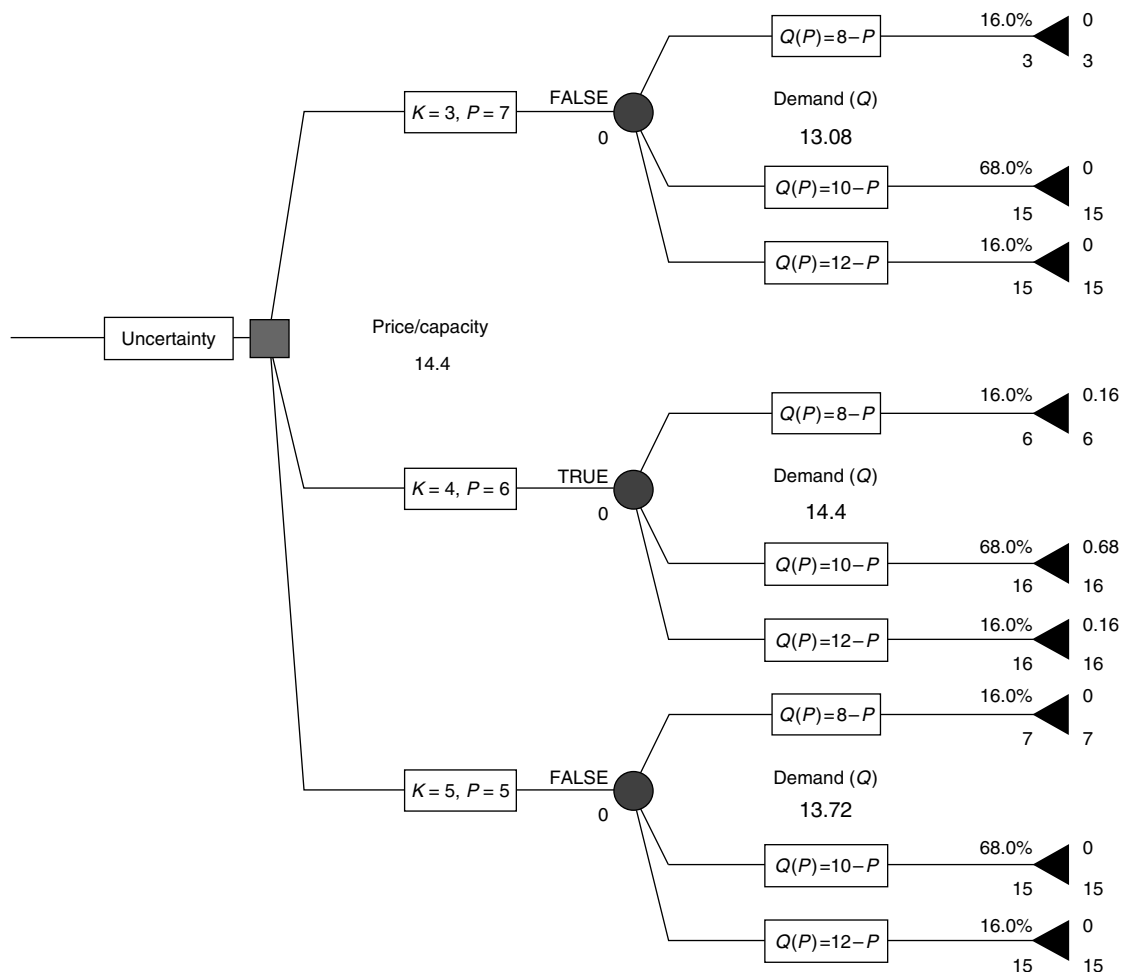
as production quantity based on the actual inverse demand function.

The optimal quantity choices are noted on the decision tree in Figure 2. For instance, if the firm observes that demand is $P(Q) = 8 - Q$, it should choose $Q = 3$, which thus dictates that $P = 5$. The other two scenarios also match the marginal-revenue-equals-marginal-cost solution, as indicated by the *TRUE* label on the corresponding branches. The firm's expected profit is

$$E(\pi(Q)) = 0.16(9) + 0.68(16) + 0.16(25) = 16.32.$$

The certainty scenario mirrors the typical assumptions made in the profit maximization techniques introduced in economics textbooks. The remainder of this section illustrates that decision trees can allow students to learn about the effect that uncertainty about the firm's actual demand may have on pricing and production quantity decisions.

Figure 3 Decision Tree Model for the Firm's Problem Under Uncertainty



3.2. Uncertainty Scenario

To utilize a decision tree to address the decision problem under uncertainty, we must limit the choice of capacity to a few discrete values. In this example, we consider possible values of $K = 3$, $K = 4$, and $K = 5$. Because the firm essentially selects its capacity and price simultaneously, the model can be simplified somewhat by assuming that the firm chooses a capacity level K and then sets a price based on the expected inverse demand function $E(P(Q)) = 10 - Q$. For instance, if the firm selects $K = 4$, price is selected by assuming that production quantity equals capacity and determining $E(P(4)) = 10 - 4 = 6$. The complete decision tree model is shown in Figure 3, where the decision and chance nodes are reversed (as opposed to those in Figure 2) to model the timing of the information concerning demand.

The profit values at each end node are determined using the function in (2). For instance, if $K = 4$ and $P = 6$ but the actual demand function is $Q(P) = 8 - P$, the firm's profit is

$$\pi(4, 6, 2) = (6 - 1) \cdot \text{Min}\{4, 2\} - 4 = 6.$$

This situation represents excess capacity for the firm. In the case where the true demand function is $Q(P) = 12 - P$, the firm's profit is

$$\pi(4, 6, 6) = (6 - 1) \cdot \text{Min}\{4, 6\} - 4 = 16.$$

The result of choosing $K = 4$ in this demand scenario is a shortage of production capacity.

The firm calculates the expected profit under each of the possible choices of a capacity/price combination. For instance, in the case where $K = 4$ and $P = 6$, the firm's expected profit is

$$E(\pi(4, 6, Q)) = 0.16(6) + 0.68(16) + 0.16(16) = 14.40.$$

The decision tree model in Figure 3 also shows that $K = 4$ (with corresponding price $P = 6$) is the firm's best choice. The lack of demand information costs the firm an expected $\$16.32 - \$14.40 = \$1.92$ in the uncertainty scenario; this is the difference in the maximum expected profits between the certainty and uncertainty models. Constructing the decision tree for both the certainty and uncertainty scenarios allows

students to see the effect that a mismatch of actual and expected demand has on the firm's profits.

The next section describes a scenario where the firm chooses price and production capacity under different information scenarios.

3.3. Partial Uncertainty Scenario

Choosing production capacity in advance of learning actual demand places an upper limit on production quantity so the firm will not necessarily be able to produce a quantity exactly equal to its actual demand (as it can in the certainty scenario). We will refer to this situation as the *partial uncertainty* case.

In the partial uncertainty scenario, the firm must choose its capacity first and then, based on the choice of capacity and eventual demand, select the price that maximizes profits. Despite very similar characteristics to the problem faced by the firm under certainty and uncertainty, the partial uncertainty scenario proves more difficult to solve with analytical methods (Göx 2002) because the downstream decision for price may depend on the previously chosen value for capacity.

To use a decision tree model to assist the firm in making its optimal capacity and pricing choices, we must limit price and capacity to a few discrete values. Suppose we allow for prices of \$5.00, \$6.00, \$7.00, or \$8.00, a set that includes the optimal prices under certainty and uncertainty, and one additional choice. In this example, the company can select from capacities of $K = 3$, $K = 4$, or $K = 5$. The decision tree model for the firm's problem under partial uncertainty is shown in Figure 4. Because of space restrictions, the portion of the tree corresponding to 5 units of capacity ($K = 5$) is collapsed, and the expected value associated with that choice of 15.32 is noted.

Each path through the tree represents a potential outcome, and the profit associated with the outcomes is calculated using the profit function in (2). For example, if the firm selects $K = 4$ and then eventually learns that $Z = -2$ and $Q(P) = 8 - P$, it earns profit of $\pi(4, 6, 2) = 5 \cdot \min\{4, 2\} - 4 = 6$ if it chooses $P = 6$. In this case, it has unused capacity because market demand is only $Q = 2$ based on $P = 6$. In another instance, if the firm selects $K = 4$ and then eventually learns that $Z = 0$ and $Q(P) = 10 - P$, it earns profit of $\pi(4, 5, 5) = 4 \cdot \min\{4, 5\} - 4 = 12$ if it chooses $P = 5$. In this case, production is limited by capacity because market demand is $Q = 5$ based on $P = 5$, but the firm only produces and sells four units.

The firm's optimal strategy is to purchase four units of capacity where this policy is denoted by *TRUE* on the $K = 4$ branch of the decision tree in Figure 4. If demand turns out to be $Q(P) = 8 - P$, the firm selects $P = 5$, whereas if true demand is $Q(P) = 10 - P$, the firm should choose $P = 6$. These prices correspond with optimal prices in the certainty scenario obtained

with the marginal-revenue-equals-marginal-cost rule, although the resulting profit in the $Z = -2$ case is not the same because of one unit of unused capacity. In the case where $Q(P) = 12 - P$ is the eventual demand, the optimal price is $P = 8$. Here, the firm raises price from the optimal value under certainty to "ration" capacity, which is fixed below demand. The expected profit of following the optimal strategy is calculated as

$$EV(K = 4) = 0.16(8) + 0.68(16) + 0.16(24) = 16.$$

To obtain maximize expected profit under partial uncertainty, the firm must make the correct pricing choice when it faces demand that exceeds its capacity.

The firm also has "off-optimal path" decision strategies that may be used if, for instance, workers quit and leave capacity at three units in the short run. In this case, the firm should price its product at $P = 5$ if $Q(P) = 8 - P$, $P = 7$ if $Q(P) = 10 - P$, and $P = 8$ if $Q(P) = 12 - P$. Again, the latter two instances represent capacity rationing scenarios where the firm raises its prices to earn additional revenue on the units it can produce with its already established labor force. The (nonoptimal) expected profit of following this strategy is $EV(K = 3) = 14.52$.

Solving the problem under uncertainty and partial uncertainty using the decision tree allows students to see that having the demand information available for the pricing decision adds $16 - 14.40 = 1.60$ to the firm's maximum expected value.

3.4. Discussion

In the managerial economics course, a decision tree template was provided to the students who then focused their effort on labeling and solving the tree. The template served to facilitate the review of the graphical components of a decision tree. Prior to its completion, the assignment was discussed in class and the calculation of profits in one of the scenarios was provided as an example. The students had the opportunity to submit decision tree solutions for feedback prior to starting the writing phase of the assignment.

After solving the problem, students in the managerial economics course created reports that described the problem, outlined solutions in each scenario, and made recommendations to the manager of the firm. In the final reports written to satisfy the assignment, students compared the optimal prices, production quantities, and profits in the certainty and partial uncertainty scenarios. In two of the three actual demand scenarios ($Q(P) = 10 - P$ and $Q(P) = 12 - P$), the profit earned under certain demand is greater than or equal to the expected profit earned in the partial uncertainty case.

Figure 4 Decision Tree Model for the Firm's Problem Under Partial Uncertainty

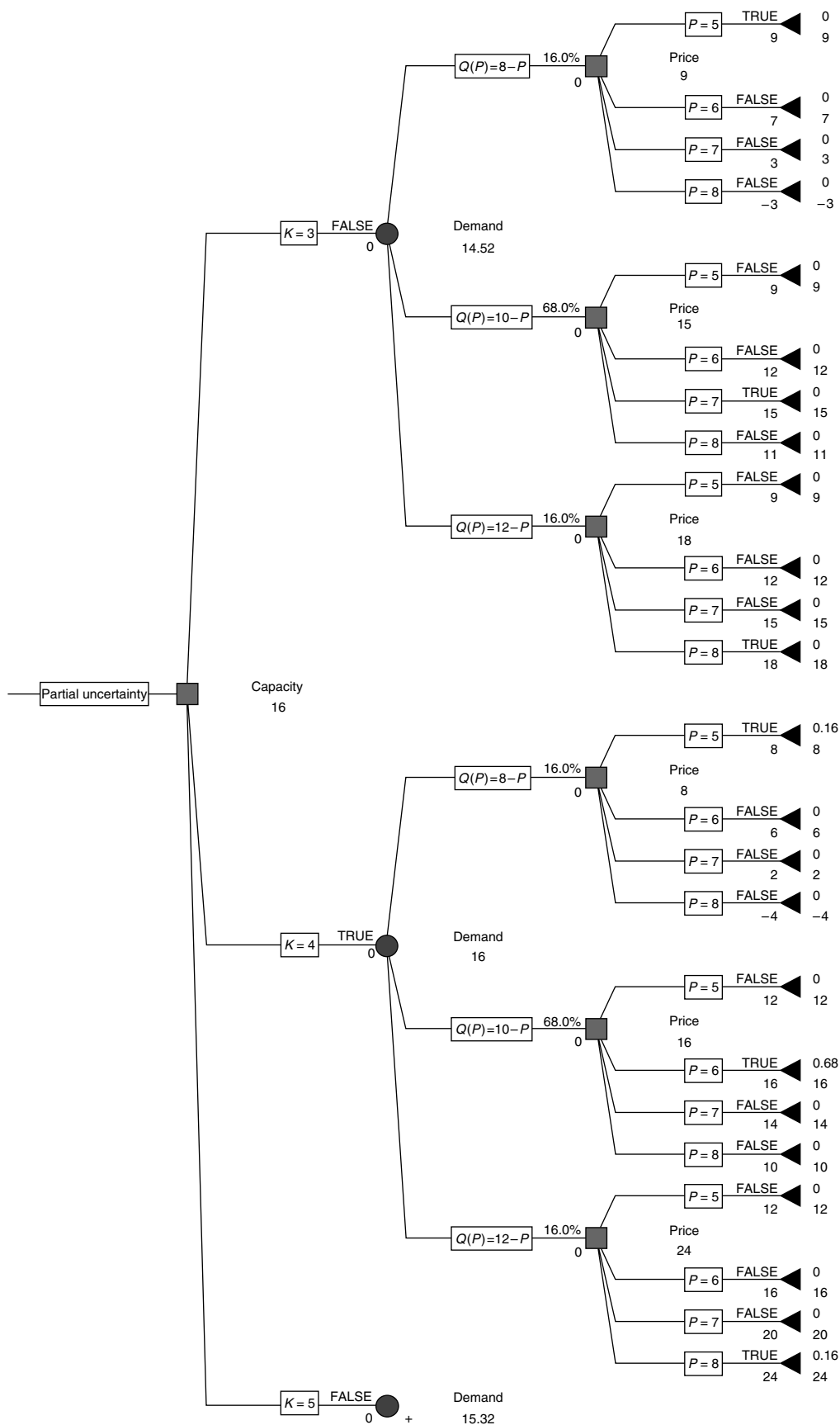
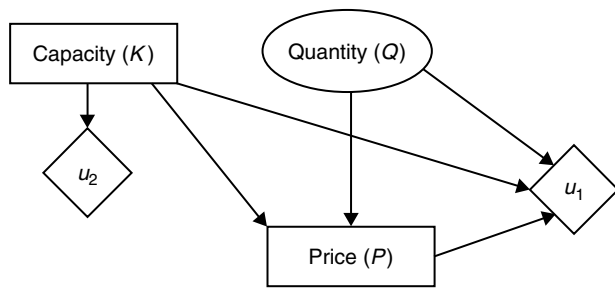


Figure 5 Influence Diagram Representation of the Monopolist's Decision Problem

Several students recognized in their reports that information regarding demand has a direct effect on the profit or expected profit earned by the firm. For instance, one of the students in the course stated that "... the firm would likely be more profitable if it knew its quantity demanded before determining its capacity and price." Another summarized the results by writing that "this problem makes clear that knowledge of demand is an essential part of successfully running a company." Although one can certainly quibble with the latter student's observation because most companies are run without certain "knowledge of demand," the point remains that the assignment encouraged the students to consider the fact that uncertainty has important implications for the decisions made by businesses.

Although numerous software packages that produce decision trees (such as the Precision Tree software used to produce the diagrams in this paper) are available, use of such software was not required of the students for this assignment because a template was provided (a 14-day trial of Precision Tree is free and a student can purchase an academic version for \$50.00).

The number of end nodes in decision trees grows exponentially when the elements in the state spaces of the chance variables and the number of alternatives available at decision nodes increase. Thus, decision trees are feasible primarily for small problems. This fact, in part, led to the development of the influence diagram model. The next section describes influence diagram solutions to the decision problem under partial uncertainty.

4. Influence Diagram Solution

As part of a term paper project, an influence diagram solution to the monopolist's decision problem was created by a student from the managerial economics class who had been introduced to influence diagrams in a management science course. A similar influence diagram solution is described in this section.

An *influence diagram* is a compact graphical representation for a decision problem under uncertainty that facilitates an understanding of the relationships among the chance and decision variables. Initially,

influence diagrams were proposed by Howard and Matheson (1984) as a front end for decision trees (in other words, the influence diagram is used to develop the model but is later converted to a decision tree to solve the problem). Subsequently, Olmsted (1983) and Shachter (1986) developed methods for evaluating influence diagrams directly without converting them to decision trees. Further advancements to these methods later allowed influence diagrams to more easily represent larger decision problems where chance and decision variables can assume large numbers of potential values (for a review of these methods, see Bielza and Shenoy 1999). Unlike the decision tree, the graphical influence diagram formulation does not limit the number of values in the state spaces of the variables.

As in the decision tree representation of the monopolist's decision problem under partial uncertainty, we assume that $P(Z = -2) = 0.16$, $P(Z = 0) = 0.68$, and $P(Z = 2) = 0.16$. The company's utility (profit) function from Equation (2) is decomposed into additive factors $u_1(K, P, Q) = (P - 1) \cdot \text{Min}\{K, Q\}$ and $u_2(K) = -K$.

The graphical representation of the influence diagram for the capacity planning and pricing problem in the partial uncertainty scenario is shown in Figure 5. The oval represents a random variable, the rectangles represent decision variables, and the diamonds represent additive factors of the joint utility function. The influence diagram is solved via dynamic programming by marginalizing the variables in the sequence P, Q, K . In this case, the influence diagram is solved using the algorithm of Tatman and Shachter (1990), which permits an additive factorization of the joint utility function.

The influence diagram is solved via dynamic programming by marginalizing the variables in the sequence P, Q, K . In this case, the influence diagram is solved using the algorithm of Tatman and Shachter (1990), which permits an additive factorization of the joint utility function.

4.1. Initial Solution

To delete P we maximize the utility function factor $u_1(K, P, Q) = (P - 1) \cdot \text{Min}\{K, Q\}$ by choosing the best value of P for each combination of K and Q and noting the associated decision strategy. The results are denoted by utility function u_3 and are shown in Table 2.

Next, we eliminate Q by calculating the expectation of the utility function u_3 as follows:

$$u_4(K = 3) = 0.16(12) + 0.68(18) + 0.16(21) = 17.52,$$

$$u_4(K = 4) = 0.16(12) + 0.68(20) + 0.16(28) = 20,$$

$$u_4(K = 5) = 0.16(12) + 0.68(20) + 0.16(30) = 20.32.$$

Table 2 Maximum Utility and Pricing Strategies for Each Combination of Capacity and Demand

K	$Q(P)$	P^*	$u_3(K, Q)$
3	$8 - P$	5	12
3	$10 - P$	7	18
3	$12 - P$	8	21
4	$8 - P$	5	12
4	$10 - P$	6	20
4	$12 - P$	8	28
5	$8 - P$	5	12
5	$10 - P$	5 or 6	20
5	$12 - P$	7	30

The variable K is removed using the following maximization operation:

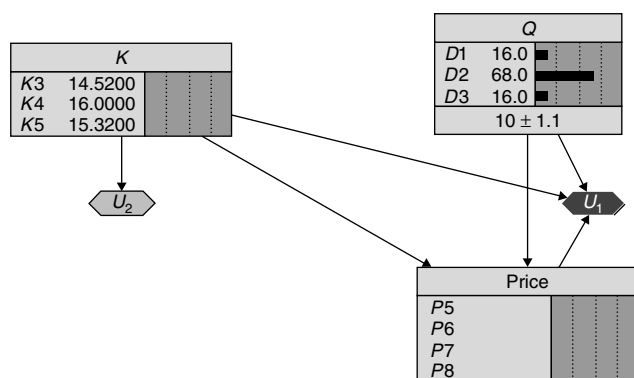
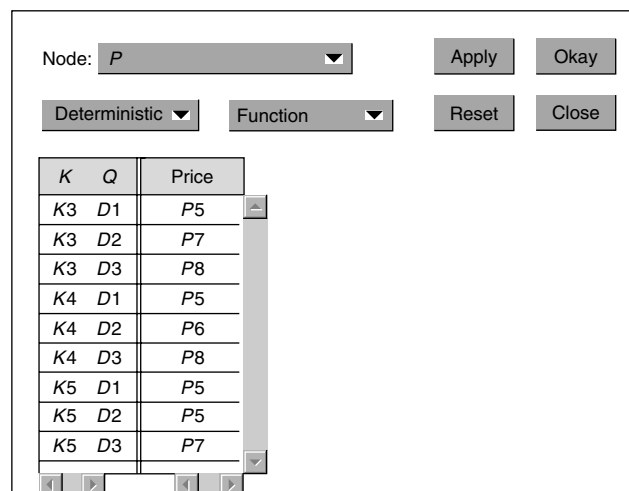
$$\max_K (u_4(K) - u_2(K)).$$

The resulting expected value of 16 is obtained by choosing 4 units of capacity ($K = 4$) and is the same as in the decision tree solution.

Several commercial software packages are available for solving influence diagram models. The solution for this example was obtained using Netica software "<http://www.norsys.com>." A limited version of this package, adequate for solving problems such as the one shown here, is available at no cost. The graphical depiction of the solution to the problem is displayed in Figure 6. The solution shows the expected value associated with each capacity option so that the user can infer the correct decision strategy, which is to choose $K = 4$ and obtain an expected value of 16. Netica also displays the optimal decision strategy for the price decision in a table associated with that node, shown in Figure 7. The demand functions $Q(P) = 8 - P$, $Q(P) = 10 - P$, and $Q(P) = 12 - P$ are denoted by $D1$, $D2$, and $D3$, respectively, in this table.

4.2. Increasing the Number of States

The influence diagram representation of the decision problem is more tractable than the decision

Figure 6 Netica Solution to the Monopolist's Decision Problem**Figure 7** Netica Display of Optimal Decision Strategy for the Price Decision in the Monopolist's Decision Problem

tree model when the number of options for price is increased, particularly because Netica offers the options of populating a table of utility values using formulas or importing values from Excel. For instance, suppose the possible values for price are expanded to include the following set: {4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8, 8.5}. Additionally, the distribution of the random demand shock will now be modeled with an eight-point discrete approximation to the $N(0, 1)$ distribution, which will create eight corresponding potential demand functions.

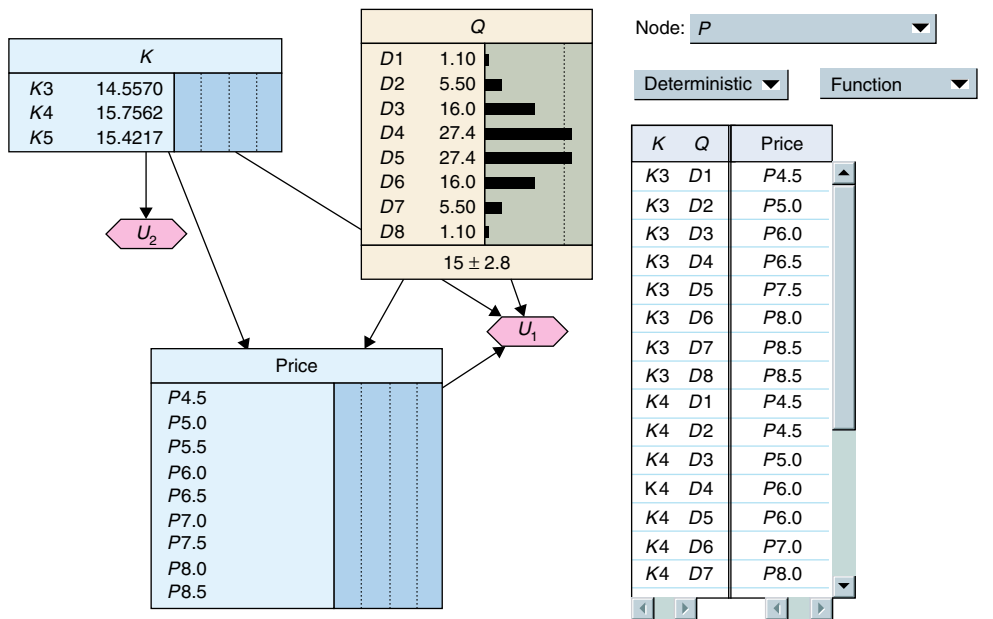
New values for the utility function u_1 can be calculated easily in Excel and imported into Netica. The revised solution and decision strategies appear in Figure 8. In two cases, the prices added to the set of possible choices are included in the optimal solution. Thus, increasing the number of possible prices provides a more refined decision rule and a better estimate (albeit lower) of maximum expected value as compared to the solution in the previous section.

Though the influence diagram solution to the capacity planning and pricing problem was first implemented as part of a managerial economics term paper project, it could potentially provide a valuable application problem for students in a management science course that teaches influence diagrams, particularly if the course is taught to students who are well-versed in the marginal analysis concepts of economics.

4.3. Continuous Price Variable

Recent advances in influence diagram models accommodate continuous chance and decision variables (Cobb 2007). With regard to the monopolist's decision problem, this allows the firm to determine a decision strategy for price as a continuous function of the random demand shock Z . The application of

Figure 8 Netica Display of the Solution and Optimal Decision Strategy for the Monopolist's Decision Problem with Eight States for Price and Demand



the influence diagram with a continuous price decision variable may be used as part of future managerial economics lectures. This model can provide some graphical intuition to help students understand that as the value of the demand shock increases, the optimal price increases.

The influence diagram model is revised in Figure 9, where the source of uncertainty is now the random demand shock Z , which is handled directly as a continuous variable.

The continuous decision variable model operates using the following joint utility function:

$$\begin{aligned}
 u_1(P, Z, K=3) &= \begin{cases} (10 - P + Z) \cdot (P - 1) - 3, & \text{if } 10 - P + Z \leq 3, \\ 3 \cdot (P - 1) - 3, & \text{if } 10 - P + Z > 3, \end{cases} \\
 u_1(P, Z, K=4) &= \begin{cases} (10 - P + Z) \cdot (P - 1) - 4, & \text{if } 10 - P + Z \leq 4, \\ 4 \cdot (P - 1) - 4, & \text{if } 10 - P + Z > 4, \end{cases} \\
 u_1(P, Z, K=5) &= \begin{cases} (10 - P + Z) \cdot (P - 1) - 5, & \text{if } 10 - P + Z \leq 5, \\ 5 \cdot (P - 1) - 5, & \text{if } 10 - P + Z > 5. \end{cases}
 \end{aligned}$$

The joint utility function (as opposed to its additive factors) is required to use the solution technique of Shenoy (1993) that is used for this type of influence diagram model.

Continuous variables are difficult to model directly in influence diagrams because closed-form mathematical operations are not possible with many probability density functions and some common forms for

utility functions. The models developed by Shachter and Kenley (1989), Poland and Shachter (1993), and Madsen and Jensen (2005) permit normal probability density functions but place restrictions on the configuration of discrete and continuous variables in the diagram and depend on a quadratic form for utility functions. The model developed by Cobb (2007) removes these restrictions by approximating probability distributions and utility functions using a format that allows the operations of combination and marginalization to be performed in closed form.

The deletion sequence in the continuous price model is P, Z, K . This model finds three piecewise linear decision rules for price corresponding to the cases where capacity is set at $K=3$, $K=4$, and $K=5$. These decision rules are shown graphically in Figure 10. For a given value of Z , the price is set highest when $K=3$ to provide rationing of a lower level of capacity through manipulation of market demand.

Because price is now completely determined as a function of Z , the piecewise linear function can be substituted for P in the utility function u_1 to obtain

Figure 9 Influence Diagram Model with Price as a Continuous Decision Variable

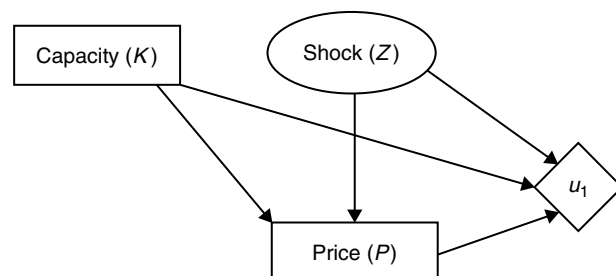
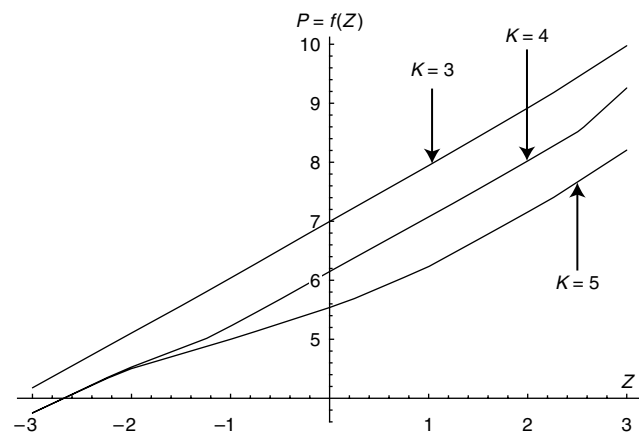


Figure 10 Piecewise Linear Decision Rules for Price



a new utility function u_2 for $\{K, Z\}$. This removes P from the influence diagram. This utility function has three parts with separate functions defined for the three possible capacities. As seen in Figure 11, for values below $Z \approx -1$, $K = 3$ is optimal whereas $K = 4$ is optimal for values between $Z \approx -1$ and $Z \approx 1$. For larger values of Z , $K = 5$ is optimal. Unfortunately, under partial uncertainty, the firm must choose capacity prior to knowing true demand.

To remove Z , the utility function u_2 for the cases where $K = 3$, $K = 4$, and $K = 5$ is combined with the pdf f_Z for Z . The results are shown graphically in Figure 12, and the function for $K = 5$ lies between the other two functions. Integrating the result over the state space of Z gives expected utilities for the two cases. This results in a larger expected utility value for the case where $K = 4$ of 15.90, so this is chosen as the optimal capacity.

We can appeal to marginal analysis techniques from economics to confirm the validity of the decision rules shown in Figure 10. By fixing the random demand shock to a value $Z = z$, we can determine the firm's

Figure 11 Utility Functions for the Cases Where $K = 3$, $K = 4$, and $K = 5$

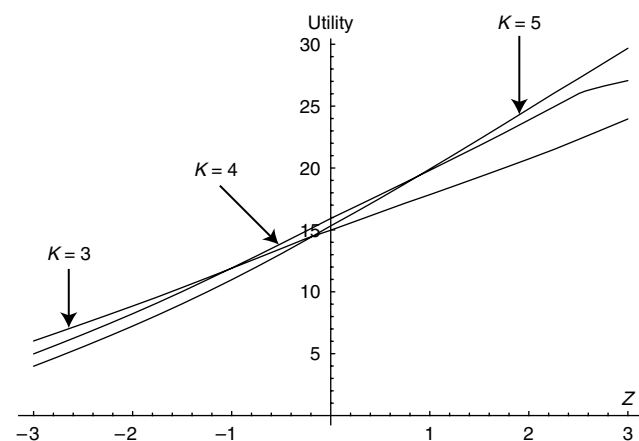
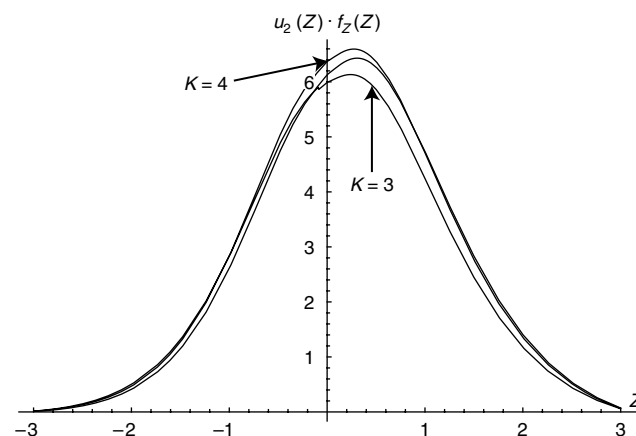


Figure 12 Functions Resulting from the Combination of the Utility Function u_2 and the pdf f_Z for Z



marginal profit functions as

$$\frac{du_1(P, z, K=3)}{dP} = \begin{cases} 11 + z - 2P, & \text{if } 10 - P + z \leq 3, \\ 3, & \text{if } 10 - P + z > 3. \end{cases}$$

$$\frac{du_1(P, z, K=4)}{dP} = \begin{cases} 11 + z - 2P, & \text{if } 10 - P + z \leq 4, \\ 4, & \text{if } 10 - P + z > 4. \end{cases}$$

$$\frac{du_1(P, z, K=5)}{dP} = \begin{cases} 11 + z - 2P, & \text{if } 10 - P + z \leq 5, \\ 5, & \text{if } 10 - P + z > 5. \end{cases}$$

For $K = 3$ and an observation of $Z = -1$, the discontinuous marginal profit function is shown in Figure 13. Of course, the firm would like to set its price where $du_1(P, -1, K=3)/dP = 0$; however, a suboptimal level of capacity prevents the firm from achieving the maximum profit it would receive with unconstrained capacity.

The firm should choose to set a price of $P = 6$ at the point where marginal profit is as close to zero as possible, a point that clearly occurs at the discontinuity in the marginal profit function. Note that at $P = 6$, the firm utilizes all of its available capacity because

Figure 13 Marginal Profit Function for $K = 3$ and $Z = -1$

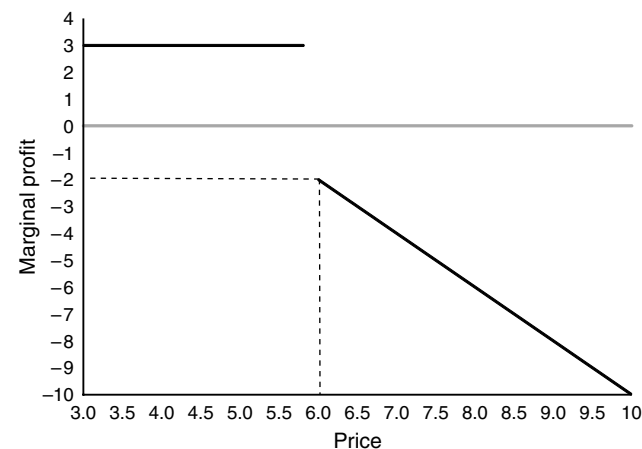
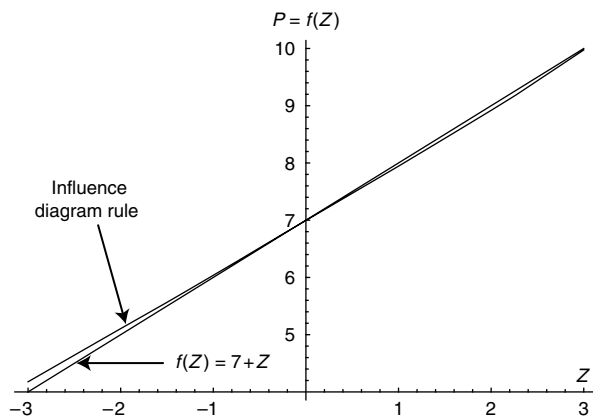


Figure 14 Function $P(Z) = 7 + Z$ Overlaid on the Decision Rule for P as a Function of Z from the Influence Diagram with $K = 3$



demand at $P = 6$ and $Z = -1$ is $Q = 3$. In fact, this condition holds for any arbitrary level of established capacity $K = k$ and observed value $Z = z$, and the firm should set $P = 10 - k + z$. For $K = 3$, the function $f(Z) = 7 + Z$ is shown in Figure 14 along with the influence diagram decision rule. This picture verifies that the influence diagram model determines a decision rule that is approximately consistent with the marginal analysis approach. In future applications of this influence diagram, an interactive software application could be developed to allow students to change the parameters in the model and observe how this affects the continuous decision rules.

5. Discussion

This paper has proposed utilizing decision trees and influence diagrams for incorporating demand uncertainty into economics decision problems. These models have been applied in business decision making for some time. The reason for introducing these models as tools for finding solutions to economics problems is to better understand how uncertainty affects profit and revenue maximization in problems where most economics textbooks present deterministic solutions that assume perfect knowledge of demand. The decision analysis models aid students in understanding the assumptions behind traditional profit maximization rules introduced in economics textbooks and help them see the results of relaxing these assumptions.

Most students in business or economics curriculum are taught to solve deterministic profit maximization problems in economics principles courses (such as the one in the certainty scenario from §3.1). In parallel, students often learn linear regression techniques and use them to estimate linear demand and cost functions in statistics and/or econometrics courses. At best, most upper-level undergraduate economics texts would connect these two concepts by teaching students that maximizing profit in the face of uncertain demand involves equating *expected*

marginal revenue with *expected* marginal cost (this would address the uncertainty scenario from §3.2). This paper further integrates profit maximization and demand uncertainty by teaching students to use decision trees and influence diagrams to incorporate demand information appropriately based on the time it is received. The partial uncertainty scenario in §3.3 demonstrates that this may involve situations other than the completely deterministic or completely uncertain scenarios.

The managerial economics course where the assignment we described was implemented had an enrollment of twelve students. One measure of the learning of the concepts and techniques was the understanding demonstrated by 4 of the 12 students in solving more complex problems in their term projects for the course (students were allowed to select term project topics from among a variety of concepts covered in the class). One student incorporated the observation of demand survey results received prior to making the capacity decision in the partial uncertainty scenario. A second student adapted the model to include a *soft capacity constraint* (as defined by Göx 2002), where the firm can increase capacity at a penalty cost if its initial capacity is too low to meet demand. Another single-product application where production can be augmented at an additional cost in response to updated demand information is the two-stage newsvendor model described by Donohue (2000). A third student increased the sample spaces for the decision variables and solved the resulting models. Finally, a fourth student who had been introduced to influence diagram models in a management science course compared the decision tree approach to the influence diagram approach, which inspired §4 of this paper.

The four extensions briefly described here demonstrated that these students learned two important ideas from the initial assignment. First, uncertainty affects decisions. The first three papers implemented three important strategies for managing uncertainty in capacity planning decisions: gathering more information, increasing the flexibility of the firm to adjust capacity, and increasing the number of decision alternatives available. Second, graphical models are useful for understanding and communicating business decisions. All of these projects required students to construct and solve more complicated models and thus demonstrate that they grasped the decision analysis concepts taught in the original assignment. The writing assignment required the students to describe the models as they would to a customer or manager.

To the extent that the students in the course learned about the effects of demand uncertainty on price-

ing and production quantity decisions, a reasonable question to ask is whether this learning occurred because graphical decision analysis models were used to introduce the topic or whether students could have grasped the concepts through purely mathematical approaches. An experiment that could verify that the graphical models contributed to the students' learning of these concepts could be designed by having students calculate expected profits under various demand scenarios in one section of a course, then requiring students in another section to use decision trees to ascertain these expected values. This may be the subject of future research if course schedules and class sizes permit. The elective course where the assignment was used did not have adequate enrollment or multiple sections available to perform such an experiment.

Marginal analysis concepts from economics and quantitative decision analysis models that capture uncertainty can complement each other to aid managers in making better decisions. In the capacity planning and pricing decisions presented in this paper, a manager might first use the traditional marginal-revenue-equals-marginal-cost profit maximization rule to determine the optimal price and production quantity under certainty.

Understanding that the certainty solution represents a best-case scenario, a manager may seek either additional information or determine when more information on actual demand will become available. If the manager finds that the capacity and pricing decisions will be made under different information constraints, a decision tree or influence diagram model can be used to incorporate the appropriate timing of the additional knowledge about demand.

As seen in §4.3, knowledge of marginal analysis concepts can help a manager check the validity of a decision model. Thus, traditional approaches from economics and graphical techniques from decision analysis can be complementary. Decision trees and influence diagrams can also be used to incorporate sample information, survey results, or other signals that might be observed by the firm prior to making price and capacity choices in the absence of complete knowledge of the true demand function.

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Appendix. Assignment for a Writing-Intensive Managerial Economics Course

A monopoly faces the demand function

$$Q = 10 - P + Z$$

for its single product where Z is a random demand "shock." The demand shock will be $Z = -2$ with probability 0.16,

$Z = 0$ with probability 0.68, or $Z = 2$ with probability 0.16. The firm can feasibly hire labor only once per period, so when it sets the size of its labor force, it establishes its production capacity. The firm can choose a labor force to allow production capacity K of either 4, 5, or 6 units. Each unit of production capacity costs \$1.00 per period and the firm incurs \$1.00 of variable materials costs per unit.

The firm has to make decisions on production capacity K and price P . The company will not know the value of the random demand shock Z prior to choosing its capacity; however, it will know the true value of the demand shock prior to choosing its price P , which can be set at either \$4.00, \$5.00, or \$6.00.

Solution Required

1. Suppose first that the firm's total cost function is $TC(Q) = 2Q$ (note from earlier that each unit incurs \$1.00 in capacity costs and \$1.00 in materials costs). If the firm can choose any capacity and price, determine the company's profit maximizing quantity at each possible level of the "demand shock." Determine the optimal price that would be realized in each case.

2. Construct a decision tree model for the monopolist's decision problem. Label all branches with payoffs (profits) and probabilities where required. Solve the decision tree to determine the complete optimal strategy and expected profit. NOTE: A decision tree template that can be used to solve this problem is posted on the course website.

Writing Required

Write a 750–1,000 word descriptive report. The following is a suggestion how to organize your report (using bold-faced or underlined section headings is recommended) and provides the details of the important points that must be addressed in the report.

Introduction and Problem Description. A description of the problem, the decision variables, the random variable, and the parameters for the model. Give a background on profit optimization for monopolists and describe your solution to Part 1 (above).

Methods. An explanation of the decision tree and the solution technique used. Describe the solution to Part 2 (above).

Results. The overall results of your solution and related observations.

Conclusions and Recommendations for Managers.

Audience

You are a decision analyst for the firm. The content should be written for the production manager, who has knowledge of the problem but not necessarily the models you will use to solve the problem. Approach the assignment as if you are an analyst for the firm making this decision and have been asked to prepare this report for your manager. Preparing useful, concise reports is a tremendously useful skill in many types of jobs that you may be interested in pursuing.

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