

Low Complexity Expert Dependent Noise Filtration Algorithm

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Abstract- In this paper, a flexible and robust wavelet based image denoising algorithm is proposed, which adapts itself to various and unknown types of noise as well as to the preference of the medical expert: a single tuning parameter is used to balance the preservation of relevant details against the degree of noise reduction. We employ a preliminary coefficient classification technique to empirically estimate the statistical distributions of the coefficients that represent useful image features on the one hand and mainly noise on the other. The proposed algorithm is of low-complexity, both in its implementation and execution time. The results show that its usefulness for denoising and enhancement of the CT, Ultrasound and Magnetic Resonance images.

KEYWORDS: filtering, Rician noise, speckle noise, Detection and Estimation.

I. INTRODUCTION

The image denoising plays a significant role in modern applications in various fields, including medical imaging and preprocessing for computer vision. Medical imaging acquisition technologies and systems introduce noise and artifacts in the images that should be attenuated by denoising algorithms. The denoising process, however, should not destroy anatomical details relevant from a clinical point of view. So, it is very difficult to suggest a robust method for noise removal which works equally well for different modalities of medical images. Also biomedical images show extreme variability and it is necessary to operate on a case by case basis. This motivates us the construction of robust and versatile denoising methods that are applicable to various circumstances, rather than being optimal under very specific conditions [1], [3]. In this paper, we propose a robust method that adapts itself to various types of image noise as well as to the preference of the medical expert: a single

tuning parameter can be used to balance the preservation of relevant details against the degree of noise reduction.

In image denoising one often faces uncertainty about the presence of a given “feature of interest” (e.g., an image edge) in a noisy observation. Due to the sparsity of the wavelet representation, the Middleton’s optimum coupled detection and estimation approach [2] seems well suited for wavelet domain image denoising. Bayesian methods [4], [5] take the uncertainty of the signal presence into account implicitly, assuming a Bernoulli process on the wavelet coefficients [6] and using Gaussian mixture models for the probability density functions of the wavelet coefficients. Hidden Markov tree models [7], [8] or Markov random field prior models [9], [10] are spatially adaptive methods usually employ complex algorithms. Other algorithms such as spatially adaptive thresholding and locally adaptive Wiener filtering can be found in [11] and [12].

In this paper, we propose a related, but more flexible method, which is applicable to various and unknown types of image noise. We employ a preliminary detection of the wavelet coefficients that represent the features of interest in order to *empirically estimate* the conditional pdf’s of the coefficients given the useful features and given background noise. At the same time, the preliminary coefficient classification is also exploited to empirically estimate the corresponding conditional pdf’s of the local spatial activity indicator. The preliminary classification step in the proposed method relies on the persistence of useful wavelet coefficients across the scales [13], and is related to the one in [38], but avoids its iterative procedure. In contrast to [2], and related methods like [14], where the inter-scale correlations between wavelet coefficients are used for a “hard” selection of the coefficients from which the denoised image is reconstructed, our algorithm performs a soft modification of the coefficients adapted to the spatial image context. The classification step of the proposed method involves an adjustable parameter that is

related to the notion of the expert-defined “relevant image features”. In certain applications the optimal value of this parameter can be selected as the one that maximizes the signal-to-noise ratio (SNR) and the algorithm can operate as fully automatic. However, we believe that in most medical applications the tuning of this parameter leading to a gradual noise suppression may be advantageous. The proposed algorithm is simple to implement and fast. We demonstrate its usefulness for denoising and enhancement of the ultrasound and the magnetic resonance images.

The paper is organized as follows. In Section II, the theoretical concept behind the proposed method and the new, practical algorithm are described. The application of the proposed method to ultrasound images is demonstrated and discussed in Section III. In Section IV, noise removal from the magnetic resonance images is addressed. The results are also discussed in Sections III and IV, and the concluding remarks are given at the end.

II. THE EXPERT NOISE FILTERING TECHNIQUE

A BASIC THEORY

A general noise model is defined as $y_k = w_k \oplus n_k$, where w_k is the unknown noise-free wavelet coefficient, \oplus a point-wise mathematical operation (addition in the case of additive noise and multiplication in the case of speckle noise) and n_k an arbitrary noise contribution. Our wavelet domain estimation approach relies on the joint detection and estimation theory and is related to the problem of the spectral amplitude estimation in [15]. The algorithm is implemented using the quadratic spline wavelets [13].

Let x_k denote a random variable, which takes values y_k from the binary label set $\{0,1\}$. The hypothesis “the wavelet coefficient y_k represents a signal of interest” is equivalent to the event $x_k = 1$, and the opposite hypothesis is equivalent to $x_k = 0$. The wavelet coefficients representing the signal of interest in a given sub band are identically distributed random variables with the probability density function $\hat{p}_{Y_k|X_k}(w_k|1)$. Similarly, the coefficients in the same sub band, corresponding to the absence of the signal of interest, are random variables with the pdf $\hat{p}_{Y_k|X_k}(w_k|0)$.

Under the model assumptions, the minimum mean squared error estimate (the conditional mean) of w_k is $\hat{w}_k = E(w_k | y_k, X_k = 1)P(X_k = 1 | y_k) + E(w_k | y_k, X_k = 0)P(X_k = 0 | y_k)$ where $E(\cdot)$ stands for the expected value. If

the signal of interest is surely absent in a given wavelet coefficient, then $w_k = 0$ and $E(w_k | y_k, X_k = 0) = 0$. In the case where the signal of interest is surely present, we approximate $E(w_k | y_k, X_k = 1) = y_k$ which accounts for the fact that vast majority of the coefficient magnitudes representing the signal of interest are highly above the noise level. Applying Bayes’ rule, one can express $P(X_k = 1 | y_k)$ as a generalized likelihood ratio, and our estimate becomes

$$\hat{w}_k = \frac{\xi_k \mu_k}{1 + \xi_k \mu_k} y_k \quad (1)$$

$$\text{where } \xi_k = \frac{p_{Y_k|X_k}(y_k|1)}{p_{Y_k|X_k}(y_k|0)} \text{ and } \mu_k = \frac{P(X_k = 1 | P)}{P(X_k = 0 | P)} \quad (2)$$

and P symbolically denotes the prior knowledge that is used to estimate the probability of signal presence. In [16], Pizurica proposed a method to estimate this probability for each wavelet coefficient from its local surrounding, using a chosen indicator e_k of the local spatial activity. In particular, since our estimate of the probability of signal presence is a function of e_k , we write $P(X_k = 1 | P) = P(X_k = 1 | e_k)$, and replace μ_k in (2) by

$$\mu_k = \frac{P(X_k = 1 | e_k)}{P(X_k = 0 | e_k)} = r \frac{p_{Y_k|X_k}(e_k|1)}{p_{Y_k|X_k}(e_k|0)} \quad (3)$$

where r is the ratio of unconditional prior probabilities

$$r = \frac{P(X_k = 1)}{P(X_k = 0)} \quad (4)$$

For a given type of noise, one can derive the complete estimator analytically. In such approaches where the required conditional densities need to be expressed analytically, the choice of the local spatial activity indicator is usually restricted to simple forms: even when e_k is defined simply as the locally averaged coefficient magnitude, certain simplifying assumptions about the statistical properties of the wavelet coefficients are needed in order to derive $p_{E_k|X_k}(e_k|x_k)$ analytically. The algorithm that we propose in this paper is applicable to various noise types, and allows an arbitrary choice of e_k .

The idea behind the proposed algorithm is to empirically estimate the probabilities and the probability density functions that specify the estimator. Let N denote the number of wavelet coefficients in a detail image. For each detail image $\mathbf{Y}_j^D = \{y_{1,j}^D, \dots, y_{N,j}^D\}$, we first estimate the *mask*

$\hat{\mathbf{X}}_j^D = \{x_{1,j}^D \dots x_{N,j}^D\}$ which indicates the positions of significant wavelet coefficients (representing the signal of interest). As usual, we relate the notion of significant wavelet coefficients to the standard deviation of the noise. Also, we rely on the persistence of significant wavelet coefficients across resolution scales. In particular, we extend our robust coarse-to-fine classification method from as follows:

$$\hat{x}_{k,j}^D = \begin{cases} 0, & \text{if } |y_{k,j}^D| \|\hat{w}_{k,j}^D\| < K(\hat{\sigma}_j^D)^2 \\ 1, & \text{if } |y_{k,j}^D| \|\hat{w}_{k,j}^D\| \geq K(\hat{\sigma}_j^D)^2 \end{cases} \quad (5)$$

where $\hat{\sigma}_j^D$ is an estimate of the noise standard deviation in the detail image \mathbf{Y}_j^D , and K is a heuristic, tunable parameter that controls the notion of the signal of interest. We estimate the standard deviation of the input noise $\hat{\sigma}$ as the median absolute deviation of the wavelet coefficients in the HH subband at the finest resolution scale, divided by 0.6745.

In estimating $\hat{\sigma}_j^D$, we follow $(\hat{\sigma}_j^D)^2 = S_j^D \hat{\sigma}^2$, where for each subband the constant S_j^D is calculated from the filter coefficients of the highpass filter \mathbf{g} and the lowpass filter \mathbf{h} of the discrete wavelet transform, as $S_j^{LH,HL} = \left(\sum_k g_k^2\right) \left(\sum_l h_l^2\right)$ and $S_j^{HH} = \left(\sum_k g_k^2\right)^2 \left(\sum_l h_l^2\right)^{2(j-1)}$. To initialize the classification (5), we start from $\hat{\mathbf{W}}_J^D = \mathbf{Y}_J^D$, where J is the coarsest resolution level in the wavelet decomposition.

Now we address the estimation of the wavelet coefficients \mathbf{Y}_j^D using the estimated mask \mathbf{x}_j^D . The estimator requires the conditional densities $p_{Y_k|X_k}(y_k | x_k)$ and $p_{E_k|X_k}(e_k | x_k)$. Since $p_{Y_k|X_k}(y_k | x_k)$ is usually highly symmetrical around 0, in practice we shall rather estimate the conditional pdf's $p_{M_k|X_k}(m_k | x_k)$ of the coefficient *magnitudes* $m_k = |y_k|$. As the local spatial activity indicator e_k , we use the averaged energy of the neighboring coefficients of y_k where the neighbors are the surrounding coefficients in a square window at the same scale and the “parent” (i.e., the coefficient at the same spatial position at the first coarser scale). Having the estimated mask $\hat{\mathbf{x}} = \{\hat{x}_1 \dots \hat{x}_N\}$, let $S_0 = \{k : \hat{x}_k = 0\}$, and $S_1 = \{k : \hat{x}_k = 1\}$. The empirical estimates $\hat{p}_{M_k|X_k}(m_k | 0)$ and $\hat{p}_{E_k|X_k}(e_k | 0)$ are computed from the histograms of $\{m_k : k \in S_0\}$ and $\{e_k : k \in S_0\}$

respectively (by normalizing the area under the histogram). Similarly, $\hat{p}_{M_k|X_k}(y_k | 1)$ and $\hat{p}_{E_k|X_k}(e_k | 1)$ are computed from the corresponding histograms for $k \in S_1$.

Our estimation approach still requires the probability ratio. Reasoning that $P(X_k = 1)$ can be estimated as the fractional number of labels for which $\hat{x}_k = 1$, we estimate the parameter r as

$$\hat{r} = \frac{\sum_{k=1}^N \hat{x}_k}{N - \sum_{k=1}^N \hat{x}_k} \quad (6)$$

Then the final estimation is defined as

$$\hat{w}_k = \frac{\hat{r} \hat{\xi}_k \hat{\eta}_k}{1 + \hat{r} \hat{\xi}_k \hat{\eta}_k} y_k \quad (7)$$

where

$$\hat{\xi}_k = \frac{\hat{p}_{M_k|X_k}(m_k | 1)}{\hat{p}_{M_k|X_k}(m_k | 0)} \quad \text{and} \quad \hat{\eta}_k = \frac{\hat{p}_{E_k|X_k}(e_k | 1)}{\hat{p}_{E_k|X_k}(e_k | 0)} \quad (8)$$

In Fig.1, we show an example of the empirical densities $\hat{p}_{M_k|X_k}(m_k | x_k)$ and $\hat{p}_{E_k|X_k}(e_k | x_k)$. The direct computation of the ratios $\hat{\xi}_k$ and $\hat{\eta}_k$ from the normalized histograms shown in Fig.1 is not appropriate due to errors in the tails. One solution is to first fit a certain distribution to the histogram. Here we apply a simpler approach, observing that both $\log(\hat{\xi}_k)$ and $\log(\hat{\eta}_k)$ can be approximated well by fitting a piece-wise linear curve as illustrated in Fig.1. Formally, we approximate

$$\log(\hat{\xi}_k) \square \begin{cases} a_1 + b_1 m_k, \hat{\xi}_k < 1 \\ a_2 + b_2 m_k, \hat{\xi}_k \geq 1 \end{cases} \quad (9)$$

$$\log(\hat{\eta}_k) \square \begin{cases} c_1 + d_1 m_k, \hat{\eta}_k < 1 \\ c_2 + d_2 m_k, \hat{\eta}_k \geq 1 \end{cases} \quad (10)$$

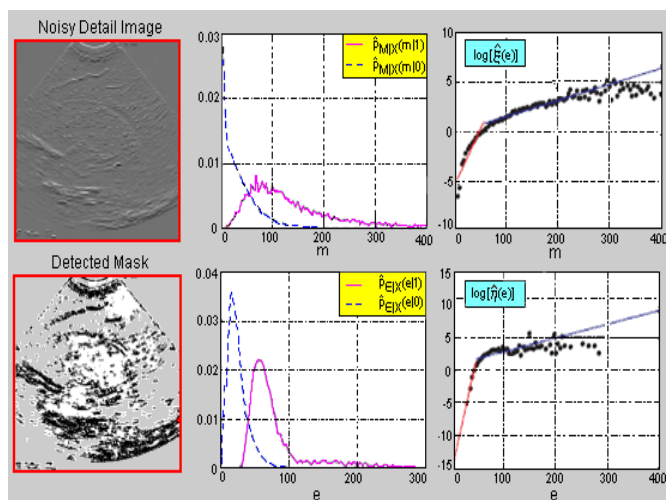


Fig 1 Examples of the empirical pdf's and fitted log-ratios in the proposed method, for the top left ultrasound image.

II. APPLICATION TO ULTRASOUND IMAGES

Ultrasound images are corrupted by speckle noise, which affects all coherent imaging systems. We compare the performance of the proposed method to one conventional approach in speckle filtering: the homomorphic Wiener filter. In particular, we apply Matlab's spatially adaptive Wiener filter to the image logarithm and subsequently perform the exponential transformation on the filtered output. The window size of the Wiener filter was experimentally optimized to produce the maximum output SNR for each test image and for each amount of noise used in the simulations.

Table 1 shows the quantitative comparison of widely used metrics, signal to noise ratio (SNR), and peak signal to noise ratio (PSNR). The computation time for each algorithm is also included in this table. We can notice that our proposed filter exhibits more than 2dB improvement in both SNR and PSNR over Homomorphic Wiener filter. The results clearly demonstrate that the proposed filter outperforms the homomorphic spatially adaptive Wiener filtering both in terms of SNR and PSNR.

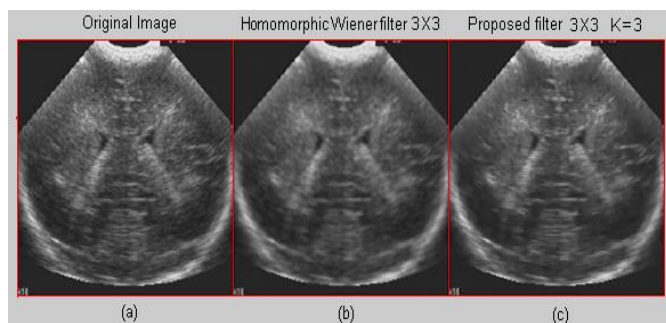


Fig. 2 Application to the real noisy image-2 (a) Real speckle noised Ultrasound image2 (b) Wiener filter (c) proposed filter

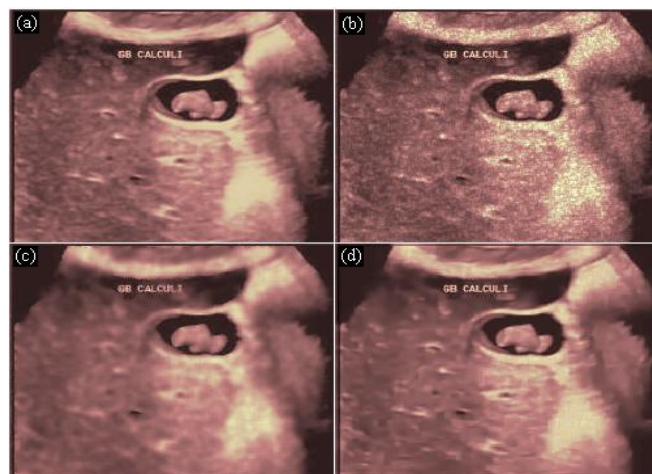


Fig.3. Visual Comparison for Ultrasound Gallbladder image (a) Original Image (b) artificially speckled images, the results of the homomorphic spatially adaptive Wiener filter, and the results of the proposed method, for $K=3$ and window size 5×5 .

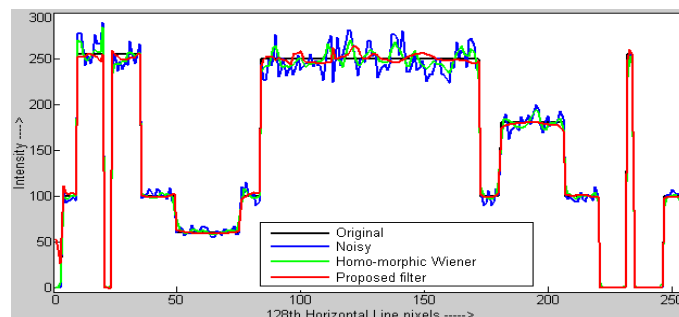


Fig.4. Horizontal profile comparison for 'synth' image.

Table 1 Signal to Noise Comparisons for the three Test Images.

Reconstruction Method	Test Image1, 5X5, K=3			Test Image2, 5X5, K=3			Test Image3, 5X5, K=7		
	SNR (dB)	PSNR (dB)	Comp.Time (Sec)	SNR (dB)	PSNR (dB)	Comp.Time (Sec)	SNR (dB)	PSNR (dB)	Comp.Time (Sec)
Noisy Image	17.75	25.06	---	17.71	27.31	---	23.89	32.60	---
Homomorphic Wiener Filter	17.86	25.16	0.078	19.05	28.65	0.078	26.96	35.67	0.016
Proposed Filter	19.51	26.81	2.562	20.73	30.34	2.312	29.02	37.72	0.75

IV APPLICATION TO MRI IMAGES

In magnetic resonance imaging (MRI) the practical limits of the acquisition time impose a trade-off between the signal to noise ratio and the image resolution. The MRI image is commonly reconstructed by computing the inverse discrete Fourier transform of the raw data. Most commonly, the magnitude of the reconstructed image is used for visual inspection and for computer analysis. Noise in the MRI image magnitude is Rician, having a signal dependent mean.

It was noted that, due to the signal-dependent mean of the Rician noise, both wavelet and scaling coefficients of a noisy MRI image are *biased* estimates of their noise-free counterparts. It was shown that one can efficiently overcome this problem by filtering the *square* of the MRI magnitude image in the wavelet domain. In the squared magnitude image, data are non-central chi-square distributed, and the wavelet coefficients are no longer biased estimates of their noise-free counterparts. The bias still remains in the scaling coefficients, but is not signal dependent and it can be removed easily at the resolution scale 2^j , from each scaling coefficient $2^{j+1}\sigma_c$ should be subtracted, where σ_c^2 is the under lying complex Gaussian noise variance. We therefore apply our method to the squared magnitude of the MRI image, subtract the constant bias from the scaling coefficients, and subsequently compute the square root of the denoised squared magnitude image.

Three clinical MR Images, a Pelvic MR Image of size 644X626, a Brain MR Image of size 471X341, and a Spine MR Image of size 490X486 are used for the experimental evaluation purpose. In simulations, complex zero mean white Gaussian noise with standard deviation $\sigma_c = 30$ was added to these images. Following figure shows the denoising result of the proposed method comparison with the spatially adaptive wiener filter. The results show that our proposed algorithm out performs with that of the spatially adaptive Wiener filtering. Table 2 shows the quantitative comparison of widely used metrics, signal to noise ratio (SNR), and peak signal to noise ratio (PSNR). The computation time for each algorithm is also included in this table. We can observe that 5.35 dB improvement in both SNR and PSNR.

V CONCLUSIONS

A flexible and robust wavelet domain method for noise filtering in medical images is designed and presented. This method adapts itself to various and unknown types of image noise as well as to the preference of the medical expert: a single tuning parameter can be used to balance the preservation of relevant details against the degree of noise reduction. The presented algorithm is a low-complexity, both in its implementation and execution time. The simulation results show that our algorithm performs better in terms of signal to noise ratio in relation to the existing methods in reducing the various types of noise such as Speckle noise and Rician noise by adjusting a single parameter.

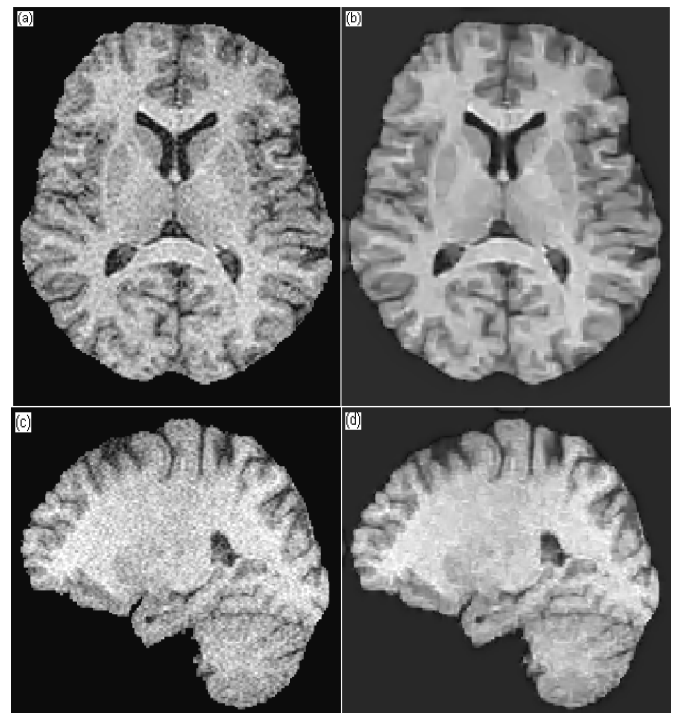


Fig. 5. Application to real noisy MRI images. (a) and (c) original images, (b) and (d) the corresponding results of the proposed method for $K = 2$ and a 3x3 window size.

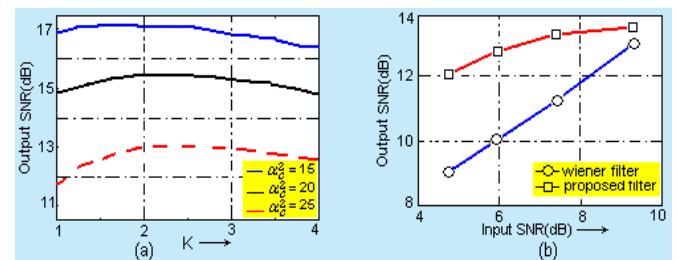


Fig.6. Quantitative performance of the proposed method. (a) Influence of the parameter K for different values α_c^2 of the underlying complex Gaussian noise. (b) Noise suppression performance (for $K = 2$) in comparison to the spatially adaptive Wiener filtering.

Table 2 Signal to Noise Ratio comparison for three images. Pelvic MR Image, Brain MR Image and Spine MR Image, window size 3X3, $K = 2$.

Reconstruction Method	Pelvic MR Image size 644 X 626 $\sigma = 30$			Brain MR Image size 471 X 341 $\sigma = 30$			Spine MR Image size 490 X 486 $\sigma = 30$		
	SNR (dB)	PSNR (dB)	Comp.Time (sec)	SNR (dB)	PSNR (dB)	Comp.Time (sec)	SNR (dB)	PSNR (dB)	Comp.Time (sec)
Noisy Image	11.66	18.59	---	08.32	17.68	---	13.00	18.86	---
Spatially Adaptive Wiener Filter	16.23	23.16	0.325	10.99	20.35	0.187	19.12	24.98	0.297
Proposed Filter	20.33	27.26	11.344	16.34	25.70	5.672	19.49	25.35	9.813

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