

Optimization of Charging Current and SOH Estimation for Lead Acid Batteries

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Abstract

In this paper a new model-based approach is used to optimize the charging current of lead acid batteries for use in hybrid electric. The used model is a dynamical nonlinear model and so steepest descent, as a nonlinear optimization technique, is used to design the desired current profile. To verify the results, Unscented Kalman Filter is used to estimate battery capacity as a criterion of the state of health of the battery. Simulation results show that in comparison with multi level charging current, the proposed approach improves the state of health of the battery, up to 2.5% in the first 100 charge/discharge cycle.

1. Introduction

In recent years, batteries play an important role in many fields, such as portable equipments, hybrid electric vehicles (HEVs), etc. Battery life is one of major factors presently limiting the realization of economically viable HEVs [1].

Different charging methods exist in order to increase battery lifetime and reduce the charging time. The simplest charging method is constant current charging in which constant current is used to charge the series batteries. However battery over charging will result in the degradation of battery life and small charging current will prolong the charging time. To overcome this problem, two step charging method (CC-CV) is used, which combines the constant current and constant voltage charging. In the first stage of charging, the batteries are charged by constant current until the battery voltage reaches a preset value and in the second stage, a constant voltage is applied for battery charging [1]. CC-CV is still not suitable for rapid charging since constant voltage charging seriously extends the charging time. In [2] a fuzzy controlled active state of charge controller is proposed to replace the constant voltage stage in order to reduce the charging time. The proposed charger can adaptively provided suitable charging current for the battery. Also, other charging algorithms exist for the reduction of charging time. These methods include pulse charging algorithm [1,3], adaptable multilevel current controller [4], etc. In [5], a genetic algorithm approach to optimize a fuzzy-rule-based system was presented; however, the drawback of this approach is that a microcontroller is required to implement the

proposed algorithm. The multistage constant-current charging algorithm has the advantages such as prolonged cycle lives, enhanced discharge/ charge energy efficiency, and reduced charging time [6]. To determine the optimal control value in each stage, all possible combination of charging current values should be tested, that is not economical for the manufacturer. In [7], ant colony system algorithm is used to determine the charging current in each stage.

In this paper we introduce a model-based method to optimize the charging current for the battery. To show that the designed current is the optimized charging current, the SOH of the battery should be measured. The most reliable way of measuring SOH of batteries is to take them off-line and perform a load test on them. However, this is a costly, labor-intensive approach that requires the batteries to be taken off-line and is therefore typically performed at off-peak hours. Also a discharge test is detrimental to batteries, since routine deep discharges can reduce the life of the battery. Considerable savings could be made if the battery testing could be performed on-line in a reliable manner [8]. Because of these shortcomings of the discharge test, users are increasingly turning to the use of much simpler methods such as: (a) partial battery discharge tests coupled with algorithms to calculate battery state of charge and (b) ohmic techniques including impedance, resistance and conductance measurements to calculate the state of health. However, the accuracy of these techniques largely depends on the depth of discharge for (a), and the way the contact is made between the battery terminals and the leads of the ohmic meter for (b) [9]. A different approach involves the intelligent inspection of ac impedance data and using the magnitude and phase angle at different frequencies as inputs to a fuzzy logic model to estimate battery SOH [8]. Another way of accurately estimating the state of health of a lead acid battery is measuring the coup de fouet voltage appearing in the early minutes of the battery discharge. The method involves measuring the trough voltage (low voltage point) in the coup de fouet region and using it in determining the battery state of health [9]. In [10], EKF is used to estimate the SOH. Although the EKF is a widely used filtering strategy, it is difficult to implement, difficult to tune, and only reliable for systems which are almost linear on the time scale of the update intervals

In this paper a dynamical model of the battery is used. Because of nonlinearity of the model, steepest descent method is applied to it to determine the optimal charging current and then Unscented Kalman Filter (UKF), a new technique for estimating the states in nonlinear systems, is used to estimate the SOH of the battery in order to verify the results.

The paper is organized as followed: in section 2, a dynamical model of the battery is described. Steepest descent method for charging current control is applied to the model in section 3 and in section 4, UKF is used to estimate the SOH.

2. Battery model

There exist several battery models with different degrees of complexity for different applications. For our purpose, we need a dynamical battery model in the form of state variable equations.

In this study, the model described in [10] is used-see Fig.1. This model consists of a bulk capacitor C_{bulk} to characterize the ability of the battery to store charge, a capacitor to model surface capacitance and diffusion effects within the cell $C_{surface}$, a terminal resistance R_t , surface resistance R_s and end resistance R_e .

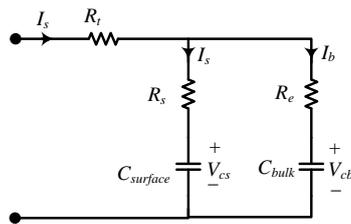


Figure 1. Battery RC model

State variable description of the network is as (for more details see [10]):

$$\begin{bmatrix} \dot{V}_{Cb} \\ \dot{V}_{Cs} \\ \dot{V}_o \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_{bulk}(R_e + R_s)} & \frac{1}{C_{bulk}(R_e + R_s)} & 0 \\ \frac{1}{C_{surface}(R_e + R_s)} & -\frac{1}{C_{surface}(R_e + R_s)} & 0 \\ A(3,1) & 0 & A(3,3) \end{bmatrix} \begin{bmatrix} V_{Cb} \\ V_{Cs} \\ V_o \end{bmatrix} + B \cdot I(1) \quad (1)$$

$$A(3,1) = -\frac{R_s}{C_{bulk}(R_e + R_s)^2} + \frac{R_e}{C_{surface}(R_e + R_s)^2} - \frac{R_s^2}{C_{bulk}R_e(R_e + R_s)^2} + \frac{R_s}{C_{surface}(R_e + R_s)^2}$$

$$A(3,3) = \frac{R_s}{C_{bulk}R_e(R_e + R_s)} - \frac{1}{C_{surface}(R_e + R_s)}$$

$$B = \begin{bmatrix} \frac{R_s}{C_{bulk}(R_e + R_s)} \\ \frac{R_e}{C_{surface}(R_e + R_s)} \\ \frac{R_e^2}{C_{surface}(R_e + R_s)^2} - \frac{R_s R_t}{C_{bulk}R_e(R_e + R_s)} \\ + \frac{R_t}{C_{surface}(R_e + R_s)} + \frac{R_s R_s}{C_{bulk}(R_e + R_s)^2} \end{bmatrix}$$

In these equations, C_{bulk} is assumed to be constant, but in practice this is not the case. The ability of the battery to store energy decreases with cell usage. We model this, by reduction of C_{bulk} in the model. Also experiments show that this reduction will be more serious if the charging current is high. To consider the effect of both time and charging current, C_{bulk} is modeled as:

$$C_{bulk} = k I t + C_0 \quad (2)$$

where k is a constant to be determined. To determine k , C_{bulk} should be estimated in different times and with different charging currents. This is done in section 4. Here we use the results and the following values are considered for k and C_0 : $k = -0.002$, $C_0 = 1000$ so

$$C_{bulk} = -0.002 I t + 1000 \quad (3)$$

The obtained equation for C_{bulk} causes the state equations to be nonlinear. Also we will have an extra state equation, C_{bulk} , which was assumed to be constant in the described model. New state variable description of the system is as:

$$\dot{x} = f(x, u) \quad (4)$$

$$x = [V_{Cb}, V_{Cs}, V_o, C_{bulk}]^T$$

$$f(x, u) = \begin{bmatrix} -\frac{V_{Cb}}{C_{bulk}(R_e + R_s)} + \frac{V_{Cs}}{C_{bulk}(R_e + R_s)} + \frac{I R_s}{C_{bulk}(R_e + R_s)} \\ \frac{1}{C_{surface}} \left[\frac{V_{Cb}}{(R_e + R_s)} - \frac{V_{Cs}}{(R_e + R_s)} + \frac{I R_e}{(R_e + R_s)} \right] \\ V_{Cb} \cdot f_1 + V_o \cdot f_2 + I \cdot f_3 \\ -0.002 I \end{bmatrix}$$

$$f_1 = -\frac{R_s}{C_{bulk}(R_e + R_s)^2} + \frac{R_e}{C_{surface}(R_e + R_s)^2}$$

$$-\frac{R_s^2}{C_{bulk}R_e(R_e + R_s)^2} + \frac{1}{C_{surface}(R_e + R_s)^2}$$

$$f_2 = \frac{R_s}{C_{bulk}R_e(R_e + R_s)} - \frac{1}{C_{surface}(R_e + R_s)}$$

$$f_3 = \frac{R_e^2}{C_{surface}(R_e + R_s)^2} - \frac{R_s R_t}{C_{bulk}R_e(R_e + R_s)} + \frac{R_t}{C_{surface}(R_e + R_s)} + \frac{R_e R_s}{C_{surface}(R_e + R_s)^2}$$

3. Charging current optimization

Because of nonlinearity of the described battery model, we should use a nonlinear optimization method in order to design the charging current. In this study, the method of steepest descent is applied to the model.

3.1. Steepest Descent method

Steepest descent is an iterative numerical

technique for determining optimal controls and trajectories [12]. It determines an open-loop optimal control, which is the optimal control history associated with a specified set of initial conditions. The Hamiltonian is used in this technique, which is defined as:

$$\dot{x}(t) = a(x(t), u(t), t) \quad (5)$$

$$J = h(x(t_f), t_f) + \int_{t_0}^{t_f} g(x(t), u(t), t) dt \quad (6)$$

$$H(x(t), u(t), p(t), t) = g(x(t), u(t), t) + p^T(t)[a(x(t), u(t), t)] \quad (7)$$

where $p(t)$ is the costate and the necessary conditions for optimization are:

$$\dot{p}(t) = -\frac{\partial H}{\partial x}(x^*(t), u^*(t), p^*(t), t) \quad (8)$$

$$\frac{\partial H}{\partial u}(x^*(t), u^*(t), p^*(t), t) = 0 \quad (9)$$

As these equations are nonlinear, they can not be solved analytically to obtain the optimal control law. For solving this problem, the following iterative method is used. First a discrete approximation to the nominal control history, $u^{(0)}(t)$, $t \in [t_0, t_f]$ is selected. Using the nominal control history, $u^{(i)}$, the state equations from t_0 to t_f are integrated with initial conditions $x(t_0) = x_0$. Then $p^{(i)}(t_f)$ is calculated from $x^{(i)}(t_f)$. Using this value of $p^{(i)}(t_f)$ as the initial condition and $x^{(i)}$, the costate equations from t_f to t_0 are integrated and $\frac{\partial H^{(i)}(t)}{\partial u}$, $t \in [t_0, t_f]$ is evaluated. if

$$\left\| \frac{\partial H^{(i)}}{\partial u} \right\| \leq \gamma \quad (10)$$

the iterative procedure is terminated, unless we should generate a new control function given by:

$$u^{(i+1)}(t) = u^{(i)}(t) - \tau \frac{\partial H^{(i)}}{\partial u}(t) \quad t \in [t_0, t_f] \quad (11)$$

where τ is the step size. $u^{(i)}(t)$ is replaced with $u^{(i+1)}(t)$ and the above procedure will be repeated until (10) is satisfied. When (11) is used to generate the new control function, it is guaranteed that in every stage, the cost function is smaller than the previous stage (for more details see [12]) and finally reaches its minimum value.

3.2. Simulation Results

The following cost function is considered for designing the optimized charging current in order to

reduce the charging time, improve the battery lifetime and limit the input current.

$$J = \int_{t_0}^{t_f} \left\{ (V_{cb, \max} - V_{cb})^2 Q_1 + (C_{b, \max} - C_{bulk})^2 Q_2 + I^2 R \right\} dt \quad (12)$$

where $V_{cb, \max}$ is the voltage at the end of the charging process, and $C_{b, \max}$ is the maximum battery capacity at the beginning of the charging process. The values of parameters given in table 1 are used to solve the optimization problem.

Q_1 , Q_2 and R are selected due to the importance of the charging time, battery lifetime and input current for the specified application. In this study the following values are used: $Q_1=100$, $Q_2=1$ and $R=0.1$. Simulation results are shown in Fig (2). Fig (3) shows the results if Q_1 and Q_2 are increased, respectively. Increasing Q_1 results in higher voltage in the same time and increasing Q_2 results in lower voltage, but improved capacity, as will be shown in the next section.

Table 1. Parameters for the Cell Model

| R_e | R_s | R_t | $C_{surface}$ | V_{Cbmax} | $C_{b,max}$ |
|----------|----------|-----------|---------------|-------------|-------------|
| 0.00375Ω | 0.00375Ω | 0.002745Ω | 82.11F | 2V | 1000F |

4. Battery SOH Estimation

In this paper, UKF, a new method for estimation of nonlinear dynamical systems models, is used to estimate the battery SOH.

The Extended Kalman Filter has been one of the most widely used methods for estimation of nonlinear systems through linearization non-linear models. In recent several decades people have realized that there are a lot of constraints in application of the EKF for its hard implementation and intractability. The UKF addresses the approximation issues of the EKF. The state distribution is again represented by a Gaussian Random Variable (GRV), but is now specified using a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the GRV, and when propagated through the true nonlinear system, captures the posterior mean and covariance accurately to the 3rd order (Taylor series expansion) for any nonlinearity. To elaborate on this, we start by first explaining the unscented transformation.[14]

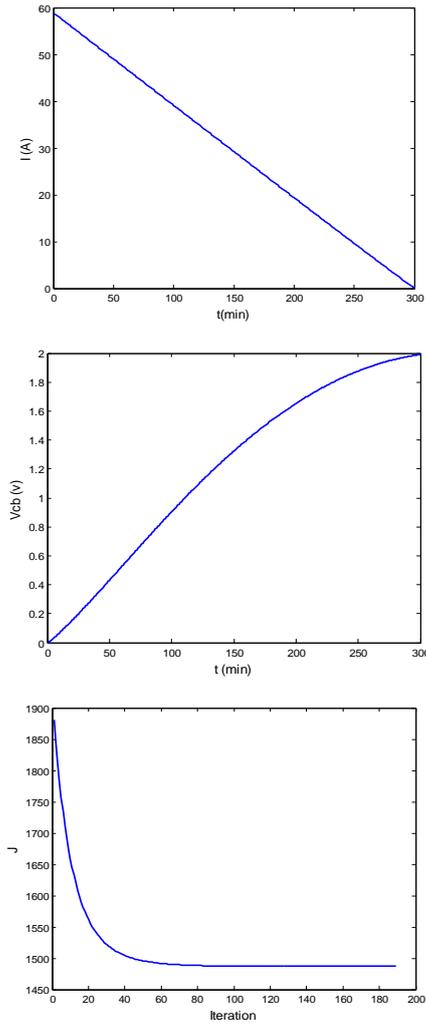


Figure 2. Simulation results: battery charging current, battery voltage and cost function

4.1. Unscented transformation

The unscented transformation (UT) is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation [13]. Consider propagating a random variable (dimension L) through a nonlinear function, $y = g(x)$. Assume x has mean \bar{x} and covariance P_x . To calculate the statistics of y , we form a matrix χ of $2L+1$ sigma vectors χ (with corresponding weights W_i), according to the following:

$$\begin{aligned} \chi_0 &= \bar{x} \\ \chi_i &= \bar{x} + (\sqrt{(L+\lambda)P_x})_i \quad i = 1, \dots, L \\ \chi_i &= \bar{x} - (\sqrt{(L+\lambda)P_x})_{i-L} \quad i = L+1, \dots, 2L \\ W_0^{(m)} &= \lambda / (L + \lambda) \\ W_0^{(c)} &= \lambda / (L + \lambda) + (1 - \alpha^2 + \beta) \\ W_i^{(m)} &= W_i^{(c)} = 1 / \{2(L + \lambda)\} \quad i = 1, \dots, 2L \end{aligned} \quad (13)$$

Where $\lambda = \alpha^2(L + \kappa) - L$ is a scaling parameter. α determines the spread of the sigma points around \bar{x}

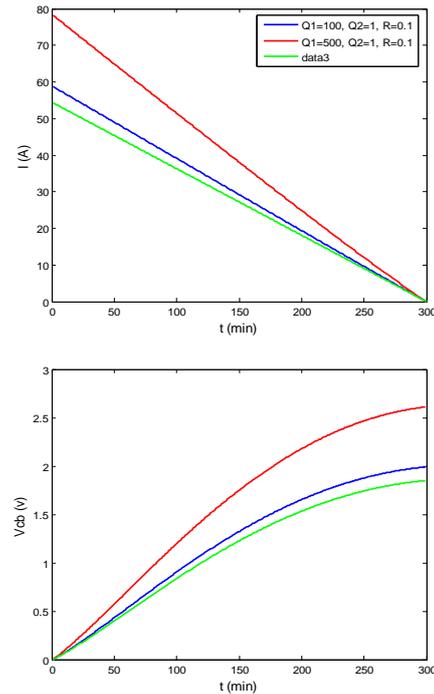


Figure 3. Battery charging current and voltage for different weighting matrices

and is usually set to a small positive value (e.g., $1e-3$). κ is a secondary scaling parameter which is usually set to 0, and β is used to incorporate prior knowledge of the distribution of x (for Gaussian distributions, $\beta = 2$ is optimal). $(\sqrt{(L+\lambda)P_x})_i$ is the i 'th row of the matrix square root. These sigma vectors are propagated through the nonlinear function, $y_i = g(\chi_i) \quad i = 1, \dots, 2L$ and the mean and covariance for y are approximated using a weighted sample mean and covariance of the posterior sigma points,

$$\begin{aligned} \bar{y} &\approx \sum_{i=0}^{2L} W_i^{(m)} y_i \\ P_y &\approx \sum_{i=0}^{2L} W_i^{(c)} \{y_i - \bar{y}\} \{y_i - \bar{y}\}^T \end{aligned} \quad (14)$$

Note that this method differs substantially from general "sampling" methods (e.g., Monte-Carlo methods such as particle filters [14]) which require orders of magnitude more sample points in an attempt to propagate an accurate (possibly non-Gaussian) distribution of the state. The deceptively simple approach taken with the UT results in approximations that are accurate to the third order for Gaussian inputs for all nonlinearities. For non-Gaussian inputs, approximations are accurate to at least the second-order, with the accuracy of third and higher order moments determined by the choice of α and β (See [13] for a detailed discussion of the UT).

The basic framework for the EKF involves estimation of the state of a discrete-time nonlinear dynamic system,

$$\begin{aligned} x_{k+1} &= F(x_k, v_k) \\ y_k &= H(x_k, n_k) \end{aligned} \quad (15)$$

Where x_k represent the unobserved state of the system and y_k is the only observed signal. The process noise v_k drives the dynamic system, and the observation noise is given by n_k . A recursive estimation for x_k can be expressed in the form

$$\hat{x}_k = (\text{prediction of } x_k) + \kappa_k \cdot [y_k - (\text{prediction of } y_k)] \quad (16)$$

The Unscented Kalman Filter (UKF) is a straightforward extension of the UT to the recursive estimation in Equation 16, where the state RV is redefined as the concatenation of the original state and noise variables: $x_k^a = [x_k^T \ v_k^T \ n_k^T]^T$. The UT sigma point selection scheme (Equation 15) is applied to this new augmented state RV to calculate the corresponding sigma matrix, χ_k^a . The UKF equations are given in the Algorithm1. Note that no explicit calculations of Jacobians or Hessians are necessary to implement this algorithm. Furthermore, the overall numbers of computations are the same order as the EKF.

4.2. Simulation Results

Using the model described in section 2 and the parameters given in table 1, UKF is applied to estimate the SOH of the battery. Simulation result for the current profile obtained in section 3 is shown in Fig (4).

Fig (5) shows the estimated capacity with increased Q_1 and Q_2 . According to the estimated curves, increasing Q_2 will improve the capacity (as was mentioned in the previous section).

To compare the results with constant current charging and multilevel current charging methods, battery capacity is estimated using these current profiles. The results are shown in Fig. 6.

5. Conclusion

In this paper, steepest descent technique is applied to a dynamical model of a battery to obtain the optimum charging current. The values of weighting matrices in the cost function can be chosen due to the importance of charging time and battery life time for the user. A new approach for battery SOH estimation, using UKF is also employed to verify the results. Simulations results show that battery lifetime will be improved using the obtained optimum current profile

| |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Initialized Values:</p> $\hat{x}_0 = E[x_0]$ $P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$ $\hat{x}_0^a = E[x^a] = [\hat{x}_0^T \ 0 \ 0]^T$ $P_0^a = E[(x_0^a - \hat{x}_0^a)(x_0^a - \hat{x}_0^a)^T] = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & P_v & 0 \\ 0 & 0 & P_n \end{bmatrix}$ |
| <p>for $k \in \{1, \dots, \infty\}$</p> <p>Calculate sigma points:</p> $\chi_{k-1}^a = [\hat{x}_{k-1}^a \ \hat{x}_{k-1}^a \pm \sqrt{(L + \lambda)P_{k-1}^a}]$ |
| <p>Time update:</p> $\chi_{k k-1}^x = F[\chi_{k-1}^x, \chi_{k-1}^v]$ $\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \chi_{i,k k-1}^x$ $P_k^- = \sum_{i=0}^{2L} W_i^{(c)} [\chi_{i,k k-1}^x - \hat{x}_k^-][\chi_{i,k k-1}^x - \hat{x}_k^-]^T$ $Y_{k k-1} = H[\chi_{k k-1}^x, \chi_{k-1}^n]$ $\hat{y}_k^- = \sum_{i=0}^{2L} W_i^{(m)} Y_{i,k k-1}$ |
| <p>Measurement update equation:</p> $P_{\hat{y}_k \hat{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} [Y_{i,k k-1} - \hat{y}_k^-][Y_{i,k k-1} - \hat{y}_k^-]^T$ $P_{x_k y_k} = \sum_{i=0}^{2L} W_i^{(c)} [\chi_{i,k k-1}^x - \hat{x}_k^-][Y_{i,k k-1} - \hat{y}_k^-]^T$ $\kappa = P_{x_k y_k} P_{\hat{y}_k \hat{y}_k}^{-1}$ $P_k = P_k^- - \kappa P_{\hat{y}_k \hat{y}_k} \kappa^T$ |
| <p>Where,</p> $x_k^a = [x_k^T \ v_k^T \ n_k^T]^T$ $\chi^a = [(\chi^x)^T \ (\chi^v)^T \ (\chi^n)^T]^T$ <p>λ = composite scaling parameter L= dimension of augmented state P_v= process noise cov. P_n= measurement noise cov W_i= weight as calculated in Eq.13</p> |

Algorithm1. UKF Algorithm for Implementation

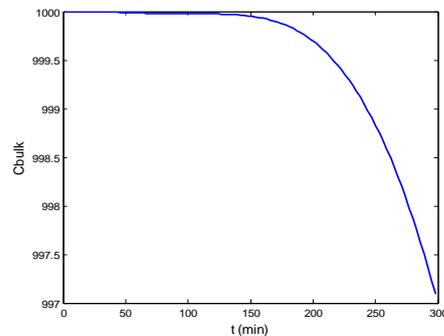


Fig4. The estimated capacity for optimal current

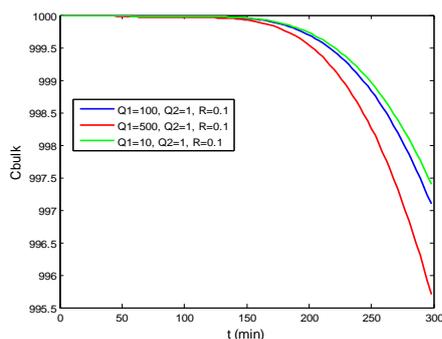
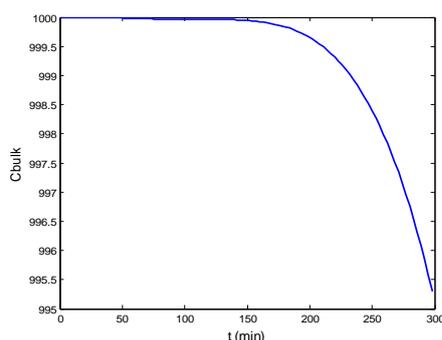
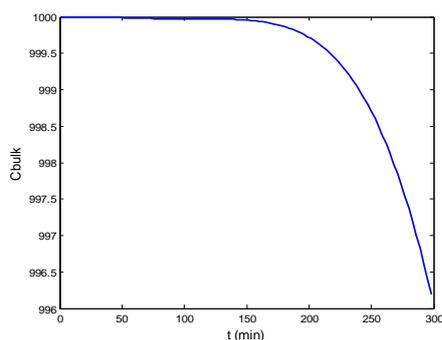


Fig5. Estimated capacity for different weighting matrices



(a)



(b)

Figure 6. The estimated capacity for (a) constant current charging, (b) multilevel current charging

6. References

- [1] C. C. Hua, M. Y. Lin, "A study of charging control of lead-acid battery for electric vehicles", *Proceedings of the 2000 IEEE International Symposium on Industrial Electronics*, ISIE'2000, Vol. 1, 2000, pp. 135-140.
- [2] G. C. Hsieh, L. R. Chen, K. S. Huang, "Fuzzy-controlled li-ion battery charge system with active state-of-charge controller", *IEEE Trans. Ind. Electron.*, vol. 48, no. 3, pp. 583-593, Jun. 2001.
- [3] B. Tsenter and F. Schwartzmiller, "Universal charge algorithm for telecommunication batteries", in *Proc. Twelfth Annu. Battery Conf. Applications and Advances*, 1997, pp. 233-238.
- [4] S. Dilip and D. Stoltzka, "Advanced algorithms and hardware for intelligent batteries", In *Proc. Tenth Annu.*

Battery Conf. Applications and Advances, 1995, pp. 177-182.

[5] H. Surmann, "Genetic optimization of a fuzzy system for charging batteries", *IEEE Trans. Ind. Electron.*, vol. 43, no. 5, pp. 541-548, Oct. 1996.

[6] T. Ikeya, N. Sawada, S. Takagi, J. Murakami, and K. Kobayashi et al., "Multi-step constant-current charging method for electric vehicle, valve-regulated, lead/acid batteries during night time for load-leveling", *J. Power Sources*, vol. 75, no. 1, Sep. 1998, pp. 101-107.

[7] Y. H. Lio, J. H. Teng, Y. C. Lin, "Search for an optimal rapid charging pattern for lithium-ion batteries using ant colony system algorithm," *IEEE Trans. Ind. Electron.*, vol. 52, no. 5, Oct 2005, pp. 1328-1336.

[8] P. Singh, S. Kaneria, J. Broadhead, X. Wang, "Fuzzy logic estimation of SOH of 125 Ah VRLA batteries" *Telecommunications Energy Conference. INTELEC 2004. 26th Annual International*, PP. 524-531, Sep 2004.

[9] C. S. C. Bose, F. C. Laman, "Battery state of health estimation through coup the fouet" *Telecommunications Energy Conference*, 2000. INTELEC. Twenty-second International, pp. 597-601, Sep 2000.

[10] B. S. Bhangu, P. Bentley, D. A. Stone, C. M. Bingham, "Nonlinear observers for predicting state-of-charge and state-of-health of lead-acid batteries for hybrid-electric vehicles," *IEEE Trans. Vehicular technology*, vol. 54, no. 3, pp. 783-794, May 2005.

[11] D. E. Kirk, "Optimal control theory" *Prentice-Hall, Inc. Englewood Cliffs*, New Jersey, pp. 329-343, 1970.

[12] Simon J. Julier and Jeffrey K. Uhlmann "A New Extension of the Kalman Filter to Nonlinear Systems", The Robotics Research Group, Department of Engineering Science, University of Oxford.

[13] S. J. Julier, "The Scaled Unscented Transformation," To appear in *Automatica*, February 2000.

[14] Eric A. Wan and Rudolph van der Merwe, "The Unscented Kalman Filter for Nonlinear Estimation," Oregon Graduate Institute of Science & Technology.