

Frequency Domain Soft-Decision Feedback Equalization for SC-FDMA with Insufficient Cyclic Prefix

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Abstract

Single-carrier frequency-division multiple access (SC-FDMA) system suffers performance degradation when the length of the cyclic prefix (CP) is less than the channel delay spread. In this paper, based on MMSE criterion, we proposed an efficient frequency domain soft-decision feedback equalization (FD-SDFE) scheme for SC-FDMA system. The proposed scheme employs residual inter-symbols cancellation (RISIC) algorithm, combining *a priori* information to reduce inter-block interference (IBI) and inter-carrier interference (ICI) component caused by the absence of CP in multipath fading channel. In addition, an FD-SDFE is derived to further mitigate the residual interference. Simulation results show that the proposed scheme has a significant performance improvement.

Keywords: SC-FDMA, Frequency domain (FD), soft-decision feedback equalization (SDFE), cyclic prefix (CP)

1. Introduction

Single-carrier frequency-division multiple access (SC-FDMA) has been adopted as the uplink transmission in Third-Generation Partnership Project Long-Term Evolution (3GPP-LTE), due to its low peak-to-average power ratio (PAPR) and high frequency diversity gain [1], [2]. SC-FDMA is a combination of FDMA and single-carrier modulation with frequency domain equalization (SC-FDE), which has similar implementation complexity and bit error rate (BER) performance, compared with orthogonal frequency-division multiple access (OFDMA) [3]. SC-FDMA should have a cyclic prefix (CP) long enough to cope with the maximum channel delay spread, thus, the inter-block interference (IBI) is avoided, at the same time, the linear convolution in the time domain can be converted into circular convolution in the frequency domain. However, the use of CP reduces the spectral efficiency of SC-FDMA.

In recent years, several approaches have been proposed to overcome the performance degradation due to the insufficient CP, such as residual inter-symbol interference cancellation (RISIC) [4], Turbo FDE technique [5]. In [5], Turbo equalization scheme achieves performance improvement by iteratively exchanging the soft extrinsic information between detector and soft-input soft-output (SISO) decoder. [6], [7]

In this paper, we propose a new, non-linear, low-complexity MMSE-based frequency domain soft-decision feedback equalization (FD-SDFE), a turbo equalization scheme for SC-FDMA with insufficient CP, which shares some similarities with soft-feedback equalizer (SFE) in [8], [9], but unlike [8], we implement the feedforward filter and feedback filter both in frequency domain, thus computational complexity is reduced. In our scheme, we first use previously detected soft symbols to reduce IBI and ICI as in [4], [10], and then FD-SDFE is performed to cancel residual interference. Moreover, equalizer coefficients are dynamically formulated with the help of the *a priori* mean and the variance of the symbols, thus, the cross-correlation function between the detected symbols and the transmitted symbols in [5] is avoided. Simulation results show that the proposed scheme can achieve good performance.

The remainder of the paper is organized as follows. The signal model and interference analysis of SC-FDMA is described in section II. Section III derives the FD-SDFE turbo equalization. Error performance and computational complexity are presented in section IV. Finally, the conclusions are drawn in Section V

Notations: In this paper, capital letters denote entities in the frequency domain and lowercase letters denote entities in the time domain. Bold symbols denote matrices or

vector. \mathbf{A}^{-1} , \mathbf{A}^T , \mathbf{A}^H denote the inverse, transpose, and conjugate transpose of matrix \mathbf{A} , respectively. $\mathbb{C}^{M \times N}$ denotes the space of all $M \times N$ complex matrices and \mathbf{I}_N is the $N \times N$ identity matrix. The operator $\text{diag}\{\cdot\}$ to be applied to a length N vector denotes an $N \times N$ squared matrix with the vector elements along the diagonal. \mathbf{F}_N is the normalized $N \times N$ DFT matrix with its entry $F_{p,q} = \frac{1}{\sqrt{N}} \exp(-j2\pi pq/N)$ for $p, q = 0, 1, \dots, N-1$. Finally, $\mathbb{E}\{\cdot\}$ denotes the expectation operation and $\text{tr}\{\cdot\}$ represents the trace of a matrix.

2. SYSTEM MODEL AND INTERFERENCE ANALYSIS

We consider a SC-FDMA system with U users that may transmit simultaneously. Each user is equipped with single transmit antenna and the base station has single receive antenna. Assume that there are totally N subcarriers, the number of subcarriers for each user is M ($M = N/U, N \gg M$).



Fig.1. The transmitter structure of SC-FDMA

2.1 SC-FDMA Signal

The structure of the SC-FDMA is depicted in Fig. 1. We assume perfect time synchronization. As illustrated in Fig. 1, at the communication transmitter, the binary information, $b_n \in \{0, 1\}$, is passed through channel encoder followed by an interleaver. The output of the interleaver is divided into blocks with length $K \cdot M$, where $K = \log_2 Q$, Q is the modulation level. The output of the interleaver can be denoted in vector form as $\mathbf{c} = [\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_M]^T \in \mathcal{B}^{KM \times 1}$, where $\mathbf{c}_n = [c_{n,1}, c_{n,2}, \dots, c_{n,K}] \in \mathcal{B}^{1 \times K}$. The symbol mapper maps K -bit data, \mathbf{c}_n , to one modulation symbol, $x_n \in \mathcal{S}$, where $\mathcal{S} = \{\alpha_1, \alpha_2, \dots, \alpha_{2^k}\}$ is the modulation constellation set with $\alpha_i \in \mathbb{C}$, then grouped into block of the transmission. Without loss of generality, each block of symbols can be expressed as $\mathbf{x} = [x(0), x(1), \dots, x(M-1)]^T$. After that, \mathbf{x} is transformed to frequency domain signal \mathbf{X} by a normalized M -point DFT. Through corresponding subcarrier mapping scheme, we can get N -point frequency domain signal $\mathbf{S} = [S(0), S(1), \dots, S(N-1)]^T$,

subsequently, time domain data block $\mathbf{s} = [s(0), s(1), \dots, s(N-1)]^T$ is obtained by a normalized N -point inverse DFT (IDFT).

After G symbols are inserted as CP in the front of \mathbf{s} , the SC-FDMA signal block is transmitted over the multipath fading channel. The length of the multipath channel is defined as L , and it is assumed that $L-1 \geq G$.

2.2 Interference Analysis

At the receiver, the received time domain signal after removing CP can be represented in matrix format as

$$\mathbf{r}_i = \mathbf{H}_0 \mathbf{s}_i + \mathbf{H}_1 \mathbf{s}_{i-1} + \mathbf{v}_i \quad (1)$$

where $\mathbf{r}_i = [r_{i,0}, r_{i,1}, \dots, r_{i,N-1}]^T \in \mathbb{C}^{N \times 1}$, $\mathbf{v}_i = [v_{i,0}, v_{i,1}, \dots, v_{i,N-1}]^T \in \mathbb{C}^{N \times 1}$ are the i th block receive data and noise vector with variance $\sigma_v^2 \mathbf{I}_N$, respectively. \mathbf{H}_0 denotes lower triangular matrix of $N \times N$ circulant Toeplitz channel impulse response (CIR) matrix \mathbf{H}_c

$$\mathbf{H}_0 = \begin{bmatrix} h_0 & 0 & \dots & 0 & \dots & 0 \\ h_1 & \ddots & \vdots & & \ddots & \vdots \\ \vdots & \ddots & h_0 & 0 & \dots & 0 \\ h_{L-1} & \dots & h_1 & \ddots & \dots & 0 \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ 0 & \dots & h_{L-1} & \dots & \dots & h_0 \end{bmatrix} \in \mathbb{C}^{N \times N} \quad (2)$$

\mathbf{H}_1 is upper triangular matrix of \mathbf{H}_c .

$$\mathbf{H}_1 = \begin{bmatrix} 0 & 0 & \dots & h_{L-1} & \dots & h_{G+1} \\ 0 & \ddots & \vdots & & \ddots & \vdots \\ \vdots & \ddots & 0 & 0 & \dots & h_{L-1} \\ 0 & \dots & 0 & \ddots & \dots & 0 \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & \dots & \dots & 0 \end{bmatrix} \in \mathbb{C}^{N \times N} \quad (3)$$

where h is the time domain channel CIR. Since circulant Toeplitz matrix \mathbf{H}_c is the sum of matrices \mathbf{H}_0 and \mathbf{H}_1 , we can rewrite (1) as follows

$$\mathbf{r}_i = \mathbf{H}_c \mathbf{s}_i - \mathbf{H}_1 \mathbf{s}_i + \mathbf{H}_1 \mathbf{s}_{i-1} + \mathbf{v}_i \quad (4)$$

where $\mathbf{H}_1 \mathbf{s}_{i-1}$ denotes the IBI component caused by the previous block, and $\mathbf{H}_1 \mathbf{s}_i$ represents the ICI component.

3. FD-SDFE DESIGN

For SC-FDMA system, equalizer design is essential to

$$\begin{aligned} \Delta &= \bar{\mathbf{R}} - \mathbf{X} = \mathbf{W}\hat{\mathbf{R}} - \mathbf{B}\bar{\mathbf{X}} - \mathbf{X} \\ &= \mathbf{W}\hat{\mathbf{H}}\mathbf{X} - \mathbf{B}\bar{\mathbf{X}} - \mathbf{X} \end{aligned} \quad (11)$$

The MSE is

$$\begin{aligned} \text{MSE} &= \text{tr} \left\{ \mathbf{E} \left\{ \Delta \Delta^H \right\} \right\} = \text{tr} \left\{ \mathbf{W} \left(\sigma_x^2 \|\hat{\mathbf{H}}\|^2 + \sigma_v^2 \right) \mathbf{W}^H \right\} \\ &\quad - \text{tr} \left\{ \sigma_x^2 \left(\mathbf{W}\hat{\mathbf{H}}\mathbf{B}^H + \mathbf{B}\hat{\mathbf{H}}^H\mathbf{W}^H \right) \right\} \\ &\quad + \text{tr} \left\{ -\sigma_x^2 \left(\mathbf{W}\hat{\mathbf{H}} + \hat{\mathbf{H}}^H\mathbf{W}^H \right) + \mathbf{B}\sigma_x^2\mathbf{B}^H \right\} \\ &\quad + \text{tr} \left\{ \sigma_x^2 \left(\mathbf{B} + \mathbf{B}^H \right) + \sigma_x^2 \right\} \end{aligned} \quad (12)$$

To avoid self-subtraction of the desired symbol by its previous estimate, the MSE is minimized with respect to \mathbf{W} and \mathbf{B} subject to the constraint [12]

$$\text{tr} \{ \mathbf{B} \} = 0 \quad (13)$$

We use Lagrange multiplier method to solve the minimum of MSE. Construct the cost function $f(\mathbf{W}, \mathbf{B}, \lambda)$

$$\begin{aligned} f(\mathbf{B}, \lambda) &= \text{tr} \left\{ \mathbf{W} \left(\sigma_x^2 \|\hat{\mathbf{H}}\|^2 + \sigma_v^2 \right) \mathbf{W}^H \right\} \\ &\quad - \text{tr} \left\{ \sigma_x^2 \left(\mathbf{W}\hat{\mathbf{H}}\mathbf{B}^H + \mathbf{B}\hat{\mathbf{H}}^H\mathbf{W}^H \right) \right\} \\ &\quad + \text{tr} \left\{ -\sigma_x^2 \left(\mathbf{W}\hat{\mathbf{H}} + \hat{\mathbf{H}}^H\mathbf{W}^H \right) + \mathbf{B}\sigma_x^2\mathbf{B}^H \right\} \\ &\quad + \text{tr} \left\{ \sigma_x^2 \left(\mathbf{B} + \mathbf{B}^H \right) + \sigma_x^2 \right\} + \lambda \text{tr} \{ \mathbf{B} \} \end{aligned} \quad (14)$$

where λ is the Lagrange multiplier. By setting the gradient of (14) with respect to \mathbf{W} , \mathbf{B} and λ to zero, respectively. We obtain

$$\frac{\partial f(\mathbf{W}, \mathbf{B}, \lambda)}{\partial \mathbf{W}} = \left(\sigma_x^2 \|\hat{\mathbf{H}}\|^2 + \sigma_v^2 \right) \mathbf{W} - \sigma_x^2 \mathbf{B}\hat{\mathbf{H}}^H - \sigma_x^2 \hat{\mathbf{H}}^H = 0 \quad (15)$$

$$\frac{\partial f(\mathbf{B}, \lambda)}{\partial \mathbf{B}} = \mathbf{W}\hat{\mathbf{H}} - \left(1 + \frac{\lambda}{2\sigma_x^2} \right) \mathbf{I} = 0 \quad (16)$$

$$\frac{\partial f(\mathbf{B}, \lambda)}{\partial \lambda} = \text{tr} \{ \mathbf{B} \} = 0 \quad (17)$$

From (15), (16) and (17), we can get

$$\Gamma = \left(1 - \frac{\sigma_x^2}{\sigma_x^2} \right) \|\hat{\mathbf{H}}\|^2 + \frac{\sigma_v^2}{\sigma_x^2} \quad (18)$$

$$\kappa = \frac{1}{N} \sum_{n=0}^{M-1} \|\hat{\mathbf{H}}\|^2 \Gamma^{-1} \quad (19)$$

$$\mu = \frac{\kappa}{1 + \frac{\sigma_x^2}{\sigma_x^2} \kappa} \quad (20)$$

$$\mathbf{W} = \left(1 - \frac{\sigma_x^2}{\sigma_x^2} \mu \right) \hat{\mathbf{H}}^H \Gamma^{-1} \quad (21)$$

$$\mathbf{B} = \mathbf{W}\hat{\mathbf{H}} - \mu \mathbf{I} \quad (22)$$

3.3 Equivalent AWGN channel assumption

At the output of FD-SDFE, in order to demodulate the transmitted symbols, we assume that the estimate $\hat{\mathbf{x}}$ is the output of an equivalent AWGN channel having \mathbf{x} as its input [7], [13]

$$\hat{\mathbf{x}} = \mu \mathbf{x} + \eta \quad (23)$$

where μ is the equivalent amplitude of the signal at the output, η is a complex white Gaussian noise with zero mean and variance δ_η^2 . The parameters μ and δ_η^2 are calculated at each iteration as a function of FD-SDFE structure. The variance δ_η^2 can be expressed as

$$\delta_\eta^2 = \mu(1 - \mu)\sigma_x^2 \quad (24)$$

4. SIMULATION RESULTS

4.1 Performance simulation

In this section, the bit error rate (BER) performance of the proposed FD-SDFE turbo equalization scheme for SC-FDMA system is evaluated. We consider an coded SC-FDMA system with 5MHz system bandwidth, 512 subcarriers ($N = 512$), the number of users $U = 4$, then $M = 128$ subcarriers can be allocated to each user with interleaver subcarrier mapping scheme, $subband = 2$. No CP ($G = 0$), and a 1/2 -rate convolutional code with constraint of 3. The encoded binary bits are interleaved randomly and mapped to QPSK symbols. To illustrate the performance of SC-FDMA under practical system, the 6-path frequency-selective fading channel is generated based on the International Telecommunication Union (ITU) Vehicular A (ITU V-A) channel model [14] listed in Table 1. The channel decoder employs log-MAP algorithm. It is assumed that the channel information is not known at the transmitter, and perfect channel estimation is available at the receiver.

Table 1: ITU vehicular A channel [14]

	0	310	710	1090	1730	2510
Delay (ns)	0	310	710	1090	1730	2510
Power (dB)	0	-1.0	-9.0	-10.0	-15.0	-20.0

Fig. 3 shows the BER performance of the proposed FD-SDFE turbo equalization scheme as functions of signal-to-

noise ratio (SNR). In this figure, the performance curves of frequency domain approximate linear MMSE turbo equalization (FD-TEQ) [13], and high complexity maximum *a posteriori* probability (MAP) equalization [6] are also provided for comparison. The output of a non-iterative equalizer corresponds to the MMSE-based FD linear equalization (MMSE-FD-LE).

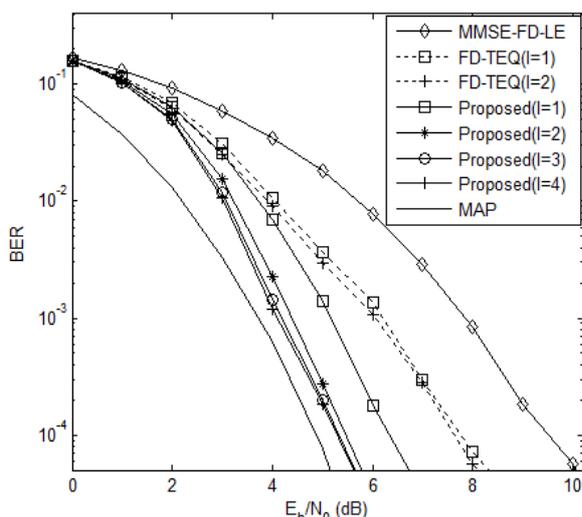


Fig.3. Performance of FD-SDFE turbo equalization scheme

From Fig. 3, it is clear that the first iteration ($l = 1$) yields a significant performance improvement with respect to MMSE-FD-LE, for example, at $BER = 10^{-3}$, the proposed FD-SDFE turbo equalization scheme achieves $2.7dB$ gain, whereas the gain is $1.6dB$ for FD-TEQ. At the same time, three iterations were sufficient for FD-TEQ to converge, because it cannot fully exploit the *a priori* information from SISO decoder. However, the proposed scheme can adaptively update the feedforward and feedback coefficients by taking advantage of the *a priori* information from SISO decoder, thus, as we can see, with two iterations, the performance is around $0.7dB$ better than the first iteration, then the gain of further iterations tends to decrease. After four iterations, the proposed FD-SDFE converges, and is about $0.4dB$ worse than the optimal MAP algorithm at $BER = 10^{-3}$.

4.2 Complexity analysis

The complexity analysis is concentrated in equalization computation. We quantify the complexity in terms of the number of complex multiplications per block. Similarly to the frequency domain turbo equalization in [5], [9], the total of the proposed FD-SDFE is dominated by the DFT, and is shown to be of order $\mathcal{O}(M \log_2 M)$, which is only linear in the block length. Compared with MAP algorithm

complexity $\mathcal{O}(MQ^L)$, the proposed FD-SDFE significantly reduces computational complexity. At the same time, it avoids the calculation of the cross-correlation function between the detected symbols and the transmitted symbols in each iteration in [5], thus achieves the better tradeoff between the performance and computational complexity.

5. Conclusions

A MMSE-based frequency domain soft-decision feedback turbo equalization scheme has been proposed for SC-FDMA system with insufficient CP. The scheme first use RISIC algorithm and soft-decision symbols to cancel IBI and ICI interference, then FD-SDFE equalization is performed to further suppress residual interference caused by frequency-selective fading channel. These two steps are iterative until desired receiver performance is achieved. Simulation results show that the FD-SDFE turbo equalization scheme can improve the BER performance without CP, thus increase spectral efficiency of SC-FDMA system.

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