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Particle filter in power system state estimation – bad measurement data and branch disconnection

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Abstract: An approach to power system state estimation using a particle filter has been proposed in the paper. Two problems have been taken into account during research, namely bad measurements data and a network structure modification with rapid changes of the state variables. For each case the modification of the algorithm has been proposed. It has also been observed that anti-zero bias modification has a very positive influence on the obtained results (few orders of magnitude, in comparison to the standard particle filter), and additional calculations are quite symbolic. In the second problem, used modification also improved estimation quality of the state variables. The obtained results have been compared to the extended Kalman filter method.

Key words: bad data, branch disconnection measurement errors, particle filter, power system, state estimation

1. Introduction

The current problem on the borderline between automation and electrical engineering is the Power System State Estimation (PSSE). The information obtained by the estimation are used for other calculations (e.g. a control [1] or an optimal power flow [2, 3]). Therefore, the estimation is an important step, because the use of the erroneous estimation in subsequent calculations will also return incorrect results.

The most commonly used method for this task is the Weighted Least Squares (WLS), however, it has been proposed nearly 50 years ago by Schweppe and Rom in [4]. One can find a modification of this method in the literature, i.e., Extended WLS (EWLS) [5]. The results obtained by the authors suggest that the use of EWLS slightly increases the estimation quality.

The dynamic estimation methods, i.e. the methods which utilize the history of the state variables (the history is usually limited to the previous state) are alternatives. Such methods include Extended Kalman Filter (EKF) [6], Unscented Kalman Filter (UKF) [7], and also Particle Filter (PF).

The authors of this article are interested in PF methods for PSSE problem solution ([8-10]). This article is focused on the two issues that may occur in the power system, i.e. large measurement errors and line disconnection – the modifications of PF method to the both problems have been proposed.

In the second Section, PF principle of operation has been described. The third Section is devoted to the power system used in the simulations. In the next Section, PF modifications have been presented. In the fifth Section, the performed simulations and the obtained results have been described. Conclusions and summary of the article have been presented in the last Section.

2. Particle filter

PF principle of operation is based on the Bayes filtering, with the Bayes rule described by

$$p\left(\mathbf{X}^{(k)} \mid \mathbf{Y}^{(k)}\right)=\frac{p\left(\mathbf{Y}^{(k)} \mid \mathbf{X}^{(k)}\right)p\left(\mathbf{X}^{(k)}\right)}{p\left(\mathbf{Y}^{(k)}\right)}, \tag{1}$$

where $p(\cdot|\cdot)$ is a conditional probability, whereas $\mathbf{X}^{(k)}$ and $\mathbf{Y}^{(k)}$ are, in general, random variables (in this case $\mathbf{X}^{(k)}$ is a set of state vectors from first to the k-th time step and $\mathbf{Y}^{(k)}$ is a set of measurement vectors from first to the k-th time step).

Assuming the use of the marginalized posterior Probability Density Function (PDF), instead of the joint posterior PDF, one can write the same as in (1) [11]

$$p\left(\mathbf{x}^{(k)} \mid \mathbf{Y}^{(k)}\right)=\frac{p\left(\mathbf{Y}^{(k)} \mid \mathbf{x}^{(k)}\right)p\left(\mathbf{x}^{(k)}\right)}{p\left(\mathbf{Y}^{k}\right)}, \tag{2}$$

where $\mathbf{x}^{(k)}$ is a vector of the state variables in the k-th time step, $\mathbf{Y}^{(k)}$ is a set of the measurements vectors, from first to the k-th time step $\mathbf{Y}^{(k)}=\{\mathbf{y}^{(1)}, \mathbf{y}^{(2)}, ..., \mathbf{y}^{(k)}\}$.

Based on the basic concepts in probability, i.e. [12]

$$p(A \mid B)=\frac{p(A \cap B)}{p(B)}, \tag{3}$$

$$p(A \cap B)=p(B \cap A), \tag{4}$$

Bayes rule, which results from above expressions

$$p(A \mid B)=\frac{p(B \mid A)p(A)}{p(B)}, \tag{5}$$

and also on

$$p(A \cap B \mid C) = \frac{p(A \cap B \cap C)}{p(C)}, \tag{6}$$

$$p(A \mid B \cap C) = \frac{p(A \cap B \cap C)}{p(B \cap C)}, \tag{7}$$

one can obtain expression of the Bayes filtering, i.e.

$$\underbrace{p\left(\mathbf{x}^{(k)} \mid \mathbf{Y}^{(k)}\right)}_{\text{Posterior}} = \frac{\overbrace{p\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{(k)}\right)}^{\text{Likelihood}}\overbrace{p\left(\mathbf{x}^{(k)} \mid \mathbf{Y}^{(k-1)}\right)}^{\text{Prior}}}{\underbrace{p\left(\mathbf{y}^{(k)} \mid \mathbf{Y}^{(k-1)}\right)}_{\text{Evidence}}}, \tag{8}$$

assuming that the object is a first order Hidden Markov Model (HMM). An interested reader can find full derivation in literature, e.g. [13, 14].

PF is one of the possible Bayes filter implementations. There has been made an assumption that the posterior PDF is composed of particles. The i-th particle has a certain value $\mathbf{x}^i$ and the weight q^i. Greater weight q^i affects on greater probability that $\mathbf{x}^i$ is a correct value. When the number of particles N tends to the infinity, one can write that

$$p\left(\mathbf{x}^{(k)} \mid \mathbf{Y}^{(k)}\right) \overset{N\to\infty}{=} \hat{p}\left(\mathbf{x}^{(k)} \mid \mathbf{Y}^{(k)}\right) = \sum_{i=1}^{N} q^{i,(k)} \delta_D\left(\mathbf{x}^{(k)} - \mathbf{x}^{i,(k)}\right), \tag{9}$$

where $\delta_D(\cdot)$ is a Dirac delta.

Particle filtering is one of the three main Sequential Monte Carlo (SMC) method types, in which PDF $p(\mathbf{x}^{(k)}|\mathbf{Y}^{(k)})$ is calculated. Two other variants are a smoothing $p(\mathbf{x}^{(k)}|\mathbf{Y}^{(k+n)})$ and a prediction $p(\mathbf{x}^{(k+n)}|\mathbf{Y}^{(k)})$, where n is a natural number [15].

The first particle filter was proposed by Gordon, Salmond and Smith in 1993 and has been called the Bootstrap Filter [16]. Operation principle of PF has been presented below.

Algorithm 1: Bootstrap filter
1) Initialization. Draw N particles from the initial PDF $\mathbf{x}^{i,(0)} \sim p(\mathbf{x}^{(0)})$, set the time step value $k = 1$.
2) Prediction. Draw N new particles from the transition model $\mathbf{x}^{i,(k)} \sim p(\mathbf{x}^{(k)}|\mathbf{x}^{i,(k-1)})$.
3) Update. Calculate particle weights based on measurement model $q^{i,(k)} = p(\mathbf{y}^{(k)}|\mathbf{x}^{i,(k)})$.
4) Normalization. Normalize weights so that their sum be equal to 1.
5) Resampling. Draw N new particles based on the posterior PDF, obtained in previous steps.
6) Calculate an estimated value, increase the time step $k = k + 1$, go to the second step.

A general form of PF is the Sequential Importance Resampling (SIR) algorithm, which has been described below.

Algorithm 2: Sequential importance resampling

1) Initialization. Draw N particles from the initial PDF $\mathbf{x}^{i,(0)} \sim p(\mathbf{x}^{(0)})$, set the time step value $k = 1$.
2) Prediction. Draw N new particles from the importance density $\mathbf{x}^{i,(k)} \sim g(\mathbf{x}^{(k)}|\mathbf{x}^{i,(k-1)}, \mathbf{y}^{(k)})$.
3) Update. Calculate particle weights based on the measurement model, the transition model and the importance density

$$q^{i,(k)} = q^{i,(k-1)} \frac{p\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{i,(k)}\right) p\left(\mathbf{x}^{i,(k)} \mid \mathbf{x}^{i,(k-1)}\right)}{g\left(\mathbf{x}^{i,(k)} \mid \mathbf{x}^{i,(k-1)}, \mathbf{y}^{(k)}\right)}.. \tag{10}$$

4) Normalization. Normalize weights so that their sum be equal to 1.
5) Check condition for a resampling execution – if it is not met, go to the 7th step.
6) Resampling. Draw N new particles based on the posterior PDF, obtained in the previous steps; set the new weights values equal to $^1/_N$.
7) Calculate an estimated value, increase the time step $k = k + 1$, go to the second step.

The importance density, which has been used in steps 2 and 3 of the Algorithm 2, is an arbitrary function, which can be dependent on the measurement values (what is not obligatory). This should be used, when the values can be easily calculated from the transition model $p(\mathbf{x}^{(k)}|\mathbf{x}^{i,(k-1)})$, but there are difficulties in drawing from this PDF. This can also be applied when any additional information are known, which can allow for narrow the consecutive draws.

As one can see, there may be many different types of PF, e.g., Auxiliary PF [17], Marginal PF [18], Gaussian PF [19]. One can also find algorithms, which combine two different methods, e.g. Extended Kalman PF [20], Unscented PF [21], Hybrid Kalman PF [22] or Iterated Extended Kalman PF [23].

These PF variations mainly relate to the steps 2 and 3 in the Algorithm 2. Whereas the element that appears in every PF is resampling. In this article, the systematic resampling has been used for simulations.

Algorithm 3: Systematic resampling

1) Initialization. Set value $j = 1$ and sum of weights $S = q^{1,(k)}$, generate a one random value $u \sim \mathrm{U}[0,^1/_N)$.
2) Steps 3-5 perform for $i = 1 \ldots N$.
3) While $S < u$ do $j = j + 1$ and $S = S + q^{j,(k)}$.
4) Choose value $\mathbf{x}^{j,(k)}$ for replication.
5) Calculate $u = u + ^1/_N$.

There are also many other types of resampling steps. Interested readers are recommended to study the contents of the publication [24] where more than 20 different types of resampling have been described.

As one can easily see, most of the required in PF calculations can be performed simultaneously. For this reason, one can find proposals for PF implementation on GPU [25] or on FPGA [26] in the literature. This is important, because only with a parallel implementation PF can compete in terms of the execution time with other estimation methods.

For readers interested in the particle filtering subject, references [27] and [14] are recommended.

3. Power system

An object, which has been chosen (power system) is a multidimensional nonlinear system. It consists of nodes (buses) and lines (branches) which connect the nodes. A π quadripole has been selected as the line model (see Fig. 1),

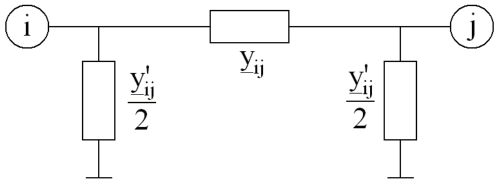

Fig. 1. Branch model in power system

where $\underline{y}'_{ij}/2$ is a half total line charging susceptance [28], and $\underline{y}_{ij}$ is a line admittance calculated from

$$\underline{y}_{ij} = \frac{1}{R_{ij} + \mathrm{s}X_{ij}}, \tag{11}$$

where R_{ij} is a resistance and X_{ij} is a reactance.

Based on the line parameters, the admittance matrix $\underline{\mathbf{Y}}$, which is required for subsequent calculations, is created. The diagonal values are calculated based on expression

$$\underline{Y}_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{B} \left(\frac{\underline{y}'_{ij}}{2} + \underline{y}_{ij} \right), \tag{12}$$

and the remaining matrix values, based on the equation

$$\underline{Y}_{ij} = -\underline{y}_{ij} \qquad i \neq j. \tag{13}$$

Elements of $\underline{\mathbf{Y}}$ matrix can represented in exponential notation

$$\underline{Y}_{ij} = Y_{ij} \cdot \exp(\mathrm{j}\mu_{ij}). \tag{14}$$

The power system is composed of B buses and L branches. The unequivocal network state can be determined by phasors in all nodes, where each phasor can be represented by the voltage magnitude U and phase angle δ. However, a reference node is needed, wherein the phase value will be a constant (usually equal to 0). Based on the latter, one can conclude that the power system consisting of B nodes is a multidimensional object with $2B$-1 state variables

$$\mathbf{x} = [x_1 \ \cdots \ x_{2B-1}]^T = [U_1 \ \cdots \ U_B \ \ \delta_2 \ \cdots \ \delta_{2B-1}]^T. \tag{15}$$

In (15) it is assumed that the first node is reference.

5 measurement types have been specified in the power system, i.e. a voltage magnitude and 4 types of power:

- active (P) and reactive (Q) power injections

$$P_i(\mathbf{U},\boldsymbol{\delta}) = P_i = \sum_{j=1}^{B} U_i U_j Y_{ij} \cos(\delta_i - \delta_j - \mu_{ij}), \tag{16}$$

$$Q_i(\mathbf{U},\boldsymbol{\delta}) = Q_i = \sum_{j=1}^{B} U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \mu_{ij}), \tag{17}$$

- active and reactive power flows

$$P_{ij}(\mathbf{U},\boldsymbol{\delta}) = P_{ij} = U_i^2 Y_{ij} \cos(-\mu_{ij}) - U_i U_j Y_{ij} \cos(\delta_i - \delta_j - \mu_{ij}), \tag{18}$$

$$Q_{ij}(\mathbf{U},\boldsymbol{\delta}) = Q_{ij} = U_i^2 Y_{ij} \sin(-\mu_{ij}) - U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \mu_{ij}) + U_i^2 \frac{y'_{ij}}{2}. \tag{19}$$

The 5-nodal power system has been selected for simulations, the same as in [29]. The assumption has been made that the all possible measurements are performed (i.e. 24 power flows, 10 power injections and 5 voltage magnitudes – total 39 measurements). In the case with bad data, five of the measurements were completely wrong. Errors which have been assumed are:

- the measured value is continuously the same, significantly overstated,
- the measured value has the opposite sign,
- the measured value is biased by constant value,
- the measured value always is equal to 0.

In the case with the line disconnection, all measurements were correct, however line 1-5 was disconnected in the half of the simulation.

In the Figure 2 scheme of the power system, with marked places of the erroneous measurements, has been presented. In the Figure 3 one can see the explanation of the used designations.

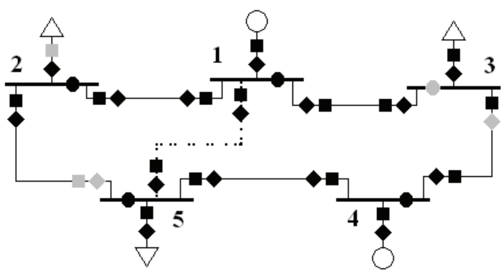

Fig. 2. Scheme of the power system. The bad measurements for first case are marked as grey elements. The disconnected line for second case is marked by dotted line

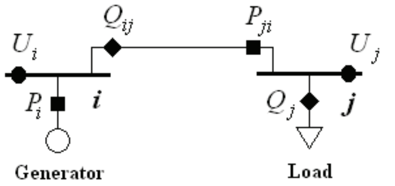

Fig. 3. The explanation of the designations in the Figure 2

The power system has been modelled as a first order Hidden Markov Model (HMM)

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{v}^{(k)}, \tag{20}$$

$$\mathbf{y}^{(k)} = \mathbf{h}\left(\mathbf{x}^{(k)}\right) + \mathbf{n}^{(k)}, \tag{21}$$

where $\mathbf{v}$ and $\mathbf{n}$ are vectors of, respectively, a system noise and a measurement noise (both have been taken as the Gaussian noise), and $\mathbf{h}(\cdot)$ is a vector of measurement functions (expressions (16-19) or the voltage magnitude).

For more information about power system, the books [29] and [30] are recommended.

4. Proposed modifications

4.1. Anti-zero bias

This modification is concentrated on the update step in PF method (step 3 in the Algorithm 1). The measurement model $p(\mathbf{y}^{(k)}|\mathbf{x}^{i,(k)})$ determines what is the probability that with the given measurement values $\mathbf{y}^{(k)}$, the state vector is equal to $\mathbf{x}^{i,(k)}$. In practice, this operation is distributed into simpler calculations, i.e.

$$p\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{i,(k)}\right) = p\left(y_{(1)}^{(k)} \mid \mathbf{x}^{i,(k)}\right) \cdot p\left(y_{(2)}^{(k)} \mid \mathbf{x}^{i,(k)}\right) \cdot \ldots \cdot p\left(y_{(m)}^{(k)} \mid \mathbf{x}^{i,(k)}\right), \tag{22}$$

where m is a number of all measurements (size of $\mathbf{y}^{(k)}$ vector). To illustrate the problem, the situation for continuous PDF has been shown in the Figure 4 (in the expression (22) it is the same, but in a one point, i.e. for $\mathbf{x}^{i,(k)}$).

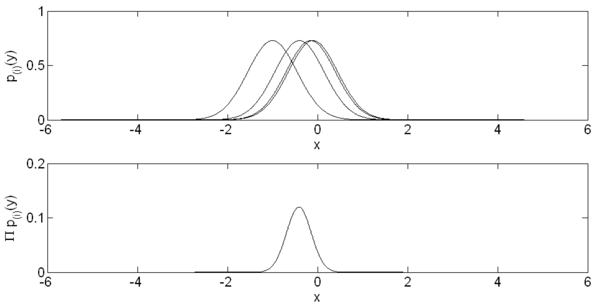

Fig. 4. Probability density functions and their product. The case with correct measurements

In the case with bad measurement data, the expected value of the PDF product will significantly change (see Fig. 5, the solid line). Additionally, for some $\mathbf{x}^{i,(k)}$ vectors the value of expression (22) may be equal to the zero. It is possible in the case where bad measurements would result in a lower value of probability than the minimum value in the floating-point system.

The modification of the likelihood calculation from the measurement model has been proposed to prevent this problem:

$$p\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{i,(k)}\right) = \left(p\left(y_{(1)}^{(k)} \mid \mathbf{x}^{i,(k)}\right) + b_0\right) \cdot \ldots \cdot \left(p\left(y_{(m)}^{(k)} \mid \mathbf{x}^{i,(k)}\right) + b_0\right), \tag{23}$$

where b_0 is the anti-zero bias.

The use of the anti-zero bias prevents resetting a likelihood to zero and causes that bad measurement data have a negligible effect on the calculated expected value of PDF (see Fig. 5, the dashed line).

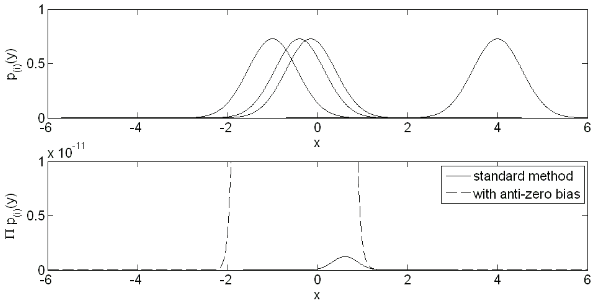

Fig. 5. Probability density functions and their product. The case with bad measurement

4.2. Modification during line disconnection

The nature of changes of the state variables in successive iterations is described by the transition model $p(\mathbf{x}^{(k)}|\mathbf{x}^{i,(k\text{-}1)})$. In the case where a factual change of the state variable is much greater than based on the transition model, the correct estimation is not possible.

On the other hand, the assumption that the variance in the transition model is greater, is inefficient if everything works properly.

In the case of detection of the line disconnection, the increasing of the variance in the transition model would help, however, it is not known how much this variance should be increased.

Therefore, the modification of Algorithm 1 has been proposed, which should be used only in the case of the line disconnection. The modification depends only on the last step of the PF algorithm.

Algorithm 4: Bootstrap Filter modification for the line disconnection

6) Calculate an estimated value. If the huge error in the estimation is not detected, increase the time step $k = k + 1$. Go to the second step.

As one can see, during one time step k, the calculations can be performed few times.

The line disconnection (and more generally – the change of the state variables greater than this is possible with a chosen transition model) can be easily checked by comparing the unnormalized weights (step 3 in the Algorithm 1). In the performed simulations, the maximum weight value decreased by more than 50 orders of magnitude, after the disconnection.

The condition in the Algorithm 4 can take a several different forms, e.g. an iteration can be repeated until a maximum unnormalized weight reaches a certain threshold value. One can also set up a number of iteration repetitions. In this article the authors assumed that the iteration is repeated as long as the increase of the maximum unnormalized weight is "significant" (greater than 5 orders of magnitude – it has been considered as the huge error in the Algorithm 4). When the maximum unnormalized weight returns to an earlier level, the modification is disabled.

The additional assumption has been made that since the modification is enabled, the standard deviation in the transition model is 3 times higher.

5. Obtained results

Simulations have been performed for the power system presented in the Figure 2 for the same set of data, with length of $M = 100$ time steps.

For a small number of particles $N \leq 10{,}000$ simulations have been repeated 1,000 times. For $N \leq 100{,}000$, 100 simulations have been performed, and 10 simulations in other cases.

The estimation quality has been described by the indicator D, which contains information about the mean square errors (MSE) of the all state variables. It can be written as

$$D = 10^6 \cdot \sum_{i=1}^{2B-1} \left(MSE_i\right)^2, \tag{24}$$

where B is a number of buses in the power system. Coefficient 10^6 has been used for obtaining „pleasing to the eye" results.

5.1. Results for bad data measurements (anti-zero bias)

Value of the anti-zero bias has been chosen to be $b_0 = 10^{-6}$. The simulation results have been presented in the Figure 6.

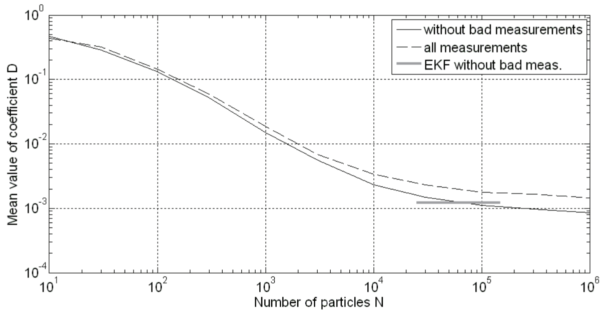

Fig. 6. Simulation results for case with bad data measurements

Results presented in the chart concern only on the PF method with the anti-zero bias. It is because without this modification results were about 5 orders of magnitude higher. The situation with the EKF method results were similar. In the case of all (including wrong) measurements, the value $D = 552$ has been obtained.

5.2. Results for line disconnection

The line disconnection in the $k = 51$ time step has been assumed. However, an effect of a network structure alteration is not only visible in the power flows and the state variables in buses, but also in the parameters that have not been taken into account. One of the problems is the active power balance, which affects the frequency in the power system.

In the prepared simulation the simplified approach has been used. This is justified, because the most important effect (the rapid change in values of the state variables) has been achieved. After the branch disconnection the estimated state vector values have been calculated using the Weighted Least Squares (WLS) method, assuming that the power injections can not change rapidly. Obtained in this way state vector has been considered as genuine.

As a result of these calculations, a significant increase of the voltage occurred in the power system, and in the next few steps it has been reduced. The remainder part of the simulation (to $k = 100$) has been held without significant changes. In such a way, the waveforms without noise have been created.

The effects (a chart of the state variables) are shown in the Figure 7.

The simulation results have been presented in the Figure 8.

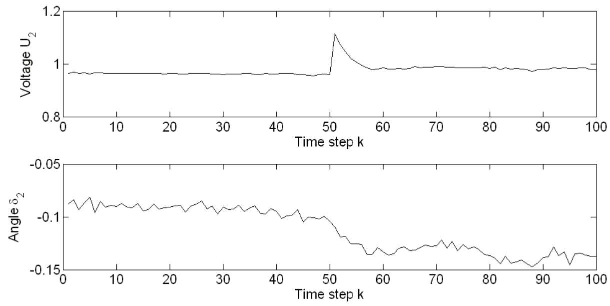

Fig. 7. Plot of the state variables U_2 (above) and δ_2 (below) – the values without noises

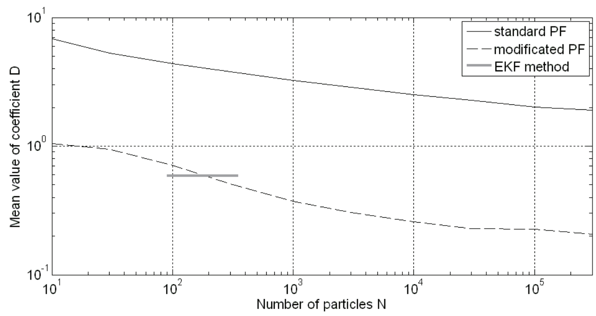

Fig. 8. Simulation results for the case with the branch disconnection

6. Conclusions

Based on the performed simulations one can say that thanks to the anti-zero bias PF can be used for cases in which bad measurements data may occur. The modification does not affect the computation time, and the obtained results are little worse than in the case without bad measurements.

PF algorithm with the anti-zero bias has a higher performance in the comparison to the EKF method. Only assuming that a detection of bad data measurements in EKF is implemented, this method is competitive. However, it should be noted that some errors may be higher than assumed in the algorithm and small enough to be unnoticed. In such cases, the versatility of PF method is undoubted advantage.

The second proposed modification has a noticeable effect on the obtained results (almost 10-times smaller D value – see Fig. 8). In the comparison to the EKF method, the standard PF algorithm is far worse. Whereas the modificated PF method gives better results already for about $N = 200$ particles.

It should be noted that the characteristics of the state variables may look very different for disconnections of various lines, as well as for a different state of the power system. Despite this, the obtained results are satisfactory, because it has been shown that the approach, which allows to track rapid changes of the state variables, is possible. However, this issue requires further study in the future.

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