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Designing a group of single-branch filters taking into account their mutual influence

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Abstract: Single-branch filters are still popular and are commonly used for power quality improvement purposes. Analysis of a single-branch filter is a relatively simple task. Although individual filters tuned to specific harmonics can be easily designed, after connecting them into a group it turns out that the capacitance and inductance mutually influence each other, distorting the resulting frequency characteristics. This article presents a matrix method for design a group of single-branch filters, so that the resultant frequency characteristic satisfies the design requirements including the requirements for location of the frequency characteristic maxima. Designer indicates the frequencies of the parallel resonances.

Key words: passive filter, compensation of reactive power, harmonics

1. Introduction

Due to the ever-growing number of industrial nonlinear, large power loads the high harmonics passive filters are still a commonly used mean of mitigating the voltage distortion at the point of load connection [1-5]. Passive harmonic filters are still very commonly used to improve the quality of electricity supply (reducing distortion of voltage and current, and fundamental harmonic power factor correction) [6-8]. There are several passive filter systems, with different structures and different operating characteristics [9-12]. The primary is still a single-branch filter, which is predominant in industrial applications and certainly is the basis for understanding the operation of more complex filter structures [13-14].

The essential data, necessary for filter design are:

- data about the source of harmonics: amplitude-frequency spectrum of a nonlinear load obtained from measurements or from technical specification of the filtered load, reactive power of the fundamental harmonic to be compensated, etc.,

- data on the supply network: the frequency characteristic of the power system impedance at the point of the filter common coupling (PCC) or, if this data is not available, the short-circuit capacity, diagrams and technical data of the nearest neighbourhood of the considered point of connection, the voltage distortion spectrum at this point, total voltage harmonic distortion (THD_U) set forth in the technical conditions of connection, harmonic ratio for specific harmonics, etc.,
- data about the filter: location of installation, technical specification of passive elements to be used, etc.

2. Single-branch passive filter

Most theoretical considerations of filter design using analytical methods are carried out with the following simplifying assumptions:
- high harmonics source is a current source,
- inductance L_F and capacitance C_F of the filter are centred and have a constant value in the considered frequency range.
- filter is loaded only with fundamental harmonic and the harmonic to which it is tuned,
- resistance $R_F = 0$.

Power passive filter can have different configurations, it can also be a combination of several single-branch filters or other filters to eliminate certain harmonics. Impedance characteristic of the filter is a function of the frequency. At the frequency of eliminated harmonic the filter impedance reaches its minimum, thus the major portion of the current harmonics (of this frequency) flows through the filter instead of a supply network. In simple terms the filter design can be reduced to shaping its impedance characteristic.

The basic structure selected for further analysis was a single-frequency single-branch filter, its diagram shown in Figure 1.

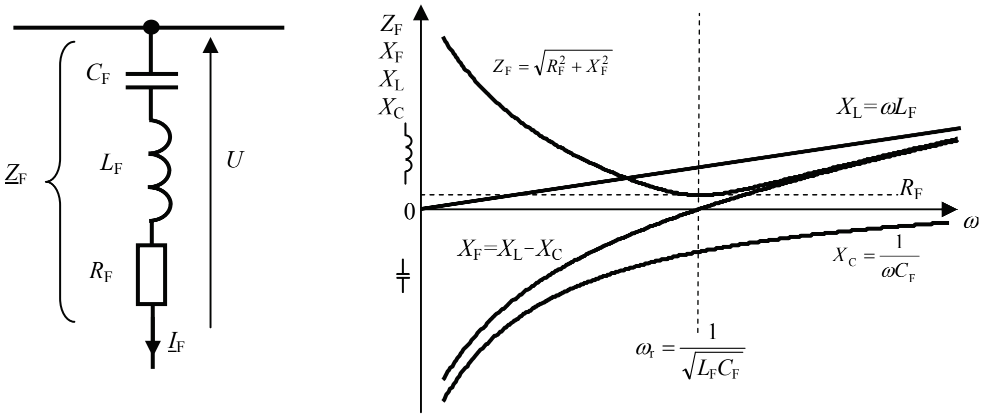

Fig. 1. The equivalent circuit for the single-branch filter and its characteristics

The filter equivalent impedance is given in the form (1)

$$\underline{Z}_{\mathrm{F}}(\omega) = R_{\mathrm{F}} + \mathrm{j}\omega L_{\mathrm{F}} - \mathrm{j}\frac{1}{\omega C_{\mathrm{F}}}, \tag{1}$$

where: R_F, L_F, C_F – the filter resistance, inductance and capacitance, respectively.

In order to simplify the analysis it is assumed $R_F = 0$. The single-branch filter is used for elimination of unwanted harmonics and to power factor correction of the power system. The filter resonant angular frequency is described by (2)

$$\omega_{\mathrm{r}} = \frac{1}{\sqrt{L_{\mathrm{F}} C_{\mathrm{F}}}} = n_{\mathrm{r}}\omega_1, \tag{2}$$

where: ω_r – resonant angular frequency, n_r – order of the resonant frequency, ω_1 – fundamental harmonic angular frequency.

Finding the L_F value from the formula (3) for resonant angular frequency:

$$L_{\mathrm{F}} = \frac{1}{C_{\mathrm{F}} n_{\mathrm{r}}^2 \omega_1^2} \tag{3}$$

and substituting (3) into (1), we get

$$\underline{Z}_{\mathrm{F}}(\omega) = -\mathrm{j}\frac{1}{C_{\mathrm{F}}}\frac{n_{\mathrm{r}}^2\omega_1^2 - \omega^2}{n_{\mathrm{r}}^2\omega_1^2\omega}. \tag{4}$$

Now the capacitance value C_F should be selected, such as to minimise the reactive power generated by the nonlinear load (5)

$$\mathrm{Im}\{\underline{Z}_{\mathrm{F}}(\omega_1)\} = \frac{U^2}{Q_{\mathrm{F}}}, \tag{5}$$

where: U – operating voltage of the filter capacitors, Q_F – the filter capacitive reactive power (fundamental harmonic).

Substituting (4) into (5) and considering $\omega = \omega_1$.

$$C_{\mathrm{F}} = \frac{1 - n_{\mathrm{r}}^2}{n_{\mathrm{r}}^2}\frac{1}{\omega_1}\frac{Q_{\mathrm{F}}}{U^2}. \tag{6}$$

Parameters of a single-branch filter can be determined from the formulas (3) and (6).

In practical applications a passive filter design problem consists in designing a filter that will reduce the voltage total harmonic distortion ($\mathrm{THD_U}$) below the allowable limit. This means, that reduction of one harmonic may not be sufficient (the factor $\mathrm{THD_U}$ may still be higher than the allowed limit). In such cases, consideration should be given the opportunity to design additional filters for subsequent harmonics present in the system. The project task will be to design a group of single-branch filters. Each filter is tuned to a different harmonic present in the system in order to reduce it.

3. A group of single-branch filters

The main task in designing a filters group is to divide the total reactive power of the fundamental harmonic between various branches of the filter (7):

$$Q_{\mathrm{F}} = \sum_{i=1}^{k} Q_{\mathrm{Fn_r}(i)}, \tag{7}$$

where: $Q_{\mathrm{Fnr}(i)}$ – the i-th filters passive power (fundamental harmonic).

This task can be accomplished using the assumption that passive powers of individual filters are inversely proportional to the order of the harmonic to be eliminated (8)

$$n_{\mathrm{r}(i)} Q_{\mathrm{Fn_r}(i)} = n_{\mathrm{r}(i+1)} Q_{\mathrm{Fn_r}(i+1)} \qquad i = 1, .. \ , d-1, \tag{8}$$

where: $n_{\mathrm{r}(i)}$ – order of harmonic tuned of filter, d – number of designed filters

Example 1. For non-linear load connected to the supply line voltage 6 kV (S_C = 500 MVA), required reactive power of the filters group equals –1 Mvar and on the condition of the exceeding of the limit for $\mathrm{THD_U}$ value implicitly require the need design four single-branch filters to reduce harmonic orders: 5, 7, 11 and 13$^{\mathrm{th}}$.

Applying (3), (6) – (8) the filters' parameters have been found (Tab. 1).

Table 1. Parameters of the group of 4 single-branch filters

$n_{\mathrm{r}(i)}$		**5**	**7**	**11**	**13**
$Q_{\mathrm{Fnr}(i)}$	kvar	–391.63	–279.73	–178.01	–150.63
$C_{\mathrm{Fnr}(i)}$	μF	33.24	24.23	15.61	13.24
$L_{\mathrm{Fnr}(i)}$	mH	12.2	8.5	5.4	4.5

The resultant characteristic of the group of 4 filters, designed according to the above method, is shown in Figure 2. As seen from the characteristic, the impedance reaches its minimum for the eliminated harmonics but for the 6th, 9$^{\mathrm{th}}$ and 12$^{\mathrm{th}}$ harmonic the impedance is relatively low, whereas it is required to attain its maxima for these harmonics.

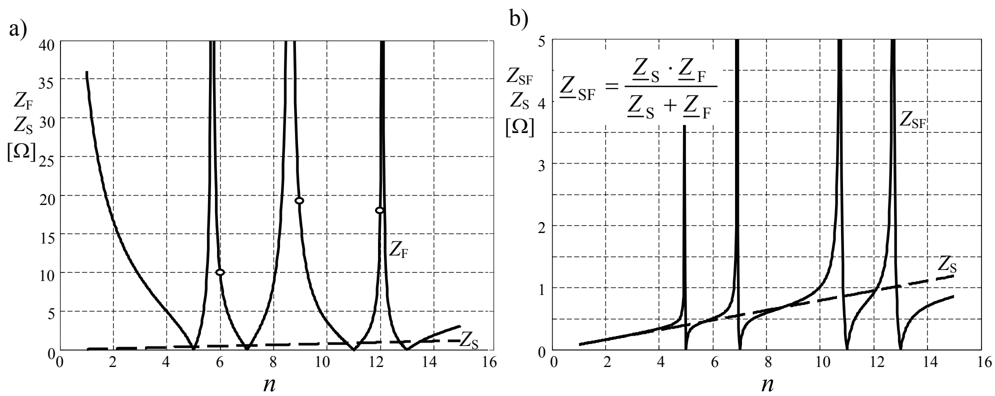

Fig. 2. The graph of impedance: a) group of 4 filters, b) viewed from the load

4. Matrix method of designing the group of single-branch filters

The impedance of a single-branch filter is given from the formula (4); the resultant admittance of the group of filters is given by the relationship:

$$\underline{Y}_{\mathrm{F}}(\omega) = \frac{1}{\underline{Z}_{\mathrm{F}}(\omega)} = \sum_{i=1}^{d} \frac{1}{\underline{Z}_{\mathrm{Fnr}(i)}(\omega)}. \tag{9}$$

For eliminated harmonics

$$Z_{\mathrm{F}}\left(\omega_{\mathrm{rn}(i)}\right) = 0 \Rightarrow \mathrm{Im}\left(\underline{Z}_{\mathrm{F}}\left(\omega_{\mathrm{rn}(i)}\right)\right) = 0. \tag{10}$$

For frequencies for which the characteristics of impedance reaches maxima

$$Z_{\mathrm{F}}\left(\omega_{\mathrm{pm}(j)}\right) = \infty \Rightarrow \frac{1}{Z_{\mathrm{F}}\left(\omega_{\mathrm{pm}(j)}\right)} = 0 \Rightarrow \mathrm{Im}\left(\frac{1}{Z_{\mathrm{F}}\left(\omega_{\mathrm{pm}(j)}\right)}\right) = 0, \tag{11}$$

where: $\omega_{\mathrm{pm}(j)}$ – angular frequencies for which the characteristics of impedance reaches maxima

$$m_{(j)} = \frac{\omega_{\mathrm{pm}(j)}}{\omega_1}. \tag{12}$$

From (5) and (11) and considering the assumption $R_{\mathrm{Fnr}(i)} = 0$

$$-\,\mathrm{Im}\left(\frac{1}{\underline{Z}_{\mathrm{F}}(\omega_1)}\right) = \frac{Q_{\mathrm{F}}}{U^2}. \tag{13}$$

Based on (11) and (13) linear equations can be formulated

$$\begin{cases} \mathrm{Im}\left\{\dfrac{1}{\underline{Z}_{\mathrm{F}}(\omega_1)}\right\} = -\dfrac{Q_{\mathrm{F}}}{U^2} \\ \mathrm{Im}\left\{\dfrac{1}{\underline{Z}_{\mathrm{F}}\left(\omega_{\mathrm{pm}(2)}\right)}\right\} = 0 \\ \qquad\vdots \\ \mathrm{Im}\left\{\dfrac{1}{\underline{Z}_{\mathrm{F}}\left(\omega_{\mathrm{pm}(d)}\right)}\right\} = 0. \end{cases} \tag{14}$$

Taking into account Equations (9) and (14) leads to the equation

$$\begin{bmatrix} \operatorname{Im}\left\{\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(1)}(\omega_1)}+\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(2)}(\omega_1)}+\ldots+\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(d)}(\omega_1)}\right\} \\ \operatorname{Im}\left\{\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(1)}(\omega_{\mathrm{pm}(2)})}+\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(2)}(\omega_{\mathrm{pm}(2)})}+\ldots+\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(d)}(\omega_{\mathrm{pm}(2)})}\right\} \\ \operatorname{Im}\left\{\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(1)}(\omega_{\mathrm{pm}(3)})}+\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(2)}(\omega_{\mathrm{pm}(3)})}+\ldots+\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(d)}(\omega_{\mathrm{pm}(3)})}\right\} \\ \vdots \\ \operatorname{Im}\left\{\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(1)}(\omega_{\mathrm{pm}(d)})}+\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(2)}(\omega_{\mathrm{pm}(d)})}+\ldots+\dfrac{1}{\underline{Z}_{\mathrm{Fnr}(d)}(\omega_{\mathrm{pm}(d)})}\right\} \end{bmatrix} = \begin{bmatrix} -\dfrac{Q_{\mathrm{F}}}{U^2} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{15}$$

Transforming (15) we get the matrix equation:

$$\underbrace{\begin{bmatrix} \dfrac{n_{\mathrm{r}(1)}^2 m_{(1)}\omega_1}{n_{\mathrm{r}(1)}^2 - m_{(1)}^2} & \dfrac{n_{\mathrm{r}(2)}^2 m_{(1)}\omega_1}{n_{\mathrm{r}(2)}^2 - m_{(1)}^2} & \cdots & \dfrac{n_{\mathrm{r}(d)}^2 m_{(1)}\omega_1}{n_{\mathrm{r}(d)}^2 - m_{(1)}^2} \\ \dfrac{n_{\mathrm{r}(1)}^2 m_{(2)}}{n_{\mathrm{r}(1)}^2 - m_{(2)}^2} & \dfrac{n_{\mathrm{r}(2)}^2 m_{(2)}}{n_{\mathrm{r}(2)}^2 - m_{(2)}^2} & \cdots & \dfrac{n_{\mathrm{r}(d)}^2 m_{(2)}}{n_{\mathrm{r}(d)}^2 - m_{(2)}^2} \\ \dfrac{n_{\mathrm{r}(1)}^2 m_{(3)}}{n_{\mathrm{r}(1)}^2 - m_{(3)}^2} & & \cdots & \dfrac{n_{\mathrm{r}(d)}^2 m_{(3)}}{n_{\mathrm{r}(d)}^2 - m_{(3)}^2} \\ \vdots & & \ddots & \vdots \\ \dfrac{n_{\mathrm{r}(1)}^2 m_{(d)}}{n_{\mathrm{r}(1)}^2 - m_{(d)}^2} & & \cdots & \dfrac{n_{\mathrm{r}(d)}^2 m_{(d)}}{n_{\mathrm{r}(d)}^2 - m_{(d)}^2} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} C_{\mathrm{Fnr}(1)} \\ \\ \vdots \\ \\ C_{\mathrm{Fnr}(d)} \end{bmatrix}}_{C_{Fn}} = \underbrace{\begin{bmatrix} -\dfrac{Q_{\mathrm{F}}}{U^2} \\ 0 \\ 0 \\ \\ 0 \end{bmatrix}}_{W} \tag{16}$$

From matrix Equation (16) can be determined the capacitance of each filter:

$$\begin{gathered} \mathbf{A}\cdot\mathbf{C}_{\mathrm{Fnr}} = \mathbf{W} \\ \mathbf{C}_{\mathrm{Fnr}} = \mathbf{A}^{-1}\cdot\mathbf{W}, \end{gathered} \tag{17}$$

where:

$$1 = m_1 < n_1 < m_2 < n_2 < \cdots < m_d < n_d. \tag{18}$$

By solving the matrix Equation (17) by Cramer's rule, we get

$$C_{\mathrm{Fnr}(i)} = \frac{W_i}{|A|} = \frac{-\dfrac{Q_{\mathrm{F}}}{U^2}\cdot A_{\mathrm{dop1},i}}{\sum\limits_{j=1}^{d}\dfrac{n_j^2 m_1\omega_{(1)}}{n_j^2 - m_1^2}A_{\mathrm{dop1},j}}. \tag{19}$$

The determinant of the matrix **A** (with respect to first row)

$$|\mathbf{A}| = \begin{vmatrix} \dfrac{n_1^2 m_1 \omega_{(1)}}{n_1^2 - m_1^2} & \dfrac{n_2^2 m_1 \omega_{(1)}}{n_2^2 - m_1^2} & \cdots & \dfrac{n_d^2 m_1 \omega_{(1)}}{n_d^2 - m_1^2} \\ \dfrac{n_1^2 m_2}{n_1^2 - m_2^2} & \dfrac{n_2^2 m_2}{n_2^2 - m_2^2} & \cdots & \dfrac{n_d^2 m_2}{n_d^2 - m_2^2} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{n_1^2 m_d}{n_1^2 - m_d^2} & \dfrac{n_2^2 m_d}{n_2^2 - m_d^2} & \cdots & \dfrac{n_d^2 m_d}{n_d^2 - m_d^2} \end{vmatrix} = \sum_{j=1}^{d} \frac{n_j^2 m_1 \omega_{(1)}}{n_j^2 - m_1^2} A_{\mathrm{dop}1,j}, \tag{20}$$

where $A_{\mathrm{dop}1,j}$ – algebraic complement of the matrix $\mathbf{A}$ with respect to element $A_{1,j}$

$$W_i = \begin{vmatrix} \dfrac{n_1^2 m_1 \omega_{(1)}}{n_1^2 - m_1^2} & \cdots & -\dfrac{Q_{\mathrm{F}}}{U^2} & \cdots & \dfrac{n_d^2 m_1 \omega_{(1)}}{n_d^2 - m_1^2} \\ \dfrac{n_1^2 m_2}{n_1^2 - m_2^2} & \cdots & 0 & \cdots & \dfrac{n_d^2 m_2}{n_d^2 - m_2^2} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \dfrac{n_1^2 m_d}{n_1^2 - m_d^2} & \cdots & 0 & \cdots & \dfrac{n_d^2 m_d}{n_d^2 - m_d^2} \end{vmatrix} = -\frac{Q_{\mathrm{F}}}{U^2} \cdot A_{\mathrm{dop}1,i}. \tag{21}$$

Since in the Equation (19)

$$-\frac{Q_{\mathrm{F}}}{U^2} \text{ is greater than 0, and } \frac{n_j^2 m_1 \omega_{(1)}}{n_j^2 - m_1^2}$$

is greater than 0 for every j, thus $C_{\mathrm{Fnr}(i)}$ is greater than 0 if and only if all $A_{\mathrm{dop}1,i}$ are of the same sign. The complement $A_{\mathrm{dop}1,i}$ can be written as

$$A_{\mathrm{dop}1,i} = (-1)^{1+i} \begin{vmatrix} \dfrac{n_1^2 m_2}{n_1^2 - m_2^2} & \cdots & \dfrac{n_{i-1}^2 m_2}{n_{i-1}^2 - m_2^2} & \dfrac{n_{i+1}^2 m_2}{n_{i+1}^2 - m_2^2} & \cdots & \dfrac{n_d^2 m_2}{n_d^2 - m_2^2} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \dfrac{n_1^2 m_d}{n_1^2 - m_d^2} & \cdots & \dfrac{n_{i-1}^2 m_d}{n_{i-1}^2 - m_d^2} & \dfrac{n_{i+1}^2 m_d}{n_{i+1}^2 - m_d^2} & \cdots & \dfrac{n_d^2 m_d}{n_d^2 - m_d^2} \end{vmatrix} =$$

$$= (-1)^{1+i} \left(\prod_{\substack{k=1 \\ k \neq i}}^{d} n_k^2 \right) \cdot \left(\prod_{k=2}^{d} m_k \right) \begin{vmatrix} \dfrac{1}{n_1^2 - m_2^2} & \cdots & \dfrac{1}{n_{i-1}^2 - m_2^2} & \dfrac{1}{n_{i+1}^2 - m_2^2} & \cdots & \dfrac{1}{n_d^2 - m_2^2} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \\ \dfrac{1}{n_1^2 - m_d^2} & \cdots & \dfrac{1}{n_{i-1}^2 - m_d^2} & \dfrac{1}{n_{i+1}^2 - m_d^2} & \cdots & \dfrac{1}{n_d^2 - m_d^2} \end{vmatrix} = \tag{22}$$

$$= (-1)^{1+i} \left(\prod_{\substack{k=1 \\ k \neq i}}^{d} n_k^2 \right) \left(\prod_{k=2}^{d} m_k \right) \frac{1}{\displaystyle\prod_{k=2}^{d} \prod_{\substack{j=1 \\ j \neq i}}^{d} \left(n_j^2 - m_k^2\right)} \begin{vmatrix} \displaystyle\prod_{\substack{j=1 \\ j \neq i \\ j \neq 1}}^{d} \left(n_j^2 - m_2^2\right) & \cdots & \displaystyle\prod_{\substack{j=1 \\ j \neq i \\ j \neq l}}^{d} \left(n_j^2 - m_2^2\right) & \cdots & \displaystyle\prod_{\substack{j=1 \\ j \neq i \\ j \neq d}}^{d} \left(n_j^2 - m_2^2\right) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \displaystyle\prod_{\substack{j=1 \\ j \neq i \\ j \neq 1}}^{d} \left(n_j^2 - m_d^2\right) & \cdots & \displaystyle\prod_{\substack{j=1 \\ j \neq i \\ j \neq l}}^{d} \left(n_j^2 - m_d^2\right) & \cdots & \displaystyle\prod_{\substack{j=1 \\ j \neq i \\ j \neq d}}^{d} \left(n_j^2 - m_d^2\right) \end{vmatrix}$$

where l is column number.

Finally complement $A_{\mathrm{dop}1,i}$ defines the relationship

$$A_{\mathrm{dop}1,i} = (-1)^{1+i}\left(\prod_{\substack{k=1\\k\neq i}}^{d} n_k^2\right)\cdot\left(\prod_{k=2}^{d} m_k\right)\cdot\left(\prod_{\substack{j,k=1,\\j<k,j\neq i,k\neq i}}^{d}\left(n_k^2-n_j^2\right)\right)\frac{\prod\limits_{\substack{j,k=2\\j<k}}^{d}\left(m_j^2-m_k^2\right)}{\prod\limits_{k=2}^{d}\prod\limits_{\substack{j=1\\j\neq i}}^{d}\left(n_j^2-m_k^2\right)}. \tag{23}$$

Regarding (18), it is evident that the elements $(m_j^2 - m_k^2)$ are smaller then 0 and $(n_k^2 - n_j^2)$ are smaller then 0, because for these components j is smaller than k. Also, considering (18), for j smaller than k, elements $(n_j^2 - m_k^2)$ are smaller then 0, and for j greater than or equal to 0 elements $(n_j^2 - m_k^2)$ are greater than 0.

Number of components $(m_j^2 - m_k^2)$ having a negative value in formula (23) is equal to

$$\frac{(d-1)(d-2)}{2},$$

and the number of components $(n_j^2 - m_k^2)$ having negative value depending on the i is equal to

$$\frac{(d-1)(d-2)}{2}+i-1.$$

Taking into account the formula (23) we obtain

$$A_{\mathrm{dop}1,i} = (-1)^{1+i}\left(\prod_{\substack{k=1\\k\neq i}}^{d} n_k^2\right)\cdot\left(\prod_{k=2}^{d} m_k\right)\cdot\left(\prod_{\substack{j,k=1,\\j<k,j\neq i,k\neq i}}^{d}\left(n_k^2-n_j^2\right)\right)\frac{(-1)^{\frac{(d-1)(d-2)}{2}}\prod\limits_{\substack{j,k=2\\j<k}}^{d}\left|m_j^2-m_k^2\right|}{(-1)^{\frac{(d-1)(d-2)}{2}+i-1}\prod\limits_{k=2}^{d}\prod\limits_{\substack{j=1\\j\neq i}}^{d}\left|n_j^2-m_k^2\right|} \tag{24}$$

hence

$$A_{\mathrm{dop}1,i} = (-1)^{2}\left(\prod_{\substack{k=1\\k\neq i}}^{d} n_k^2\right)\cdot\left(\prod_{k=2}^{d} m_k\right)\cdot\left(\prod_{\substack{j,k=1,\\j<k,j\neq i,k\neq i}}^{d}\left(n_k^2-n_j^2\right)\right)\frac{\prod\limits_{\substack{j,k=2\\j<k}}^{d}\left|m_j^2-m_k^2\right|}{\prod\limits_{k=2}^{d}\prod\limits_{\substack{j=1\\j\neq i}}^{d}\left|n_j^2-m_k^2\right|} \tag{25}$$

In the formula (25) all components are positive, that means all complements are positive i.e. there are of same sign. This allows concluding that the relation (19) allows to determine all positive values of $C_{\mathrm{Fnr}(i)}$, for all $i = 1, ..., d$.

Example 2. For the data from Example 1 design a filter group using the matrix method. The impedance characteristic should reach maximums for frequencies 300 Hz, 450 Hz and 600 Hz that means $m_1 = 6$, $m_2 = 9$ and $m_3 = 12$, ($n_1 = 5$, $n_2 = 7$, $n_3 = 11$ and $n_4 = 13$).

Substituting numerical data into Equation (16)

$$\begin{bmatrix} \frac{5^2\cdot 100\cdot\pi}{5^2-1} & \frac{7^2\cdot 100\cdot\pi}{7^2-1} & \frac{11^2\cdot 100\cdot\pi}{11^2-1} & \frac{13^2\cdot 100\cdot\pi}{13^2-1} \\ \frac{5^2\cdot 6}{5^2-6^2} & \frac{7^2\cdot 6}{7^2-6^2} & \frac{11^2\cdot 6}{11^2-6^2} & \frac{13^2\cdot 6}{13^2-6^2} \\ \frac{5^2\cdot 9}{5^2-9^2} & \frac{7^2\cdot 9}{7^2-9^2} & \frac{11^2\cdot 9}{11^2-9^2} & \frac{13^2\cdot 9}{13^2-9^2} \\ \frac{5^2\cdot 12}{5^2-12^2} & \frac{7^2\cdot 12}{7^2-12^2} & \frac{11^2\cdot 12}{11^2-12^2} & \frac{13^2\cdot 12}{13^2-12^2} \end{bmatrix} \begin{bmatrix} C_{\mathrm{Fnr5}} \\ C_{\mathrm{Fnr7}} \\ C_{\mathrm{Fnr11}} \\ C_{\mathrm{Fnr13}} \end{bmatrix} = \begin{bmatrix} -\frac{-10^6}{\left(6\cdot 10^3\right)^2} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{23}$$

Equation (23) could be easy solved using Matlab software (Tab. 2).

Table 2. Parameters of the group of 4 filters designed using the matrix method

$n_{r(i)}$		5	7	11	13
$Q_{\mathrm{Fnr}(i)}$	kvar	−533.94	−230.3	−113.93	−121.79
$C_{\mathrm{Fnr}(i)}$	μF	45.33	19.95	10	10.7
$L_{\mathrm{Fnr}(i)}$	mH	8.9	10.4	8.4	5.6

Figure 3 shows the resultant characteristic of a group of filters designed using the matrix method. The shape of the impedance characteristic complies with the assumed one and with other design requirements.

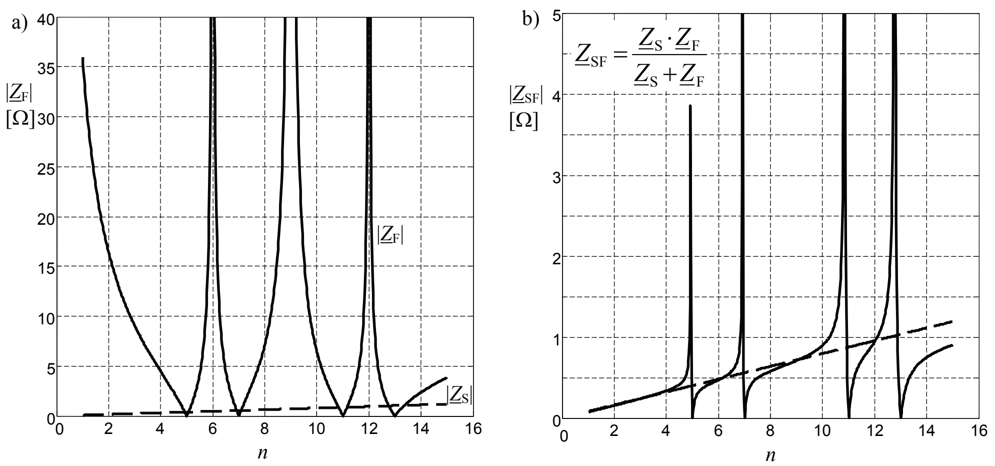

Fig. 3. The graph of impedance: a) group of 4 filters, b) viewed from the load

5. Conclusions

Graphs of the designed filters group power and powers of individual filters are shown in Figure 4. Powers of individual filters determined using the matrix method and those determined by means of the method in the third section are compared for a given harmonic (consecutively). The power of the 5th harmonic filter, determined using the matrix method is greater than the power using the former method. For other harmonics the powers determined by means of the matrix method are smaller. The power of the last filter (Q_{Fn13}) is greater then that of the previous filter (Q_{Fn11}).

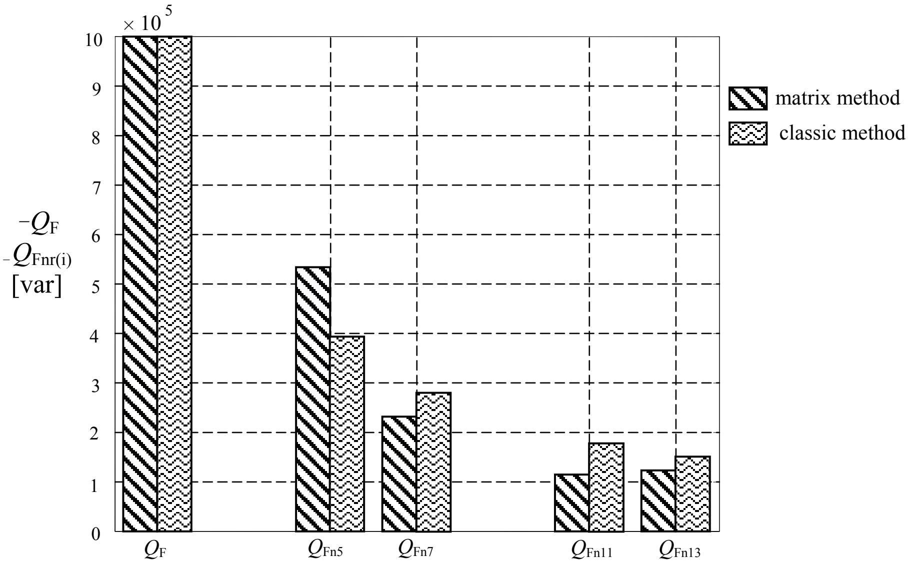

Fig. 4. Comparison of the powers of filters designed with both methods

The advantage of the proposed method is that the frequency characteristic of the group of filters can be shaped in the way given a priori. The shape of the frequency characteristic does not result from selection of the filters' group, but is based on the made assumption and the set of parameters selected for optimisation.

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