

Coupling electromagnetic (FE) models to multidomain simulator to analyse electrical drives and complex control systems

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Abstract: This work presents the co-simulation approach to the analysis of control systems containing detailed models of electromagnetic and electromechanical converters. In this method of analysis the attention is paid to the whole system and not only to its electromagnetic part. The latter is described by equations resulted from the two-dimensional finite element discretisation of the Maxwell equations, and is coupled weakly with the remaining part of the system. The simulation is carried out in Matlab/Simulink environment wherein the coupling is realised through the S-function. Example results regarding simulation of the operation of the control system of an electrical machine and the operation of a power electronic converter are presented and compared with available reference data.

Key words: co-simulation, electrical drives and electromagnetic systems, mathematical modeling, finite element methods

1. Introduction

The electromagnetic and electromechanical systems such as electrical machines, transducers and actuators are often only the “pieces” of more composite structures. This point of view is natural, bearing in mind that currently the majority of electromagnetic systems and electric drives are supplied from the frequency converters containing switching elements whose operation is controlled using sophisticated control algorithms. This is also related with the principal points of view of scientific disciplines such as automatic control and mechatronics where central attention is focused on the operation of the entire system rather than on the particular part of it.

Creating reliable mathematical models of the complex systems, in general can be realised on the basis of the identification of their parameters. If the electromagnetic and the electromechanical systems are the point of interest the basic mathematical models used are in the form of the equivalent electric circuits. These models can be executed in a fraction of time required for execution by more detailed models, and are best choice from the point of view of analysis

and system synthesis. This approach often provides enough precise results, but there are many cases wherein these models are not adequate. One example of such the systems, are those containing the magnetic cross-couplings, such as magnetic bearings or doubly-fed machines. Also, converters containing solid electromagnetically coupled elements, such as electrical machines with the solid rotors, are the significant challenge for the circuit modelling. With respect to the analysis of this type models the finite element method offers an excellent performance [1-8]. The finite element models easily handle magnetic nonlinearity and the magnetic cross-couplings. The eddy current phenomenon can also be relatively easily accounted for. The solutions of the electromagnetic problems by the time-stepping analysis are the most informative ones. To date, only the two-dimensional field-circuit time-stepping models find wide applicability as the overall cost involved with the usage of the three-dimensional models is still considerable.

To analyse the operation of the electromagnetic system, as the part of the more composite structure one will need to take account of the description of the model using the partial differential equations as well as the state-space equations. This problem has already been investigated and has found several successful implementations, for example in [1-5]. The need for using this type models in the designing of various systems has recently motivated the manufacturers of the commercial packages for the electromagnetic analysis to incorporate the co-simulation facility to their products, e.g. [6].

Solution of the equations of the two types together, using a strong coupling, has many limitations due to practical reasons. When the state equations describing, e.g. the system controllers are coupled to the equations resulted from the finite element discretisation, the time-constants attributed to these problems can be different by a few orders. In such the case the system of strongly coupled equations must be solved using a very small time-step required by the smallest time-constant existing within the system, which is not a cost-effective approach. In the modelling, another problem appears when the control algorithms become algorithmically complex requiring description at different abstraction levels. In the design of control systems various approaches are used for that purpose, e.g. the block diagrams or the finite state automata. Inclusion of different abstraction levels in the model requires an appropriate environment providing a logical consistency and the data flow in different directions between the levels. An ergonomics is also very important from the point of view of the designer's productivity. To solve the above mentioned problems in this work the algorithm that allows coupling the finite element models of the electromagnetic systems, to the equations of the models at system level, is presented. It is based on a weak coupling between the models and is implemented in the environment of Matlab/Simulink to create the opportunity of analysing systems of complex structures.

2. Theory

2.1. Equations of transient finite element model

The description of the electromagnetic system is based on the Maxwell equations. Using the magnetic the Ampere, the Faraday, and the continuity laws the equation governing the

electromagnetic field in term of the magnetic vector potential $\mathbf{A} = (A_x, A_y, A_z)$ can be written as:

$$\mathbf{curl} \nu \mathbf{curl} \mathbf{A} = \mathbf{J}_s - \sigma \left(\frac{d\mathbf{A}}{dt} + \mathbf{grad} V \right), \quad (1)$$

where: ν is the magnetic reluctivity, $\mathbf{J}_s = (J_{sx}, J_{sy}, J_{sz})$ is the vector of source current density σ is an electric conductivity, and V is the electric scalar potential. From the point of view of the problems currently analysed, the considerations are applicable only to the 2-D models. Consequently, equation (1) will be simplified to:

$$\nu \left(\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} \right) = -J_{sz} + \sigma \left(\frac{dA_z}{dt} - \frac{u}{\ell} \right), \quad (2)$$

where: u stands for the voltage drop across the conductor terminals and ℓ is a depth of the system in the direction of axis z of the Cartesian system of coordinates. In many cases this equation must be solved together with the circuit equations that are associated with the windings existing within the electromagnetic system. In the analysis of the electromagnetic systems the typical distinction between types of winding is related with the way on how the magnetic field penetrates conductors. Following this distinction, generally there are stranded windings having infinitely thin conductors where a single coil is described by equation:

$$u_s = Ri_s + L_e \frac{di_s}{dt} + \frac{d}{dt} \oint_C A_z dl \quad (3)$$

with: u_s being a voltage source, i_s a current through conductors, R a resistance of coil, L_e a leakage inductance of the end-winding, C a curve along conductors. Considering a coil containing only one turn of a solid conductor driven from the voltage source the equations governing the continuity law for the electric current are:

$$i_m = Gu_m^+ - \frac{d}{dt} \sigma \iint_{S_m^+} A_z dS, \quad (4a)$$

$$-i_m = Gu_m^- - \frac{d}{dt} \sigma \iint_{S_m^-} A_z dS, \quad (4b)$$

$$u_m^+ - u_m^- = R_e i_m + L_e \frac{di_m}{dt}, \quad (4c)$$

where: i_m is a current through conductor, G is a conductor conductance, u_m^+ and u_m^- are respectively, voltage drop across conductor oriented towards and backwards the z - axis of the Cartesian coordinate system, and S_m^+ and S_m^- are their corresponding cross-section areas, R_e and L_e are a resistance and a leakage inductance of the coil end-regions. Considering coils' interconnections the equations become more complex. Detailed considerations on the forma-

tion of circuit equation for the time-stepping finite element analysis can be found in e.g. [6], and consequently these will not be carried out here.

A traditional process of discretising (1)-(4) relies upon usage of weighted residual criterion and the Galerkin method. Evaluation of coefficients that results from the finite element discretisation is rather standard procedure which can be found in e.g. [7]. The form of equations being the result of evaluation of these coefficients is:

$$\mathbf{G} \frac{d}{dt} \mathbf{X} + \mathbf{H} \mathbf{X} = \mathbf{F}. \quad (5)$$

In this equation the matrices \mathbf{G} , \mathbf{H} hold coefficients related with the discretisation of the finite element space and also the coefficients due to circuit equations (1)-(4). The vector \mathbf{F} normally holds excitations existing within the system such as those due to current, voltage sources or a permanent magnetisation. The elements of matrices \mathbf{G} , \mathbf{H} , \mathbf{F} are usually dependent on certain values of vector \mathbf{X} as the nonlinearity is incorporated into the model via magnetic materials or elements of the electric circuits. The vector \mathbf{X} is associated with unknown variables. In a typical two-dimensional electromagnetic finite element model it is in form:

$$\mathbf{X} = \begin{bmatrix} \boldsymbol{\phi} \\ \mathbf{u} \\ \mathbf{i} \end{bmatrix}, \quad (6)$$

where $\boldsymbol{\phi}$ is the vector of nodal values of the magnetic vector potential whilst \mathbf{u} and \mathbf{i} , represent the vectors of branch voltages and currents through windings, respectively. The majority of the algorithms for the time-domain analysis rely upon solving system of equations (5) using one-step discretisation methods. Typically the backward Euler or the Crank-Nicolson schema, are used [1, 2, 5-8]. After discretising (5) in the domain of time using the former, it takes form:

$$(\mathbf{G}_n + \Delta t \mathbf{H}_n) \mathbf{X}_n = \Delta t \mathbf{F}_{n-1} + \mathbf{G}_{n-1} \mathbf{X}_{n-1}, \quad (7)$$

where Δt and n are the time-step size and the time-step index, respectively. In the case of the rotating machines the algorithm also requires modeling the movement. In this work this is realized by means of the sliding surface technique in which the finite element mesh is discontinuous through the air-gap [8-10]. A rudimentary second-order ordinary differential equation of motion can also be taken into account. This equation is considered in form:

$$\frac{d\alpha}{dt} = \omega, \quad (8a)$$

$$I \frac{d\omega}{dt} = T_e - T_l(\omega), \quad (8b)$$

where: α is a mechanical angle, I is a moment of inertia, ω is an angular velocity of the rotor, T_e is the electromagnetic torque and $T_l(\omega)$ is a total load torque. Equations (8) are discretised in the domain of time, using explicit formula, to obtain:

$$\alpha_n = \Delta t \omega_{n-1} + \alpha_{n-1}, \quad (9a)$$

$$\omega_n = \frac{\Delta t}{I} (T_{en-1} - T_l(\omega_{n-1})) + \omega_{n-1}. \quad (9b)$$

The electromagnetic torque in (9) is computed by means of the Maxwell stress tensor method.

2.2. Algorithm of coupling and its implementation

Assume an arbitrary electromagnetic system described by the system of equations (7). Suppose that the winding of such the system is driven by the voltage source whose frequency, phase and magnitude are adjusted by the system controller. The controller sets up the voltage according to the algorithm implemented, which can be either continuous or discrete, linear or nonlinear, and to the actual values of controlled variables like current, force, torque, or more frequently – mechanical angle, position or velocity. The voltage developed by the controller affects the operation of the electromagnetic system in the usual way, but the operation of this system has no effect on how the controller develops the control voltage. This is very close to the operation of real systems because the system controllers are always insulated from the objects. This also supports the way on how the coupling between the electromagnetic system and the system controller is realized numerically in this work. From the computational point of view both models (the electromagnetic model and the model of a control system) require integration in the domain of time. As it was already mentioned, the time constants related with these two models usually are different by two orders or more. Consequently, the equations of the system controller require integration with a very small time-step whilst the time-step required by the electromagnetic models can be a few orders higher. From this point of view it is convenient to consider the whole model to be comprised of two submodels – one being an outer one associated with the system controller, and the second being an inner one associated with the electromagnetic part of the model. Each submodel has an integration process involved respectively, the outer one the integration time-step δt and the inner one the time-step Δt , such that $\delta t \ll \Delta t$.

Solution of the system of equations (7) results in obtaining electromagnetic quantities at time $t = n \cdot \Delta t$. The time-varying quantities obtained from these equations are considered as the signals of controlled variables in the entire model. For example, if a simple inductive converter is considered, such as an inductor or a transformer, the current computed from (7) can be considered a controlled variable whilst voltage applied to the coil terminals a control variable. The controlled variables can also be determined in different ways. For example, the power attributed to the winding can be determined from computed currents and voltages. The input voltages are generated by external sources or controllers whose equations are not present in (7). To couple the submodels the signals computed in the outer integration process are sampled with the time-period $T_s = \Delta t$. This enables using at time $n \cdot T_s$ the instantaneous values of these signals as an input data to the inner process. The quantities computed in the inner processes are returned back to the outer process after Δt . In the meantime the parameters of the

electromagnetic system, whose equation are solved by the inner process, in the outer process are seen constant. The flowchart diagram illustrating this algorithm is depicted in Fig. 1. Formally, this coupling is weak because there is no continuity of energy through the submodels, but this is an analogous to real systems where the control part of the system is insulated from the object.

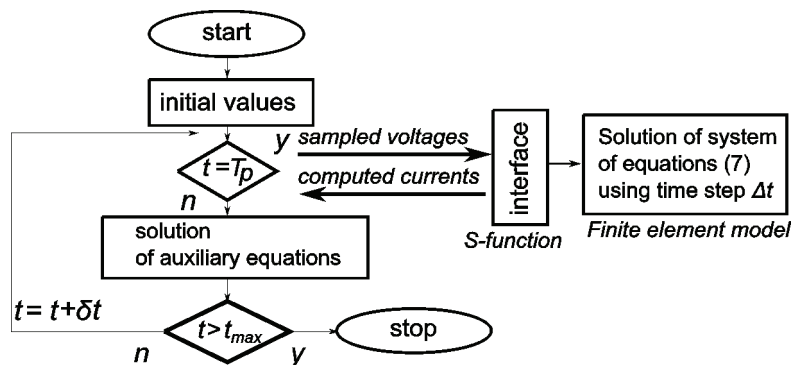


Fig. 1. Flowchart diagram illustrating the algorithm of computations

Implementation of this algorithm requires an appropriate data management in order to provide data flow between the submodels. In this work this is realized in the environment of Matlab/Simulink which is one of the most widely available and efficient system simulators for multidomain analysis. It provides an elegant way of interfacing external codes, written in various programming languages, called the *S*-function [11, 13]. The description of models in Simulink is realized by means of the block-diagram modeling. However, the *S*-function is a purely programming interface being a piece of structured code. By a principle, the *S*-function provides description of the complete state-space model. The Simulink works so that the subfunctions nested in the master *S*-function are called sequentially. The environment sets up the value of flag variable which is captured by the *S*-function to realize appropriate computations in order to determine state of the model. This is explained by means of the following pseudocode:

```

switch(flag) {
    case 0: basic settings and initializations;
    case 1: S-function outputs derivatives of continuous states;
    case 2: S-function updates discrete states;
    case 3: S-function returns values at the block outputs;
    case 4 to 8: not used
    case 9: termination of simulation;
}

```

In this work the *S*-function is used in such a way as no states are defined when calling flag value equal to 1. At flag equal to 0 the block sample time is set equal to Δt . The *S*-function

computes only the outputs, at flag value equal to 3, which are determined by solving (7). In this way the algorithm does not require approximation of derivatives in order to determine states. Computation of states would be the most time-consuming part of the simulation as approximation of derivatives of some electromagnetic quantities usually involves considerable effort related with the need for using values of the electromagnetic quantities from previous time-steps. However, a disadvantageous consequence of this simplification is an inconstancy of the algorithm related with the need for the sampled voltages and the computed currents to be known at the same time. This inconsistency is called the algebraic loop and cannot be avoided without delaying the computed currents by one integration step Δt with respect to the sampled voltages. This delay influences the accuracy of computations which can be increased by reducing Δt .

3. Examples of calculations

The analysis using the elaborated computational tools is demonstrated on the sample models. First computational problem regards an implementation and testing a speed controller for a special doubly-fed induction motor with two stator windings and a brushless rotor (see Fig. 2a) [8, 12]. One of the stator windings – the 4-pole power winding (UVW terminals in Fig. 2a) is driven from 3-phase mains whilst the other – the 8-pole control winding (uvw terminals in Fig. 2a) is driven from the frequency converter. The operation principle of the motor relies upon coupling the stator winding, having different numbers of pole-pairs $p_1 = 2$ and $p_2 = 4$, by the rotor with special cage winding, having the number of poles N_b equal to $p_1 + p_2$. This machine has the potential for being used both as a motor and generator. Its control requires only a fraction of power required to control the conventional induction machine of the same nominal power. It is a significant challenge for the modeling, designing and control due to the complex principle of operation, existence of the magnetic cross-couplings between the stator windings and complex distribution of the air-gap magnetic flux density (see Fig. 2c) that are responsible for a few unusual features of this machine. In this work the controller that uses the linear control law is considered. The scalar technique of torque control is realized by adjusting magnitude and frequency of the voltage applied to the uvw terminals. To stabilize the rotor speed the phase of the control voltage is adjusted too.

In the simulation the system is tested under heavy variations of load torque in order to check the system stability around the given point of operation. In this case the two-directional frequency converter depicted in Fig. 2b is not modeled and the voltage produced by the controller is applied directly to the machine terminals. Figure 2d depicts the Simulink implementation of the model. In Fig. 3 the results of simulation, using linear (constant permeability) and nonlinear (variable permeability) finite element models, are compared with the results obtained by means of the equivalent circuit model whose parameters were identified using the experimental method.

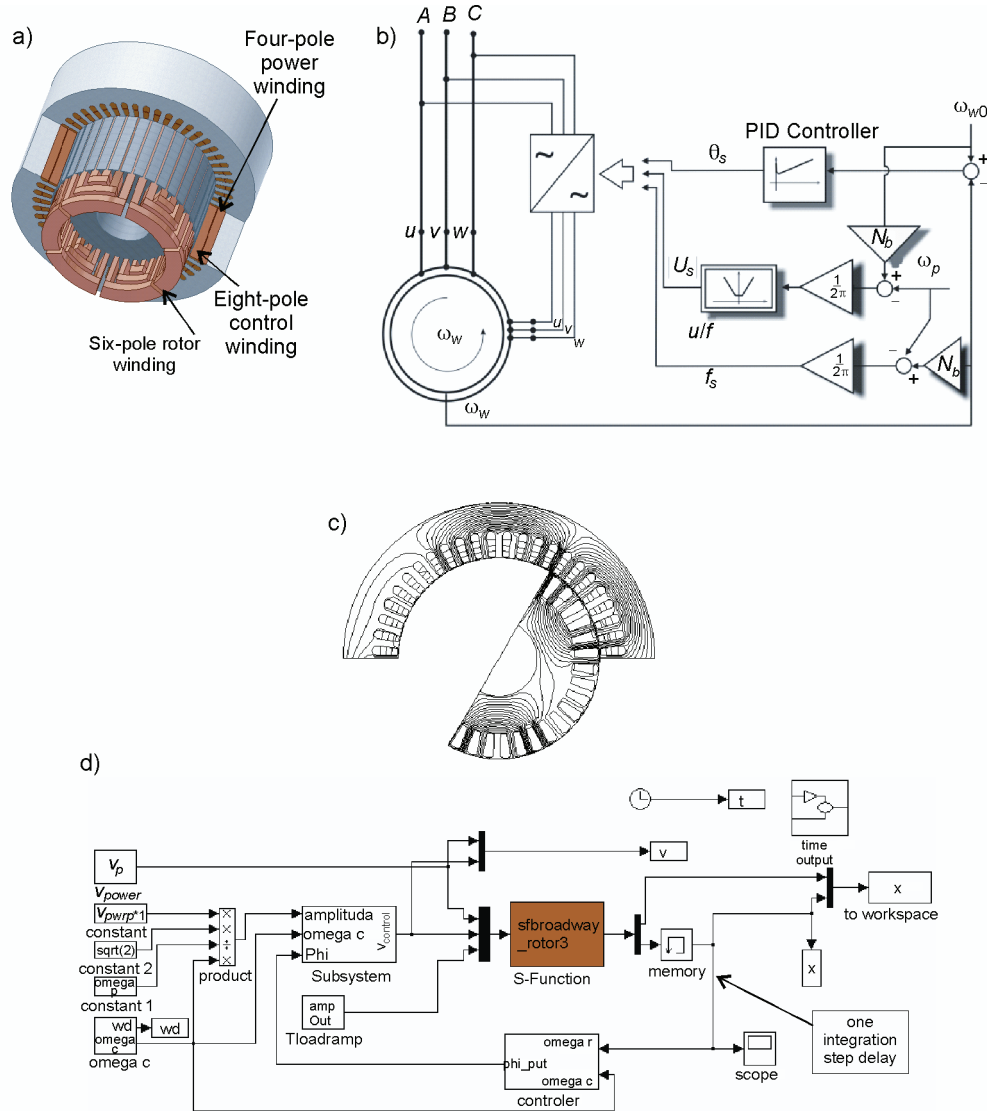


Fig. 2. Brushless doubly-fed induction motor: a) motor structure and winding configuration, b) control system using scalar torque control technique: θ_s – phase angle of control voltage, $|U_s|$ – magnitude of control voltage, f_s – frequency of control voltage, ω_0 – prescribed rotor angular speed, ω_w – instantaneous rotor angular speed, N_b – number of rotor poles, ω_p – angular frequency of mains voltage, c) plot of magnetic flux distribution at the steady-state operation considering periodic symmetry of the model, d) Simulink implementation of the entire model: shaded block implements the coupling of the finite element motor model through the S-function.

As one can notice the agreement is satisfactory with some visible differences regarding transient conditions. The results obtained from the equivalent circuit model are closer to those obtained from the linear finite element model than to those obtained from the nonlinear model.

This is natural as the parameters of the circuit model, determined experimentally, are constants and so allow modeling the machine performance only in the average sense.

The overall time of simulation using time step Δt equal to $200 \mu\text{s}$ was approximately 200 hours on Pentium IV computer with 2666 MHz CPU clock frequency and 1 GB of operating memory. Further reduction of Δt , and so the sampling time T_s , significantly intensify the cost of computation with small gain in accuracy.

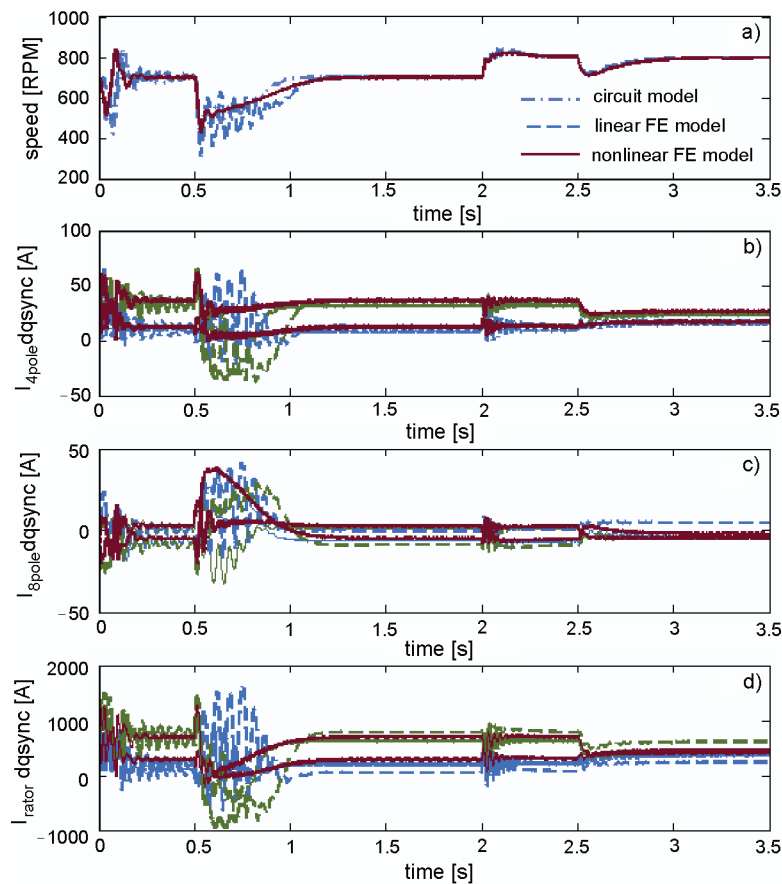


Fig. 3. Results of computations regarding speed control of a brushless doubly-fed induction motor, a) rotor speed, b) UVW (power) winding currents in dq frame of reference, c) uvw (control) winding currents in dq frame of reference, d) rotor winding currents in dq frame of reference. The control loop is closed for $t > 0.5$ s. The load torque equals, respectively: $-25 \text{ N}\cdot\text{m}$ at time $t \leq 0.5$ s, and $100 \text{ N}\cdot\text{m}$ at time $t > 2.5$ s. The prescribed rotor speed equals 700 RPM at time $t \leq 2$ s, and 800 RPM for $t > 2$ s

The second problem presented here involves the analysis of a switched mode solid-state converter being the part of the power supply with voltage stabilization. The analysis regards the need for considering the frequency effects in the winding of a choke inductor [13]. The power supply analysed is based on a buck-type, step-down converter with the structure of power train depicted in Fig. 4a. The converter operates controlled in the closed loop. The

pulse width modulation (PWM) and linear PI controller are used to stabilize the output voltage by switching transistor S between “on” and “off” states. The choke inductor has a ferrite core made of the 3c90 material. The diameter of the wire is equal to 1.5 mm. The converter operates with the switching frequency equal to 31 kHz. The depth of wire penetration, corresponding with the fundamental harmonic of the PWM waveform, is 0.27 mm. However, the waveform of the current through the winding of an inductor contains large DC and AC components so that evaluation of power loss considering the skin and the proximity effects would be difficult using the conventional methods of analysis. From the point of view of the numerical computations, modeling the high-frequency effects requires high-quality interpolation of the magnetic potential in the area of conductors, preferably using higher order elements. However, the usage and postprocessing on first order elements are much simpler, and the problem can be solved by means of high density meshing (see Fig. 4b). The finite element model of the inductor is considered in a planar 2D system. Figure 4b depicts a sample distribution of magnetic flux and current density in the area of winding of the inductor for the given conditions of operation.

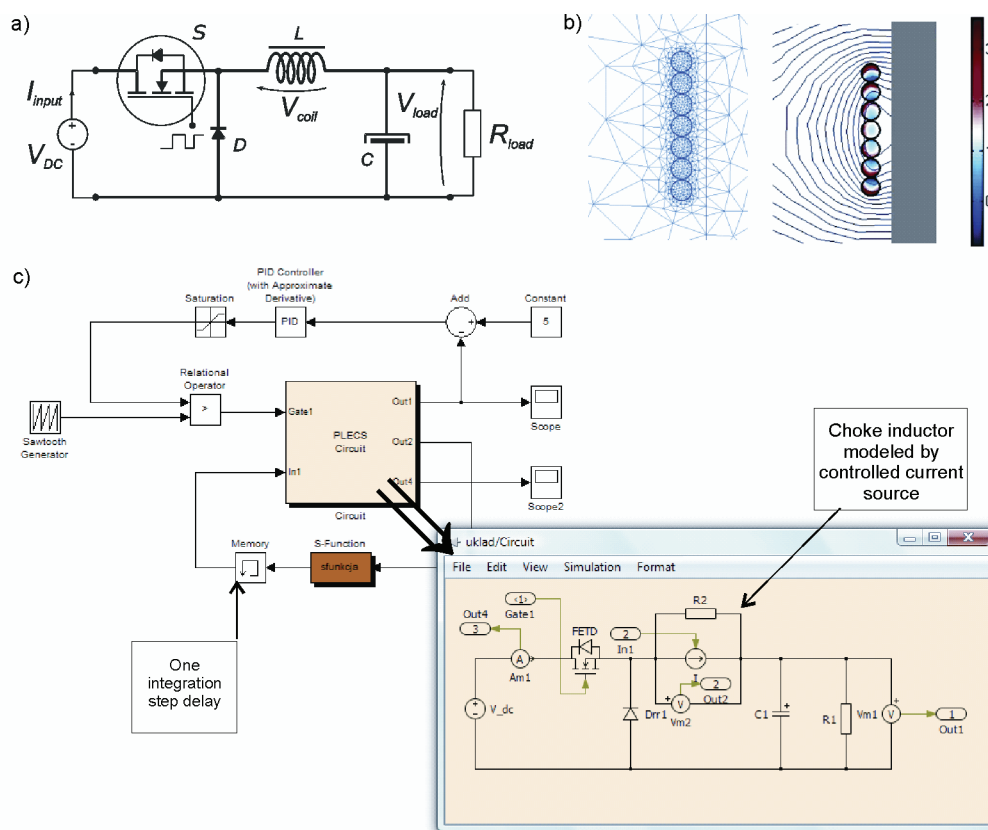


Fig. 4. Voltage-controlled power supply based on the buck-type switched mode converter: a) structure of power train, b) finite element mesh over the region of the winding and plot of magnetic flux and current distribution in the area of winding of the choke inductor, c) Simulink implementation of the model: shaded block implements the coupling of the finite element motor model through the S-function

In this case the model has been extended as to consider the converter structure composed of power electronic components. A third-party Simulink library called PLECS is used for that purpose [14]. This library provides a flexible modeling technique based on the piecewise-linear models of the switching elements and a Spice-like modeling environment. It provides a broad range of detailed models of the solid-state elements for large-signal analysis.

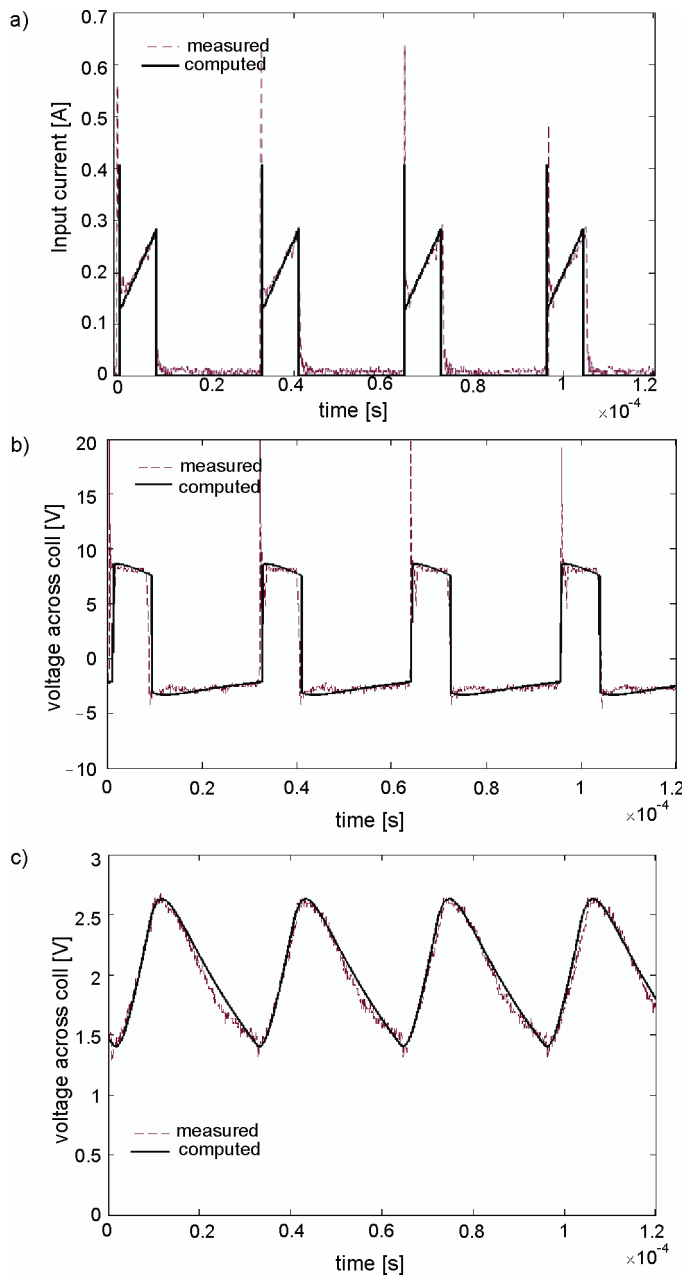


Fig. 5. Results of computations compared with measurements of the switched mode power supply: a) converter input current (I_{input}), b) voltage across winding of choke inductor (V_{coil}), c) converter output voltage (V_{load})

In the case presented the finite element model of a choke inductor is used in a different way. The inductor in the model of a switched mode converter is represented by the current source (see Fig. 4c). The voltage computed across this element is passed to the finite element model and is used as an excitation in the system of equations (7). The current computed from (7) is used to control the current source.

The results compared with measurements, at the given conditions of steady-state operation, corresponding with the transistor duty cycle equal to 26%, are shown plotted in Fig. 5. As it can be observed the converter operates in the continuous mode, but the model enables determining winding loss both in the continuous and the discontinuous mode. As it can be noticed, in the test example the agreement obtained is very good. The visible differences regard the switching processes which are more “spiky” for the measured waveforms. This is due to existence of additional wiring, related with the imperfect assembly of elements on the printed circuit board of the physical converter, which is difficult to account for.

The time step Δt , and so the sampling time, in simulation was equal to $0.25 \mu\text{s}$ which is approximately 130 times smaller than the frequency of fundamental harmonic of the PWM waveform used. Reducing the sampling time to $0.1 \mu\text{s}$ did not cause any relevant change of the result. The overall time of execution of this model, corresponding to obtaining the steady-state solution, was equal to 6 minutes on a laptop computer equipped with Pentium T3400 dual core processor and 4 GB of operating memory.

4. Conclusions

The approach to modeling complex systems containing electromagnetic parts presented in this work may be helpful in particular in better understanding their operation. It may be especially advantageous in cases when creation or identification of low-order models causes significant technical problems or these models are too approximate. The applicability of the approach to problems of such types has been confirmed by physical validation of the results. Due to significant computational costs involved it is unlikely to be applicable to the computational tasks related with the control systems synthesis, but can be used in the cases where a deep insight into the operation of particular components of the systems is required. Although it has not been realised to date, using this approach, the inclusion of more than a single finite element model of the electromagnetic system in a Simulink block-diagram model should be possible because the simulator environment does not incorporate any restrictions regarding the number of S -functions used.

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