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A New Topology of Solutions of Chemical Equations

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ABSTRACT. In this work is induced a new topology of solutions of chemical equations by virtue of point-set topology in an abstract stoichiometrical space. Subgenerators of this topology are the coefficients of chemical reaction. Complex chemical reactions, as those of direct reduction of hematite with a carbon, often exhibit distinct properties which can be interpreted as higher level mathematical structures. Here we used a mathematical model that exploits the stoichiometric structure, which can be seen as a topology too, to derive an algebraic picture of chemical equations. This abstract expression suggests exploring the chemical meaning of topological concept. Topological models at different levels of realism can be used to generate a large number of reaction modifications, with a particular aim to determine their general properties. The more abstract the theory is, the stronger the cognitive power is.

Key words: Topology, Subgenerators, Basis, Dimension, Equations

INTRODUCTION

In this work for the first time in scientific literature is induced a topology of solutions of chemical equations. This topology is developed by virtue of a new algebraic analysis of subgenerators of coefficients of chemical reaction and theory of point-set topology.[1,2]

Why did we do it? Simply speaking, it was necessary because the theory of balancing chemical equations worked only on determination of coefficients of reactions, without taking into account interactions among them. Was it correct? No! It was an artificial approach, which was used by chemists, only in order to determine quantification relations among reaction molecules and nothing more.

That so-called *traditional approach* with a minor scientific meaning, did not provide complete information for reaction character, just it represented only a rough reaction quantitative picture. Chemists by that approach, or more accurately speaking so-called *chemical techniques* balanced only very simple chemical equations. Their procedures were inconsistent and produced illogical results. The author of this work refuted all of them in his previous comprehensive work.[3]

We open this algebraic analysis by examining three senses which the word *topology* has in our discourse.

• The first sense is that proposed when we say that *topology is the constructive theory of relations among sets.* We notice that we draw constructive conclusions from our topological data. Progressively we become aware that constructive topological calculations conducted according to certain norms can be depended on if the data are correct. The study of these norms, or principles has always been considered as a branch of applied topology. In order to distinguish topology of this sense from other senses introduced later, we shall call it *applied topology.*

• In the study of *applied topology* it has been found productive to use mathematical methods, i.e., to construct mathematical systems having some connection therewith. What such a system is, and the nature of the connections, are questions which we shall consider later. The systems so formed are obviously a proper subject for study in themselves, and it is usual to apply the term *topology* to such a study. Topology in this sense is a branch of mathematics. To distinguish it from other senses, it will be called *mathematical topology.*

• In both of its preceding senses, *topology* was used as a proper name. The word is also frequently used as a common noun, and this usage is a third sense of the word distinct from the first two. In this sense a topology is a system, or theory, such as one considers in mathematical or applied topology. Thus we may have algebraic topology, geometric topology, differential topology, etc.

This is as far as it is desirable to go, at present, in defining *mathematical topology.* As a matter of reality, it is ineffective to try to define any branch of science by delimiting accurately its boundaries; rather, one states the essential idea or purpose of the subject and leaves the boundaries to fall where they can. It is a benefit that the definition of topology is broad enough to admit different shades of opinion. Also, it will be allowable to speak of *topological systems*, *topological algebras*, without giving an accurate criterion for deciding whether a given system is such.

There are, however, several remarks which it is suitable to make now to intensify and illuminate the above discussion.

In the first place, we can and do consider topologies as formal structures, whose interest from the standpoint of applied topology may lie in some formal analogy with other systems which are more directly applicable.

In the second place, although the distinction between the different senses of *topology* has been stressed here as a means of clarifying our thinking, it would be a mistake to suppose that applied and mathematical topology are completely separate subjects. In fact, there is a unity between them. Mathematical topology, as has been said, is productive as a means of studying applied topology. Any sharp line between the two aspects would be arbitrary.

Finally, mathematical topology has a regular relation to the rest of mathematics. For mathematics is a deductive science, at least in the sense that a concept of exact proof is fundamental to all part of it. The question of what constitutes an exact proof is a topological question in the sense of the preceding discussion. The question therefore falls within the area of topology; since it is relevant to mathematics, it is expedient to consider it in mathematical topology. Thus, the task of explaining the nature of mathematical strictness falls to mathematical topology, and indeed may be regarded as its most essential problem. We understand this task as including the explanation of mathematical truth and the nature of mathematics generally. We express this by saying that *mathematical topology includes the study of the foundation of chemistry, as that of abstract balancing chemical equations.*

The part of topology which is selected for treatment may be described as the constructive theory of point-set topological calculus. That this topological calculus is central in modern topology does not need to be argued. Also, the constructive aspects of this topological calculus are fundamental for its higher study. Moreover, it is becoming more and more obvious that mathematicians in general need to be conscious of the difference between the constructive and nonconstructive, and there is hardly any better manner of increasing this consciousness than by giving a separate treatment of the former.

The conventional approach to the topological calculus is that it is a formal system like any other; it is unusual only in that it must be formalized more strictly, since we cannot take *topology* for granted, and in that it can be explained in the statements of usual discussion. Here the point of view is taken that we can explain our systems in the more restricted set of statements which we form in dealing with some other (unspecified) formal system.

Since in the study of a formal system we can form assertions which can not be decided by the expedients of that system, this brings in possibilities which did not arise, or seemed merely pathological, in the conventional theories.

It is an explanation for the word *topology* used in our discourse.

PRELIMINARIES

How are things right now in the theory of balancing chemical equations? We shall try to give a comprehensive reply to this question from the view point of chemistry as well as mathematics.

There are two approaches, competing with each other, for balancing chemical equations: chemical and mathematical.

1^{o} First, we shall explain the chemical approach for balancing chemical equations.

In chemistry, there are lots of particular procedures for balancing chemical equations, but unfortunately all of them are inconsistent. In order to avoid reference repetition, we intentionally neglected to mention chemical references here, because a broad list with them is given in.[4,5] These informal procedures were founded by virtue of so-called *traditional chemical principles* and experience, but not on genuine principles. Since, these *principles* were not formalized, they very often generated wrong results. It was a main cause for the appearance of a great number of paradoxes in theory of balancing chemical equations. These paradoxes were discovered and analyzed in detail in.[3] Furthermore, it is true that from the beginning of chemistry to date chemists did not develop their own general consistent method for balancing chemical equations. Why? Probably they must ask themselves!

2^{o} Next we shall explain the mathematical approach for balancing chemical equations.

Simply speaking, that which chemists did not do, mathematicians did.

The earliest reference with a mathematical method (frequently referred to as the *algebraic method* or the *method of undetermined coefficients*) of balancing chemical equations is that of Bottomley[6] published in 1878. A textbook written by Barker[7] in 1891 has devoted some space to this topic too. Unfortunately, the method proposed by Bottomley, more than fifty years, was out of usage, because it and his author both were forgotten. Endslow[8] illustrated this method again in 1931. It is not surprising, therefore, that even today the method is not broadly familiar to chemistry teachers.

The next very important step which mathematicians made is the transfer of problem of balancing of chemical equations from the field of chemistry to the field of mathematics. Jones[9] by virtue of the Crocker's article[10] in 1971 proposed the general problem of balancing chemical equations. He formalized the century old problem in a compact linear operator form as a Diophantine matrix equation. Actually it is the first formalized approach in theory of balancing chemical equations.

The Jones' problem waited for its solution *only* thirty-six years. In 2007, the author of this article by using a reflexive g-inverse matrix gave an elegant solution[4] of this problem, which generalized all known results in chemistry and mathematics.

Krishnamurthy[11] in 1978 gave a mathematical method for balancing chemical equations founded by virtue of a generalized matrix inverse. He considered some elementary chemical equations, which were well-known in chemistry for a long time.

Das[12] in 1986 offered a method of partial equations for balancing chemical equations. He described his method by elementary examples.

In 1989 Yde[13] criticized the half reaction method and proved that half equations can not be defined mathematically, so that they correspond exactly to the chemists' idea of half reaction. He wrote: *The half reaction method of balancing chemical equations has severe disadvantages compared to alternative methods. It is difficult to define a half reaction exactly, and thus to define a corresponding mathematical concept. Furthermore, existence and uniqueness proofs of the solutions* (*balanced chemical equations*) *require advanced mathematics.* It shows that balancing chemical equations is not a *piece of cake* as some chemists think or as they like it to be. Yde by this article announced the need for the formalization of chemistry. Baby is on the way, but is not born!

Also, Yde in his article[14] offered a mathematical interpretation of the *gain-loss rule.* He wrote: *It does not look elegant! Neither does the proof of it! But there is hardly anything we can do about this, if we demand a full mathematical presentation. In fact, the point is that* '*it is complicated*'. Actually, he made a modern version of Johnson's derivation of the oxidation number method.[15]

Subramaniam, Goh and Chia in[16] showed that a chemical equation is equivalent to a class of linear Diophantine equations.

A new general nonsingular matrix method for balancing chemical equations is developed in.[17] It is a formalized method, which include stability criteria for the general chemical equation.

The most general results for balancing chemical equations by using the Moore-Penrose pseudoinverse matrix[18,19] are obtained in.[5] Also, this method is formalized method, which belongs to the class of consistent methods.

In[20] is developed a completely new generalized matrix inverse method for balancing chemical equations. The offered method is founded by virtue of the solution of a homogeneous matrix equation by using von Neumann pseudoinverse matrix.[21–23] The method has been tested on many typical chemical equations and found to be very successful for all equations. Chemical equations treated by this method possessed atoms with fractional oxidation numbers. Furthermore, in this work are analyzed some necessary and sufficient criteria for stability of chemical equations over stability of their reaction matrices. By this method is given a formal way for balancing general chemical equation with a matrix analysis.

Other new singular matrix method for balancing chemical equations which reduce them to an $n \times n$ matrix form is obtained in.[24] This method is founded by virtue of the solution of a homogeneous matrix equation by using Drazin pseudoinverse matrix.[25]

The newest mathematical method for balancing chemical equations is proved in.[26] This method is founded by virtue of the theory of n-dimensional complex vector spaces.

Such looks the picture for balancing chemical equations that mathematicians painted.

We would like to emphasize here that all of the previously mentioned contemporary matrix methods[4,5,17,20,24,26] are rigorously formalized and consistent. Only such formalized methods are not contradictory and work successfully without any limitations. All other techniques or procedures known in chemistry have a limited usage and hold only for balancing simple chemical equations and nothing more. Most of them are inconsistent and produce only paradoxes.

Into a mathematical model must be introduced a whole set of auxiliary definitions to make the chemistry work consistently. Just this kind of set will be constructed in the next section.

Only on this way chemistry will be consistent and resistant to paradoxes appearance.

A NEW CHEMICAL FORMAL SYSTEM

In this section we shall develop a new chemical formal system founded by virtue of principles of a point-set topology.

Let $\mathscr{X}$ is a finite set of molecules.

Definition 2.1. *A chemical reaction on $\mathscr{X}$ is a pair of formal linear combinations of elements of $\mathscr{X}$, such that*

$$\rho: \sum_{j=1}^{r} a_{ij} x_j \to \sum_{j=1}^{s} b_{ij} y_j, (1 \le i \le m) \tag{2.1}$$

with a_{ij}, $b_{ij} \geq 0$.

The coefficients x_j, y_j satisfy three basic principles (corresponding to a closed input-output static model)

- the law of conservation of atoms,
- the law of conservation of mass, and
- the reaction time-independence.

What does it mean a chemical equation? The reply of this question lies in the following descriptive definition given in a compact form.

Definition 2.2. *Chemical equation is a numerical quantification of a chemical reaction.*

In[5] is proved the following proposition.

Proposition 2.3. *Any chemical equation may be presented in this algebraic form*

$$\sum_{j=1}^{s} x_j \prod_{i=1}^{m} \Psi^i a_{ij} = \sum_{j=s+1}^{n} x_j \prod_{i=1}^{m} \Psi^i b_{ij}, \tag{2.2}$$

where x_j, $(1\le j\le n)$ are unknown rational coefficients, $\Psi^i a_{ij}$ and $\Psi^i b_{ij}$, $(1\le i\le m)$ are chemical elements in reactants and products, respectively, a_{ij} and b_{ij}, $(1\le i\le m;\ 1\le j\le n;\ m<n)$ are numbers of atoms of elements $\Psi^i a_{ij}$ and $\Psi^i b_{ij}$, respectively, in *j-th* molecule.

Definition 2.4. *Each chemical reaction ρ has a domain*

$$Dom\rho = \{x \in \mathscr{X} \mid a_{ij} > 0\}. \tag{2.3}$$

Definition 2.5. *Each chemical reaction ρ has an image*

$$Im\rho = \{y \in \mathscr{X} \mid b_{ij} > 0\}. \tag{2.4}$$

Definition 2.6. *Chemical reaction ρ is generated for some $x\in\mathscr{X}$, if both $a_{ij} > 0$ and $b_{ij} > 0$.*

Definition 2.7. *For the case as the previous definition, we say x is a generator of ρ.*

Definition 2.8. *The set of generators of ρ is thus $Dom\rho \cap Im\rho$.*

Often chemical reactions are modeled like pairs of multisets, corresponding to integer stoichiometric constants.

Definition 2.9. *A stoichiometrical space is a pair* ($\mathscr{X}$, $\mathscr{R}$), where $\mathscr{R}$ is a set of chemical reactions on $\mathscr{X}$. It may be symbolized by an arc-weighted bipartite directed graph $\Gamma(\mathscr{X}, \mathscr{R})$ with vertex set $\mathscr{X}\cup\mathscr{R}$, arcs $x\to\rho$ with weight a_{ij} if $a_{ij} > 0$, and arcs $\rho\to y$ with weight b_{ij} if $b_{ij} > 0$.

Let us now consider an arbitrary subset $\mathscr{A}\subseteq\mathscr{X}$.

Definition 2.10. *A chemical reaction ρ may take place in a reaction combination composed of the molecules in $\mathscr{A}$ if and only if $Dom\rho \subseteq \mathscr{A}$.*

Definition 2.11. *The collection of all possible reactions in the stoichiometrical space ($\mathscr{X}$, $\mathscr{R}$), that can start from $\mathscr{A}$ is given by*

$$\mathscr{R}_{\mathscr{A}} = \{\rho\in\mathscr{R} \mid Dom\rho \subseteq \mathscr{A}\}. \tag{2.5}$$

Definition 2.12. *Subgenerators of the chemical reaction* (2.1) *are the coefficient of its general solution*

$$x_i = d_{i1}x_{k1} + d_{i2}x_{k2} + \cdots + d_{i,n-r}x_{k,n-r}, (1\le i\le r), \tag{2.6}$$

where $x_{k1}, x_{k2}, \ldots, x_{k,n-r}$, $(n>r)$ are free variables.

Definition 2.13. *For any subgenerator holds*

$$x_j > 0, (1\le j\le r). \tag{2.7}$$

Definition 2.14. *A sequence of vectors* $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_k\}$ *is a basis of the chemical reaction* (2. 1) *if the vectors of solutions $\boldsymbol{x}_i$, $(1\le i\le k)$ of* (2. 2) *are linearly independent and $\boldsymbol{x}_i$, $(1\le i\le k)$ generate the vector space $\boldsymbol{W}$ of the solutions $\boldsymbol{x}_i$, $(1\le i\le k)$.*

Definition 2.15. *The vector space $\boldsymbol{W}$ of the vectors of solutions $\boldsymbol{x}_i$, $(1\le i\le k)$ of* (2. 2) *is said to be of finite dimension k, written* dim $\boldsymbol{W} = k$, *if $\boldsymbol{W}$ contains a basis with k elements.*

Definition 2.16. *If $\boldsymbol{W}$ is a subspace of $\boldsymbol{V}$, then the orthogonal complement $\boldsymbol{W}^{\perp}$ of* (2.2) *is*

$$\boldsymbol{W}^{\perp} = \{\boldsymbol{x}\in\boldsymbol{V} \mid \langle\boldsymbol{x}, \boldsymbol{y}\rangle = 0, \forall\boldsymbol{y}\in\boldsymbol{W}\}. \tag{2.8}$$

Definition 2.17. *The set $\boldsymbol{X}\subset\mathbb{R}$ is a set of all the coefficients x_j, $(1\le j\le n)$ of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.18. *Cardinality of the set $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$ of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1) *is*

$Card\boldsymbol{X} = |\boldsymbol{X}| = n.$

Definition 2.19. *If $\boldsymbol{X}\subset\mathbb{R}$ is a set of the coefficients x_j, $(1\le j\le n)$ of the chemical equation* (2.2) *of the reaction* (2.1), *then the power set of $\boldsymbol{X}$, denoted by $\mathscr{P}(\boldsymbol{X})$, is the set of all subsets of $\boldsymbol{X}$.*

Definition 2.20. *Cardinality of the power set $\mathscr{P}(\boldsymbol{X})$ of the set $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$ of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1) is

$Card\mathscr{P}(\boldsymbol{X}) = 2^{|\boldsymbol{X}|} = 2^n.$

Definition 2.21. *The set $\boldsymbol{X} \subset \mathbb{R}$ of the coefficients x_j, (1≤*

$j \le n$) *of the chemical equation* (2.2) *of the reaction* (2.1), *is open, if it is a member of the topology.*

Definition 2.22. *The set* $\mathbf{X} \subset \mathbb{R}$ *of the coefficients* x_j, ($1 \le j \le n$) *of the chemical equation* (2.2) *of the reaction* (2.1), *is called closed, if the complement* $\mathbb{R} \setminus \mathbf{X}$ *of* $\mathbf{X}$ *is an open set.*

Definition 2.23. *The interior of* $\mathbf{X}$ *is the union of all open sets contained in* $\mathbf{X}$,

$Int\{\mathbf{X}\} = \cup \{\mathbf{Y} \subset \mathbf{X} \mid \mathbf{Y} \text{ open}\} = \mathbf{X}^{o}$.

Definition 2.24. *The exterior of* $\mathbf{X}$ *is the interior of the complement of* $\mathbf{X}$,

$Ext\{\mathbf{X}\} = Int\{\mathbf{X}^{c}\}$.

Definition 2.25. *The closure of* $\mathbf{X}$ *is the intersection of all closed sets containing* $\mathbf{X}$,

$Cl\{\mathbf{X}\} = \cap \{\mathbf{Y} \supset \mathbf{X} \mid \mathbf{Y} \text{ closed}\} = \mathbf{X}^{-}$.

Definition 2.26. *The boundary of* $\mathbf{X}$ *is*

$\partial\mathbf{X} = Cl\{\mathbf{X}\} - Int\{\mathbf{X}\} = \mathbf{X}^{-} - \mathbf{X}^{o} = Bd\{\mathbf{X}\}$.

Definition 2.27. *A point* $x \in \mathbf{X}$ *is called isolated point of* $\mathbf{X}$ *if there exists a neighborhood* $\mathbf{Y}$ *of* x *such that* $\mathbf{Y} \cap \mathbf{X} = \{x\}$.

Definition 2.28. *A point* $x \in \mathbf{X}$ *is called accumulation point or limit point of a subset* $\mathbf{A}$ *of* $\mathbf{X}$ *if and only if every open set* $\mathbf{Y}$ *containing* x *contains a point of* $\mathbf{A}$ *different from* x, i.e.

$\mathbf{Y}$ *open*, $x \in \mathbf{Y} \Rightarrow \{\mathbf{Y} \setminus \{x\}\} \cap \mathbf{A} \neq \emptyset$.

Definition 2.29. *The set of accumulation points of* $\mathbf{X}$, *denoted by* $\mathbf{X}'$, *is called the derived set of* $\mathbf{X}$.

Definition 2.30. *A class* $\mathscr{T}$ *of subsets of* $\mathbf{X}$, *whose elements are referred as the open sets, is called topology of the chemical equation* (2.2) *of the reaction* (2.1) *if the following axioms are satisfied*

1° $\emptyset$, $\mathbf{X} \in \mathscr{T}$, *where* $\emptyset$ *is an empty set,*

2° *if* $\{\mathbf{X}_i \mid i \in I\} \subset \mathscr{T}$, *then* $\cup_{i \in I} \mathbf{X}_i \in \mathscr{T}$,

3° *if* $\mathbf{X}_i$, $\mathbf{X}_j \in \mathscr{T}$, *then* $\mathbf{X}_i \cap \mathbf{X}_j \in \mathscr{T}$.

The pair $(\mathbf{X}, \mathscr{T})$ *is called a topological space of solutions of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.31. *A subset* $\mathbf{Y}$ *of the topological space* $(\mathbf{X}, \mathscr{T})$, *of solutions of the chemical equation* (2.2) *of the reaction* (2.1), *is said to be dense in* $\mathbf{Z} \subset (\mathbf{X}, \mathscr{T})$ *if* $\mathbf{Z}$ *is contained in the closure of* $\mathbf{Y}$, i.e., $\mathbf{B} \subset Cl\{\mathbf{X}\}$.

Definition 2.32. *Let* x *be point in the topological space* $(\mathbf{X}, \mathscr{T})$, *of solutions of the chemical equation* (2.2) *of the reaction* (2.1). *A subset* $\mathscr{N}$ of $(\mathbf{X}, \mathscr{T})$ *is a neighborhood of* x *if and only if* $\mathscr{N}$ *is a superset of an open set* $\mathbf{Y}$ *containing* x, i.e.,

$x \in \mathbf{Y} \subset \mathscr{N}$, $\mathbf{Y}$ *open.*

Definition 2.33. *The class of all neighborhoods of* $x \in (\mathbf{X}, \mathscr{T})$, *denoted by* $\mathscr{N}_p$, *is called the neighborhood system of* x.

Definition 2.34. *Let* $\mathbf{Y}$ *be a non-empty subset of a topological space* $(\mathbf{X}, \mathscr{T})$. *The class* $\mathscr{T}_{\mathbf{Y}}$ *of all intersections of* $\mathbf{Y}$ *with* $\mathscr{T}$*-open subsets of* $\mathbf{X}$ *is a topology on* $\mathbf{Y}$; *it is called the relative topology on* $\mathbf{Y}$ *or the relativization of* $\mathscr{T}$ *to* $\mathbf{Y}$, *and the topological space* $(Y, \mathscr{T}_Y)$ *is called a subspace of* $(\mathbf{X}, \mathscr{T})$.

Definition 2.35. *The discrete topology of the chemical equation* (2.2) *of the reaction* (2.1) *is the topology* $\mathscr{D} = \mathscr{P}(\mathbf{X})$ *on* $\mathbf{X}$, *where* $\mathscr{P}(\mathbf{X})$ *denotes the power set of* $\mathbf{X}$. *The pair* $(\mathbf{X}, \mathscr{D})$ *is called a discrete topological space of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.36. *The indiscrete topology of the chemical equation* (2.2) *of the reaction* (2.1) *is the topology* $\mathscr{I} = \{\emptyset, \mathbf{X}\}$. *The pair* $(\mathbf{X}, \mathscr{I})$ *is called an indiscrete topological space of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.37. *The n-th complete Bell polynomial*[27] *is defined by*

$$Y_n(fx_1, fx_2, \ldots, fx_n) = \sum [n!\, f_r/(r_1!\, r_2! \cdots r_n!)] \times (x_1/1!)^{r1} (x_2/2!)^{r2} \cdots (x_n/n!)^{rn}, \tag{2.9}$$

where $f^r \equiv f_r = (-1)^{r-1}(r-1)!$ and the summation is over all non negative integers satisfying the following conditions

$$r_1 + 2r_2 + \cdots + nr_n = n, \quad r_1 + r_2 + \cdots + r_n = r, \tag{2.10}$$

where r_i, $(1 \le i \le n)$ are the numbers of parts of size i.

Definition 2.38. *Let* $Y_n(x_1, x_2, \ldots, x_n)$ *denote the Bell polynomial with all* f_i *set at unity. This particular Bell polynomial can be interpreted as an ordered-cycle indicator.*

Definition 2.39. *Let* $\mathscr{Q}(n)$ *be the number of quasi-orders on the set* $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ *of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.40. *Let* $\mathscr{Q}^c(n)$ *be the number of connected quasi-orders on the set* $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ *of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.41. *Let* $\mathscr{P}(n)$ *be the number of partial orders on the set* $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ *of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.42. *Let* $\mathscr{P}^c(n)$ *be the number of connected partial orders on the set* $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ *of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.43. *Let* $\mathscr{T}(\mathbf{X})$ *be the set of all topologies that can be defined on the set* $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ *of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.44. *Let* $\mathscr{T}^c(\mathbf{X})$ *be the set of all connected topologies that can be defined on the set* $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ *of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.45. *Let* $\mathscr{T}_0(\mathbf{X})$ *be the set of all* $\mathscr{T}_0$*-topologies that can be defined on the set* $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ *of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1).

Definition 2.46. *Let* $\mathscr{T}_0^{\,c}(\mathbf{X})$ *be the set of all connected*

$\mathscr{T}_0$-topologies that can be defined on the set $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$ of the coefficients of the chemical equation (2.2) *of the reaction* (2.1).

Definition 2.47. *Let* $\mathscr{T}(n) = |\mathscr{T}(\boldsymbol{X})|$, $\mathscr{T}^c(n) = |\mathscr{T}^c(\boldsymbol{X})|$, $\mathscr{T}_0(n) = |\mathscr{T}_0(\boldsymbol{X})|$ *and* $\mathscr{T}_0{}^c(n) = |\mathscr{T}_0{}^c(\boldsymbol{X})|$.

MAIN RESULTS

In this section we shall present our newest research results.

Theorem 3.1. *Echelon form of the chemical equation* (2.2) *of the reaction* (2.1) *has one solution for each specification of $n - r$ free variables if $r < n$.*

Proof. According to the Theorem 4.2 from[5] the chemical reaction (2.1) reduces to (2.2), i.e., this system of linear equations

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \cdots + a_{1s}x_s = b_{1,s+1}x_{s+1} \\
&\quad + b_{1,s+2}x_{s+2} + \cdots + b_{1n}x_n, \\
&a_{21}x_1 + a_{22}x_2 + \cdots + a_{2s}x_s = b_{2,s+1}x_{s+1} \\
&\quad + b_{2,s+2}x_{s+2} + \cdots + b_{2n}x_n, \\
&\qquad\vdots \\
&a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{ms}x_s = b_{m,s+1}x_{s+1} \\
&\quad + b_{m,s+2}x_{s+2} + \cdots + b_{mn}x_n,
\end{aligned} \tag{3.1}$$

The echelon form of the system (3.1) is

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = 0, \\
&a_{2j2}x_{j2} + a_{2,j2+1}x_{j2+1} + \cdots + a_{2n}x_n = 0, \\
&\qquad\vdots \\
&a_{rjr}x_{jr} + a_{r,jr+1}x_{jr+1} + \cdots + a_{rn}x_n = 0,
\end{aligned} \tag{3.2}$$

where $1 < j_2 < \cdots < j_r$ and $a_{11} \neq 0$, $a_{2j2} \neq 0, \ldots, a_{rjr} \neq 0$, $r < n$.

If we use mathematical induction for $r = 1$, then we have a single, nondegenerate, linear equation to which (3.2) applies when $n > r = 1$. Thus the theorem holds for $r = 1$.

Now, suppose that $r > 1$ and that the theorem is true for a system of $r - 1$ equations. We shall consider the $r - 1$ equations.

$$\begin{aligned}
&a_{2j2}x_{j2} + a_{2,j2+1}x_{j2+1} + \cdots + a_{2n}x_n = 0, \\
&\qquad\vdots \\
&a_{rjr}x_{jr} + a_{r,jr+1}x_{jr+1} + \cdots + a_{rn}x_n = 0,
\end{aligned} \tag{3.3}$$

as a system in the unknowns $x_{j2}, \ldots, x_n$. Note that the system (3.3) is in echelon form. By the induction hypothesis, we may arbitrary assign values to the $(n - j_2 + 1) - (r - 1)$ free variables in the reduced system to obtain a solution $x_{j2}, \ldots, x_n$. As in case $r = 1$, these values and arbitrary values for the additional $j_2 - 2$ free variables $x_2, \ldots, x_{j2-1}$, yield a solution of the first equation with

$$x_1 = (-\, a_{12}x_2 - \cdots - a_{1n}x_{j2-1})/a_{11}. \tag{3.4}$$

Note that there are $(n - j_2 + 1) - (r - 1) + (j_2 - 2) = n - r$ free variables.

Furthermore, these values for $x_1, \ldots, x_n$ also satisfy the other equations since, in these equations, the coefficients $x_1, \ldots, x_{j2-1}$ are zero.

Theorem 3.2. *Let echelon form of the chemical equation* (2.2) *of the reaction* (2.1) *has v free variables. Let $\boldsymbol{x}_i$,* ($1 \leq i \leq v$) *be the solutions obtained by setting one of the free variables equal to one* (*or any nonzero constant*) *and the remaining free variables equal to zero. Then the solutions $\boldsymbol{x}_i$,* ($1 \leq i \leq v$) *form a basis for solution space $\boldsymbol{W}$ of the chemical equation* (2.2) *of the reaction* (2.1).

Proof. This means that any solution of the system (3.2) can be expressed as a *unique* linear combination of $\boldsymbol{x}_i$, ($1 \leq i \leq v$). Thus, the dimension of $\boldsymbol{W}$ is $dim\boldsymbol{W} = v$.

Theorem 3.3. *Let chemical equation* (2.2) *of the reaction* (2.1) *is in echelon form* (3.2). *The basis of solution space $\boldsymbol{W}$ of chemical equation* (2.2) *of the reaction* (2.1) *are the solutions $\boldsymbol{x}_i$,* ($1 \leq i \leq n - r$), *such that* $\dim \boldsymbol{W} = n - r$.

Proof. The system (3.2) has $n - r$ free variables $x_{i1}, x_{i2}, \ldots, x_{i,n-r}$. The solution $\boldsymbol{x}_j$ is obtained by setting $x_{ij} = 1$ (or any nonzero constant) and the remaining free variables are equal to zero. Then the solutions $\boldsymbol{x}_i$, ($1 \leq i \leq n - r$) form a basis of $\boldsymbol{W}$ and so $dim\boldsymbol{W} = n - r$.

Theorem 3.4. *Let $x_{i1}, x_{i2}, \ldots, x_{ir}$, be the free variables of the homogeneous system* (3.2) *of the chemical equation* (2.2) *of the reaction* (2.1). *Let $\boldsymbol{x}_j$ be the solution for which $x_{ij} = 1$ and all other free variables are equal to zero. Solutions $\boldsymbol{x}_i$,* ($1 \leq i \leq r$) *are linearly independent.*

Proof. Let $\boldsymbol{A}$ be the matrix whose rows are the $\boldsymbol{x}_i$, respectively. We interchange column 1 and column i_1, then the column 2 and column $i_2, \ldots$, and then column r and column i_r, and obtain $r \times n$ matrix

$$\boldsymbol{B} = (\boldsymbol{I}, \boldsymbol{C}) = \begin{bmatrix}
1\,0\,0\,\ldots\,0\,0 & c_{1,r+1} & \ldots & c_{1n} \\
0\,1\,0\,\ldots\,0\,0 & c_{2,r+1} & \ldots & c_{2n} \\
0\,0\,1\,\ldots\,0\,0 & c_{3,r+1} & \ldots & c_{3n} \\
\vdots & & & \\
0\,0\,0\,\ldots\,0\,1 & c_{r,r+1} & \ldots & c_{rn}
\end{bmatrix}.$$

The above matrix $\boldsymbol{B}$ is in echelon form and so its rows are independent, hence $rank\boldsymbol{B} = r$. Since $\boldsymbol{A}$ and $\boldsymbol{B}$ are column equivalent, they have the same rank, i.e., $rank\boldsymbol{A} = r$. But $\boldsymbol{A}$ has r rows, hence these rows, i.e., the $\boldsymbol{x}_i$ are linearly independent as claimed.

Theorem 3.5. *The dimension of the solution space $\boldsymbol{W}$ of the chemical equation* (2.2) *of the reaction* (2.1) *is $n - r$, where n is the number of molecules and r is the rank of the reaction matrix $\boldsymbol{A}$.*

Proof. If we take into account that

$$r = rank\boldsymbol{A} = dim(Im\boldsymbol{A})$$

and

$$n = dim\mathbb{R}^n = dim(Dom\boldsymbol{A}),$$

then immediately follows

$$dim\boldsymbol{W} = dim(Ker\boldsymbol{A}) = dim(Dom\boldsymbol{A}) - dim(Im\boldsymbol{A}) = n - r.$$

Corollary 3.6. *If $n = r$, then $dim\boldsymbol{W} = 0$, that means reaction* (2.1) *is impossible.*

Corollary 3.7. *If $n = r + 1$, then $dim\boldsymbol{W} = r + 1 - r = 1$, that means that chemical equation* (2.2) *of the reaction* (2.1) *has a unique set of coefficients.*

Corollary 3.8. *If $n > r + 1$, then $dim\boldsymbol{W} > r + 1 - r > 1$, that means that chemical equation* (2.2) *of the reaction* (2.1) *has an infinite number of sets of coefficients.*

Remark 3.9. *Those chemical reactions with properties of Corollary* 3.8*, we shall call continuum reactions, because they can be reduced to the Cantor's continuum problem.*[28]

It shows that the balancing of chemical equations is neither simple nor easy matter. To date, these reactions were not seriously considered in scientific literature, or more accurately speaking these reactions were simply neglected, because their research looks for a very sophisticated and multidisciplinary approach. Just it was a challenge and main motive of the author of this work, to dedicate his research on these reactions.

Theorem 3.10. *If $\mathscr{T}$ is the class of subsets of $\mathbb{N}$ consisting of $\varnothing$ and all subsets of $\mathbb{N}$ of the form $\mathscr{E}_n = \{n, n+1, n+2, \ldots\}$ with $n \in \mathbb{N}$, then $\mathscr{T}$ is a topology on $\mathbb{N}$ and n open sets containing the positive integer n.*

Proof. Since $\varnothing$ and $\mathscr{E}_1 = \{1, 2, 3, \ldots\} = \mathbb{N}$, belong to $\mathscr{T}$, $\mathscr{T}$ satisfies 1° of Definition 2.30. Furthermore, since $\mathscr{T}$ is totally ordered by set inclusion, $\mathscr{T}$ also satisfies 3° of Definition 2.30.

Now, let $\mathscr{A}$ be a subclass of $\mathscr{T} \setminus \{\mathbb{N}, \varnothing\}$, i.e., $\mathscr{A} = \{\mathscr{E}_n |\ n \in I\}$ where I is some set positive integers. Note that I contains a smallest positive integer n_0 and

$$\cup \{\mathscr{E}_n |\ n \in I\} = \{n_0, n_0 + 1, n_0 + 2, \ldots\} = \mathscr{E}n_0,$$

which belongs to $\mathscr{T}$. We want to show that $\mathscr{T}$ also satisfies 2° of Definition 2.30, i.e., that $\cup \{\mathscr{E}_n | \mathscr{E}_n \in \mathscr{A}\} \in \mathscr{T}$.

Case 1. If $\boldsymbol{X} \in \mathscr{A}$, then

$\cup \{\mathscr{E}_n | \mathscr{E}_n \in \mathscr{A}\} = \boldsymbol{X}$, and therefore belongs to $\mathscr{T}$ by 1° of Definition 2.30.

Case 2. If $\boldsymbol{X} \notin \mathscr{A}$, then

$$\cup \{\mathscr{E}_n | \mathscr{E}_n \in \mathscr{A}\} = \cup \{\mathscr{E}_n | \mathscr{E}_n \in \mathscr{A} \setminus \{\boldsymbol{X}\}\}.$$

But the empty set $\varnothing$ does not contribute any elements to union of sets; hence

$$\cup \{\mathscr{E}_n | \mathscr{E}_n \in \mathscr{A}\} = \cup \{\mathscr{E}_n | \mathscr{E}_n \in \mathscr{A} \setminus \{\boldsymbol{X}\}\} = \cup \{\mathscr{E}_n | \mathscr{E}_n \in \mathscr{A} \setminus \{\boldsymbol{X}, \varnothing\}\}.$$

Since $\mathscr{A}$ is a subclass of $\mathscr{T}$, $\mathscr{A} \setminus \{\boldsymbol{X}, \varnothing\}$ is a subclass of $\mathscr{T} \setminus \{\boldsymbol{X}, \varnothing\}$, so the union of any number of sets in $\mathscr{T} \setminus \{\boldsymbol{X}, \varnothing\}$ belongs to $\mathscr{T}$. Hence $\mathscr{T}$ satisfies $\mathscr{T} \setminus \{\mathbb{N}, \varnothing\}$, and so $\mathscr{T}$ is a topology on $\mathbb{N}$.

Since the non-empty open sets are of the form $\mathscr{E}_n = \{n, n+1, n+2, \ldots\}$ with $n \in \mathbb{N}$, the open sets contain the positive integer n are the following

$$\begin{aligned} \mathscr{E}_1 &= \mathbb{N} = \{1, 2, 3, \ldots, n, n+1, \ldots\}, \\ \mathscr{E}_2 &= \{2, 3, \ldots, n, n+1, \ldots\}, \\ &\ \vdots \\ \mathscr{E}_n &= \{n, n+1, \ldots\}. \end{aligned}$$

Theorem 3.11. *Let $\mathscr{T}$ be the topology on which consists of $\varnothing$ and all subsets of $\mathbb{N}$ of the form $\mathscr{E}_n = \{n, n+1, n+2, \ldots\}$ with $n \in \mathbb{N}$, then the derived set of $\boldsymbol{Y} = \{y_1, y_2, \ldots, y_n\}$, $(y_1 < y_2 < \cdots < y_n)$ of the coefficients of the chemical equation* (2.2) *of the reaction* (2.1) *is $\boldsymbol{Y}' = \{1, 2, \ldots, y_n\}$.*

Proof. Observe that the open sets containing any point $x \in \mathbb{N}$ are the sets $\mathscr{E}_i$ where $i \le x$. If $n_0 \le y_n - 1$, then every open set containing n_0 also contains $y_n \in \boldsymbol{Y}$ which is different from n_0; hence $n_0 \le y_n - 1$ is a limit point of $\boldsymbol{Y}$. On the other hand, if $n_0 \ge y_n - 1$ then the open set $\mathscr{E}n_0 = \{n_0, n_0 + 1, n_0 + 2, \ldots\}$ contains no point of $\boldsymbol{Y}$ different from n_0. So $n_0 \ge y_n - 1$ is not a limit point of $\boldsymbol{Y}$. Accordingly, the derived set of $\boldsymbol{Y}$ is $\boldsymbol{Y}' = \{1, 2, \ldots, y_n\}$.

Theorem 3.12. *If $\boldsymbol{Y}$ is any subset of a discrete topological space $(\boldsymbol{X}, \mathscr{D})$, then derived set $\boldsymbol{Y}'$ of $\boldsymbol{Y}$ is empty.*

Proof. Let x be any point in $\boldsymbol{X}$. Recall that every subset of a discrete space is open. Hence, in particular, the singleton set $\boldsymbol{G} = \{x\}$ is an open subset of $\boldsymbol{X}$. But

$$x \in \boldsymbol{G} \wedge \boldsymbol{G} \cap \boldsymbol{Y} = \{\{x\} \cap \boldsymbol{Y} \subset \{x\}\}.$$

Hence, $x \notin \boldsymbol{Y}'$ for every $x \in \boldsymbol{X}$, i.e., $\boldsymbol{Y}' = \varnothing$.

Theorem 3.13. *If $\boldsymbol{Y}$ is a subset of $\boldsymbol{X}$, then every limit point of $\boldsymbol{Y}$ is also a limit point of $\boldsymbol{X}$.*

Proof. Recall that $y \in \boldsymbol{Y}'$ if and only if $\{\boldsymbol{G} \setminus \{y\}\} \cap \boldsymbol{Y} \neq \varnothing$ for every open set $\boldsymbol{G}$ containing y. But $\boldsymbol{X} \supset \boldsymbol{Y}$; therefore

$$\{\boldsymbol{G} \setminus \{y\}\} \cap \boldsymbol{X} \supset \{\boldsymbol{G} \setminus \{y\}\} \cap \boldsymbol{Y} \neq \varnothing.$$

So $y \in \boldsymbol{Y}'$ implies $y \in \boldsymbol{X}'$, i.e., $\boldsymbol{Y}' \subset \boldsymbol{X}'$.

Theorem 3.14. *A subset $\boldsymbol{Y}$ of a topological space $(\boldsymbol{X}, \mathscr{T})$ is closed if and only if $\boldsymbol{Y}$ contains each of its accumulation points.*

Proof. Suppose $\boldsymbol{Y}$ is closed, and let $y \notin \boldsymbol{Y}$, i.e., $y \in \boldsymbol{Y}^c$. But

$\boldsymbol{Y}^c$, the complement of a closed set, is open; therefore $y \notin \boldsymbol{Y}'$ for $\boldsymbol{Y}^c$ is an open set such that

$$y \in \boldsymbol{Y}^c \wedge \boldsymbol{Y}^c \cap \boldsymbol{Y} = \varnothing.$$

Thus $\boldsymbol{Y}' \subset \boldsymbol{Y}$ if $\boldsymbol{Y}$ is closed.

Now assume $\boldsymbol{Y}' \subset \boldsymbol{Y}$; we show that $\boldsymbol{Y}^c$ is open. Let $y \in \boldsymbol{Y}^c$; then $y \notin \boldsymbol{Y}'$, so $\exists$ an open set $\boldsymbol{G}$ such that

$$y \in \boldsymbol{G} \wedge \{\boldsymbol{G} \setminus \{y\}\} \cap \boldsymbol{Y} = \varnothing.$$

But, $y \notin \boldsymbol{Y}$; hence $\boldsymbol{G} \cap \boldsymbol{Y} = \{\boldsymbol{G} \setminus \{y\}\} \cap \boldsymbol{Y} = \varnothing$.

So, $\boldsymbol{G} \subset \boldsymbol{Y}^c$. Thus y is an interior point of $\boldsymbol{Y}^c$, and so $\boldsymbol{Y}^c$ is open.

Theorem 3.15. *If $\boldsymbol{Z}$ is a closed superset of any set $\boldsymbol{Y}$, then $\boldsymbol{Y}' \subset \boldsymbol{Z}$.*

Proof. By Theorem 3.13, $\boldsymbol{Y} \subset \boldsymbol{Z}$ implies $\boldsymbol{Y}' \subset \boldsymbol{Z}'$. But, $\boldsymbol{Z}' \subset \boldsymbol{Z}'$ by Theorem 3. 14, since $\boldsymbol{Z}$ is closed. Thus $\boldsymbol{Y}' \subset \boldsymbol{Z}' \subset \boldsymbol{Z}$, which implies $\boldsymbol{Y}' \subset \boldsymbol{Z}$.

The last case, given by the Corollary 3.8., will be an object of research in the next section.

APPLICATION OF THE MAIN RESULTS

Let's consider the reaction

$$x_1\ Fe_2O_3 + x_2\ C \rightarrow x_3\ Fe_3O_4 + x_4\ FeO + x_5\ Fe + x_6\ Fe_3C + x_7\ CO + x_8\ CO_2. \tag{4.1}$$

This reaction was an object of research in theory of metallurgical processes. There it was considered only from thermodynamic point of view.[29,30] This reaction was studied broadly, but only in some particular cases. Its general case will be an object of study just in this section.

On one hand, at once we would like to emphasize that this reaction belongs to the class of continuum reactions. It is according to the Remark 3.9. On other hand, it shows that it is a *juicy* problem which deserves to be studied and solved in whole.

Since the reaction (4.1) is very important for metallurgical engineering, chemistry and mathematics, just here we shall consider it from this multidisciplinary aspect. That aspect looks for a strict topological approach toward on total solution of (4.1). This total solution gives an opportunity to be seen both general solution of (4.1) and its particular solutions generated by the reaction subgenerators.

First, we shall look for its *minimal solution* which is crucial in theory of fundamental stoichiometric calculations and foundation of chemistry. For that goal, let construct its scheme.

	Fe_2O_3	C	Fe_3O_4	FeO	Fe	Fe_3C	CO	CO_2
Fe	2	0	–3	–1	–1	–3	0	0
O	3	0	–4	–1	0	0	–1	–2
C	0	1	0	0	0	–1	–1	–1

From the above scheme immediately follows reaction matrix

$$\boldsymbol{A} = \begin{bmatrix} 2 & 0 & -3 & -1 & -1 & -3 & 0 & 0 \\ 3 & 0 & -4 & -1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

with a $rank\boldsymbol{A} = 3$.

It is well-known[11] that the reaction (4.1) can reduce in this matrix form

$$\boldsymbol{Ax} = \boldsymbol{0}. \tag{4.2}$$

where $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)^{\mathrm{T}}$ is the unknown vector of the coefficients of (4.1), $\boldsymbol{0} = (0, 0, 0)^{\mathrm{T}}$ is the zero vector and T denoting transpose.

The general solution of the matrix equation (4.2) is given by the following expression

$$\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A}^{+}\boldsymbol{A})\boldsymbol{a}, \tag{4.3}$$

where $\boldsymbol{I}$ is a unit matrix and $\boldsymbol{a}$ is an arbitrary vector.

The Moore-Penrose generalized inverse matrix, for the chemical reaction (4.1), has this format

$$\boldsymbol{A}^{+} = \boldsymbol{A}^{\mathrm{T}}(\boldsymbol{A}\,\boldsymbol{A}^{\mathrm{T}})^{-1}$$

$$= (1/1379) \times \begin{bmatrix} 29 & 127 & -117 \\ -36 & -15 & 383 \\ -77 & -147 & 168 \\ -48 & -20 & 51 \\ -115 & 67 & 36 \\ -309 & 216 & -275 \\ 103 & -72 & -368 \\ 170 & -159 & -353 \end{bmatrix}.$$

For instance, by using the vector

$$\boldsymbol{a} = (1, 1, 1, 1, 1, 1, 1, 1)^{\mathrm{T}},$$

as an arbitrary chosen vector, $\boldsymbol{A}$ and $\boldsymbol{A}^{+}$ determined previously, by virtue of (4.3) one obtains the minimal solution of the matrix equation (4.2) given by

$$\boldsymbol{x}_{min} = (1/1379) \times (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)^{\mathrm{T}},$$

where $x_1 = 1954$, $x_2 = 1854$, $x_3 = 518$, $x_4 = 1093$, $x_5 = 1096$, $x_6 = 55$, $x_7 = 901$ and $x_8 = 898$.

Balanced chemical reaction (4.1) with minimal coefficients has this form

$$1954\ Fe_2O_3 + 1854\ C \rightarrow 518\ Fe_3O_4 + 1093\ FeO + 1096\ Fe + 55\ Fe_3C + 901\ CO + 898\ CO_2.$$

Now, we shall look for sets of solutions of the reaction (4.1). From (4.1) immediately follows this system of linear equations

$$\begin{aligned} 2x_1 &= 3x_3 + x_4 + x_5 + 3x_6,\\ 3x_1 &= 4x_3 + x_4 + x_7 + 2x_8,\\ x_2 &= x_6 + x_7 + x_8, \end{aligned} \tag{4.4}$$

which general solution is

$$\begin{aligned} x_6 &= 2x_1/3 - x_3 - x_4/3 - x_5/3,\\ x_7 &= -13x_1/3 + 2x_2 + 6x_3 + 5x_4/3 + 2x_5/3,\\ x_8 &= 11x_1/3 - x_2 - 5x_3 - 4x_4/3 - x_5/3, \end{aligned} \tag{4.5}$$

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers. Actually, the expressions (4.5) represent the set $\{x_6, x_7, x_8\}$ of subgenerators of the reaction (4.1).

If we substitute (4.5) into (4.1), then one obtains balanced reaction

$$3x_1\ Fe_2O_3 + 3x_2\ C \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + 3x_5\ Fe + (2x_1 - 3x_3 - x_4 - x_5)\ Fe_3C + (-13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5)\ CO + (11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5)\ CO_2, \tag{4.6}$$

in its general form, where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

According to the Definition 2.17, the set of the coefficients of (4.1) is

$$\boldsymbol{X} = \{3x_1, 3x_2, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \tag{4.7}$$

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

If we take into account the Definition 2.18, then the cardinality of the set $\boldsymbol{X}$, given by (4.7), is $Card\boldsymbol{X} = |\boldsymbol{X}| = 8$, and according to the Definition 2.20, follows $Card\mathscr{P}(\boldsymbol{X}) = 2^{|\boldsymbol{X}|} = 2^8 = 256$, that means that the power set $\mathscr{P}(\boldsymbol{X})$ of the set $\boldsymbol{X}$ of the coefficients of chemical reaction (4.1) contains 256 members, which are subsets of $\boldsymbol{X}$, i.e.,

$\mathscr{P}(\boldsymbol{X}) = \{\varnothing, \{3x_1\}, \{3x_2\}, \{3x_3\}, \{3x_4\}, \{3x_5\}, \{2x_1 - 3x_3 - x_4 - x_5\}, \{-13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, 3x_2\}, \{3x_1, 3x_3\}, \{3x_1, 3x_4\}, \{3x_1, 3x_5\}, \{3x_1, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_1, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_2, 3x_3\}, \{3x_2, 3x_4\}, \{3x_2, 3x_5\}, \{3x_2, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_2, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_2, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_3, 3x_4\}, \{3x_3, 3x_5\}, \{3x_3, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_3, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_3, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_4, 3x_5\}, \{3x_4, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_4, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_4, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_5, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{-13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, 3x_2, 3x_3\}, \{3x_1, 3x_2, 3x_4\}, \{3x_1, 3x_2, 3x_5\}, \{3x_1, 3x_2, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_1, 3x_2, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 3x_2, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, 3x_3, 3x_4\}, \{3x_1, 3x_3, 3x_5\}, \{3x_1, 3x_3, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_1, 3x_3, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 3x_3, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, 3x_4, 3x_5\}, \{3x_1, 3x_4, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_1, 3x_4, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 3x_4, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_1, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_2, 3x_3, 3x_4\}, \{3x_2, 3x_3, 3x_5\}, \{3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_2, 3x_3, 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_2, 3x_3, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_2, 3x_4, 3x_5\}, \{3x_2, 3x_4, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_2, 3x_4, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_2, 3x_4, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_2, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_2, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_2, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_2, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_3, 3x_4, 3x_5\}, \{3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_3, 3x_4, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_3, 3x_4, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_3, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_3, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_3, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_4, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_4, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_4, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_4, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_4, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, 3x_2, 3x_3, 3x_4\}, \{3x_1, 3x_2, 3x_3, 3x_5\}, \{3x_1, 3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_1, 3x_2, 3x_3, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\},$

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4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_4, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_2, 3x_4, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_2, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_4, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_2, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_4, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_3, 3x_4, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_4, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_4, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2,$

$3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_4, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_2, 3x_3, 3x_4, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_2, 3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_4, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_2, 3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_2, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_4, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_3, 3x_4, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_2, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_2, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_4, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_4, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_4, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 3x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_2, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_2, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}\}$, (4.8)

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

According to the Definition 2.35, the discrete topology of the reaction (4.1) is the topology $\mathscr{D} = \mathscr{P}(\boldsymbol{X})$ on the set $\boldsymbol{X}$, where $\boldsymbol{X}$ and $\mathscr{P}(\boldsymbol{X})$ are given by (4.7) and (4.8), respectively. The pair $(\boldsymbol{X}, \mathscr{D})$ is the discrete topological space of the chemical reaction (4.1).

If we take into account the Definition 2.36, then the indiscrete topology of the chemical reaction (4.1) is

$$\mathscr{I} = \{\varnothing, \boldsymbol{X}\} = \{\varnothing, \{3x_1\}, \{3x_2\}, \{3x_3\}, \{3x_4\}, \{3x_5\}, \{2x_1 - 3x_3 - x_4 - x_5\}, \{- 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}\}, \quad (4.9)$$

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

The pair

$$(\boldsymbol{X}, \mathscr{I}) = (\{3x_1, 3x_2, 3x_3, 3x_4, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{\varnothing, \{3x_1\}, \{3x_2\}, \{3x_3\}, \{3x_4\}, \{3x_5\}, \{2x_1 - 3x_3 - x_4 - x_5\}, \{- 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}\}), \quad (4.10)$$

is an indiscrete topological space of the chemical reaction (4.1), where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

Now, we shall consider the following class of subsets

$$\mathscr{T} = \{\boldsymbol{X}, \varnothing, \{3x_2\}, \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}\}, \quad (4.11)$$

of

$$\boldsymbol{X} = \{3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \quad (4.12)$$

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

Let's check axioms of the Definition 2.30.

1° Since $\boldsymbol{X}$ and $\varnothing$ belong to $\mathscr{T}$, it means that the first axiom of the Definition 2.30 is satisfied.

2° From the Definition 2.30, follows

$\boldsymbol{X} \cup \varnothing = \boldsymbol{X} \in \mathscr{T}$,

$\boldsymbol{X} \cup \{3x_2\} = \boldsymbol{X} \in \mathscr{T}$,

$\boldsymbol{X} \cup \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \boldsymbol{X} \in \mathscr{T}$,

$\boldsymbol{X} \cup \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \boldsymbol{X} \in \mathscr{T}$,

$\boldsymbol{X} \cup \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \boldsymbol{X} \in \mathscr{T}$,

$\varnothing \cup \{3x_2\} = \{3x_2\} \in \mathscr{T}$,

$\varnothing \cup \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$,

$\varnothing \cup \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$,

$\varnothing \cup \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} \in \mathscr{T}$,

$\{3x_2\} \cup \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$,

$\{3x_2\} \cup \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$,

$\{3x_2\} \cup \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \{3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \boldsymbol{X} \in \mathscr{T}$,

$\{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \cup \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$,

$\{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \cup \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} \in \mathscr{T}$,

$\{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \cup \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \{3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \boldsymbol{X} \in \mathscr{T}$.

We obtain that arbitrary union of any pair of sets of $\mathscr{T}$ belongs to $\mathscr{T}$. It shows that axiom 2° of Definition 2.30 is satisfied.

3° Now, we shall determine arbitrary intersections of any number of sets of the class $\mathscr{T}$.

$\boldsymbol{X} \cap \varnothing = \varnothing \in \mathscr{T}$,

$\boldsymbol{X} \cap \{3x_2\} = \{3x_2\} \in \mathscr{T}$,

$\boldsymbol{X} \cap \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$,

$\boldsymbol{X} \cap \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$,

$\boldsymbol{X} \cap \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} \in \mathscr{T}$,

$\varnothing \cap \{3x_2\} = \varnothing \in \mathscr{T}$,

$\varnothing \cap \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \varnothing \in \mathscr{T}$,

$\varnothing \cap \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \varnothing \in \mathscr{T}$,

$\varnothing \cap \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \varnothing \in \mathscr{T}$,

$\{3x_2\} \cap \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \varnothing \in \mathscr{T}$,

$\{3x_2\} \cap \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{3x_2\} \in \mathscr{T}$,

$\{3x_2\} \cap \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \varnothing \in \mathscr{T}$,

$\{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \cap \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$,

$\{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \cap \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$,

$\{3x_2, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \cap \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \in \mathscr{T}$, where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

We obtain that arbitrary intersection of any number of sets of the class $\mathscr{T}$ belongs to $\mathscr{T}$. That means, that is satisfied the axiom 3° of Definition 2.30.

By this we showed that the class $\mathscr{T}$ is topology on $\boldsymbol{X}$ since it satisfies the necessary three axioms of Definition 2.30.

Consider the topology $\mathscr{T}$ on $\boldsymbol{X}$, given by (4.11) and (4.12), respectively and the subset $\boldsymbol{Y} = \{3x_2, 3x_3, 2x_1 - 3x_3 - x_4 - x_5\}$ of $\boldsymbol{X}$. Observe that $3x_3 \in \boldsymbol{X}$ is a limit point of $\boldsymbol{Y}$ since the open sets containing $3x_3$ are $\{3x_3, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ and $\boldsymbol{X}$, and each contains a point of $\boldsymbol{Y}$ different from $3x_3$, i.e., $2x_1 - 3x_3 - x_4 - x_5$. On the other hand, the point $3x_2 \in \boldsymbol{X}$ is not a limit point of $\boldsymbol{Y}$ since the open set $\{3x_2\}$, which contains $3x_2$, does not contain a point of $\boldsymbol{Y}$ different from $3x_2$. Similarly, the points $- 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5$ and $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$ are limit points of $\boldsymbol{Y}$ and the point $2x_1 - 3x_3 - x_4 - x_5$ is not limit point of $\boldsymbol{Y}$.

So

$$\boldsymbol{Y}' = \{3x_3, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\},$$

is the derived set of $\boldsymbol{Y}$, where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

The closed subsets of $\boldsymbol{X}$ are

$$\varnothing, \boldsymbol{X}, \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_2, 3x_3, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_3, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_2\}$$

that is, the complements of the open subsets of $\boldsymbol{X}$. Note that there are subsets of $\boldsymbol{X}$, such as $\{3x_3, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, which are both open and closed, and there are subsets of $\boldsymbol{X}$, such as $\{3x_2, 3x_3\}$, which are neither open nor closed.

Accordingly

$$Cl\{3x_3\} = \{3x_3, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, Cl\{3x_2, 2x_1 - 3x_3 - x_4 - x_5\} = \boldsymbol{X},$$

$$Cl\{3x_3, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\},$$

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

Therefore the set $\{3x_2, 2x_1 - 3x_3 - x_4 - x_5\}$ is a dense subset of $\boldsymbol{X}$, but the set $\{3x_3, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$ is not, where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

Consider the topology (4.11) on (4.12) and the subset $\boldsymbol{Y} = \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$ of $\boldsymbol{X}$.

The points $2x_1 - 3x_3 - x_4 - x_5$ and $- 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5$ are each interior points of $\boldsymbol{Y}$ since $2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5 \in \{2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \subset \boldsymbol{Y}$, where $\{2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$ is an open set. The point $3x_3 \in \boldsymbol{Y}$ is not an interior point of $\boldsymbol{Y}$, so $Int\{\boldsymbol{Y}\} = \{2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$. Only the point $3x_2 \in \boldsymbol{X}$ is exterior to $\boldsymbol{Y}$, i.e., interior to the complement $\boldsymbol{Y}^c = \{3x_2, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ of $\boldsymbol{Y}$; hence $Ext\{\boldsymbol{Y}\} = Int\{\boldsymbol{Y}^c\} = \{3x_2\}$. Accordingly the boundary of $\boldsymbol{Y}$ consists of the points $3x_3$ and $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$, i.e., $Bd\{\boldsymbol{Y}\} = \{3x_3, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

Consider the topology $\mathscr{T}$ on $\boldsymbol{X}$, given by (4.11) and (4.12), respectively and the subset $\boldsymbol{Y} = \{3x_2, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ of $\boldsymbol{X}$.

Observe that

$$\boldsymbol{X} \cap \boldsymbol{Y} = \boldsymbol{Y}, \{3x_2\} \cap \boldsymbol{Y} = \{3x_2\}, \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \cap \boldsymbol{Y} = \{3x_2, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}.$$

$$\varnothing \cap \boldsymbol{Y} = \varnothing, \{2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \cap \boldsymbol{Y} = \{- 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_3, 2x_1 - 3x_3 - x_4 - x_5, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} \cap \boldsymbol{Y} = \{- 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}.$$

Hence the relativization of $\mathscr{T}$ to $\boldsymbol{Y}$ is

$$\mathscr{T}_{\boldsymbol{Y}} = \{\boldsymbol{Y}, \varnothing, \{3x_2\}, \{- 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{\{3x_2\}, \{- 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}\}, \{\{- 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}\}\},$$

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

From (4.5) and the Definition 2.13, follows this system of inequalities

$$2x_1 - 3x_3 - x_4 - x_5 > 0, - 13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5 > 0, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5 > 0. \tag{4.13}$$

From (4.13), one obtains the relation

$$3x_2 < 11x_1 - 15x_3 - 4x_4 - x_5 < 6x_2. \tag{4.14}$$

The expression (4.14) is a necessary and sufficient condition to hold the general reaction (4.6).

Example 4.1. For $x_3 = 1/3$, $x_4 = 1$ and $x_5 = 1$ from the first inequality of (4.13) follows $x_1 = 2$, then from (4.14) immediately follows $x_2 = 3$ and from (4.5) one obtains $x_6 = 1/3$, $x_7 = 5/3$ and $x_8 = 1$, such that the particular reaction of (4.6) has a form

$$6\ Fe_2O_3 + 9\ C \rightarrow Fe_3O_4 + 3\ FeO + 3\ Fe + Fe_3C + 5\ CO + 3\ CO_2.$$

Now we shall find the dimension and the basis of the solution space $\boldsymbol{W}$ of the system (4.4) generated by the chemical reaction (4.1).

The system (4.4) reduces to this form

$$x_1 - 3x_3/2 - x_4/2 - x_5/2 - 3x_6/2 = 0,\ x_2 - x_6 - x_7 - x_8 = 0,$$
$$x_3 + x_4 + 3x_5 + 9x_6 - 2x_7 - 4x_8 = 0. \quad (4.15)$$

The system (4.15) has a three (nonzero) equations in eight unknowns; and hence the system has $8 - 3 = 5$ free variables which are x_4, x_5, x_6, x_7 and x_8. Thus $dim\boldsymbol{W} = 5$.

To obtain a basis for $\boldsymbol{W}$, one sets

1° $x_4 = 1, x_5 = x_6 = x_7 = x_8 = 0$ in (4.15), such that the solution is $\boldsymbol{x}_1 = (-1, 0, -1, 1, 0, 0, 0, 0)$,

2° $x_4 = 0, x_5 = 1, x_6 = x_7 = x_8 = 0$ in (4.15), such that the solution is $\boldsymbol{x}_2 = (-4, 0, -3, 0, 1, 0, 0, 0)$,

3° $x_4 = x_5 = 0, x_6 = 1, x_7 = x_8 = 0$ in (4.15), such that the solution is $\boldsymbol{x}_3 = (-12, 1, -9, 0, 0, 1, 0, 0)$,

4° $x_4 = x_5 = x_6 = 0, x_7 = 1, x_8 = 0$ in (4.15), such that the solution is $\boldsymbol{x}_4 = (3, 1, 2, 0, 0, 0, 1, 0)$,

5° $x_4 = x_5 = x_6 = x_7 = 0, x_8 = 1$ in (4.15), such that the solution is $\boldsymbol{x}_5 = (6, 1, 4, 0, 0, 0, 0, 1)$.

The set $\boldsymbol{W} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4, \boldsymbol{x}_5\}$ is a basis of the solution space $\boldsymbol{W}$.

Since $\boldsymbol{A}\boldsymbol{W}^T$ and $\boldsymbol{W}\boldsymbol{A}^T$ are zero matrices of format 3×5 and 5×3, respectively, that means that the orthogonal complement $\boldsymbol{W}^\perp$ of (4.1) is

$$\boldsymbol{W}^\perp = \begin{bmatrix} -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ -4 & 0 & -3 & 0 & 1 & 0 & 0 & 0 \\ -12 & 1 & -9 & 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 6 & 1 & 4 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, we shall consider some particular cases of the reaction (4.6) generated by its generators.

I. *Particular cases of* (4.6) *generated by the subgenerator* $2x_1 - 3x_3 - x_4 - x_5$

Here, we would like to emphasize that with considered particular cases the chemical reaction (4.6) do not lose its generality.

1° If $x_1 = (3x_3 + x_4 + x_5)/2$, then the reaction (4.6) transforms into

$$(3x_3 + x_4 + x_5)\ Fe_2O_3 + 2x_2\ C \to 2x_3\ Fe_3O_4 + 2x_4\ FeO + 2x_5\ Fe + (4x_2 - x_3 - x_4 - 3x_5)\ CO + (-2x_2 + x_3 + x_4 + 3x_5)\ CO_2, \quad (4.16)$$

where x_i, $(2 \le i \le 5)$ are arbitrary real numbers.

From (4.16) follow these inequalities

$4x_2 - x_3 - x_4 - 3x_5 > 0$

and

$-2x_2 + x_3 + x_4 + 3x_5 > 0,$

or

$$2x_2 < x_3 + x_4 + 3x_5 < 4x_2. \quad (4.17)$$

Reaction (4.16) is possible if and only if (4.17) holds.

Example 4.2. For $x_3 = x_4 = 1$ and $x_5 = 2/3$ from (4.17) one obtains $x_2 = 3/2$, such that particular reaction of (4.16) has a form

$$14\ Fe_2O_3 + 9\ C \to 6\ Fe_3O_4 + 6\ FeO + 4\ Fe + 6\ CO + 3\ CO_2. \quad (4.18)$$

2° If $x_2 = (x_3 + x_4 + 3x_5)/4$, then the reaction (4.16) becomes

$$(6x_3 + 2x_4 + 2x_5)\ Fe_2O_3 + (x_3 + x_4 + 3x_5)\ C \to 4x_3\ Fe_3O_4 + 4x_4\ FeO + 4x_5\ Fe + (x_3 + x_4 + 3x_5)\ CO_2,\ (\forall\ x_3, x_4, x_5 \in \mathbb{R}) \quad (4.19)$$

where x_i, $(3 \le i \le 5)$ are arbitrary real numbers.

3° If $x_2 > (x_3 + x_4 + 3x_5)/4$, then the reaction (4.16) transforms into

$$(3x_3 + x_4 + x_5)\ Fe_2O_3 + 2x_2\ C + (2x_2 - x_3 - x_4 - 3x_5)\ CO_2\ 2x_3\ Fe_3O_4 + 2x_4\ FeO + 2x_5\ Fe + (4x_2 - x_3 - x_4 - 3x_5)\ CO, \quad (4.20)$$

where x_i, $(2 \le i \le 5)$ are arbitrary real numbers.

From (4.20) one obtains

$$3x_3 + x_4 + x_5 > 0,\ 2x_2 - x_3 - x_4 - 3x_5 > 0,\ 4x_2 - x_3 - x_4 - 3x_5 > 0. \quad (4.21)$$

The system of inequalities (4.21) holds if and only if

$$x_4 < 2x_2 - x_3 - 3x_5. \quad (4.22)$$

Expression (4.22) is a necessary and sufficient condition to hold (4.20).

Example 4.3. For $x_2 = 3$, $x_3 = 1$ and $x_5 = 1$, from (4.22) one obtains $x_4 = 1$, such that particular reaction of (4.20) has a form

$$5\ Fe_2O_3 + 6\ C + CO_2 \to 2\ Fe_3O_4 + 2\ FeO + 2\ Fe + 7\ CO. \quad (4.23)$$

4° If $x_2 = (x_3 + x_4 + 3x_5)/2$, then (4.20) becomes

$$(3x_3 + x_4 + x_5)\ Fe_2O_3 + (x_3 + x_4 + 3x_5)\ C \to 2x_3\ Fe_3O_4 + 2x_4\ FeO + 2x_5\ Fe + (x_3 + x_4 + 3x_5)\ CO,\ (\forall\ x_3, x_4, x_5 \in \mathbb{R}) \quad (4.24)$$

where x_i, $(3 \le i \le 5)$ are arbitrary real numbers.

5° If $x_2 > (x_3 + x_4 + 3x_5)/2$, then (4.20) holds.

6^o If $x_2 < (x_3 + x_4 + 3x_5)/2$, then (4.20) becomes

$(3x_3 + x_4 + x_5)\ Fe_2O_3 + 2x_2\ C \rightarrow 2x_3\ Fe_3O_4 + 2x_4\ FeO + 2x_5\ Fe + (4x_2 - x_3 - x_4 - 3x_5)\ CO + (-2x_2 + x_3 + x_4 + 3x_5)\ CO_2$, (4.25)

where x_i, $(2 \le i \le 5)$ are arbitrary real numbers.

7^o If $x_2 = (x_3 + x_4 + 3x_5)/4$, then from (4.20) follows (4.19).

8^o If $x_2 > (x_3 + x_4 + 3x_5)/4$, then from (4.20) one obtains (4.16).

9^o If $x_2 < (x_3 + x_4 + 3x_5)/4$, then (4.20) becomes

$(3x_3 + x_4 + x_5)\ Fe_2O_3 + 2x_2\ C + (-4x_2 + x_3 + x_4 + 3x_5)\ CO \rightarrow 2x_3\ Fe_3O_4 + 2x_4\ FeO + 2x_5\ Fe + (-2x_2 + x_3 + x_4 + 3x_5)\ CO_2$. (4.26)

10^o If $x_2 < (x_3 + x_4 + 3x_5)/4$, then the reaction (4.16) transforms into (4.26).

11^o For $x_2 = (x_3 + x_4 + 3x_5)/2$, then (4.16) becomes (4.24).

12^o If $x_2 > (x_3 + x_4 + 3x_5)/2$, then (4.16) transforms into (4.20).

13^o If $x_2 < (x_3 + x_4 + 3x_5)/2$, then (4.16) holds.

14^o If $x_1 > (3x_3 + x_4 + x_5)/2$, then (4.6) becomes

$3x_1\ Fe_2O_3 + 3x_2\ C + (13x_1 - 6x_2 - 18x_3 - 5x_4 - 2x_5)\ CO \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + 3x_5\ Fe + (2x_1 - 3x_3 - x_4 - x_5)\ Fe_3C + (11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5)\ CO_2$, (4.27)

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

Reaction (4.27) is possible if and only if holds this system of inequalities

$13x_1 - 6x_2 - 18x_3 - 5x_4 - 2x_5 > 0,\ 2x_1 - 3x_3 - x_4 - x_5 > 0,\ 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5 > 0$, (4.28)

i.e.,

$x_1 > (6x_2 + 18x_3 + 5x_4 + 2x_5)/13$.

The above inequality is a necessary and sufficient condition to hold reaction (4.27).

Example 4.4. For $x_3 = 2/3$, $x_4 = x_5 = 1$ and $x_1 = 3$, from (4.17) one obtains $x_2 = 2$, such that particular reaction of (4.27) has a form

$9\ Fe_2O_3 + 6\ C + 8\ CO \rightarrow 2\ Fe_3O_4 + 3\ FeO + 3\ Fe + 2\ Fe_3C + 12\ CO_2$. (4.29)

15^o If $x_1 = (6x_2 + 18x_3 + 5x_4 + 2x_5)/13$, then (4.27) becomes

$(6x_2 + 18x_3 + 5x_4 + 2x_5)\ Fe_2O_3 + 13x_2\ C \rightarrow 13x_3\ Fe_3O_4 + 13x_4\ FeO + 13x_5\ Fe + (4x_2 - x_3 - x_4 - 3x_5)\ Fe_3C + (9x_2 + x_3 + x_4 + 3x_5)\ CO_2$, (4.30)

where x_i, $(2 \le i \le 5)$ are arbitrary real numbers.

From the above reaction (4.30) one obtains this system of inequalities

$6x_2 + 18x_3 + 5x_4 + 2x_5 > 0,\ 4x_2 - x_3 - x_4 - 3x_5 > 0,\ 9x_2 + x_3 + x_4 + 3x_5 > 0$. (4.31)

From (4.31) immediately follows this expression

$x_3 + x_4 + 3x_5 < 4x_2$, (4.32)

which is a necessary and sufficient condition to hold (4.30).

Example 4.5. For $x_3 = x_4 = x_5 = 1$, from (4.32) one obtains $x_2 = 3/2$, such that particular reaction of (4.30) has a form

$68\ Fe_2O_3 + 39\ C \rightarrow 26\ Fe_3O_4 + 26\ FeO + 26\ Fe + 2\ Fe_3C + 37\ CO_2$. (4.33)

16^o If $x_2 = (x_3 + x_4 + 3x_5)/4$, then (4.30) transforms into (4.19).

17^o If $x_2 > (x_3 + x_4 + 3x_5)/4$, then (4.30) holds.

18^o If $x_2 < (x_3 + x_4 + 3x_5)/4$, then (4.30) becomes

$(6x_2 + 18x_3 + 5x_4 + 2x_5)\ Fe_2O_3 + 13x_2\ C + (-4x_2 + x_3 + x_4 + 3x_5)\ Fe_3C\ 13x_3\ Fe_3O_4 + 13x_4\ FeO + 13x_5\ Fe + (9x_2 + x_3 + x_4 + 3x_5)\ CO_2$, (4.34)

where x_i, $(2 \le i \le 5)$ are arbitrary real numbers.

19^o If $x_2 = (x_3 + x_4 + 3x_5)/4$, then (4.34) transforms into (4.19).

20^o If $x_1 > (6x_2 + 18x_3 + 5x_4 + 2x_5)/13$, then (4.27) holds.

21^o If $x_1 < (6x_2 + 18x_3 + 5x_4 + 2x_5)/13$, then (4.27) becomes

$3x_1\ Fe_2O_3 + 3x_2\ C + (-2x_1 + 3x_3 + x_4 + x_5)\ Fe_3C + (-11x_1 + 3x_2 + 15x_3 + 4x_4 + x_5)\ CO_2 \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + 3x_5\ Fe + (-13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5)\ CO$, (4.35)

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

22^o If $x_1 = (3x_3 + x_4 + x_5)/2$, then from (4.27) follows

$(3x_3 + x_4 + x_5)\ Fe_2O_3 + 6x_2\ C + (-12x_2 + 3x_3 + 3x_4 + 9x_5)\ CO \rightarrow 6x_3\ Fe_3O_4 + 6x_4\ FeO + 6x_5\ Fe + (-6x_2 + 3x_3 + 3x_4 + 9x_5)\ CO_2$, (4.36)

where x_i, $(2 \le i \le 5)$ are arbitrary real numbers.

23^o If $x_1 > (3x_3 + x_4 + x_5)/2$, then holds (4.27).

24^o If $x_1 < (3x_3 + x_4 + x_5)/2$, then (4.27) becomes (4.35).

25^o If $x_1 = (3x_2 + 15x_3 + 4x_4 + x_5)/11$, then from (4.27) follows

$(3x_2 + 15x_3 + 4x_4 + x_5)\ Fe_2O_3 + 11x_2\ C \rightarrow 11x_3\ Fe_3O_4 +$

$11x_4$ FeO + $11x_5$ Fe + $(2x_2 - x_3 - x_4 - 3x_5)$ Fe_3C + $(9x_2 + x_3 + x_4 + 3x_5)$ CO, (4.37)

where x_i, $(2 \leq i \leq 5)$ are arbitrary real numbers.

The reaction (4.37) holds if and only if

$$2x_2 > x_3 + x_4 + 3x_5. \quad (4.38)$$

26° If $x_2 = (x_3 + x_4 + 3x_5)/2$, then (4.37) transforms into (4.24).

27° If $x_2 > (x_3 + x_4 + 3x_5)/2$, then (4.37) holds.

28° If $x_2 < (x_3 + x_4 + 3x_5)/2$, then (4.37) becomes

$(3x_2 + 15x_3 + 4x_4 + x_5)$ Fe_2O_3 + $(-2x_2 + x_3 + x_4 + 3x_5)$ Fe_3C + $11x_2$ C $\to$ $11x_3$ Fe_3O_4 + $11x_4$ FeO + $11x_5$ Fe + $(9x_2 + x_3 + x_4 + 3x_5)$ CO, (4.39)

where x_i, $(2 \leq i \leq 5)$ are arbitrary real numbers.

29° If $x_1 > (3x_2 + 15x_3 + 4x_4 + x_5)/11$, then (4.27) holds.

30° If $x_1 < (3x_2 + 15x_3 + 4x_4 + x_5)/11$, then (4.27) becomes (4.35).

The reaction (4.35) holds if and only if the following system of inequalities is satisfied

$$-2x_1 + 3x_3 + x_4 + x_5 > 0, -11x_1 + 3x_2 + 15x_3 + 4x_4 + x_5 > 0, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5 > 0. \quad (4.40)$$

From (4.40) immediately follows this inequality

$x_1 < (6x_2 + 18x_3 + 5x_4 + 2x_5)/13$.

The last inequality is a necessary and sufficient condition the reaction (4.35) to be possible.

31° If $x_1 = (3x_3 + x_4 + x_5)/2$, then (4.35) becomes (4.20).

32° If $x_1 = (3x_2 + 15x_3 + 4x_4 + x_5)/11$, then from (4.35) one obtains (4.39).

33° If $x_1 = (6x_2 + 18x_3 + 5x_4 + 2x_5)/13$, then (4.35) transforms into (4.34).

34° If $x_1 < (3x_3 + x_4 + x_5)/2$, then (4.6) becomes (4.35).

II. *Particular cases of* (4.6) *generated by* $-13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5$

35° If $x_1 = (6x_2 + 18x_3 + 5x_4 + 2x_5)/13$, then (4.6) transforms into (4.30).

36° If $x_1 > (6x_2 + 18x_3 + 5x_4 + 2x_5)/13$, then from (4.6) one obtains (4.27).

37° If $x_1 < (6x_2 + 18x_3 + 5x_4 + 2x_5)/13$, then (4.6) becomes (4.35).

III. *Particular cases of* (4.6) *generated by the subgenerator* $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$

38° If $x_1 = (3x_2 + 15x_3 + 4x_4 + x_5)/11$, then (4.6) transforms into (4.37).

39° If $x_1 > (3x_2 + 15x_3 + 4x_4 + x_5)/11$, then from (4.6) one obtains

$3x_1$ Fe_2O_3 + $3x_2$ C + $(-2x_1 + 3x_3 + x_4 + x_5)$ Fe_3C $\to$ $3x_3$ Fe_3O_4 + $3x_4$ FeO + $3x_5$ Fe + $(-11x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5)$ CO, + $(-11x_1 + 3x_2 + 15x_3 + 4x_4 + x_5)$ CO_2. (4.41)

40° If $x_1 < (3x_2 + 15x_3 + 4x_4 + x_5)/11$, then (4.6) becomes (4.35).

Next, we shall analyze some particular cases of the general reaction (4.6) for $x_i = 0$, $(1 \leq i \leq 5)$.

As particular reactions of (4.6) we shall derive the following cases.

If $x_1 = 0$, then from (4.6) one obtains this particular reaction

$(3x_3 + x_4 + x_5)$ Fe_3C + $(3x_2 + 15x_3 + 4x_4 + x_5)$ CO_2 + $3x_2$ C $\to$ $3x_3$ Fe_3O_4 + $3x_4$ FeO + $3x_5$ Fe + $(6x_2 + 18x_3 + 5x_4 + 2x_5)$ CO, $(\forall\ x_2, x_3, x_4, x_5 \in \mathbb{R})$. (4.42)

If $x_2 = 0$, then from (4.6) follows

$3x_1$ Fe_2O_3 $3x_3$ Fe_3O_4 + $3x_4$ FeO + $3x_5$ Fe + $(2x_1 - 3x_3 - x_4 - x_5)$ Fe_3C + $(-13x_1 + 18x_3 + 5x_4 + 2x_5)$ CO + $(11x_1 - 15x_3 - 4x_4 - x_5)$ CO_2, (4.43)

where x_1, x_3, x_4 and x_5 are arbitrary real numbers.

In the reaction (4.43) the molecules arrangement is given in an implicit form. To be find their explicit arrangement, from (4.43) one obtains this system of inequalities

$$2x_1 - 3x_3 - x_4 - x_5 > 0, -13x_1 + 18x_3 + 5x_4 + 2x_5 > 0, 11x_1 - 15x_3 - 4x_4 - x_5 > 0,$$

or

$$x_1 = (15x_3 + 4x_4 + x_5)/11, \quad (4.44)$$

where x_3, x_4 and x_5 are arbitrary real numbers.

If we substitute (4.44) in (4.43), then one obtains

$(15x_3 + 4x_4 + x_5)$ Fe_2O_3 + $(x_3 + x_4 + 3x_5)$ Fe_3C $\to$ $11x_3$ Fe_3O_4 + $11x_4$ FeO + $11x_5$ Fe + $(x_3 + x_4 + 3x_5)$ CO, $(\forall\ x_3, x_4, x_5 \in \mathbb{R})$. (4.45)

41° If $x_1 > (15x_3 + 4x_4 + x_5)/11$, then from (4.43) follows this reaction

$3x_1$ Fe_2O_3 + $(-2x_1 + 3x_3 + x_4 + x_5)$ Fe_3C + $(13x_1 - 18x_3 - 5x_4 - 2x_5)$ CO $\to$ $3x_3$ Fe_3O_4 + $3x_4$ FeO + $3x_5$ Fe + $(11x_1 - 15x_3 - 4x_4 - x_5)$ CO_2, (4.46)

where x_1, x_3, x_4 and x_5 are arbitrary real numbers.

From (4.46) one obtains this system of inequalities

$$-2x_1 + 3x_3 + x_4 + x_5 > 0, 13x_1 - 18x_3 - 5x_4 - 2x_5 > 0, 11x_1 - 15x_3 - 4x_4 - x_5 > 0,$$

or

$x_1 > (18x_3 + 5x_4 + 2x_5)/13$,

where x_1, x_3, x_4 and x_5 are arbitrary real numbers.

Last inequality is a necessary and sufficient condition to hold (4.46).

42° If $x_1 = (3x_3 + x_4 + x_5)/2$, then from (4.46) follows this reaction

$$(3x_3 + x_4 + x_5)\ Fe_2O_3 + (x_3 + x_4 + 3x_5)\ CO\ 2x_3\ Fe_3O_4 + 2x_4\ FeO + 2x_5\ Fe + (x_3 + x_4 + 3x_5)\ CO_2,\ (\forall\ x_3, x_4, x_5 \in \mathbb{R}). \tag{4.47}$$

43° If $x_1 > (3x_3 + x_4 + x_5)/2$, then from (4.46) transforms into

$$3x_1\ Fe_2O_3 + (13x_1 - 18x_3 - 5x_4 - 2x_5)\ CO \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + 3x_5\ Fe + (2x_1 - 3x_3 - x_4 - x_5)\ Fe_3C + (11x_1 - 15x_3 - 4x_4 - x_5)\ CO_2, \tag{4.48}$$

where x_1, x_3, x_4 and x_5 are arbitrary real numbers.

From (4.48) one obtains this system of inequalities

$$13x_1 - 18x_3 - 5x_4 - 2x_5 > 0,\ 2x_1 - 3x_3 - x_4 - x_5 > 0,\ 11x_1 - 15x_3 - 4x_4 - x_5 > 0,$$

or

$$x_1 > (3x_3 + x_4 + x_5)/2.$$

where x_1, x_3, x_4 and x_5 are arbitrary real numbers.

Last inequality is necessary and sufficient condition to hold (4.48).

44° If $x_1 < (3x_3 + x_4 + x_5)/2$, then from (4.46) follows this reaction

$$3x_1\ Fe_2O_3 + (-2x_1 + 3x_3 + x_4 + x_5)\ Fe_3C + (-11x_1 + 15x_3 + 4x_4 + x_5)\ CO_2 \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + 3x_5\ Fe + (-13x_1 + 18x_3 + 5x_4 + 2x_5)\ CO, \tag{4.49}$$

where x_1, x_3, x_4 and x_5 are arbitrary real numbers.

From (4.49) one obtains this system of inequalities

$$-2x_1 + 3x_3 + x_4 + x_5 > 0,\ -11x_1 + 15x_3 + 4x_4 + x_5 > 0,\ -13x_1 + 18x_3 + 5x_4 + 2x_5 > 0,$$

or

$$x_1 < (3x_3 + x_4 + x_5)/2,$$

where x_1, x_3, x_4 and x_5 are arbitrary real numbers.

Last inequality is a necessary and sufficient condition to hold (4.49).

45° If $x_1 = (18x_3 + 5x_4 + 2x_5)/13$, then from (4.46) follows this reaction

$$(18x_3 + 5x_4 + 2x_5)\ Fe_2O_3 + (x_3 + x_4 + 3x_5)\ Fe_3C\ 13x_3 \rightarrow Fe_3O_4 + 13x_4\ FeO + 13x_5\ Fe + (x_3 + x_4 + 3x_5)\ CO_2,\ (\forall\ x_3, x_4, x_5 \in \mathbb{R}). \tag{4.50}$$

46° If $x_1 > (18x_3 + 5x_4 + 2x_5)/13$, then from (4.46) follows (4.48).

47° If $x_1 < (18x_3 + 5x_4 + 2x_5)/13$, then from (4.46) one obtains (4.49).

48° If $x_1 = (15x_3 + 4x_4 + x_5)/11$, then (4.46) transforms into

$$(15x_3 + 4x_4 + x_5)\ Fe_2O_3 + (x_3 + x_4 + 3x_5)\ Fe_3C \rightarrow 11x_3\ Fe_3O_4 + 11x_4\ FeO + 11x_5\ Fe + (x_3 + x_4 + 3x_5)\ CO,\ (\forall\ x_3, x_4, x_5 \in \mathbb{R}). \tag{4.51}$$

49° If $x_1 > (15x_3 + 4x_4 + x_5)/11$, then from (4.46) follows (4.48).

50° If $x_1 < (15x_3 + 4x_4 + x_5)/11$, then from (4.46) one obtains (4.49).

51° If $x_1 = (18x_3 + 5x_4 + 2x_5)/13$, then (4.48) transforms into (4.50).

52° If $x_1 > (18x_3 + 5x_4 + 2x_5)/13$, then holds (4.48).

53° If $x_1 < (18x_3 + 5x_4 + 2x_5)/13$, then from (4.48) follows (4.49).

54° If $x_1 = (3x_3 + x_4 + x_5)/2$, then from (4.48) one obtains (4.47).

55° If $x_1 > (3x_3 + x_4 + x_5)/2$, then holds (4.48).

56° If $x_1 < (3x_3 + x_4 + x_5)/2$, then from (4.48) follows (4.49).

57° If $x_1 = (15x_3 + 4x_4 + x_5)/11$, then (4.48) transforms into (4.51).

58° If $x_1 > (15x_3 + 4x_4 + x_5)/11$, then holds (4.48).

59° If $x_1 < (15x_3 + 4x_4 + x_5)/11$, then from (4.48) follows (4.49).

60° If $x_1 = (3x_3 + x_4 + x_5)/2$, then (4.49) transforms into (4.47).

61° If $x_1 > (3x_3 + x_4 + x_5)/2$, then from (4.49) one obtains (4.48).

62° If $x_1 < (3x_3 + x_4 + x_5)/2$, then holds (4.49).

63° If $x_1 = (15x_3 + 4x_4 + x_5)/11$, then (4.49) becomes (4.51).

64° If $x_1 > (15x_3 + 4x_4 + x_5)/11$, then from (4.49) one obtains (4.48).

65° If $x_1 < (15x_3 + 4x_4 + x_5)/11$, then holds (4.49).

66° If $x_1 = (18x_3 + 5x_4 + 2x_5)/13$, then from (4.49) follows (4.50).

67° If $x_1 > (18x_3 + 5x_4 + 2x_5)/13$, then from (4.49) one obtains (4.48).

68° If $x_1 < (18x_3 + 5x_4 + 2x_5)/13$, then holds (4.49).

69° If $x_1 = (3x_3 + x_4 + x_5)/2$, then from (4.43) follows (4.47).

70° If $x_1 > (3x_3 + x_4 + x_5)/2$, then from (4.43) one obtains (4.48).

71° If $x_1 < (3x_3 + x_4 + x_5)/2$, then (4.43) becomes (4.49).

72° If $x_1 = (18x_3 + 5x_4 + 2x_5)/13$, then (4.43) transforms into (4.50).

73° If $x_1 > (18x_3 + 5x_4 + 2x_5)/13$, then from (4.43) follows (4.48).

74° If $x_1 < (18x_3 + 5x_4 + 2x_5)/13$, then from (4.43) one obtains (4.49).

75° If $x_1 = (15x_3 + 4x_4 + x_5)/11$, then from (4.43) follows (4.51).

76° If $x_1 > (15x_3 + 4x_4 + x_5)/11$, then (4.43) transforms into (4.46).

77° If $x_1 < (15x_3 + 4x_4 + x_5)/11$, then from (4.43) follows (4.49). If $x_3 = 0$, then from (4.6) one obtains this particular reaction

$$3x_1\ Fe_2O_3 + 3x_2\ C \rightarrow 3x_4\ FeO + 3x_5\ Fe + (2x_1 - x_4 - x_5)\ Fe_3C + (-13x_1 + 6x_2 + 5x_4 + 2x_5)\ CO + (11x_1 - 3x_2 - 4x_4 - x_5)\ CO_2, \quad (4.52)$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

From (4.52) one obtains this system of inequalities

$$2x_1 - x_4 - x_5 > 0, -13x_1 + 6x_2 + 5x_4 + 2x_5 > 0, 11x_1 - 3x_2 - 4x_4 - x_5 > 0, \quad (4.53)$$

or

$$3x_2 < 11x_1 - 4x_4 - x_5 < 6x_2, \quad (4.54)$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

The inequality (4.54) is a necessary and sufficient condition to hold the reaction (4.52).

78° If $x_1 = (x_4 + x_5)/2$, then from (4.52) one obtains this particular reaction

$$(x_4 + x_5)\ Fe_2O_3 + 2x_2\ C\ 2x_4\ FeO + 2x_5\ Fe + (4x_2 - x_4 - 3x_5)\ CO + (-2x_2 + x_4 + 3x_5)\ CO_2, \quad (4.55)$$

where x_2, x_4 and x_5 are arbitrary real numbers.

79° If $x_1 > (x_4 + x_5)/2$, then from (4.52) follows

$$3x_1\ Fe_2O_3 + 3x_2\ C + (13x_1 - 6x_2 - 5x_4 - 2x_5)\ CO \rightarrow 3x_4\ FeO + 3x_5\ Fe + (2x_1 - x_4 - x_5)\ Fe_3C + (11x_1 - 3x_2 - 4x_4 - x_5)\ CO_2, \quad (4.56)$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

80° If $x_1 < (x_4 + x_5)/2$, then from (4.52) one obtains

$$3x_1\ Fe_2O_3 + (-2x_1 + x_4 + x_5)\ Fe_3C + 3x_2\ C + (-11x_1 + 3x_2 + 4x_4 + x_5)\ CO_2 \rightarrow 3x_4\ FeO + 3x_5\ Fe + (-13x_1 + 6x_2 + 5x_4 + 2x_5)\ CO, \quad (4.57)$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

81° If $x_1 = (6x_2 + 5x_4 + 2x_5)/13$, then from (4.52) follows this reaction

$$(6x_2 + 5x_4 + 2x_5)\ Fe_2O_3 + 13x_2\ C \rightarrow 13x_4\ FeO + 13x_5\ Fe + (4x_2 - x_4 - 3x_5)\ Fe_3C + (9x_2 + x_4 + 3x_5)\ CO_2, \quad (4.58)$$

where x_2, x_4 and x_5 are arbitrary real numbers.

The reaction (4.58) has the following three subgenerators

$$6x_2 + 5x_4 + 2x_5 > 0, 4x_2 - x_4 - 3x_5 > 0, 9x_2 + x_4 + 3x_5 > 0.$$

A necessary and sufficient condition to hold the above inequalities and the reaction (4.58) is

$x_2 > (x_4 + 3x_5)/4$.

82° If $x_1 > (6x_2 + 5x_4 + 2x_5)/13$, then (4.52) becomes

$$3x_1\ Fe_2O_3 + 3x_2\ C + (-2x_1 + x_4 + x_5)\ Fe_3C + (13x_1 - 6x_2 - 5x_4 - 2x_5)\ CO \rightarrow 3x_4\ FeO + 3x_5\ Fe + (11x_1 - 3x_2 - 4x_4 - x_5)\ CO_2, \quad (4.59)$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

The reaction (4.59) contains these subgenerators

$$-2x_1 + x_4 + x_5 > 0, 13x_1 - 6x_2 - 5x_4 - 2x_5 > 0, 11x_1 - 3x_2 - 4x_4 - x_5 > 0.$$

The reaction (4.59) is possible if and only if are satisfied the above three inequalities and if and only if this inequality holds

$x_2 < (11x_1 - 4x_4 - x_5)/6$.

83° If $x_1 < (6x_2 + 5x_4 + 2x_5)/13$, then from (4.52) one obtains

$$3x_1\ Fe_2O_3 + 3x_2\ C + (-2x_1 + x_4 + x_5)\ Fe_3C + (-11x_1 + 3x_2 + 4x_4 + x_5)\ CO_2 \rightarrow 3x_4\ FeO + 3x_5\ Fe + (-13x_1 + 6x_2 + 5x_4 + 2x_5)\ CO, \quad (4.60)$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

From the reaction (4.60) follows this system of inequalities

$$-2x_1 + x_4 + x_5 > 0, -11x_1 + 3x_2 + 4x_4 + x_5 > 0, -13x_1 + 6x_2 + 5x_4 + 2x_5 > 0.$$

The above system of inequalities is satisfied if and only if

$$x_1 < (x_4 + x_5)/2,$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

Last inequality is a necessary and sufficient condition to hold the reaction (4.60).

84° If $x_1 = (3x_2 + 4x_4 + x_5)/11$, then from (4.52) follows this particular reaction

$$(3x_2 + 4x_4 + x_5)\ Fe_2O_3 + 11x_2\ C \rightarrow 11x_4\ FeO + 11x_5\ Fe + (2x_2 - x_4 - 3x_5)\ Fe_3C + (9x_2 + x_4 + 3x_5)\ CO, \quad (4.61)$$

where x_2, x_4 and x_5 are arbitrary real numbers.

The reaction (4.61) holds if and only if this inequality is satisfied

$x_2 > (x_4 + 3x_5)/2$.

85° If $x_1 > (3x_2 + 4x_4 + x_5)/11$, then the reaction (4.52) transforms into (4.59).

86° If $x_1 < (3x_2 + 4x_4 + x_5)/11$, then from (4.52) one obtains (4.60).

Let's consider (4.55). This reaction contains the following two subgenerators $4x_2 - x_4 - 3x_5 > 0$ and $-2x_2 + x_4 + 3x_5 > 0$. From them immediately follows

$$2x_2 < x_4 + 3x_5 < 4x_2, \quad (4.62)$$

The inequality (4.62) is a necessary and sufficient condition to hold (4.55).

87° If $x_2 = (x_4 + 3x_5)/4$, then (4.55) transforms into

$$(4x_4 + 4x_5)\,Fe_2O_3 + (2x_4 + 6x_5)\,C \rightarrow 8x_4\,FeO + 8x_5\,Fe + (2x_4 + 6x_5)\,CO_2,\ (\forall\ x_4, x_5 \in \mathbb{R}). \tag{4.63}$$

88° If $x_2 > (x_4 + 3x_5)/4$, then from (4.55) one obtains

$$(x_4 + x_5)\,Fe_2O_3 + 2x_2\,C + (2x_2 - x_4 - 3x_5)\,CO_2 \rightarrow 2x_4\,FeO + 2x_5\,Fe + (4x_2 - x_4 - 3x_5)\,CO, \tag{4.64}$$

where x_2, x_4 and x_5 are arbitrary real numbers.

89° If $x_2 < (x_4 + 3x_5)/4$, then from (4.55) follows

$$(x_4 + x_5)\,Fe_2O_3 + 2x_2\,C + (-4x_2 + x_4 + 3x_5)\,CO \rightarrow 2x_4\,FeO + 2x_5\,Fe + (-2x_2 + x_4 + 3x_5)\,CO_2, \tag{4.65}$$

where x_2, x_4 and x_5 are arbitrary real numbers.

90° If $x_2 = (x_4 + 3x_5)/2$, then (4.55) becomes

$$(x_4 + x_5)\,Fe_2O_3 + (x_4 + 3x_5)\,C \rightarrow 2x_4\,FeO + 2x_5\,Fe + (x_4 + 3x_5)\,CO, \tag{4.66}$$

91° If $x_2 > (x_4 + 3x_5)/2$, then (4.55) transforms into (4.64).

92° If $x_2 < (x_4 + 3x_5)/2$, then from (4.55) follows (4.65).

Now, we shall consider the reaction (4.56). This reaction contains the following three subgenerators $13x_1 - 6x_2 - 5x_4 - 2x_5$, $2x_1 - x_4 - x_5$ and $11x_1 - 3x_2 - 4x_4 - x_5$. According to the Definition (2.13), immediately follows this system of inequalities

$$13x_1 - 6x_2 - 5x_4 - 2x_5 > 0,\ 2x_1 - x_4 - x_5 > 0,\ 11x_1 - 3x_2 - 4x_4 - x_5 > 0, \tag{4.67}$$

or

$$13x_1 - 5x_4 - 2x_5 > 6x_2, \tag{4.68}$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

The expression (4.68) is a necessary and sufficient condition to hold the reaction (4.56).

93° If $x_1 = (6x_2 + 5x_4 + 2x_5)/13$, then from (4.56) one obtains (4.58).

94° If $x_1 > (6x_2 + 5x_4 + 2x_5)/13$, then (4.56) transforms into (4.59).

95° If $x_1 < (6x_2 + 5x_4 + 2x_5)/13$, then (4.56) becomes (4.60).

96° If $x_1 = (x_4 + x_5)/2$, then from (4.56) one obtains (4.55).

97° If $x_1 > (x_4 + x_5)/2$, then (4.56) holds if (4.68) is satisfied.

98° If $x_1 < (x_4 + x_5)/2$, then (4.56) becomes (4.57).

99° If $x_1 = (3x_2 + 4x_4 + x_5)/11$, then from (4.56) follows (4.61).

100° If $x_1 > (3x_2 + 4x_4 + x_5)/11$, then (4.56) becomes (4.59).

101° If $x_1 < (3x_2 + 4x_4 + x_5)/11$, then from (4.56) follows (4.60).

Let's consider the reaction (4.57). From this reaction follows the system of inequalities

$$-2x_1 + x_4 + x_5 > 0,\ -11x_1 + 3x_2 + 4x_4 + x_5 > 0,\ -13x_1 + 6x_2 + 5x_4 + 2x_5 > 0. \tag{4.69}$$

The system of inequalities (4.69) holds if and only if

$$x_1 < (3x_2 + 4x_4 + x_5)/13, \tag{4.70}$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

The inequality (4.70) is a necessary and sufficient condition to hold the reaction (4.57).

Next, we shall consider particular cases of (4.57).

102° If $x_1 = (x_4 + x_5)/2$, then from (4.57) one obtains (4.64).

103° If $x_1 > (x_4 + x_5)/2$, then (4.57) transforms into

$$3x_1\,Fe_2O_3 + 3x_2\,C + (-11x_1 + 3x_2 + 4x_4 + x_5)\,CO_2 \rightarrow 3x_4\,FeO + 3x_5\,Fe + (2x_1 - x_4 - x_5)\,Fe_3C + (-13x_1 + 6x_2 + 5x_4 + 2x_5)\,CO, \tag{4.71}$$

where x_1, x_2, x_4 and x_5 are arbitrary real numbers.

104° If $x_1 < (x_4 + x_5)/2$, then the reaction (4.57) holds.

105° If $x_1 = (3x_2 + 4x_4 + x_5)/11$, then from (4.57) one obtains (4.61).

106° If $x_1 > (3x_2 + 4x_4 + x_5)/11$, then (4.57) transforms into (4.59).

107° If $x_1 < (3x_2 + 4x_4 + x_5)/11$, then from (4.57) follows (4.60).

108° If $x_1 = (6x_2 + 5x_4 + 2x_5)/13$, then from (4.57) one obtains (4.58).

109° If $x_1 > (6x_2 + 5x_4 + 2x_5)/13$, then from (4.57) follows (5.59).

110° If $x_1 < (6x_2 + 5x_4 + 2x_5)/13$, then (4.57) transforms into (4.60).

If $x_4 = 0$, then from (4.6) one obtains this particular reaction

$$3x_1\,Fe_2O_3 + 3x_2\,C \rightarrow 3x_3\,Fe_3O_4 + 3x_5\,Fe + (2x_1 - 3x_3 - x_5)\,Fe_3C + (-13x_1 + 6x_2 + 18x_3 + 2x_5)\,CO + (11x_1 - 3x_2 - 15x_3 - x_5)\,CO_2, \tag{4.72}$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

From (4.72) one obtains this system of inequalities

$$2x_1 - 3x_3 - x_5 > 0,\ -13x_1 + 6x_2 + 18x_3 + 2x_5 > 0,\ 11x_1 - 3x_2 - 15x_3 - x_5 > 0, \tag{4.73}$$

or

$$3x_2 < 11x_1 - 15x_3 - x_5 < 6x_2, \tag{4.74}$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

The inequality (4.74) is a necessary and sufficient condition to hold the reaction (4.72).

111° If $x_1 = (3x_3 + x_5)/2$, then (4.72) transforms into

$$(3x_3 + x_5)\,Fe_2O_3 + 2x_2\,C \rightarrow 2x_3\,Fe_3O_4 + 2x_5\,Fe + (4x_2 - x_3 - 3x_5)\,CO + (-2x_2 + x_3 + 3x_5)\,CO_2, \tag{4.75}$$

where x_2, x_3 and x_5 are arbitrary real numbers.

Reaction (4.75) is possible if and only if holds the following inequality

$$2x_2 < x_3 + 3x_5 < 4x_2, \tag{4.76}$$

The inequality (4.76) is a necessary and sufficient condition to hold (4.75).

112° If $x_2 = (x_3 + 3x_5)/4$, then (4.75) obtains this form

$$(6x_3 + 2x_5)\ Fe_2O_3 + (x_3 + 3x_5)\ C \rightarrow 4x_3\ Fe_3O_4 + 4x_5\ Fe + (x_3 + 3x_5)\ CO_2,\ (\forall\ x_3, x_5 \in \mathbb{R}). \tag{4.77}$$

113° If $x_2 > (x_3 + 3x_5)/4$ then (4.75) transforms into

$$(3x_3 + x_5)\ Fe_2O_3 + 2x_2\ C + (2x_2 - x_3 - 3x_5)\ CO_2\ 2x_3\ Fe_3O_4 + 2x_5\ Fe + (4x_2 - x_3 - 3x_5)\ CO, \tag{4.78}$$

where x_2, x_3 and x_5 are arbitrary real numbers.

The above reaction is possible if and only if holds this inequality

$$2x_2 > x_3 + 3x_5. \tag{4.79}$$

The inequality (4.79) is a necessary and sufficient condition to hold (4.78).

114° If $x_2 < (x_3 + 3x_5)/4$, then from (4.75) one obtains

$$(3x_3 + x_5)\ Fe_2O_3 + 2x_2\ C + (-\ 4x_2 + x_3 + 3x_5)\ CO \rightarrow 2x_3\ Fe_3O_4 + 2x_5\ Fe + (-\ 2x_2 + x_3 + 3x_5)\ CO_2, \tag{4.80}$$

where x_2, x_3 and x_5 are arbitrary real numbers.

The reaction (4.80) is possible if and only if holds the following inequality

$$x_3 + 3x_5 > 4x_2. \tag{4.81}$$

The inequality (4.81) is a necessary and sufficient condition to hold (4.80).

115° If $x_2 = (x_3 + 3x_5)/2$, then from (4.75) follows

$$(3x_3 + x_5)\ Fe_2O_3 + (x_3 + 3x_5)\ C \rightarrow 2x_3\ Fe_3O_4 + 2x_5\ Fe + (x_3 + 3x_5)\ CO,\ (\forall\ x_3, x_5 \in \mathbb{R}). \tag{4.82}$$

116° If $x_2 > (x_3 + 3x_5)/2$, then (4.75) becomes (4.78).

117° If $x_2 < (x_3 + 3x_5)/2$, then (4.75) holds.

118° If $x_1 > (3x_3 + x_5)/2$, then (4.72) holds.

119° If $x_1 < (3x_3 + x_5)/2$, then from (4.72) one obtains

$$3x_1\ Fe_2O_3 + 3x_2\ C + (-\ 2x_1 + 3x_3 + x_5)\ Fe_3C \rightarrow 3x_3\ Fe_3O_4 + 3x_5\ Fe + (-\ 13x_1 + 6x_2 + 18x_3 + 2x_5)\ CO + (11x_1 - 3x_2 - 15x_3 - x_5)\ CO_2, \tag{4.83}$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

The reaction (4.83) is possible if and only if the inequality (4.74) is satisfied.

120° If $x_1 = (6x_2 + 18x_3 + 2x_5)/13$, then (4.72) becomes

$$(6x_2 + 18x_3 + 2x_5)\ Fe_2O_3 + 13x_2\ C \rightarrow 13x_3\ Fe_3O_4 + 13x_5\ Fe + (4x_2 - x_3 - 3x_5)\ Fe_3C + (9x_2 + x_3 + 3x_5)\ CO_2, \tag{4.84}$$

where x_2, x_3 and x_5 are arbitrary real numbers.

From the above reaction (4.84) one obtains this system of inequalities

$$6x_2 + 18x_3 + 2x_5 > 0,\ 4x_2 - x_3 - 3x_5 > 0,\ 9x_2 + x_3 + 3x_5 > 0. \tag{4.85}$$

From (4.85) immediately follows this inequality

$$x_3 + 3x_5 < 4x_2, \tag{4.86}$$

which is a necessary and sufficient condition to hold (4.84).

121° If $x_2 = (x_3 + 3x_5)/4$, then (4.84) obtains this form

$$(6x_3 + 2x_5)\ Fe_2O_3 + (x_3 + 3x_5)\ C \rightarrow 4x_3\ Fe_3O_4 + 4x_5\ Fe + (x_3 + 3x_5)\ CO_2,\ (\forall\ x_3, x_5 \in \mathbb{R}). \tag{4.85}$$

122° If $x_2 > (x_3 + 3x_5)/4$, then (4.84) holds.

123° If $x_2 < (x_3 + 3x_5)/4$, then from (4.84) follows

$$(6x_2 + 18x_3 + 2x_5)\ Fe_2O_3 + 13x_2\ C + (-\ 4x_2 + x_3 + 3x_5)\ Fe_3C \rightarrow 13x_3\ Fe_3O_4 + 13x_5\ Fe + (9x_2 + x_3 + 3x_5)\ CO_2, \tag{4.86}$$

where x_2, x_3 and x_5 are arbitrary real numbers.

124° If $x_1 > (6x_2 + 18x_3 + 2x_5)/13$, then (4.72) transforms into

$$3x_1\ Fe_2O_3 + 3x_2\ C + (13x_1 - 6x_2 - 18x_3 - 2x_5)\ CO \rightarrow 3x_3\ Fe_3O_4 + 3x_5\ Fe + (2x_1 - 3x_3 - x_5)\ Fe_3C + (11x_1 - 3x_2 - 15x_3 - x_5)\ CO_2, \tag{4.87}$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

From (4.87) follows this system of inequalities

$$13x_1 - 6x_2 - 18x_3 - 2x_5 > 0,\ 2x_1 - 3x_3 - x_5 > 0,\ 11x_1 - 3x_2 - 15x_3 - x_5 > 0, \tag{4.88}$$

or

$$13x_1 - 18x_3 - 2x_5 > 6x_2, \tag{4.89}$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

The inequality (4.89) is a necessary and sufficient condition to hold the reaction (4.87).

125° If $x_1 < (6x_2 + 18x_3 + 2x_5)/13$, then from (4.72) one obtains

$$3x_1\ Fe_2O_3 + 3x_2\ C + (-\ 2x_1 + 3x_3 + x_5)\ Fe_3C + (-\ 11x_1 + 3x_2 + 15x_3 + x_5)\ CO_2 \rightarrow 3x_3\ Fe_3O_4 + 3x_5\ Fe + (-\ 13x_1 + 6x_2 + 18x_3 + 2x_5)\ CO, \tag{4.90}$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

From (4.90) one obtains this system of inequalities

$$-\ 2x_1 + 3x_3 + x_5 > 0,\ -\ 11x_1 + 3x_2 + 15x_3 + x_5 > 0,\ -\ 13x_1 + 6x_2 + 18x_3 + 2x_5 > 0. \tag{4.91}$$

or

$$x_1 < (6x_2 + 18x_3 + 2x_5)/13, \quad (4.92)$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

The inequality (4.92) is a necessary and sufficient condition to hold the reaction (4.91).

126° If $x_1 = (3x_2 + 15x_3 + x_5)/11$, then from (4.72) follows this reaction

$$(3x_2 + 15x_3 + x_5)\ Fe_2O_3 + 11x_2\ C \rightarrow 11x_3\ Fe_3O_4 + 11x_5\ Fe + (2x_2 - x_3 - 3x_5)\ Fe_3C + (9x_2 + x_3 + 3x_5)\ CO, \quad (4.93)$$

where x_2, x_3 and x_5 are arbitrary real numbers.

Reaction (4.93) is possible if and only if holds this inequality

$$x_2 > (x_3 + 3x_5)/2. \quad (4.94)$$

Actually, the inequality (4.94) is a necessary and sufficient condition to hold (4.93).

127° If $x_1 = (x_3 + 3x_5)/2$, then (4.93) becomes (4.82).

128° If (4.94) holds, then holds (4.93) too.

129° If $x_2 < (x_3 + 3x_5)/2$, then (4.93) transforms into

$$(3x_2 + 15x_3 + x_5)\ Fe_2O_3 + 11x_2\ C + (-2x_2 + x_3 + 3x_5)\ Fe_3C \rightarrow 11x_3\ Fe_3O_4 + 11x_5\ Fe + (9x_2 + x_3 + 3x_5)\ CO, \quad (4.95)$$

where x_2, x_3 and x_5 are arbitrary real numbers.

130° If $x_1 > (3x_2 + 15x_3 + x_5)/11$, then (4.72) becomes

$$3x_1\ Fe_2O_3 + 3x_2\ C + (-2x_1 + 3x_3 + x_5)\ Fe_3C \rightarrow 3x_3\ Fe_3O_4 + 3x_5\ Fe + (-13x_1 + 6x_2 + 18x_3 + 2x_5)\ CO + (11x_1 - 3x_2 - 15x_3 - x_5)\ CO_2, \quad (4.96)$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

The subgenerators of the reaction (4.96) are given by the following expression

$$-2x_1 + 3x_3 + x_5 > 0, -13x_1 + 6x_2 + 18x_3 + 2x_5 > 0, 11x_1 - 3x_2 - 15x_3 - x_5 > 0. \quad (4.97)$$

A necessary and sufficient condition to hold the inequalities (4.97) and the reaction (4.96) is

$$x_1 > (3x_2 + 15x_3 + 5x_5)/11. \quad (4.98)$$

131° If $x_1 < (3x_2 + 15x_3 + x_5)/11$, then (4.72) transforms into

$$3x_1\ Fe_2O_3 + 3x_2\ C + (-2x_1 + 3x_3 + x_5)\ Fe_3C + (-11x_1 + 3x_2 + 15x_3 + x_5)\ CO_2 \rightarrow 3x_3\ Fe_3O_4 + 3x_5\ Fe + (-13x_1 + 6x_2 + 18x_3 + 2x_5)\ CO, \quad (4.99)$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

From the reaction (4.99) immediately follows this system of inequalities

$$-2x_1 + 3x_3 + x_5 > 0, -11x_1 + 3x_2 + 15x_3 + x_5 > 0, -13x_1 + 6x_2 + 18x_3 + 2x_5 > 0. \quad (4.100)$$

The system of inequalities (4.100) holds if and only if this inequality is satisfied

$$x_1 < (6x_2 + 18x_3 + 2x_5)/13, \quad (4.101)$$

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

If $x_5 = 0$, then from (4.6) one obtains this particular reaction

$$3x_1\ Fe_2O_3 + 3x_2\ C \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + (2x_1 - 3x_3 - x_4)\ Fe_3C + (-13x_1 + 6x_2 + 18x_3 + 5x_4)\ CO + (11x_1 - 3x_2 - 15x_3 - 4x_4)\ CO_2, \quad (4.102)$$

where x_1, x_2, x_3 and x_4 are arbitrary real numbers.

From (4.102) follows this system of inequalities

$$2x_1 - 3x_3 - x_4 > 0, -13x_1 + 6x_2 + 18x_3 + 5x_4 > 0, 11x_1 - 3x_2 - 15x_3 - 4x_4 > 0. \quad (4.103)$$

or

$$3x_2 < 11x_1 - 15x_3 - 4x_4 < 6x_2, \quad (4.104)$$

where x_1, x_2, x_3 and x_4 are arbitrary real numbers.

The inequality (4.104) is a necessary and sufficient condition to hold the reaction (4.102).

Now we shall consider some particular cases of (4.102).

132° If $x_1 = (3x_3 + x_4)/2$, then (4.102) transforms into

$$(3x_3 + x_4)\ Fe_2O_3 + 2x_2\ C \rightarrow 2x_3\ Fe_3O_4 + 2x_4\ FeO + (4x_2 - x_3 - x_4)\ CO + (-2x_2 + x_3 + x_4)\ CO_2, \quad (4.105)$$

where x_2, x_3 and x_4 are arbitrary real numbers.

The reaction (4.105) is possible if and only if this inequality holds

$$2x_2 < x_3 + x_4 < 4x_2. \quad (4.106)$$

133° If $x_2 = (x_3 + x_4)/4$, then from (4.105) one obtains

$$(6x_3 + 2x_4)\ Fe_2O_3 + (x_3 + x_4)\ C \rightarrow 4x_3\ Fe_3O_4 + 4x_4\ FeO + (x_3 + x_4)\ CO_2, (\forall\ x_3, x_4 \in \mathbb{R}). \quad (4.107)$$

134° If $x_2 > (x_3 + x_4)/4$, then the reaction (4.105) holds.

135° If $x_2 < (x_3 + x_4)/4$, then the reaction (4.105) transforms into this form

$$(3x_3 + x_4)\ Fe_2O_3 + 2x_2\ C + (-4x_2 + x_3 + x_4)\ CO \rightarrow 2x_3\ Fe_3O_4 + 2x_4\ FeO + (-2x_2 + x_3 + x_4)\ CO_2, \quad (4.108)$$

where x_2, x_3 and x_4 are arbitrary real numbers.

A necessary and sufficient condition to hold the reaction (4.108) is to be satisfied this inequality

$$x_3 + x_4 > 4x_2. \quad (4.109)$$

136° If $x_2 = (x_3 + x_4)/2$, then the reaction (4.105) reduces to this particular reaction

$(3x_3 + x_4)\ Fe_2O_3 + (x_3 + x_4)\ C \rightarrow 2x_3\ Fe_3O_4 + 2x_4\ FeO + (x_3 + x_4)\ CO,\ (\forall\ x_3, x_4 \in \mathbb{R}).$ (4.110)

137° If $x_2 > (x_3 + x_4)/2$, then (4.105) becomes

$(3x_3 + x_4)\ Fe_2O_3 + 2x_2\ C + (2x_2 - x_3 - x_4)\ CO_2 \rightarrow 2x_3\ Fe_3O_4 + 2x_4\ FeO + (4x_2 - x_3 - x_4)\ CO,$ (4.111)

where x_2, x_3 and x_4 are arbitrary real numbers.

The reaction (4.111) is possible if and only if is satisfied this inequality

$x_3 + x_4 < 2x_2.$ (4.112)

138° If $x_2 < (x_3 + x_4)/2$, then from (4.105) one obtains

$(3x_3 + x_4)\ Fe_2O_3 + 2x_2\ C + (-\ 4x_2 + x_3 + x_4)\ CO \rightarrow 2x_3\ Fe_3O_4 + 2x_4\ FeO + (-\ 2x_2 + x_3 + x_4)\ CO_2,$ (4.113)

where x_2, x_3 and x_4 are arbitrary real numbers.

The reaction (4.113) is possible if and only if this inequality is satisfied

$x_3 + x_4 > 4x_2.$ (4.114)

139° If $x_1 > (3x_3 + x_4)/2$, then (4.102) holds.

140° If $x_1 < (3x_3 + x_4)/2$, then from (4.102) one obtains

$3x_1\ Fe_2O_3 + 3x_2\ C + (-\ 2x_1 + 3x_3 + x_4)\ Fe_3C + (-\ 11x_1 + 3x_2 + 15x_3 + 4x_4)\ CO_2 \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + (-\ 13x_1 + 6x_2 + 18x_3 + 5x_4)\ CO\ ,$ (4.115)

where x_1, x_2, x_3 and x_4 are arbitrary real numbers.

The reaction (4.115) holds if and only if this inequality is satisfied

$x_1 < (3x_2 + 15x_3 + 4x_4)/11,$ (4.116)

where x_1, x_2, x_3 and x_4 are arbitrary real numbers.

141° If $x_1 = (6x_2 + 18x_3 + 5x_4)/13$, then from (4.102) one obtains

$(6x_2 + 18x_3 + 5x_4)\ Fe_2O_3 + 13x_2\ C \rightarrow 13x_3\ Fe_3O_4 + 13x_4\ FeO + (4x_2 - x_3 - x_4)\ Fe_3C + (9x_2 + x_3 + x_4)\ CO_2,$ (4.117)

where x_2, x_3 and x_4 are arbitrary real numbers.

The reaction (4.117) is possible if and only if the following inequality is satisfied

$4x_2 > x_3 + x_4.$ (4.118)

142° If $x_2 = (x_3 + x_4)/4$, then (4.117) becomes (4.107).

143° If $x_1 > (6x_2 + 18x_3 + 5x_4)/13$, then from (4.102) follows

$3x_1\ Fe_2O_3 + 3x_2\ C + (13x_1 - 6x_2 - 18x_3 - 5x_4)\ CO \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + (2x_1 - 3x_3 - x_4)\ Fe_3C + (11x_1 - 3x_2 - 15x_3 - 4x_4)\ CO_2,$ (4.118)

where x_1, x_2, x_3 and x_4 are arbitrary real numbers.

The reaction (4.118) holds if and only if this inequality is satisfied

$13x_1 - 18x_3 - 5x_4 > 6x_2,$ (4.119)

where x_1, x_2, x_3 and x_5 are arbitrary real numbers.

144° If $x_1 < (6x_2 + 18x_3 + 5x_4)/13$, then (4.102) becomes

$3x_1\ Fe_2O_3 + 3x_2\ C + (-\ 2x_1 + 3x_3 + x_4)\ Fe_3C + (-\ 11x_1 + 3x_2 + 15x_3 + 4x_4)\ CO_2 \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + (-\ 13x_1 + 6x_2 + 18x_3 + 5x_4)\ CO,$ (4.120)

where x_1, x_2, x_3 and x_4 are arbitrary real numbers.

The reaction (4.120) is possible if and only if this inequality is satisfied

$x_1 < (3x_2 + 15x_3 + 4x_4)/11,$ (4.121)

where x_1, x_2, x_3 and x_4 are arbitrary real numbers.

145° If $x_1 = (3x_2 + 15x_3 + 4x_4)/11$, then from (4.102) one obtains

$(3x_2 + 15x_3 + 4x_4)\ Fe_2O_3 + 11x_2\ C \rightarrow 11x_3\ Fe_3O_4 + 11x_4\ FeO + (2x_2 - x_3 - x_4)\ Fe_3C + (9x_2 + x_3 + x_4)\ CO,$ (4.122)

where x_2, x_3 and x_4 are arbitrary real numbers.

The above reaction (4.122) has only one subgenerator $2x_2 - x_3 - x_4$, that means that it is possible if and only if

$x_2 > (x_3 + x_4)/2.$ (4.123)

146° If $x_2 = (x_3 + x_4)/2$, then (4.122) becomes (4.110).

147° If $x_1 > (3x_2 + 15x_3 + 4x_4)/11$, then (4.102) holds.

148° If $x_1 < (3x_2 + 15x_3 + 4x_4)/11$, then (4.102) becomes

$3x_1\ Fe_2O_3 + 3x_2\ C + (-\ 2x_1 + 3x_3 + x_4)\ Fe_3C + (-\ 11x_1 + 3x_2 + 15x_3 + 4x_4)\ CO_2 \rightarrow 3x_3\ Fe_3O_4 + 3x_4\ FeO + (-\ 13x_1 + 6x_2 + 18x_3 + 5x_4)\ CO,$ (4.124)

where x_1, x_2, x_3 and x_4 are arbitrary real numbers.

The reaction (4.124) holds if and only if the inequality (4.116) is satisfied.

By this we shall finish this section, where we considered some of the important particular cases.

AN EXTENTION OF THE RESULTS

In this section we shall extend topological and chemical results obtained in the previous section. Actually, here we shall develop an explicite topological calculus for the coefficients of the chemical reaction (4.6) for an another topology and will be considered balancing of the chemical reaction (5.3) which posses atoms with fractional oxidation numbers. Also for the reaction (5.6) we shall develop a comprehensive topological calculus.

First, for the reaction (4.6) we shall consider the topology

$$\mathscr{T} = \{\boldsymbol{X}, \varnothing, \{3x_1\}, \{3x_1, 3x_5\}, \{3x_1, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}\}, \tag{5.1}$$

on

$$\boldsymbol{X} = \{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \tag{5.2}$$

where x_i, $(1 \leq i \leq 5)$ are arbitrary real numbers.

Now, we shall determine

1° the derived sets of $\boldsymbol{Y} = \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ and $\boldsymbol{Z} = \{3x_5\}$,

2° the closed subsets of $\boldsymbol{X}$,

3° the closure of the sets $\{3x_1\}$, $\{3x_5\}$ and $\{2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$,

4° which sets in 3° are dense in $\boldsymbol{X}$?

5° the interior points of the subset $\boldsymbol{T} = \{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\}$ of $\boldsymbol{X}$,

6° the exterior points of $\boldsymbol{T}$,

7° the boundary points of $\boldsymbol{T}$,

8° the neighborhoods of the point $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$ and of the point $2x_1 - 3x_3 - x_4 - x_5$,

9° the members of the relative topology $\mathscr{T}_S$ on $\boldsymbol{S} = \{3x_1, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$.

Let's go forward.

1° Note that $\{3x_1, 3x_5\}$ and $\{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ are open subsets of $\boldsymbol{X}$ and that

$$3x_1, 3x_5 \in \{3x_1, 3x_5\}$$

and

$$\{3x_1, 3x_5\} \cap \boldsymbol{Y} = \varnothing, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5 \in \{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$$

and

$$\{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} \cap \boldsymbol{Y} = \{11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}.$$

Hence $3x_1$, $3x_5$ and $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$ are not limit point of $\boldsymbol{Y}$. On the other hand, every other point in $\boldsymbol{X}$ is a limit point of $\boldsymbol{Y}$ since every open set containing it also contains a point of $\boldsymbol{Y}$ different from it. Accordingly,

$$\boldsymbol{Y}' = \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}.$$

Note that $\{3x_1\}$, $\{3x_1, 3x_5\}$ and $\{3x_1, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$ are open subsets of $\boldsymbol{X}$ and that $3x_1 \in \{3x_1\}$ and $\{3x_1\} \cap \boldsymbol{Z} = \varnothing$, $3x_5 \in \{3x_1, 3x_5\}$ and $\{3x_1, 3x_5\} \cap \boldsymbol{Z} = \{3x_5\}$, $2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5 \in \{3x_1, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$ and $\{3x_1, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} \cap \boldsymbol{Z} = \varnothing$.

Hence $3x_1$, $3x_5$, $2x_1 - 3x_3 - x_4 - x_5$ and $-13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5$ are not limit point of $\boldsymbol{Z} = \{3x_5\}$. But $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$ is a limit point of $\boldsymbol{Z}$ since the open sets containing $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$ are $\{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ and $\boldsymbol{X}$ and each contains the point $3x_5 \in \boldsymbol{Z}$ different from $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$. Thus $\boldsymbol{Z}' = \{11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$.

2° A set is closed if and only if its complement is open. Hence write the complement of each set in $\mathscr{T}$:

$$\varnothing, \boldsymbol{X}, \{3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}.$$

3° The closure $Cl\{\boldsymbol{X}\}$ of any set $\boldsymbol{X}$ is the intersection of all closed supersets of $\boldsymbol{X}$.

The only closed superset of $\{3x_1\}$ is $\boldsymbol{X}$; the closed supersets of $\{3x_5\}$ are

$$\{3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} \text{ and } \boldsymbol{X};$$

and the closed supersets of $\{2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ are $\{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ and $\boldsymbol{X}$.

Thus,

$$Cl\{3x_1\} = \boldsymbol{X},\ Cl\{3x_5\} = \{3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\},\ Cl\{2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \{2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}.$$

4° A set $\boldsymbol{Y}$ is dense in $\boldsymbol{X}$ if and only if $Cl\{\boldsymbol{Y}\} = \boldsymbol{X}$; so $\{3x_1\}$ is the only dense set.

5° The points $3x_1$ and $3x_5$ are interior points of $\boldsymbol{T}$ since

$$3x_1, 3x_5 \in \{3x_1, 3x_5\} \subset \boldsymbol{T} = \{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5\},$$

where $\{3x_1, 3x_5\}$ is an open set, i.e., since each belongs to an open set contained in $\boldsymbol{T}$. Note that $2x_1 - 3x_3 - x_4 - x_5$ is not an interior point of $\boldsymbol{T}$ since $2x_1 - 3x_3 - x_4 - x_5$ does not belong to any open set contained in $\boldsymbol{T}$. Hence $Int\{\boldsymbol{T}\} = \{3x_1, 3x_5\}$ is the interior of $\boldsymbol{T}$.

6° The complement of $\boldsymbol{T}$ is

$\boldsymbol{T}^c = \{-13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5,\ 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$.

Neither $-13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5$ nor $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$ are interior points of $\boldsymbol{T}^c$ since neither belongs to any open subset of $\boldsymbol{T}^c = \{-13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5,\ 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$. Hence, $Int\{\boldsymbol{T}^c\} = \varnothing$, i.e., there are no exterior points of $\boldsymbol{T}$.

7° The boundary $Bd\{\boldsymbol{T}\}$ of $\boldsymbol{T}$ consists of those points which are neither interior nor exterior to $\boldsymbol{T}$. So

$Bd\{\boldsymbol{T}\} = \{2x_1 - 3x_3 - x_4 - x_5,\ -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5,\ 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$.

8° A neighborhood of $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$ is any superset of an open set containing $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$. The open sets containing $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$ are $\{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ and $\boldsymbol{X}$. The supersets of $\{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ are $\{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\{3x_1, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$ and $\boldsymbol{X}$; the only superset of $\boldsymbol{X}$ is $\boldsymbol{X}$. Accordingly, the class of neighborhoods of $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$, i.e., neighborhood system of $11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5$ is

$\mathscr{N}_p = \{\{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \{3x_1, 3x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \boldsymbol{X}\}$.

The open sets containing $2x_1 - 3x_3 - x_4 - x_5$ are $\{3x_1, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$, $\{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}$ and $\boldsymbol{X}$. Hence the neighborhood system of $2x_1 - 3x_3 - x_4 - x_5$ is

$\mathscr{N}_{p'} = \{\{3x_1, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\}, \{3x_1, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}, \boldsymbol{X}\}$.

9° $\mathscr{T}_S = \{\boldsymbol{S} \cap \boldsymbol{G} \mid \boldsymbol{G} \in \mathscr{T}\}$, so the members of $\mathscr{T}_S$ are

$\boldsymbol{S} \cap \boldsymbol{X} = \boldsymbol{S}$, $\boldsymbol{S} \cap \{3x_1\} = \{3x_1\}$, $\boldsymbol{S} \cap \{3x_1, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{3x_1, 2x_1 - 3x_3 - x_4 - x_5\}$, $\boldsymbol{S} \cap \{3x_1, 3x_5, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\} = \{3x_1, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}$, $\boldsymbol{S} \cap \varnothing = \varnothing$, $\boldsymbol{S} \cap \{3x_1, 3x_5\} = \{3x_1\}$, $\boldsymbol{S} \cap \{3x_1, 3x_5, 2x_1 - 3x_3 - x_4 - x_5, -13x_1 + 6x_2 + 18x_3 + 5x_4 + 2x_5\} = \{3x_1, 2x_1 - 3x_3 - x_4 - x_5\}$.

In other words

$\mathscr{T}_S = \{\boldsymbol{S}, \varnothing, \{3x_1\}, \{3x_1, 2x_1 - 3x_3 - x_4 - x_5\}, \{3x_1, 11x_1 - 3x_2 - 15x_3 - 4x_4 - x_5\}\}$,

where x_i, $(1 \le i \le 5)$ are arbitrary real numbers.

Observe that $\{3x_1, 2x_1 - 3x_3 - x_4 - x_5\}$ is not open in $\boldsymbol{X}$, but is relatively open in $\boldsymbol{S}$, i.e., is $\mathrm{T}_{\boldsymbol{S}}$ – open.

Now, we shall determine a *minimal solution* of the equation

$$x_1\ Fe_2O_3 + x_2\ C \to x_3\ Fe_3O_4 + x_4\ Fe_{0.987}O + x_5\ Fe + x_6\ Fe_3C + x_7\ CO + x_8\ CO_2, \tag{5.3}$$

The scheme for the reaction (5.3) is

	Fe_2O_3	C	Fe_3O_4	$Fe_{0.987}O$	Fe	Fe_3C	CO	CO_2
Fe	2	0	−3	−0.987	−1	−3	0	0
O	3	0	−4	−1	0	0	−1	−2
C	0	1	0	0	0	−1	−1	−1

From the above scheme, follows reaction matrix

$$\boldsymbol{A} = \begin{bmatrix} 2 & 0 & -3 & -0.987 & -1 & -3 & 0 & 0 \\ 3 & 0 & -4 & -1.000 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0.000 & 0 & -1 & -1 & -1 \end{bmatrix},$$

with a $rank\boldsymbol{A} = 3$.

The Moore-Penrose matrix has this form

$$\boldsymbol{A}^+ = \boldsymbol{A}^{\mathrm{T}}(\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}})^{-1}$$

$$= \begin{bmatrix} -0.021161720706833494352 & 0.092028392366251401914 & -0.084892584804813672200 \\ -0.026157471963011808991 & -0.010859213626263351406 & 0.277762514191956370300 \\ -0.056038349990849239674 & -0.106507343867935870450 & 0.121909270394088832600 \\ -0.033791543111127966681 & -0.015110641493880042493 & 0.036676638453756006880 \\ -0.083468167145213741613 & 0.048591537861197996292 & 0.026157471963011808991 \\ -0.224247029472629415850 & 0.156633827209857340280 & -0.199290098302920943320 \\ 0.074749009824209805283 & -0.052211275736619113427 & -0.266903300565693018890 \\ 0.123340547685407801570 & -0.115281765099501578260 & -0.256044086939429667490 \end{bmatrix}$$

By using the vector

$\boldsymbol{a} = (1, 1, 1, 1, 1, 1, 1, 1)^T$,

as an arbitrary chosen vector, $\boldsymbol{A}$ and $\boldsymbol{A}^+$ determined previously, by virtue of (4.3) one obtains the minimal solution of the chemical equation (5.3) given by

$(1/1377.770759) \times (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)^T$,

where

$x_1 = 1952.372829$, $x_2 = 1852.583871$,
$x_3 = 517.7389990$, $x_4 = 1096.003000$,
$x_5 = 1096.083759$, $x_6 = 57.89664700$,
$x_7 = 899.2149570$, $x_8 = 895.4722670$.

Balanced equation (5.3) with minimal coefficients is

1952.372829 Fe_2O_3 + 1852.583871 C $\rightarrow$ 517.738999 Fe_3O_4 + 1096.003 $Fe_{0.987}O$ + 1096.083759 Fe + 57.896647 Fe_3C + 899.214957 CO + 895.472267 CO_2.

A particular case of reaction (5.3) for $x_1 = x_2 = 0$, $x_3 = -c_1$, $x_4 = c_3$, $x_5, = x_6 = 0$, $x_7 = -c_2$ and $x_8 = c_4$ is considered in[31].

Next, we shall look for sets of solutions of the reaction (5.3).

From (5.3) immediately follows this system of linear equations

$$2x_1 = 3x_3 + 0.987x_4 + x_5 + 3x_6,$$
$$3x_1 = 4x_3 + x_4 + x_7 + 2x_8,$$
$$x_2 = x_6 + x_7 + x_8, \tag{5.4}$$

which general solution is

$$x_1 = -0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8, x_2 = x_6 + x_7 + x_8,$$
$$x_3 = -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, \tag{5.5}$$

where x_4, x_5, x_6, x_7 and x_8 are arbitrary real numbers.

Balanced chemical reaction (5.3) has this form

$(-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8)$ Fe_2O_3 + $(x_6 + x_7 + x_8)$ C $\rightarrow$ $(-0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8)$ Fe_3O_4 + x_4 $Fe_{0.987}O$ + x_5 Fe + x_6 Fe_3C + x_7 CO + x_8 CO_2, (5.6)

where x_4, x_5, x_6, x_7 and x_8 are arbitrary real numbers.

The reaction (5.6) holds if and only if

$$x_7 + 2x_8 > 0.4805x_4 + 1.5x_5 + 4.5x_6, \tag{5.7}$$

i.e.,

$$0 < x_7 < 0.4805x_4 + 1.5x_5 + 4.5x_6, \tag{5.8}$$

where x_4, x_5, x_6 and x_7 are arbitrary real numbers.

For instance, if $x_4 = x_5 = x_6 = 1$, then from (5.8) one obtains $x_7 < 6.4805$. Let it be $x_7 = 6$. Then from (5.7) follows $x_8 > 0.24025$, i.e., $x_8 = 0.3$. Now, the reaction (5.6) will have this particular form

2.852 Fe_2O_3 + 7.3 C $\rightarrow$ 0.239 Fe_3O_4 + $Fe_{0.987}O$ + Fe + Fe_3C + 6 CO + 0.3 CO_2. (5.9)

For (5.6) we shall consider the topology of closed sets

$$\mathscr{T}_{11} = \{X, \varnothing, \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8\}, \{x_6 + x_7 + x_8\}, \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8, x_6 + x_7 + x_8\}, \{x_6 + x_7 + x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4\}, \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8, x_6 + x_7 + x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4\}\}, \tag{5.10}$$

on

$$X = \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8, x_6 + x_7 + x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4, x_8\}, \tag{5.11}$$

where x_i, $(4 \le i \le 8)$ are arbitrary real numbers.

The above topology (5.10) on the set (5.11) is a collection or class of subsets that obey the axioms of the Definition 2.30.

The complements of the closed sets are defined as open sets. The open sets of the topology are the collection of subsets given by

$$\mathscr{T}_{12} = \{\varnothing, X, \{x_6 + x_7 + x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4, x_8\}, \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4, x_8\}, \{-0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4, x_8\}, \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8, x_8\}, \{x_8\}\}.$$

We would like to emphasize that the same set of all combinations of subsets can support several topologies. For instance, the subsets of the topology

$$\mathscr{T}_{21} = \{X, \varnothing, \{x_6 + x_7 + x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4, x_8\}, \{-0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4, x_8\}, \{x_4, x_8\}\}.$$

are closed. Hence $\mathscr{T}_{21}$ is a different topology made on the same set of points, X. The open sets of this topology are

$$\mathscr{T}_{22} = \{\varnothing, X, \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8\}, \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8, x_6 + x_7 + x_8\}, \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8, x_6 + x_7 + x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8\}\}.$$

Actually, there are lots of different ways to define topologies. A subset can be open, or closed, or both, or neither relative to a particular topology. For instance, with respect to the topology given by the closed sets,

$$\mathscr{T}_{31} = \{X, \varnothing, \{-0.948x_4 - 4x_5 - 12x_6 + 3x_7 + 6x_8\}, \{x_6 + x_7 + x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4, x_8\}\},$$

the set $\{x_6 + x_7 + x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8, x_4, x_8\}$ is both open and closed, and the set $\{x_6 + x_7 + x_8, -0.961x_4 - 3x_5 - 9x_6 + 2x_7 + 4x_8\}$ is neither open nor closed.

The topology of closed sets given by this collection

$\mathscr{T}_{41} = \{\boldsymbol{X}, \varnothing, \{x_6 + x_7 + x_8, -0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8, x_4, x_8\}, \{-0.948\, x_4 - 4\, x_5 - 12\, x_6 + 3\, x_7 + 6\, x_8, x_6 + x_7 + x_8, x_8\}, \{x_6 + x_7 + x_8, x_8\}, \{-0.948\, x_4 - 4\, x_5 - 12\, x_6 + 3\, x_7 + 6\, x_8\}\}$,

has its dual as the topology of open sets

$\mathscr{T}_{42} = \{\varnothing, \boldsymbol{X}, \{-0.948\, x_4 - 4\, x_5 - 12\, x_6 + 3\, x_7 + 6\, x_8\}, \{-0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8, x_4\}, \{-0.948\, x_4 - 4\, x_5 - 12\, x_6 + 3\, x_7 + 6\, x_8, -0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8, x_4\}, \{x_6 + x_7 + x_8, -0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8, x_4, x_8\}\}$.

Let $\boldsymbol{X}$ is given by (5.11) and let

$\boldsymbol{Y} = \{-0.948\, x_4 - 4\, x_5 - 12\, x_6 + 3\, x_7 + 6\, x_8, x_6 + x_7 + x_8, -0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8\}, \{-0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8, x_4\}, \{x_4, x_8\}\}$.

Now, we shall find a topology on $\boldsymbol{X}$ generated by $\boldsymbol{Y}$.

First, we shall compute the class $\boldsymbol{Z}$ of all finite intersections of sets in $\boldsymbol{Y}$:

$\boldsymbol{Z} = \{\boldsymbol{X}, \{-0.948\, x_4 - 4\, x_5 - 12\, x_6 + 3\, x_7 + 6\, x_8, x_6 + x_7 + x_8, -0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8\}, \{-0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8, x_4\}, \{x_4, x_8\}, \{-0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8\}, \{x_4\}, \varnothing\}$.

Taking unions of members of $\boldsymbol{Z}$ gives the class

$\mathscr{T} = \{\boldsymbol{X}, \{-0.948\, x_4 - 4\, x_5 - 12\, x_6 + 3\, x_7 + 6\, x_8, x_6 + x_7 + x_8, -0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8\}, \{-0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8, x_4\}, \{x_4, x_8\}, \{x_8\}, \{x_4\}, \varnothing, \{-0.948\, x_4 - 4\, x_5 - 12\, x_6 + 3\, x_7 + 6\, x_8, x_6 + x_7 + x_8, -0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8, x_4\}, \{-0.961\, x_4 - 3\, x_5 - 9\, x_6 + 2\, x_7 + 4\, x_8, x_4, x_8\}\}$,

which is a topology on $\boldsymbol{X}$ generated by $\boldsymbol{Y}$.

Here made research showed that on the set $\boldsymbol{X}$ of the coefficients x_j, $(4 \le j \le 8)$ of the chemical reaction (5.6) can be generated many topologies. A general topological problem shall be given in the next section.

AN OPEN PROBLEM

According to the obtained results in this research, we shall propose the following problem.

Problem 6.1. *How many topologies can be generated on the set* $\boldsymbol{X} \subset \mathbb{R}$ *of all the coefficients* x_j, $(1 \le j \le n)$ *of the chemical equation* (2.2) *of the reaction* (2.1)?

The above problem is a completely new problem in topology and chemistry too. Sure that this problem is not a daily particular problem, just the opposite, it is a very hard scientific problem, which we shall try to solve it in this section.

Actually, the problem reduces to finding the number of partial orders on a finite set.

Solution. Now, we shall prove the following theorem, which will be necessary for solution of the problem.

Theorem 6.2. *The following relations hold*

1° $\mathscr{P}(n) = Y_n[\mathscr{P}^c(1), \mathscr{P}^c(2), \ldots, \mathscr{P}^c(n)]$,

2° $\mathscr{P}^c(n) = Y_n[f\mathscr{P}(1), f\mathscr{P}(2), \ldots, f\mathscr{P}(n)]$,

3° $\mathscr{Q}(n) = Y_n[\mathscr{Q}^c(1), \mathscr{Q}^c(2), \ldots, \mathscr{Q}^c(n)]$,

4° $\mathscr{Q}^c(n) = Y_n[f\mathscr{Q}(1), f\mathscr{Q}(2), \ldots, f\mathscr{Q}(n)]$,

where $f_r = (-1)^{r-1}(r-1)!$

Proof. We shall only prove 1° because the proof of 3° is similar, and 2° and 4° are the inverses of 1° and 3° respectively.

Easy one can note that each partially ordered set of n elements induces a partition of n, simply by considering the cardinalities of the connected components of the given partially ordered set. There are

$n!/[r_1(1!)^{r_1}\, r_2(2!)^{r_2} \cdots r_n(n!)^{r_n}]$,

distinct ways (up to isomorphism) of distributing n distinct elements into $r(\pi)$ parts (where there are r_i parts of size i). On each of these $r(\pi)$ parts we can set up any partial ordering we wish and the resulting partial ordering on $\boldsymbol{X}$ will all be distinct, since different groups of distinct elements are involved. Thus, the theorem follows immediately.

Now, by an example we shall clarify the meaning of the above Theorem 6.2.

Example 6.3. Let's consider the case for $n = 3$.

1° $\mathscr{P}(3) = Y_3[\mathscr{P}^c(1), \mathscr{P}^c(2), \mathscr{P}^c(3)] = \mathscr{P}^c(3) + 3\mathscr{P}^c(2)\mathscr{P}^c(1) + (\mathscr{P}^c(1))^3 = 12 + 3\cdot 2\cdot 1 + (1)^3 = 19$.

2° $\mathscr{P}^c(3) = Y_3[f\mathscr{P}(1), f\mathscr{P}(2), f\mathscr{P}(3)] = f_1\mathscr{P}(3) + f_2(3\mathscr{P}(2)\mathscr{P}(1)) + f_3(\mathscr{P}(1)) = 1\cdot 19 + (-1)(3\cdot 3\cdot 1) + 2(1)^3 = 19 - 9 + 2 = 12$.

3° $\mathscr{Q}(3) = Y_3[\mathscr{Q}^c(1), \mathscr{Q}^c(2), \mathscr{Q}^c(3)] = \mathscr{Q}^c(3) + 3\mathscr{Q}^c(2)\mathscr{Q}^c(1) + (\mathscr{Q}^c(1))^3 = 19 + 3\cdot 3\cdot 1 + 1 = 29$.

4° $\mathscr{Q}^c(3) = Y_3[f\mathscr{Q}(1), f\mathscr{Q}(2), f\mathscr{Q}(3)] = f_1\mathscr{Q}(3) + f_2(3\mathscr{Q}(2)\mathscr{Q}(1)) + f_3(\mathscr{Q}(1)) = 1\cdot 29 + (-1)(3\cdot 4\cdot 1) + 2(1)^3 = 29 - 12 + 2 = 19$.

On the next four tables are presented the topologies which are calculated on same way as in Example 6.3 for $1 \le n \le 16$.

Remark 6.4. *The considered problem is solved only for some particular cases, but its complete solution really is extremely hard. Unfortunately, to date we did not find its explicit general solution which can be used for all values of n.*

***Table* 1.** Partial orders/$\mathscr{T}_0$-topologies

n	$\mathscr{T}_0(n)$
1	1
2	3
3	19
4	219
5	4 231
6	130 023
7	6 129 859
8	431 723 379
9	44 511 042 511
10	6 611 065 248 783
11	1 396 281 677 105 899
12	414 864 951 055 853 499
13	171 850 728 381 587 059 351
14	98 484 324 257 128 207 032 183
15	77 567 171 020 440 688 353 049 939
16	83 480 529 785 490 157 813 844 256 579

***Table* 2.** Connected partial orders/$\mathscr{T}_0{}^c$-topologies

n	$\mathscr{T}_0{}^c(n)$
1	1
2	2
3	12
4	146
5	3 060
6	101 642
7	5 106 612
8	377 403 266
9	40 299 722 580
10	6 138 497 261 882
11	1 320 327 172 853 172
12	397 571 105 288 091 506
13	166 330 355 795 371 103 700
14	96 036 130 723 851 671 469 482
15	76 070 282 980 382 554 147 600 692
16	82 226 869 197 428 315 925 408 327 266

***Table* 3.** Quasi-orders/$\mathscr{T}$ topologies

n	$\mathscr{T}(n)$
1	1
2	4
3	29
4	355
5	6 942
6	209 527
7	9 535 241
8	642 779 354
9	63 260 289 423
10	8 977 053 873 043
11	1 816 846 038 736 192
12	519 355 571 065 774 021
13	207 881 393 656 668 953 041
14	115 617 051 977 054 267 807 460
15	88 736 269 118 586 244 492 485 121
16	93 411 113 411 710 039 565 210 494 095

***Table* 4.** Connected quasi-orders/$\mathscr{T}^c$ topologies

n	$\mathscr{T}^c(n)$
1	1
2	3
3	19
4	233
5	4 851
6	158 175
7	7 724 333
8	550 898 367
9	56 536 880 923
10	8 267 519 506 789
11	1 709 320 029 453 719
12	496 139 872 875 425 839
13	200 807 248 677 750 187 825
14	112 602 879 608 997 769 049 739
15	86 955 243 134 629 606 109 442 219
16	91 962 123 875 462 441 868 790 125 305

CONCLUSION

Since the traditional approach of balancing chemical equation produces only paradoxes, it was abandoned and substituted with modern consistent methods.

The modern methods for balancing chemical equations work only in well-defined chemical systems. For that purpose we introduced a new formal chemical system, which is a main prerequisite of chemistry to be consistent.

The well-known classical approach of direct reduction of hematite with a carbon here is generalized by the reactions (4.1) and (5.3), which possess atoms with integers and fractional oxidation numbers, respectively. For these reactions are determined their general and minimal solutions. The minimal solutions are determined by the author's method.[5] From the reaction (4.1), its particular cases are analyzed, such that it did not lose its generality.

Also, these reactions are determined and their subgenerators analyzed by use of elementary theory of inequalities.[32]

By these chemical reactions it is showed that topological calculus is very easily applicable in chemistry and metallurgy, which gives a good opportunity for their extension toward a modern way founded by virtue of point-set topology.

Here developed topologies are generated for some subsets of solutions of reactions (4.1) and (5.3).

This article will accelerate research in the theory of chemical equations and will give topology more one application, such that the old stereotypical approach in chemistry and its foundation will be substituted with a new sophisticated topological calculus.

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