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Nonlinear Analyses by Using the ORCM*

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Abstract
In this paper, nonlinear boundary value problems are analyzed by using the over-range collocation method (ORCM). By introducing some collocation points, which are located at outside of domain of the analyzed body, unsatisfactory issue of the positivity conditions of boundary points in collocation methods can be avoided. Quite accurate numerical results of the nonlinear partial differential equations have been obtained. Because the ORCM does not demand any specific type of partial differential equations, it shows promise of wide engineering applications of the ORCM.

Key words: Nonlinear Problem, Meshless Method, Collocation Method, Positivity Conditions, Over-Range Points

1. Introduction

A lot of meshless methods have been proposed. The early representatives of meshless methods are the diffuse element method [(1)], the element free Galerkin method [(2)], the reproducing kernel particle method [(3)], the finite point method [(4)], the hp-clouds method [(5)], the partition of unity method [(6)], the meshless local Petrov-Galerkin (MLPG) approach [(7)], and the local boundary integral equation method [(8)]. Some meshless methods are based on weak form, in which background meshes are used in implementation to obtain the numerical integration. Some meshless methods are truly meshless methods. In most meshless techniques, however, complicated non-polynomial interpolation functions are used which render the integration of the weak form rather difficult. Failure to perform the integration accurately results in loss of accuracy and possibly stability of solution scheme. The integration of complicated non-polynomial interpolation function also costs much CPU time.

The collocation method is a truly meshless method, and has no issues of the integration scheme, the integration accuracy and the integration CPU time. Several collocation methods based on different types of approximations or interpolations have been proposed in the literature. Onate et al. [(4)] have proposed a finite point method based on weighted least squares interpolations for the analyses of convective transport and fluid flow problems. Aluru [(9)] has presented a point collocation method based on reproducing kernel approximations for numerical solution partial differential equations with appropriate boundary conditions. Jin, Li and Aluru [(10)] have shown the robustness of collocation meshless methods can be improved by ensuring that the positivity conditions are satisfied when constructing approximation functions and their derivatives. Atluri, Liu and Han [(11)] have presented a MLPG mixed collocation method by using the Dirac delta function as the

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test function in the MLPG method, and shown that the MPLG mixed collocation method is more efficient than the other MLPG implementations, including the MLPG finite volume method. Li and Atluri [12] have demonstrated the suitability and versatility of the MLPG mixed collocation method by solving the problem of topology-optimization of elastic structures.

But, the roughness of the collocation methods is an issue especially when scattered and random points are used. To improve the robustness of the collocation methods, Nayroles, Touzot and Villon [1] suggested that the positivity conditions could be important when using the collocation methods. Jin, Li and Aluru [10] have proposed techniques, based on modification of weighting functions, to ensure satisfaction of positivity conditions when using a scattered set of points. For boundary points, however, the positivity conditions cannot be satisfied, obviously. In this paper, nonlinear boundary value problems are analyzed by using the over-range collocation method (ORCM) [13], in which by introducing some collocation points that are located at outside of domain of the analyzed body, unsatisfactory issue of the positivity conditions of boundary points in collocation methods can be avoided.

2. Principle

2.1 Collocation Scheme

Let us assume a scalar problem governed by a partial differential equation:

$$D(u) = b\,, \quad in \;\; \Omega \tag{1}$$

with boundary conditions

$$T(u) = t\,, \quad on \;\; \Gamma_t \tag{2}$$

$$u - u_c = 0\,, \quad on \;\; \Gamma_u \tag{3}$$

to be satisfied in a domain Ω with boundary $\Gamma = \Gamma_t \cup \Gamma_u$, where D and T are appropriate differential operators, u is the problem unknown function, b and t are external forces or sources acting over Ω and along Γ_t, respectively. u_c is the assigned value of u over Γ_u.

Consider taking some collocation points in Ω, at which Eq. (1) is satisfied, and some collocation points on Γ_t, at which both Eq. (1) and Eq. (2) are satisfied, as well as some collocation points on Γ_u, at which both Eq. (1) and Eq. (3) are satisfied. Besides the collocation points over Ω, let us assume other collocation points located at outside of Ω and call them over-range points, at which no satisfaction of any governing partial differential equation or boundary condition is needed. Therefore, no over-constrained condition is imposed into the boundary value problem. While the over-range points can be used in interpolating calculation of boundary points, so that the unsatisfactory issue of the positivity conditions of boundary points in collocation methods can be avoided.

Let us assume that the number of points in domain is K_d, the number of boundary points is K_b and the number of over-range points is K_o, then the number of unknown variables is $2(K_d + K_b + K_o)$ for a 2-D problem. Because the number of equations of the ORCM is $2(K_d + K_b) + 2K_b$, by taking the same number of the equations with that of the unknown variables, we obtain that the number of the over-range points K_o must be equal to the number of boundary points K_b.

2.2 The MLS Approximation with Kronecker-Delta Property

In the classical moving least-square (MLS) approximation, the shape functions have no Kronecker-delta property, so that the essential node condition cannot be imposed on boundaries. In this paper, a modified MLS approximation is used, its shape functions have

Kronecker-delta property. Therefore, the unsatisfactory issue of the essential node condition can be avoided in the modified MLS approximation.

Consider a small domain Ω_x, the neighborhood of a point x_1, which is located in Ω or on Γ. Over a number of randomly located nodes $\{x_i\}$, $i = 1, 2, \cdots, n$, the MLS approximation u^h of u can be defined by

$$u^h = \mathbf{p}^T(\mathbf{x})\,\boldsymbol{\alpha}, \quad \forall \mathbf{x} \in \Omega_x \tag{4}$$

where $\mathbf{p}^T(\mathbf{x}) = [p_1(\mathbf{x}) \quad p_2(\mathbf{x}) \quad \cdots \quad p_m(\mathbf{x})]$ is a complete monomial basis of order m which is a function of the space coordinates $\mathbf{x} = [x \quad y \quad z]^T$. $\boldsymbol{\alpha}$ is a vector of unknown polynomial coefficients.

$$\boldsymbol{\alpha} = [\alpha_1 \quad \alpha_2 \quad \cdots \quad \alpha_m]^T \tag{5}$$

For example, for a 2-D problem,

$$\mathbf{p}^T(\mathbf{x}) = [1 \quad x \quad y \quad x^2 \quad xy \quad y^2] \tag{6}$$

this is a quadratic basis, and m=6.

A weighted least-square solution is obtained for $\boldsymbol{\alpha}$ from the following system of n equations in m unknown (n is larger than m):

$$\mathbf{u}^h = \mathbf{H}\,\boldsymbol{\alpha} \tag{7}$$

where

$$\mathbf{u}^h = [u_1^h \quad u_2^h \quad \cdots \quad u_n^h]^T \tag{8}$$

is a vector of the nodal MLS approximation of function u, and

$$\mathbf{H} = \begin{bmatrix} \mathbf{p}^T(\mathbf{x}_1) \\ \mathbf{p}^T(\mathbf{x}_2) \\ \vdots \\ \mathbf{p}^T(\mathbf{x}_n) \end{bmatrix}_{n\times m} \tag{9}$$

The classical least-square solution of the above over-constrained system does not guarantee exact satisfaction of any of the equations of Eq. (7). Non-satisfaction of the first equation would then mean $u_1^h \neq p^T(x_1)\alpha$. Hence, a different approach to weighted least-squares solution can be adopted: Out of the n equations of Eq. (7), let the first equation (corresponding to node 1) be satisfied exactly and the rest in the least-square sense. This is done by using the first equation to eliminate α_1 from the rest of equations:

$$\alpha_1 = u_1^n - (\alpha_2 x_1 + \alpha_3 y_1 + \alpha_3 x_1^2 + \alpha_5 x_1 y_1 + \alpha_6 y_1^2) \tag{10}$$

Substituting for α_1 in Eq. (7), the reduced system of equations can be obtained:

$$\bar{\mathbf{u}}^h = \bar{\mathbf{H}}\bar{\boldsymbol{\alpha}} \tag{11}$$

where

$$\bar{\mathbf{u}}^h = [u_2^h - u_1^h \quad u_3^h - u_1^h \quad \cdots \quad u_n^h - u_1^h]^T \tag{12}$$

$$\bar{\mathbf{H}} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & x_2^2 - x_1^2 & x_2y_2 - x_1y_1 & y_2^2 - y_1^2 \\ x_3 - x_1 & y_3 - y_1 & x_3^2 - x_1^2 & x_3y_3 - x_1y_1 & y_3^2 - y_1^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n - x_1 & y_n - y_1 & x_n^2 - x_1^2 & x_ny_n - x_1y_1 & y_n^2 - y_1^2 \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{p}}^T(\mathbf{x}_2) \\ \bar{\mathbf{p}}^T(\mathbf{x}_3) \\ \vdots \\ \bar{\mathbf{p}}^T(\mathbf{x}_n) \end{bmatrix} \tag{13}$$

$$\bar{\boldsymbol{\alpha}} = [\alpha_2 \quad \alpha_3 \quad \cdots \quad \alpha_m]^T \tag{14}$$

The coefficient vector $\bar{\boldsymbol{\alpha}}$ is determined by minimizing a weighted discrete L_2 norm, defined as:

$$J = \sum_{i=2}^{n} w(\mathbf{x}_i)\ [\bar{\mathbf{p}}^T(\mathbf{x}_i)\bar{\alpha} - \bar{u}_i]^2 = [\bar{\mathbf{H}}\bar{\boldsymbol{\alpha}} - \bar{\mathbf{u}}]^T\ \mathbf{W}\ [\bar{\mathbf{H}}\bar{\boldsymbol{\alpha}} - \bar{\mathbf{u}}] \tag{15}$$

where $w(\mathbf{x})$ is the weight function, with $w(\mathbf{x}) > 0$ for all nodes in the support of $w(\mathbf{x})$ (the support is considered to be equal to Ω_x in this paper), $\mathbf{x}_i$ denotes the value of $\mathbf{x}$ at node i, and the matrices $\mathbf{W}$ is defined as

$$\mathbf{W} = \begin{bmatrix} w(\mathbf{x}_2) & 0 & \cdots & 0 \\ 0 & w(\mathbf{x}_3) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & w(\mathbf{x}_n) \end{bmatrix}_{(n-1)\times(n-1)} \tag{16}$$

$$\bar{u}_i = \hat{u}_i - \hat{u}_1,\ i = 2, 3, \cdots, n \tag{17}$$

$$\bar{\mathbf{u}} = [\hat{u}_2 - \hat{u}_1 \quad \hat{u}_3 - \hat{u}_1 \quad \cdots \quad \hat{u}_n - \hat{u}_1]^T \tag{18}$$

where $\hat{u}_i,\ i = 1, 2, \cdots, n,$ are the fictitious nodal values of the function u.

Minimizing J in Eq. (15) with respect to $\bar{\boldsymbol{\alpha}}$ yields

$$\bar{\boldsymbol{\alpha}} = \mathbf{A}^{-1}\mathbf{B}\bar{\mathbf{u}} \tag{19}$$

$$\mathbf{B} = \bar{\mathbf{H}}^T\mathbf{W} \tag{20}$$

$$\mathbf{A} = \mathbf{B}\bar{\mathbf{H}} \tag{21}$$

Substituting Eq. (19) into Eq. (11) gives a relation which may be written as the form of an interpolation function, as

$$\bar{\mathbf{u}}^h = \bar{\mathbf{H}}\mathbf{A}^{-1}\mathbf{B} \tag{22}$$

Equation (10) can be rewritten as:

$$\alpha_1 = u_1^h - \mathbf{s}(\mathbf{x}_1)\ \bar{\alpha} \tag{23}$$

$$\mathbf{s}(\mathbf{x}_1) = [x_1 \quad y_1 \quad x_1^2 \quad x_1y_1 \quad y_1^2] \tag{24}$$

Equation (4) can be written as:

$$u^h = \alpha_1 + \mathbf{s}(\mathbf{x})\ \bar{\alpha} \tag{25}$$

$$\mathbf{s}(\mathbf{x}) = [x \quad y \quad x^2 \quad xy \quad y^2] \tag{26}$$

Substituting Eq. (19) and Eq. (23) into Eq. (25), the following equation can be obtained:

$$u^h = u_1^h + \mathbf{q}(\mathbf{x})\mathbf{A}^{-1}\mathbf{B}\bar{\mathbf{u}} \tag{27}$$

$$\mathbf{q}(\mathbf{x}) = \mathbf{s}(\mathbf{x}) - \mathbf{s}(\mathbf{x}_1) \tag{28}$$

Because

$$\mathbf{q}(\mathbf{x}_1) = 0 \tag{29}$$

$$u^h(\mathbf{x}_1) = u_1^h \tag{30}$$

$\hat{\mathbf{u}}$ may be defined as

$$\hat{\mathbf{u}} = [\hat{u}_1 \quad \hat{u}_2 \quad \cdots \quad \hat{u}_n]^T \tag{31}$$

then, from Eq. (27), the following equation may be obtained:

$$u^h = \mathbf{N}(\mathbf{x})\hat{\mathbf{u}} \tag{32}$$

$$\underset{1\times n}{\mathbf{N}(\mathbf{x})} = \left[1 - \left(\underset{1\times(m-1)}{\mathbf{q}(\mathbf{x})} \quad \underset{(m-1)\times(m-1)}{\mathbf{A}^{-1}} \quad \underset{(m-1)\times(n-1)}{\mathbf{B}} \quad \underset{(n-1)\times 1}{\mathbf{1}}\right) \quad \vdots \quad \underset{1\times(m-1)}{\mathbf{q}(\mathbf{x})} \quad \underset{(m-1)\times(m-1)}{\mathbf{A}^{-1}} \quad \underset{(m-1)\times(n-1)}{\mathbf{B}}\right] \tag{33}$$

In Eq. (33), $\mathbf{1}$ is vector of dimension (n-1) with all entries being equal to unity.

Recall form Eq. (29), using this result in Eq. (33), the Kronecker-delta property of $\mathbf{N}(\mathbf{x})$ may be established:

$$\mathbf{N}(\mathbf{x}_1) = [1 \quad 0 \quad 0 \quad \cdots \quad 0] \tag{34}$$

It means that at node 1, the shape function for node 1 takes a value of unity and all other shape function take zero values. Therefore, Eq. (33) is the shape functions of the MLS approximation with Kronecker-delta property.

From Eq. (32) and Eq. (30), the following result can be obtained:

$$\hat{u}_1 = u^h(\mathbf{x}_1) = u_1^h \tag{35}$$

In this paper, the weight functions $w(\mathbf{x})$ may use a spline function as follows:

$$w(\mathbf{x}) = 1 - 6\left(\frac{d}{r}\right)^2 + 8\left(\frac{d}{r}\right)^3 - 3\left(\frac{d}{r}\right)^4, \quad 0 \le d \le r \tag{36a}$$

$$w(\mathbf{x}) = 0, \quad d \ge r \tag{36b}$$

where $d = |\mathbf{x} - \mathbf{x}_1|$ is the distance from point x to the center node x_1, and r is the radius of Ω_x, which is taken as a circle for a 2-D problem and its center is the point x_1.

2.3 The Local Coordinate System

As anisotropy of the point distribution in Ω_x, matrix $\boldsymbol{A}$ in Eq. (21) becomes ill-conditioned and the quality of the approximation deteriorates. In order to prevent such undesirable effect, a local coordinate system ξ, η is chosen with origin at the node $\mathbf{x}_1$ for a 2-D problem,

$$\xi = \frac{x - x_1}{R_x} \tag{37a}$$

$$\eta = \frac{y - y_1}{R_y} \tag{37b}$$

where R_x and R_y denote maximum distances along x and y measured from the point $\mathbf{x}_1$ to exterior nodes in Ω_x. In Eq. (36a), the spline function has now the following form in terms of the local coordinates:

$$w(\boldsymbol{\xi}) = 1 - 6\left(\frac{\xi^2+\eta^2}{\rho^2}\right) + 8\left(\frac{\xi^2+\eta^2}{\rho^2}\right)^{\frac{3}{2}} - 3\left(\frac{\xi^2+\eta^2}{\rho^2}\right)^2 \tag{38}$$

where $-1 \le \xi \le 1, -1 \le \eta \le 1$ as usual, ρ is a constant that may control weight values of different nodes in Ω_x. $\rho = 6$ is used in this paper, so it means that the weight value of the furthest node from the point $\mathbf{x}_1$ is approximately 76.2% of the weight value of the point $\mathbf{x}_1$.

The matrix $\boldsymbol{A}$ is not dependent on the dimensions of Ω_x any longer. The approximate function is also expressed in terms of the local coordinate as

$$u^h(\boldsymbol{\xi}) = \mathbf{N}(\boldsymbol{\xi})\hat{\mathbf{u}} \tag{39}$$

$\mathbf{A}^{-1}\mathbf{B}$ in Eq. (33) can be defined as $\boldsymbol{C}$:

$$\mathbf{C} = \mathbf{A}^{-1}\mathbf{B} \tag{40}$$

Then, from Eq. (33), entries of $\mathbf{N}(\mathbf{x})$ for the quadratic basis (m=6) can be written as:

$$N_1(\mathbf{x}) = 1 - [(x-x_1)\textstyle\sum_{i=1}^{n-1} C_{1i} + (y-y_1)\sum_{i=1}^{n-1} C_{2i} + (x^2-x_1^2)\sum_{i=1}^{n-1} C_{3i} \\ +(xy-x_1y_1)\textstyle\sum_{i=1}^{n-1} C_{4i} + (y^2-y_1^2)\sum_{i=1}^{n-1} C_{5i}] \tag{41}$$

$$N_{i+1}(\mathbf{x}) = (x-x_1)C_{1i} + (y-y_1)C_{2i} + (x^2-x_1^2)C_{3i} + (xy-x_1y_1)C_{4i} \\ +(y^2-y_1^2)C_{5i} \qquad (i = 1,2,\cdots,n-1) \tag{42}$$

where C_{ji}, $(j = 1,2,\cdots,5\,;\ i = 1,2,\cdots,n-1)$ are entries of $\boldsymbol{C}$.

At the point x_1, because $\xi_1 = 0$, $\eta_1 = 0$, then the first-order derivatives of the shape function with the local coordinates can be obtained from Eqs. (41) and (42):

$$\frac{\partial \mathbf{N}(\xi_1)}{\partial \xi} = [-\textstyle\sum_{i=1}^{n-1} C_{1i} \quad C_{11} \quad C_{12} \quad \cdots \quad C_{1(n-1)}] \tag{43}$$

$$\frac{\partial \mathbf{N}(\xi_1)}{\partial \eta} = [-\textstyle\sum_{i=1}^{n-1} C_{2i} \quad C_{21} \quad C_{22} \quad \cdots \quad C_{2(n-1)}] \tag{44}$$

From Eqs. (43) and (44), we may see that formulas of the shape function derivatives with the local coordinates are very simple, and in fact, it is a merit of the ORCM using the local coordinates.

2.4 The Positivity Conditions

The positivity conditions [10] on the approximation function $N_i(\mathbf{x})$ of Eq. (33) and its second-order derivatives are stated as,

$$N_i(\mathbf{x}_j) \ge 0 \tag{45}$$

$$\nabla^2 N_i(\mathbf{x}_j) \ge 0,\ j \ne i \tag{46}$$

$$\nabla^2 N_i(\mathbf{x}_i) < 0 \tag{47}$$

where $N_i(\mathbf{x}_j)$ is the approximation function of a point *i* evaluated at a point *j*.

Patankar [14] included the positivity conditions in a series of basic rules for the construction of finite differences and pointed out that the consequence of violating the positivity conditions give a physically unrealistic solution. It has been shown that the satisfaction of the positivity conditions ensures the convergence of the finite difference method with arbitrary irregular meshes for some class of elliptic problems [15]. It has been shown that the significance of the positivity conditions in meshless collocation approaches, and violation of the positivity conditions can significantly result in a large error in the numerical solution [10].

For a point $\mathbf{x}_1$ on Γ, if no over-range point is used in its Ω_x, the positivity conditions on the boundary point cannot be satisfied, obviously. But by introducing some over-range points of Ω in the Ω_x, the unsatisfactory issue of the positivity conditions of the boundary point can be avoided in the ORCM

3. Results of Analyses for the Nonlinear Partial Equation

3.1 Method of Error Estimation

For the purpose of error estimation and convergence studies, the Sobolev norm $\|u\|_0$, of function u and the norm of the first-order derivative vector of u, $\|\mathbf{q}\|_0$ are calculated. These norms are defined as

$$\|u\|_0 = \left(\int_\Omega u^2 d\Omega\right)^{1/2} \tag{48}$$

$$\|\mathbf{q}\|_0 = \left(\int_\Omega \mathbf{q}^T \cdot \mathbf{q} d\Omega\right)^{1/2} \tag{49}$$

$$\mathbf{q} = [\partial u/\partial x \quad \partial u/\partial y]^T = [q_x \quad q_y]^T \tag{50}$$

The relative errors for $\|u\|_0$ and $\|\mathbf{q}\|_0$ are defined as

$$R_0 = \frac{\|u^{num} - u^{exa}\|_0}{\|u^{exa}\|_0} \tag{51}$$

$$R_q = \frac{\|\mathbf{q}^{num} - \mathbf{q}^{exa}\|_0}{\|\mathbf{q}^{exa}\|_0} \tag{52}$$

3.2 The Nonlinear Partial Equation

Some linear boundary value problems have been analyzed by using the ORCM, and it has been shown that the ORCM works quite well for those linear boundary value problems [13].

In this paper, a nonlinear partial equation

$$\nabla^2 \mathrm{u} + \varepsilon u^3 = \varepsilon(x^4 + y^4 - x^3 y - x^2 y^2 + x y^3)^3 + 10(x^2 + y^2) \tag{53}$$

is analyzed over the 1×1 domain (see Fig. 1) by using the ORCM, and its numerical solutions are compared with the exact solutions:

$$\mathrm{u} = x^4 + y^4 - x^3 y - x^2 y^2 + x y^3 \tag{54}$$

ε is a constant and is taken as 0.1 and 1.0 in this paper.

A Dirichlet problem, for which the essential boundary condition is imposed on all sides, and two mixed problem, the first mixed problem (the essential boundary condition is imposed on left and right sides and the flux boundary condition is prescribed on top and bottom sides of the domain) and the second mixed problem (the essential boundary condition is imposed on top and bottom sides and the flux boundary condition is prescribed on left and right sides of the domain), are solved. Regular nodal models (including the over-range nodes) of 89 (9×9+8)(K_d=5×5, K_b=K_o=32) nodes, 129 (11×11+8)(K_d=7×7, K_b=K_o=40) nodes and 177 (13×13+8)(K_d=9×9, K_b=K_o=48) nodes are used to study the convergence with nodal model refinement of the ORCM. Over-range points of one layer are used, and the over-range points are regularly located at outside of the four sides of the domain.

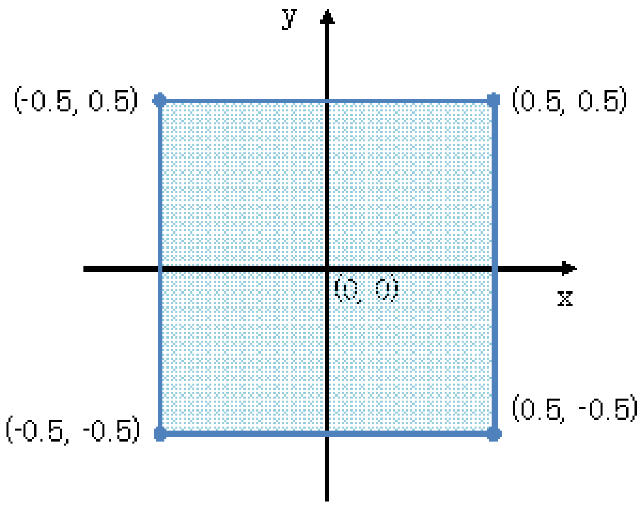

Fig. 1 Analyzed domain

For these regular nodal models, the positivity conditions are calculated. In this paper, n=3 × 3=9 is used for all node models. Because the local coordinate system is used, the values of $\nabla^2 N_1(\xi_j)$, $(j = 1, 2, \cdots, 9)$ of the three regular nodal models are the same, respectively (where subscript symbol 1 means the center node $\mathbf{x}_1$ in every small domain Ω_x of the regular nodes). The value of shape function $N_1(\xi_j)$, $(j = 1, 2, \cdots, 9)$ and the values of $\nabla^2 N_1(\xi_j)$, $(j = 1, 2, \cdots, 9)$ are shown in Table 1. From Table 1, it is seen that the positivity conditions of these nodes are satisfied.

Table 1 The values of $N_1(\xi_j)$ and $\nabla^2 N_1(\xi_j)$

j	1	2	3	4	5	6	7	8	9
$N_1(\xi_j)$	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\nabla^2 N_1(\xi_j)$	-2.443	0.389	0.222	0.389	0.222	0.389	0.222	0.389	0.222

The results of relative errors and convergences are shown in Fig. 2 and Fig. 3 for the first mixed problem of ε=0.1 and ε=1.0, respectively. These figures show that the ORCM works quite well.

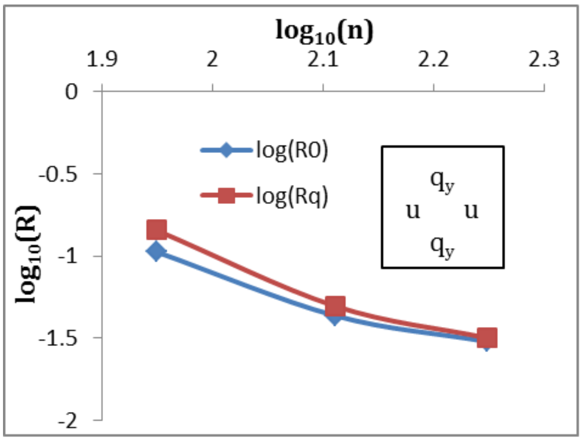

Fig. 2 Relative errors and convergences for the first mixed problem of ε=0.1 (**n** is number of the nodes)

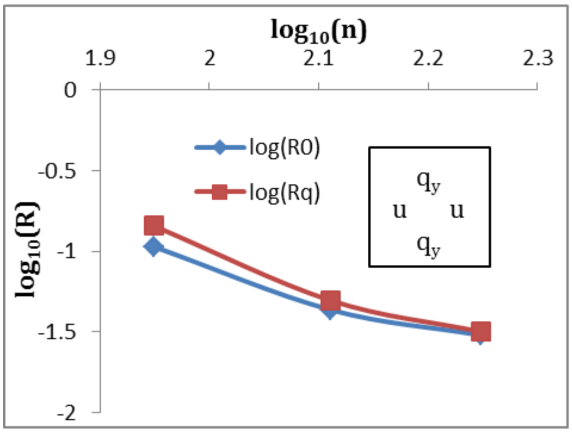

Fig. 3 Relative errors and convergences for the first mixed problem of ε=1.0 (**n** is number of the nodes)

Figures 4 and 5 show values of u at x=0.0 by regular nodal model of 177 nodes, for Dirichlet problem of ε=0.1 and ε=1.0, respectively.

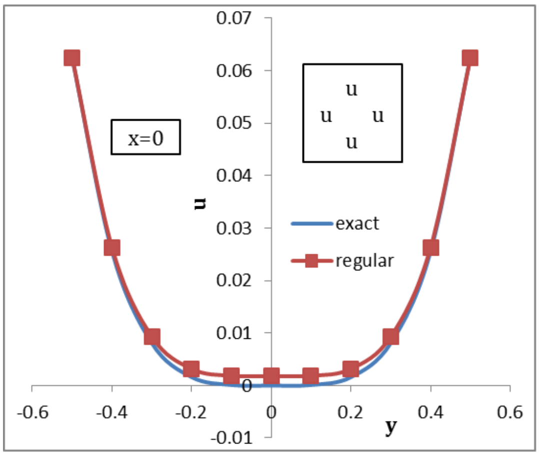

Fig. 4 Values of u at x=0.0 by regular nodal model of 177 nodes, for Dirichlet problem for ε=0.1

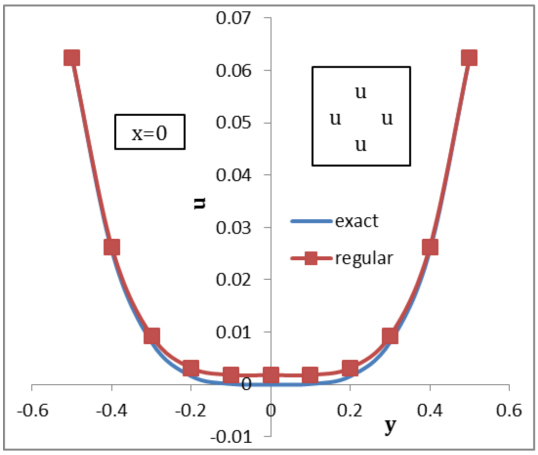

Fig. 5 Values of u at x=0.0 by regular nodal model of 177 nodes, for Dirichlet problem for ε=1.0

Figures 6 and 7 show values of ∂u/∂y at x=0.0 by regular nodal model of 177 nodes, for the first mixed problem and the second mixed problem of ε=0.1, respectively.

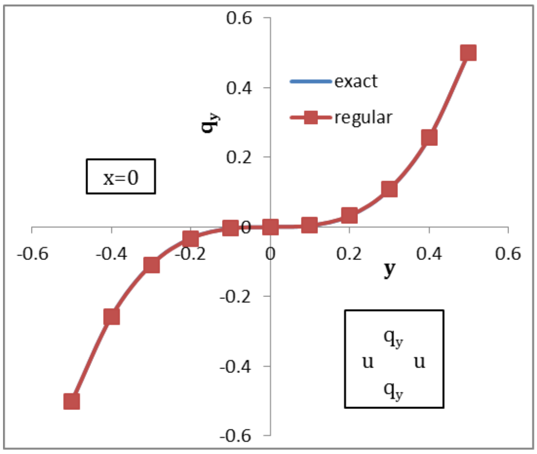

Fig. 6 Values of ∂u/∂y at x=0.0 by regular nodal model of 177 nodes, for the first mixed problem of ε=0.1

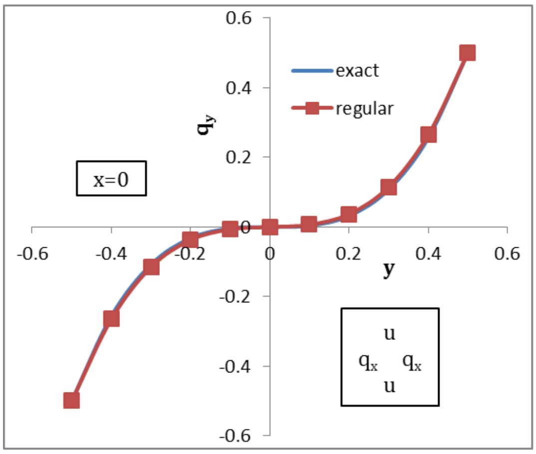

Fig. 7 Values of ∂u/∂y at x=0.0 by regular nodal model of 177 nodes, for the second mixed problem of ε=0.1

One irregular nodal model of 177 (K_d=81, K_b=K_o=48) nodes is used, too. Fig. 8 shows distribution of the nodes in domain and the boundary nodes of the irregular nodal model. Figs. 9 and 10 show values of u by the irregular nodal model for Dirichlet problem and the first mixed problem of ε=0.1, respectively. It can be seen that some accurate results for the unknown variable and its derivatives are obtained by using the irregular nodal model, too.

4. Conclusions

The nonlinear boundary value problems are analyzed by using the ORCM. By introducing some collocation points, which are located at outside of domain of the analyzed body, unsatisfactory issue of the positivity conditions of boundary points in collocation methods can be avoided. Quite accurate numerical results of the nonlinear partial differential equations have been obtained. The convergence studies show that the ORCM possesses good convergence for both the unknown variables and their derivatives. The ORCM does not demand any specific kind of partial differential equations, therefore it shows promise of wide engineering applications of the ORCM.

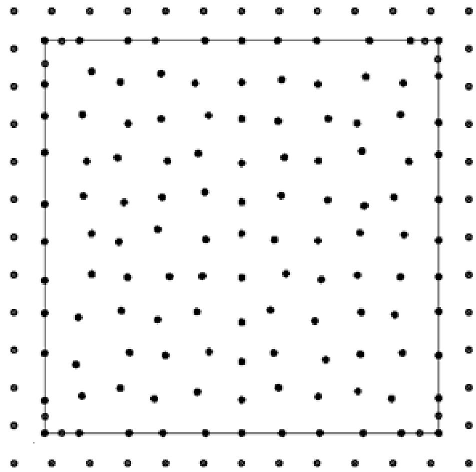

Fig. 8 Irregularly distributed nodes in domain and boundary nodes

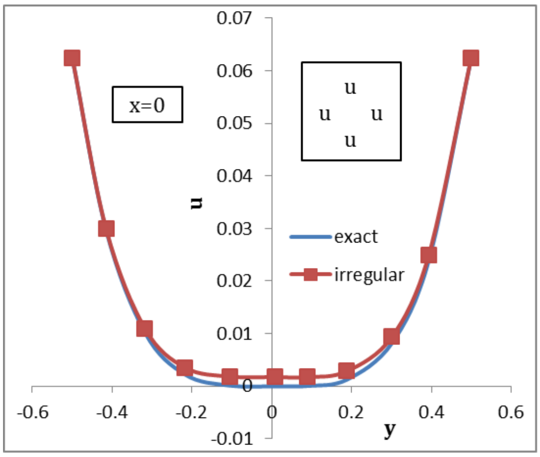

Fig. 9 Values of u at x=0.0 by irregular nodal model of 177 nodes, for Dirichlet problem

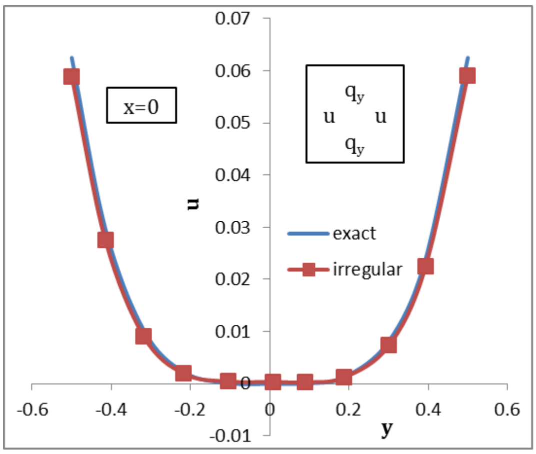

Fig. 10 Values of u at x=0.0 by irregular nodal model of 177 nodes, for the first mixed problem

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