

SHORT COMMUNICATION

## An Orthogonal Frequency Division Multiplexing Radar Waveform with a Large Time-bandwidth Product

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### ABSTRACT

This paper proposes a practical orthogonal frequency division multiplexing (OFDM) chirp diverse waveform with a large time-bandwidth product for new generation radar systems. Besides large time-bandwidth product and implementation simplicity, the OFDM chirp diverse waveform have good ambiguity function performance in range resolution and doppler resolution. Although the spectra are not uniform across the bandwidth like conventional chirp waveforms, the bandwidth is covered with no visible gaps.

**Keywords:** Orthogonal frequency division multiplexing, OFDM, waveform diversity, OFDM radar, chirp signal, time-bandwidth product

### 1. INTRODUCTION

Orthogonal waveforms are widely needed for different radars<sup>1-3</sup>. To obtain a high range resolution, the autocorrelation of the waveforms should be close to an impulse function. On the other hand, since radars are usually placed inside airplanes or satellites, a high average transmit power is required because of the long distance from the radar platform to the targets; hence, the waveforms should have a large time-bandwidth product. Certainly, the waveforms should also have good ambiguity function characteristics such as high range resolution, good doppler resolution, low adjacent-band interferences, and good matched filtering sidelobe performance.

A typical orthogonal waveform is the Barker code; however, Barker code-based waveforms usually have only a single carrier frequency<sup>4</sup> and consequently they cannot provide a high range resolution. Although waveform diversity and design for multiple-input multiple-output (MIMO) radars has received much attention in the recent year<sup>5</sup>, their performance is based primarily on theoretical analysis, neglecting real-life scenarios<sup>6</sup>. The orthogonal frequency division multiplexing (OFDM) waveforms proposed by Levanon<sup>7</sup> and Sen<sup>8</sup> can be used as a radar signal, but they lack a large time-bandwidth product. In this paper, we propose the OFDM chirp diverse waveform which has a large time-bandwidth product and good peak-to-average level performance.

### 2. PROPOSED OFDM CHIRP DIVERSE WAVEFORM

As chirp (called also as linearly frequency modulation (LFM)) waveform has been widely used in different radars due to its good properties such as high range resolution, constant modulus, Doppler tolerance and implementation simplicity<sup>9-11</sup>, from a practical point of view we think that a good waveform

should be designed based on chirp signals. For this reason, we propose the chirp signals-based OFDM chirp diverse waveform.

The OFDM concept was originally developed as a wideband communication modulation technique<sup>12-13</sup>. The primary disadvantage of using OFDM in communications lies at that time and frequency synchronization is crucial to ensure subcarrier orthogonality. At the same time, sensitivity to time and frequency synchronization is beneficial for radar systems because radar receiver usually uses a stored version of the transmitted signal to matched filtering the received signals. Since the subcarriers are orthogonal, OFDM signal spectrum can be considered as a sum of frequency-shifted sinc functions in frequency domain as illustrated in Fig. 1, where the overlapped neighboring sinc functions are spaced by  $\Delta f$ . Note that the frequency (x-axis) is normalized with respect to the carrier frequency (center frequency).

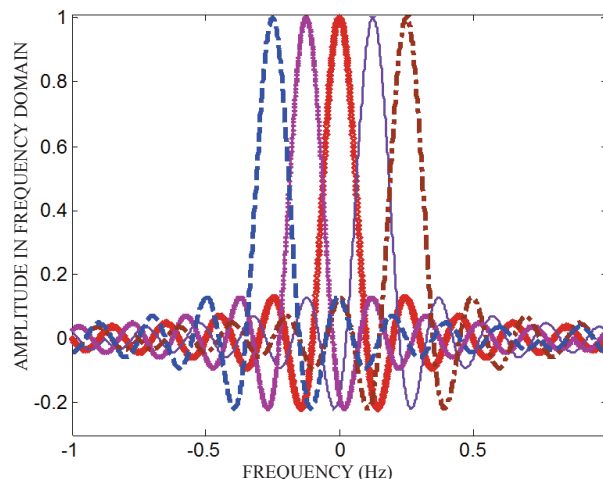


Figure 1. An illustration of an OFDM signal spectrum.

Author described the OFDM chirp diverse waveform from a general chirp signal, which can be represented by the starting frequency  $f_s$ , the chirp rate  $k_r$ , and the chirp duration  $T_p$ . Neglecting amplitude and carrier frequency terms, a chirp signal can be represented by

$$x(t) = \text{rect}\left[\frac{t}{T_p}\right] \cdot \exp\left\{j2\pi\left(f_s t + \frac{1}{2}k_r t^2\right)\right\} \quad (1)$$

As opposed to conventional OFDM waveforms, authors used chirp signals as the OFDM symbols. From a practical point of view author assumed that the chirp signals have an equal bandwidth but different chirp rates. A basic OFDM chirp diverse waveform is a signal that breaks up the chirp pulse width in time with a different frequency across each subpulse. It means that the subpulse's frequency linearly sweeps across the subpulse bandwidth. OFDM chirp diverse waveform can imitate chirp signals by linearly ordering the subpulse frequencies. However, in this paper the subpulse frequencies are randomly ordered to make the time-bandwidth region being uniformly filled.

In a manner like the general OFDM signal<sup>3</sup>, the baseband OFDM chirp waveform with  $P$  subcarriers and  $M$  temporal chips each with duration  $T_c$  can be expressed as

$$s(t) = \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} a_{p,m} u(t - mT_c) \exp[j2\pi f_{p,m}(t - mT_c)] \cdot \exp[j\pi k_{p,m}(t - mT_c)^2] \quad (2)$$

where  $f_{p,m}$  and  $k_{p,m}$  denote respectively the corresponding baseband frequency and chirp rate for the subcarriers, and  $u(t) = 1$ ,  $0 \leq t \leq T_c$ . The OFDM pulse duration  $T_p$  should satisfy  $M \cdot T_c \ll T_p$ . Otherwise, there will be overlapped frequency coverage in the time-frequency distribution of the OFDM chirp diverse waveform and consequently the waveform orthogonality will be degraded. Each temporal chip is modulated by a complex weightier

$$a_{p,m} = A_{p,m} \cdot \exp(j\phi_{p,m}) \quad (3)$$

The complex weightier's amplitude is limited to  $A_{p,m} \in [0, A_{\max}]$  with  $A_{\max}$  being the largest allowable amplitude and the weightier's phase is limited to  $\phi_{p,m} \in [0, 2\pi)$ .

To obtain a good orthogonality between the OFDM chirp diverse waveforms, the subcarrier frequency separation should be optimally chosen. It is obvious that the performance of cross-correlation suppression will improve with the increase of the separation between two starting frequencies. However, for a given bandwidth requirement, this means a wider transmitter receiver bandwidth for the radio frequency (RF) hardware system. We concluded previously that using the subchirp signals with an equal chirp rate and non-overlapped frequency coverage or the subchirp signals with an inverse chirp rate and overlapped frequency coverage offers a satisfactory suppression of the cross-correlation components<sup>14</sup>. As a compromise, the subchirp signals occupying adjacent starting frequency or overlapping frequency coverage with inverse chirp rate are used for the OFDM chirp diverse waveform. Figure 2 gives an example OFDM chirp diverse waveform, where the number of subcarriers and the number of chips are used for illustration purpose only.

The performance of the OFDM chirp diverse waveform

can be evaluated by the radar ambiguity function, which is an effective tool used to predict the matched filtering output performance when there are time-delay mismatch and/or Doppler mismatch. The radar ambiguity function of signal  $s(t)$  is defined as<sup>9</sup>

$$|\chi(\tau, \nu)| = \left| \int s(t) s^*(t - \tau) e^{j2\pi \nu t} dt \right| \quad (4)$$

where  $*$  denotes the conjugate,  $\tau$  is a temporal mismatch between the filter and the signal, and  $\nu$  is the Doppler frequency in radians per second. Substituting Eqn (2) to Eqn (4), we then get the radar ambiguity function of OFDM chirp diverse waveform

$$\begin{aligned} |\chi(\tau, \nu)| &= \left| \int_{-\infty}^{\infty} s(t) s^*(t + \tau) e^{j\nu t} dt \right| \\ &= \left| \int_{-\infty}^{\infty} \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} a_{p,m} a_{p,m}^* u(t - mT_c) u(t + \tau - mT_c) \right. \\ &\quad \times \exp[j2\pi \Delta f_{p,m}(t - mT_c)] \exp[-j2\pi \Delta f_{p,m}(t + \tau - mT_c)] \\ &\quad \cdot \exp[j\pi k_{p,m}(t - mT_c)^2] \exp[-j\pi k_{p,m}(t + \tau - mT_c)^2] e^{j\nu t} dt \left. \right| \quad (5) \end{aligned}$$

Since analytic expression cannot be easily obtained, this equation can be evaluated by numerical computation.

### 3. SIMULATION RESULTS

Suppose the OFDM chirp diverse waveform shown in Fig. 2 has the following parameters: The number of subcarriers is 16, the number of chips is 16, the subcarrier bandwidth is 10 MHz, and the chip duration is  $3\mu s$ , Fig. 3 gives the corresponding ambiguity function. As the value of  $|\chi(0,0)|$  represents the matched filtering output without any mismatch, the sharper the function  $|\chi(\tau, \nu)|$ , the better range resolution and doppler resolution can be obtained for the radars. It can be noticed that the OFDM chirp diverse pulse has a satisfactory

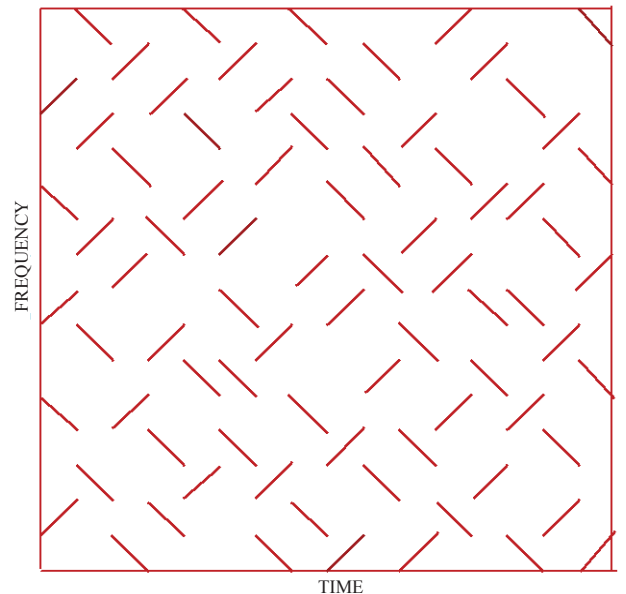


Figure 2. One example pulse of the OFDM chirp diverse waveform.

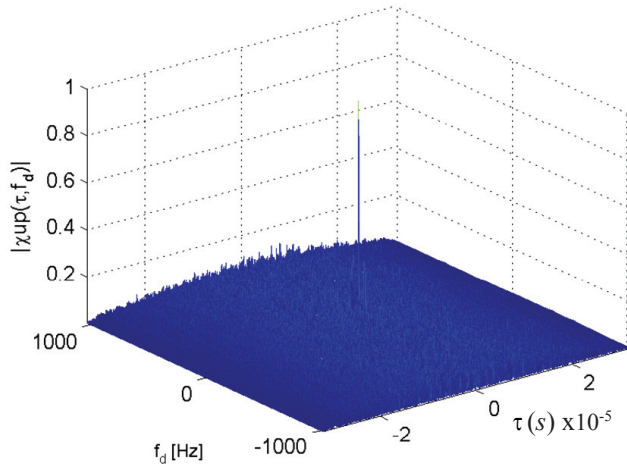


Figure 3. The ambiguity function of the OFDM chirp diverse waveform.

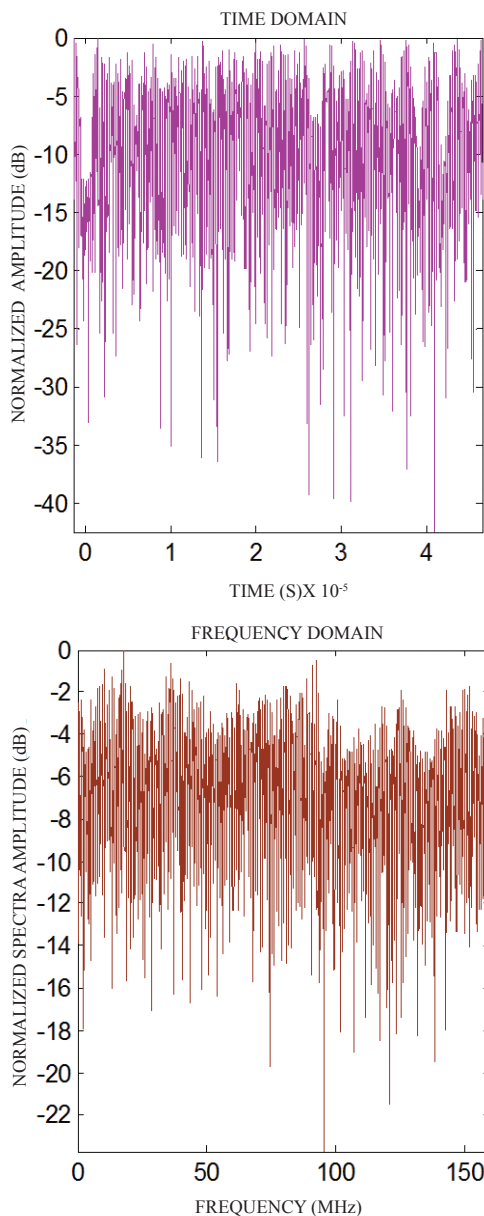


Figure 4. Amplitude and spectra of an example pulse of the OFDM diverse waveform.

ambiguity performance in both range resolution and doppler frequency resolution.

It is also worthwhile to examine the amplitude and spectra of the designed OFDM chirp diverse waveform. It can be noticed from Fig. 4 that it has a good peak-average performance which is desired for RF hardware system and good peak-average level performance. Although the spectra is not uniform across the bandwidth like conventional chirp waveforms, the bandwidth is covered with no visible gaps. Therefore, the designed OFDM chirp diverse waveform has not only a good ambiguity function but also a large time-bandwidth product. This is particularly valuable for long-range high-resolution imaging radars, e.g., airborne and spaceborne radars.

#### 4. CONCLUSION

The author has proposed a practical OFDM chirp diverse waveform. This waveform has a large time-bandwidth product providing high-resolution long-range radars, good ambiguity function performance in range resolution and Doppler resolution. Although the spectra are not uniform across the bandwidth, the bandwidth is covered with no visible gaps. Moreover, this OFDM waveform can provide a large number of orthogonal signals, so as to be used for actual radars. This OFDM waveform can be used in MIMO radars for target detection<sup>15,16</sup> and high-resolution wide-swath remote sensing<sup>17,18</sup>.

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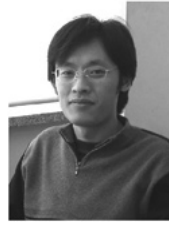
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#### Contributors



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