

A proposal of blind equalizer using kurtosis and skewness

H. Matsumoto^{1a)}, Y. Takeichi², T. Furukawa³, and H. Yashima³

¹ Maebashi institute of tech., 460 kamisatori, maebashi, Gumma, 371–0816 Japan

² Tsuruoka N. College of Tech.

³ Tokyo Univ. of science

a) matsumoto@maebashi-it.ac.jp

Abstract: In a transmission system, the conventional adaptive blind equalizers are implemented a direct estimation for the parameters of the equalizers. But both convergence rate of these algorithms and reliability of the regenerated signals degrade down.

In this paper, we propose a new equalization system and a new adaptive algorithm using the blind channel estimation for improving these two faults above. In the proposed method, first, we provide a psudo-channel and input signals to the psudo-channel in received part, and characteristics of the channel are adaptivly estimated using simplified kurtosis, skewness of output signals of the channel and the psudo-channel. Secondly, we design the equalizer using an adaptive orthogonal projection algorithm for the estimated channel impulse response matrix.

We confirm that convergence rate of the proposed algorithm and reliability of the regenrated signals are better than those of the conventional adaptive blind algorithm by numerical example.

Keywords: blind equalization, orthogonal projection, kurtosis, skewness, UD factorization

Classification: Science and engineering for electronics

References

- [1] Y. Sato, “A Method of Self-Recoverring Equalization for Multilevel Amplitude-Modulation Systems,” *IEEE Trans. Commun.*, vol. 23, no. 6, pp. 679–682, 1975.
- [2] R. A. Wiggins, “Minimum Entropy Deconvolution,” *Geoeexploration*, vol. 16, pp. 21–35, 1978.
- [3] Y. Takeichi, H. Matsumoto, and T. Furukawa, “A Consideration on the Computational Requirements of Blind Equalization Using the Orthogonal Projection,” in *Proc. of ISCAS1998 on IEEE*, 1998.
- [4] H. Matsumoto and T. Furukawa, “A Consideration on The new Blind Deconvolution Algorithm based on Probability Distribution Method,” *IEEJ Trans. EIS*, vol. 125, no. 4, pp. 551–560, 2005.
- [5] Y. Sato, “Linear Equalization Theory,” Maruzen (in Japanese), 1989.
- [6] P. Z. Peebles, Jr, “Probability, Random Variavles, And Random Signal Principles,” McGraw-Hill, 1987.

- [7] H. Ogawa, "Image and Signal Restoration," *J. IEICE*, vol. 71, no. 7, pp. 739–748, 1988.

1 Introduction

In a transmission system, blind equalization is the method to estimate the parameters of the equalizer with only the received signals, and to regenerate the transmitted signals. Even if characteristics of the channel vary, this method need not transmit the training sequence. For example, the blind equalization is useful for mobile communication systems. In these systems, it is required on-line processing with faster convergence rate of the algorithm and higher reliability of the regenerated signals. However, convergence rate of the conventional adaptive blind equalization algorithms in ref. [1, 2] is slower and reliability of the regenerated signals provided with these algorithms is lower, because these are the algorithms using simple received signals.

In order to overcome these drawbacks [3, 4, 5], we propose a new blind equalization system and a new adaptive equalization algorithm, which is the alternately repeating two steps process with block processing. Their steps are as follows:

- 1) We provide a pseudo-channel and input signals to the pseudo-channel in received part. And the estimation of the channel impulse response based on block processing is achieved by adjusting the parameters of the pseudo-channel adaptively so that simplified kurtosis, skewness of output signals of the pseudo-channel correspond with those of the transmission channel.

- 2) We design the equalizer matrix as Moore-Penrose inverse matrix of the estimated channel impulse response matrix using an adaptive orthogonal projection algorithm based on block processing.

The main advantage of the proposed method compared with the representative method in ref. [1, 2] is that the proposed method can achieve both the improvement of the regenerated signals' reliability and speed up of convergence characteristics of the adaptation rule.

2 The proposal of the algorithm

In section 2, we describe the fundamental scheme of the proposal. In Fig. 1, $x(n)$ are the transmitted signals at time n , $H(z)$ is the channel transmission function. We call $F(z)$ the pseudo-channel of $H(z)$, and $a(n)$ are input signals to $F(z)$. Signals $y(n)$, $b(n)$ are output signals of $H(z)$, $F(z)$. $W(z)$ is the equalizer and $z(n)$ are the regenerated signals. The proposed algorithm for designing the equalizer consists of repeating two steps with block processing. For the simplification of the later discussions, we assume that the channel observed noise is free.

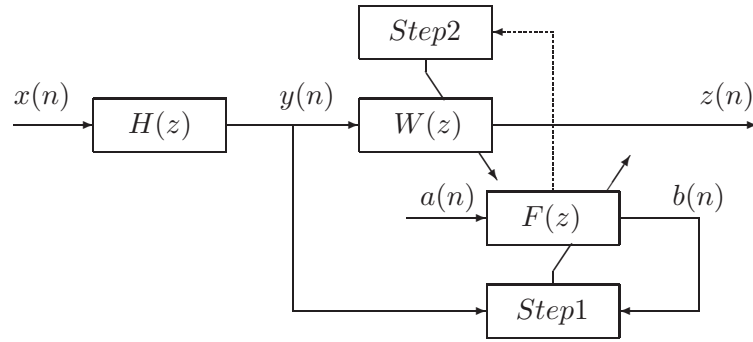


Fig. 1. The proposed model

2.1 The estimation of the channel impulse response

In section 2.1, with Fig. 1, we will explain the procedure to estimate the channel impulse response in Step1. We assume that input signals $x(n)$, $a(n)$ into $H(z)$, $F(z)$ are IID, where $x(n) \neq a(n)$. We define that $x(n)$, $a(n)$ are periodically signals with period N , which is very large value. Therefore, we discuss the estimation of the channel impulse response within the range $n_0 \leq n \leq n_0 + N$ with respect to the time n , where n_0 is arbitrary. Signals vectors $x_N^{(n)}$, $a_N^{(n)}$ are defined as $x_N^{(n)} \triangleq [x(n), x(n-1), \dots, x(n-N+1)]^T$, $a_N^{(n)} \triangleq [a(n), a(n-1), \dots, a(n-N+1)]^T$.

Following the above, the transmission function of the channel and the pseudo-channel are defined as $H(z) \triangleq \sum_{i=0}^{M-1} h_i z^{-i}$ and $F(z) \triangleq \sum_{i=0}^{M-1} f_i z^{-i}$, and we call $(N \times N)$ matrices H_{NN} , F_{NN} the impulse response matrices of $H(z)$, $F(z)$, where impulse response length M is $M \ll N$. Signals vectors $y_N^{(n)}$, $b_N^{(n)}$ of output signals $y(n)$, $b(n)$ of $H(z)$, $F(z)$ are defined as $y_N^{(n)} \triangleq H_{NN} x_N^{(n)}$, $b_N^{(n)} \triangleq F_{NN} a_N^{(n)}$, where $y_N^{(n)} = [y(n), y(n-1), \dots, y(n-N+1)]^T$, $b_N^{(n)} = [b(n), b(n-1), \dots, b(n-N+1)]^T$, and $H_{N,N}$ is nonsingular and defined as follows:

$$H_{N,N} \triangleq \begin{bmatrix} h_0 & h_1 & \cdots & \cdots & h_{M-1} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & h_0 & \cdots & \cdots & \cdots & h_{M-1} & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \cdots & h_{M-1} & 0 & \cdots & \cdots & \cdots & 0 & h_0 & h_1 \\ h_1 & \cdots & \cdots & h_{M-1} & 0 & \cdots & \cdots & \cdots & 0 & h_0 \end{bmatrix},$$

where h_i ($i = 0 \sim (M-1)$) are the parameters of $H(z)$ and $h_0 \neq 0$, $h_{M-1} \neq 0$. The definition of F_{NN} is similar to that of H_{NN} , where f_i ($i = 0 \sim (M-1)$) are the parameters of $F(z)$.

Under the conditions above, it is possible to show that $H_{NN} = F_{NN}$ is obtained when both $\text{Prob}[x_N^{(n)}] = \text{Prob}[a_N^{(n)}]$ and $\text{Prob}[y_N^{(n)}] = \text{Prob}[b_N^{(n)}]$ are satisfied [6], where $\text{Prob}[\cdot]$ denotes the joint probability density function. Therefore, based on ref. [6], we would like to adjust the parameters of $F(z)$, i.e., f_i ($i = 0 \sim (M-1)$) so that $\text{Prob}[b_N^{(n)}] = \text{Prob}[y_N^{(n)}]$ can be satisfied. In order to realize this, we should prepare the cost function including all the

joint momentums of both $y_N^{(n)}$, $b_N^{(n)}$. It is, however, very difficult to calculate their momentums.

Then, we consider to prepare the cost function including kurtosis, skewness instead of all the joint momentums of $y_N^{(n)}$, $b_N^{(n)}$, because it is known that we are able to estimate $H(z)$, $F(z)$ using skewness, kurtosis which include 2nd, 3rd, 4th order the joint momentums of $y_N^{(n)}$, $b_N^{(n)}$ [5]. Here, skewness $Sy(n)_{jk}^{qrs}$ and kurtosis $Ky(n)_{jk}^{qrst}$ are defined respectively as follows:

$$Sy(n)_{jk}^{qrs} \triangleq \frac{E[y(n-q)y(n-r)y(n-s)]}{\{E[y(n-j)y(n-k)]\}^{\frac{3}{2}}}, \quad (1)$$

$$Ky(n)_{jk}^{qrst} \triangleq \frac{E[y(n-q)y(n-r)y(n-s)y(n-t)]}{\{E[y(n-j)y(n-k)]\}^2}, \quad (2)$$

where the parameters (j , k , q , r , s and t) are zero or positive integer $0 \leq j, k, q, r, s, t \leq (N-1)$. And $E[\cdot]$ denotes $\frac{1}{T} \sum_{n=0}^{T-1} [\cdot]$, where T is the observation period and very large value, but $T \leq N$. The definitions of skewness, kurtosis of $b_N^{(n)}$, i.e., $Sb(n)_{jk}^{qrs}$, $Kb(n)_{jk}^{qrst}$ are similar to those of $y_N^{(n)}$.

With eq. (1), (2), we may consider the cost function for the channel estimation based on adaptive block processing. Yet, the usage of the cost function with eq. (1), (2) still accompanies the difficulty due to much computational complexity if we are going to get eq. (1), (2) with respect to all the parameters (j , k , q , r , s and t). To avoid such a difficulty, we will use the simplified cost function with the special forms of $Sy(n)_{jk}^{qrs}$, $Ky(n)_{jk}^{qrst}$. They are given by setting the parameters included in eq. (1), (2), i.e., j , k , q , r , s and t to all zeros, and represented as $Sy(n)$, $Ky(n)$ respectively. Therefore, for examples, $Sy(n)$, $Ky(n)$ are defined as $Sy(n) \triangleq E[y(n)^3]/\{E[y(n)^2]\}^{\frac{3}{2}}$, $Ky(n) \triangleq E[y(n)^4]/\{E[y(n)^2]\}^2$. The definitions of $Sb(n)$, $Kb(n)$ are similar to those of $Sy(n)$, $Ky(n)$. Then, the cost function $E_1[J(lL)]$ for block processing is given as follows [3, 4]:

$$E_1[J(lL)] = \frac{1}{T_1} \sum_{l=0}^{T_1-1} J(lL), \quad (3)$$

$$\text{where } J(lL) = \frac{1}{L} \sum_{i=lL-L+1}^{lL} \frac{1}{2} \{ (Sy(i) - Sb(i))^2 + (Ky(i) - Kb(i))^2 \},$$

l and L are a block number and the data length for block processing, T_1 is the total number of blocks used for processing and very large value, but $T_1 L \leq N$. And, the channel estimation is achieved by minimizing $E_1[J(lL)]$. Defining the estimated impulse response vectors $f_M^{(lL)}$ of $F(z)$ at time lL as $f_M^{(lL)} \triangleq [f_0(lL), f_1(lL), \dots, f_{M-1}(lL)]^T$, the partial differentiation of $J(lL)$ in eq. (3) with respect to f_M yields the adjustment of f_M , i.e., $f_M^{(l+1)L}$ is updated in accordance with

$$\begin{aligned} f_M^{(l+1)L} &= f_M^{(lL)} - \alpha_1 \left. \frac{\partial J(lL)}{\partial f_M} \right|_{f_M=f_M^{(lL)}} \\ &= f_M^{(lL)} - \frac{\alpha_1}{L} \sum_{i=lL-L+1}^{lL} \{ (Sy(i) - Sb(i))(Sy(i) - Sb(i))' \} \end{aligned}$$

$$+(Ky(i) - Kb(i))(Ky(i) - Kb(i))', \quad (4)$$

where α_1 is step gain. Here, we define input signals vectors to the pseudo-channel with data length M as $a_M^{(i)} \triangleq [a(i), a(i-1), \dots, a(i-M+1)]^T$, and $(Sy(i) - Sb(i))'$, $(Ky(i) - Kb(i))'$ in eq. (4) are as follows:

$$\begin{aligned} (Sy(i) - Sb(i))' &= \frac{\partial(Sy(i) - Sb(i))}{\partial f_M} \Big|_{f_M=f_M^{(i)}} \\ &= \frac{-3\{E[a_M^{(i)}b(i)^2]E[b(i)^2] - E[b(i)^3]E[a_M^{(i)}b(i)]\}}{E[b(i)^2]^{\frac{5}{2}}}, \end{aligned} \quad (5)$$

$$\begin{aligned} (Ky(i) - Kb(i))' &= \frac{\partial(Ky(i) - Kb(i))}{\partial f_M} \Big|_{f_M=f_M^{(i)}} \\ &= \frac{-4\{E[a_M^{(i)}b(i)^3]E[b(i)^2] - E[b(i)^4]E[a_M^{(i)}b(i)]\}}{E[b(i)^2]^3}. \end{aligned} \quad (6)$$

On the actual calculation of $E[y(i)^p]$, $E[b(i)^p]$ and $E[a_M^{(i)}b(i)^p]$ in eq. (4), (5) and (6), we use the equations $E[y(i)^p] = (1 - \frac{1}{T})E[y(i-1)^p] + \frac{1}{T}y(i)^p$, $E[b(i)^p] = (1 - \frac{1}{T})E[b(i-1)^p] + \frac{1}{T}b(i)^p$ and $E[a_M^{(i)}b(i)^p] = (1 - \frac{1}{T})E[a_M^{(i-1)}b(i-1)^p] + \frac{1}{T}a_M^{(i)}b(i)^p$ to reduce computational complexity, where $p = 2_{or}3_{or}4$.

2.2 The design of the equalizer

In section 2.2, we will present Step2 in Fig. 1. In Step2, we would like to design the equalizer using block processing with the result of section2.1. We design the equalizer based on the concept of ref. [7]. In this section, we present the theory and discuss the adaptive algorithm for the design of the equalizer.

2.2.1 The orthogonal projection theory for the qualizer

Let $(K \times L)$ matrices $W_{K,L}^{(lL)}$ be the equalizer matrices composed of the estimated parameters of $W(z)$ at time lL , where $K = L + M - 1$. Next, $(N \times N)$ matrices $F_{N,N}^{(lL)}$ are presented as the estimated matrices of $F_{N,N}$ at time lL . And we present $F_{L,K}^{(lL)}$ as $(L \times K)$ submatrices including first L rows and first K columns of $F_{N,N}^{(lL)}$, and $F_{L,K}^{(lL)}$ are defined as follows:

$$F_{L,K}^{(lL)} \triangleq \begin{bmatrix} f_0(lL) & \cdots & f_{M-1}(lL) & & \mathbf{0} \\ & f_{(0)}(lL) & \ddots & f_{M-1}(lL) & \\ & & \ddots & \ddots & \\ \mathbf{0} & & & f_0(lL) & \cdots & f_{M-1}(lL) \end{bmatrix},$$

where $f_i(lL)$ ($i = 0 \sim (M-1)$) are the estimated parameters of $F(z)$ using eq. (4). The transmitted signals vectors with data length K at time lL are defined as $x_K^{(lL)} \triangleq [x(lL), x(lL-1), \dots, x(lL-K+1)]^T$.

Following the above, we will describe that the equalizer matrices $W_{K,L}^{(lL)}$ with block processing are given as Moore-Penrose inverse matrices of $F_{L,K}^{(lL)}$ based on the orthogonal projection theory [7]. Based on ref. [7], since $F_{L,K}^{(lL)}$ are the operators mapping from the subspace spanned with the transmitted

signals vectors $x_K^{(lL)}$ to the subspace spanned with the received signals vectors $y_L^{(lL)}$, $y_L^{(lL)}$ are given by

$$y_L^{(lL)} = F_{L,K}^{(lL)} x_K^{(lL)}, \quad (7)$$

where $y_L^{(lL)} = [y(lL), y(lL-1), \dots, y(lL-L+1)]^T$. Next, when $W_{K,L}^{(lL)}$ are one of the inverse operators of $F_{L,K}^{(lL)}$, the regenerated signals vectors $z_K^{(lL)}$ can be given by

$$z_K^{(lL)} = W_{K,L}^{(lL)} y_L^{(lL)}, \quad (8)$$

where $z_K^{(lL)} = [z(lL), z(lL-1), \dots, z(lL-K+1)]^T$.

On the other hand, the optimum regenerated signals vectors $z_{optK}^{(lL)}$ are given by projecting orthogonally $x_K^{(lL)}$ onto the subspace $Span[F_{L,K}^{(lL)T}]$ spanned with the row vectors of $F_{L,K}^{(lL)}$ [7]. Where $Span[A]$ denotes the subspace spanned with row vectors of matrix A . Therefore, letting $(K \times K)$ matrices $P_{K,K}^{(lL)}$ be the orthogonal projection matrices onto $Span[F_{L,K}^{(lL)T}]$, $z_{optK}^{(lL)}$ are as follows:

$$z_{optK}^{(lL)} = P_{K,K}^{(lL)} x_K^{(lL)}. \quad (9)$$

In order to be satisfied that $z_K^{(lL)} = z_{optK}^{(lL)}$, with eq. (7), (8) and (9), $P_{K,K}^{(lL)}$ are given by

$$P_{K,K}^{(lL)} = W_{K,L}^{(lL)} F_{L,K}^{(lL)}. \quad (10)$$

Note that $P_{K,K}^{(lL)}$ are the orthogonal projection matrices, the following

$$W_{K,L}^{(lL)} = F_{L,K}^{(lL)+}, \quad (11)$$

are obtained, where $F_{L,K}^{(lL)+}$ are the Moore-Penrose inverse matrices of $F_{L,K}^{(lL)}$. If the rows vectors of $F_{L,K}^{(lL)}$ are assumed to be linearly independent, $F_{L,K}^{(lL)+}$ in eq. (11) are given by

$$F_{L,K}^{(lL)+} = F_{L,K}^{(lL)T} [F_{L,K}^{(lL)} F_{L,K}^{(lL)T}]^{-1}. \quad (12)$$

2.2.2 The design of the equalizer using UD Factorization

In section 2.2.2, based on eq. (12), we will consider the design method of $F_{L,K}^{(lL)+}$. On eq. (12), we can not calculate $F_{L,K}^{(lL)+}$ when $[F_{L,K}^{(lL)} F_{L,K}^{(lL)T}]$ are not nonsingular. Therefore, defining $[G_{L,L}^{(lL)}]^{-1} \triangleq [F_{L,K}^{(lL)} F_{L,K}^{(lL)T} + \delta I_{L,L}]$ as the regularized matrices of $[F_{L,K}^{(lL)} F_{L,K}^{(lL)T}]$, we calculate $F_{L,K}^{(lL)+}$ by using $G_{L,L}^{(lL)}$ which are the inverse matrices of $[G_{L,L}^{(lL)}]^{-1}$ instead of $[F_{L,K}^{(lL)} F_{L,K}^{(lL)T}]^{-1}$ in eq. (12), where $I_{L,L}$ denote $(L \times L)$ identity matrix, δ is very small value.

Based on the concept above, we present the design procedure of $F_{L,K}^{(lL)+}$. First, we define $F_{i,K}^{(lL)T}$ ($i = 2 \sim L$) as $(i \times K)$ submatrices including first i rows of $F_{L,K}^{(lL)T}$ for the calculation of $G_{L,L}^{(lL)}$ using UD Factorization. Next, we will divide $F_{i,K}^{(lL)T}$ ($i = 2 \sim L$) as follows :

$$F_{i,K}^{(lL)T} = [F_{i-1,K}^{(lL)T} | f_{K,(i)}^{(lL)}], \quad (13)$$

Table I. The design flow of $F_{L,K}^{(LL)+}$ using UD Factorization

1) $i = 1$
i) $d_{(i)}(lL) = f_{K,(i)}^{(LL)T} f_{K,(i)}^{(LL)}$
ii) $G_{i,i}^{(LL)} = d_{(i)}(lL)^{-1}$
iii) $F_{i,K}^{(LL)} = f_{K,(i)}^{(LL)T}$
2) For $i = 2, L$
i) $c_{i-1}^{(LL)} = F_{i-1,K}^{(LL)} f_{K,(i)}^{(LL)}$
ii) $g_{i-1}^{(LL)} = G_{i-1,i-1}^{(LL)} c_{i-1}^{(LL)}$
iii) $t_{K,(i)}^{(LL)} = F_{i-1,K}^{(LL)T} g_{i-1}^{(LL)}$
iv) $d_{(i)}(lL) = f_{K,(i)}^{(LL)T} [f_{K,(i)}^{(LL)} - t_{K,(i)}^{(LL)}] + \delta$
v) $D_{i-1,i-1}^{(LL)} = G_{i-1,i-1}^{(LL)} + \{d_{(i)}(lL)^{-1}\} g_{i-1}^{(LL)} g_{i-1}^{(LL)T}$
vi) $G_{i,i}^{(LL)} = \left[\begin{array}{c c} D_{i-1,i-1}^{(LL)} & -\{d_{(i)}(lL)^{-1}\} g_{i-1}^{(LL)} \\ \hline -\{d_{(i)}(lL)^{-1}\} g_{i-1}^{(LL)T} & d_{(i)}(lL)^{-1} \end{array} \right]$
vii) $F_{i,K}^{(LL)T} = [F_{i-1,K}^{(LL)T} f_{K,(i)}^{(LL)}]$
Next i
3) $F_{L,K}^{(LL)+} \triangleq F_{L,K}^{(LL)T} G_{L,L}^{(LL)}$

where $i = 2 \sim L$, the vectors with data length K as $f_{K,(i)}^{(LL)}$ denote i -th row vectors of $F_{L,K}^{(LL)T}$.

Under the conditions above, the design procedure of $F_{L,K}^{(LL)+}$ is as follows:

- 1) We calculate $[F_{i,K}^{(LL)} F_{i,K}^{(LL)T} + \delta I_{i,i}]^{-1}$ using UD Factorization of $[F_{i-1,K}^{(LL)} F_{i-1,K}^{(LL)T} + \delta I_{i-1,i-1}]^{-1}$ for $i = 2$ to L , where $I_{i,i}$ denote $(i \times i)$ identity matrices.
- 2) We calculate $F_{L,K}^{(LL)+} \triangleq F_{L,K}^{(LL)T} G_{L,L}^{(LL)}$ using $G_{L,L}^{(LL)} = [F_{L,K}^{(LL)} F_{L,K}^{(LL)T} + \delta I_{L,L}]^{-1}$ provided in 1).

From these discussions, the calculation of $F_{L,K}^{(LL)+}$ is summarized with Table I.

In the last of this section, we present the proposed algorithm based on repeating alternately the procedure of step1 and step2. Defining input signals vectors with data length K to the pseudo-channel as $a_K^{(LL)} \triangleq [a(lL), a(lL - 1), \dots, a(lL - K + 1)]^T$, the proposed algorithm is as follows:

- 1) Initialization
 - i) $l = 0$
 - ii) The set of $Sy((l-1)L)$, $Sb((l-1)L)$, $Ky((l-1)L)$, $Kb((l-1)L)$, $(Sy((l-1)L) - Sb((l-1)L))'$, $(Ky((l-1)L) - Kb((l-1)L))'$ (see eq. (4), (5), (6))
 - iii) $f_M^{(LL)} = [0, \dots, 0, 1, 0, \dots, 0]^T$
where 1 is located with the u -th element,
and u is the integer value included in $((M/2) + 1)$
- 2) Channel estimation
 - i) $b_L^{(LL)} = F_{L,K}^{(LL)} a_K^{(LL)}$
 - ii) The calculations of $Sy(i)$, $Sb(i)$, $Ky(i)$, $Kb(i)$, $(Sy(i) - Sb(i))'$, $(Ky(i) - Kb(i))'$

using the elements of $b_L^{(lL)}, y_L^{(lL)}$
for $i = lL - L + 1$ to lL (see section 2.1)

iii) Update in accordance with the following.

$$f_M^{(l+1)L} = f_M^{(lL)} - \frac{\alpha_1}{L} \sum_{i=lL-L+1}^{lL} \{(Sy(i) - Sb(i))(Sy(i) - Sb(i))' + (Ky(i) - Kb(i))(Ky(i) - Kb(i))'\}.$$

iv) $l = l + 1$

3) Equalization

i) Design $W_{K,L}^{(lL)} = F_{L,K}^{(lL)+}$ using UD Factorization. (see table I)

$$z_L^{(lL)} \leftarrow z_K^{(lL)} = W_{K,L}^{(lL)} y_L^{(lL)}.$$

(where $z_L^{(lL)}$ are given as the vectors including first L elements of $z_K^{(lL)}$)

ii) Go back to 2)

3 Computer Simulation

In this section, we confirm the equalization performance of the proposed algorithm by computer simulation. Simulation Conditions are as follows:

(1) $x(n), a(n)$ are IID under uniform distridution $[-1, 1]$, where $x(n) \neq a(n)$.

(2) A channel $H(z)$ is given by $H(z) = 0.1 - 0.3z^{-1} + z^{-2} + 0.4z^{-3} - 0.1z^{-4}$.

(3) A performance measure called MSE is defined as follows:

$$MSE \triangleq 10 \times \log_{10} \frac{E[\{z(n) - x(n)\}^2]}{E[\{z(0) - x(0)\}^2]}.$$

(4) A step gain α_1 is 0.001, and N, T, K, L and M are 10000, 50, 50, 46 and 5.

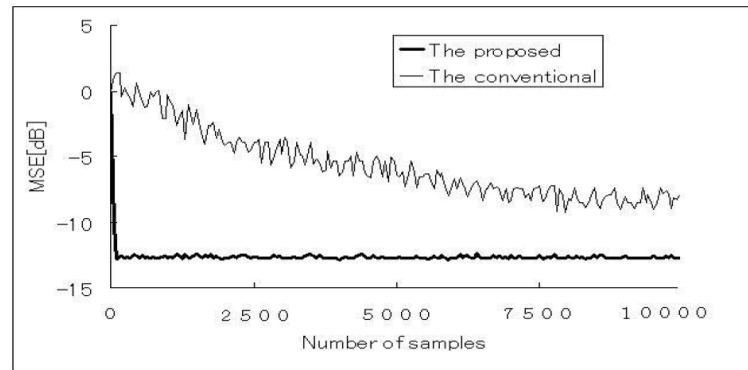


Fig. 2. Equalization performance

Fig. 2 presents the equalization performance of the proposed algorithm and well-known Sato algorithm as the conventional adaptive algorithm. It is confirmed that the convergence rate of the proposed algorithm and the reliability of the regenrated signals are better than those of sato algorithm by Fig. 2. It is clear that both the convergence rate of sato algorithm and the reliability of the regenerated signals degrade down. On the other hand,

it is shown that the proposed algorithm can achieve both improvement of the regenerated signals' reliability and speed up of convergence characteristics of the adaptation rule, because the parameters of the equalizer are calculated based on the orthogonal projection theory from the channel which is estimated using simplified kurtosis, skewness of output signals of the channel and the psude-channel.

4 Conclusions

We have proposed the adaptive blind equalization algorithm based on block processing using simplified skewness and kurtosis. The equalization performance of the proposed algorithm is better than that of Sato algorithm.

In the future, we would like to discuss the proposed algorithm under the condition that SNR of the received signals is lower.