Free space link budget estimation scheme for ultra wideband impulse radio with imperfect antennas

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Abstract: The ultra wideband (UWB) radio has been a promising technology for the short range wireless systems. The frequency-dependent behavior of the antennas are not negligible in UWB systems, especially in the impulse radio (IR).

This letter presents the extension of the Friis' transmission formula for the link budget estimation of UWB radio, considering the transmit signal waveform and the imperfect antennas. The results presented here give the upper bound of the transmission gain, irrespective of the waveform distortion due to the frequency response of the antenna.

Keywords: ultra wideband (UWB), impulse radio, Friis' transmission formula, link budget

Classification: Microwave and millimeter wave devices, circuits, and systems

References

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1 Introduction

The ultra wideband (UWB) radio has been a promising technology for the short range wireless systems, such as the personal area network (PAN). The frequency-dependent behavior of the antennas is not negligible in UWB systems, especially in the impulse radio (IR). The line-of-sight (LOS) link budget is usually evaluated by using Friis' transmission formula [1]. It is not directly applicable to the UWB-IR system, however, as the formula is expressed as a function of the frequency, assuming the narrowband transmission.

With respect to the UWB transmission, some examples are presented in ITU-R WP3K working document submitted from USA [2]. They consider the special cases of constant gain and constant aperture, but they only mention about the path loss exponent and the waveform distortion, and not about the absolute value of free space link budget.

This letter presents the comprehensive extension of the Friis' transmission formula for the link budget estimation of UWB radio, by considering the transmit waveform, and the frequency characteristics of the antennas [3]. The extension here gives the upper bound of the transmission gain, irrespective of the waveform distortion due to the frequency response of the antenna. The effect of the distortion to more realistic receiver shall be treated in a separated paper.

2 Extension of Friis' Transmission Formula for UWB

The conventional Friis' transmission formula for the narrowband system is expressed as

$$G_{\text{Friis}}(f) = \frac{P_r(f)}{P_t(f)} = G_f(f)G_r(f)G_t(f), \tag{1}$$

where $G_r(f)$ and $G_t(f)$ are Rx and Tx antenna gain,

$$G_f(f) = \left(\frac{\lambda}{4\pi d}\right)^2 \tag{2}$$

is the free space propagation gain (less than unity in practice), $\lambda = \dfrac{c}{f}$ is the wavelength, c is the velocity of the light, f is the operating frequency, and d is the separation between transmitter and receiver antennas.

It is noted, however, that Eq. (1) is satisfied only at a point frequency, and is not directly applicable to UWB systems. The Friis' transmission formula shall be extended to take into account the transmission signal waveform and the frequency characteristics of the antennas.

Let us assume that input signal $v_i(t)$ at the transmitter port is expressed as the convolution of a unit impulse input and the pulse shaping filter $h_i(t)$ as

$$v_i(t) = \delta(t) * h_i(t), \tag{3}$$

where

$$\int_{-\infty}^{\infty} h_i^2(t)\mathrm{d}t = \int_{-\infty}^{\infty} |H_i(f)|^2\, \mathrm{d}f = 1. \tag{4}$$

Note that the input energy is

$$P_t = 1. \tag{5}$$

Friis' formula is extended taking into account the transmission waveform as

$$H_{\text{e-Friis}}(f) = V_{\text{r}}(f) = H_{\text{f}}(f)H_{\text{i}}(f)\mathbf{H}_{\text{r}}(f,\ \mathbf{\Omega}_{\text{r}}) \cdot \mathbf{H}_{\text{t}}(f,\ \mathbf{\Omega}_{\text{t}}), \tag{6}$$

where $V_{\text{r}}(f)$ is the frequency spectrum of receiver output voltage,

$$\mathbf{H}_a(f,\ \mathbf{\Omega}_a) = \hat{\boldsymbol{\theta}}_a H_{a\theta}(f,\ \theta_a,\ \varphi_a) + \hat{\boldsymbol{\varphi}}_a H_{a\varphi}(f,\ \theta_a,\ \varphi_a), \tag{7}$$

$$a = \text{r } or \text{ t},$$

is a complex transfer function vector of the antenna relative to the isotropic antenna toward $\mathbf{\Omega}_a = (\theta_a,\ \varphi_a)$ direction,

$$H_{\text{f}}(f) = \frac{\lambda}{4\pi d} \exp(-\mathrm{j}kd) \tag{8}$$

is the free space transfer function where

$$k = \frac{2\pi}{\lambda} \tag{9}$$

is the propagation constant. Unit vectors $\hat{\boldsymbol{\theta}}_a$, $\hat{\boldsymbol{\varphi}}_a$ express the polarization and are defined with respect to the local polar coordinates of each of the antennas. The following relations can be easily derived.

$$\hat{\boldsymbol{\theta}}_{\text{r}} = \hat{\boldsymbol{\theta}}_{\text{t}}, \tag{10}$$

$$\hat{\boldsymbol{\varphi}}_{\text{r}} = -\hat{\boldsymbol{\varphi}}_{\text{t}}. \tag{11}$$

The energy of the received signal is expressed as

$$P_{\text{r}} = \int_{-\infty}^{\infty} |H_{\text{e-Friis}}(f)|^2 \, \mathrm{d}f. \tag{12}$$

Since the transmit energy is unity from Eq. (5), the UWB transmission gain G_{UWB} is

$$G_{\text{UWB}} = \frac{P_{\text{r}}}{P_{\text{t}}} = \int_{-\infty}^{\infty} |H_{\text{e-Friis}}(f)|^2 \, \mathrm{d}f. \tag{13}$$

Equation (13) is an extended version of the Friis' transmission formula for the UWB signal. It includes three elements, namely the frequency characteristics of the antennas, the frequency characteristics of free space propagation, and the spectrum of the transmit signal.

2.1 Feasibility of the Optimum Receiver

As is well known, the power in Eq. (13) is optimally detected over the noise by using the matched filter to $H_{\text{e-Friis}}(f)$. In the sense, the matched filter shall be designed to take into account the transfer functions. In reality, however, the design of the optimum matched filter is infeasible due to the following reasons:

- Considering the antenna directivity, the transfer function of the optimum matched filter changes from angle to angle.

- Considering the nano-second or sub-nano-second pulse, an adaptive matched filter shall run at the clock rate of tens of gigahertz.

Therefore, G_{UWB} gives the theoretical upper bound of the transmission gain of UWB signal, which can be hardly achieved. The effects of the distortion and the receiver filter are discussed in a separated paper.
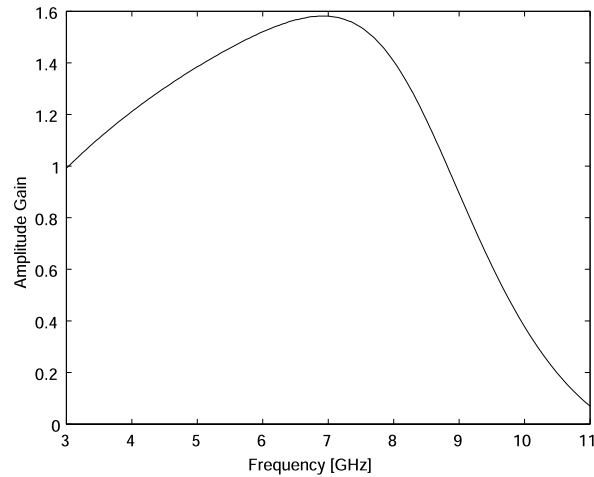
Fig. 1. Amplitude gain of dipole antenna.

3 Examples

3.1 Isotropic Antennas

Let us assume the ideal isotropic antenna which is frequency independent, and the uniform frequency spectrum between $f_{\min}$ and $f_{\max}$. The following values shall be substituted into Eq. (6).

$$H_{\mathrm{t}}(f) = H_{\mathrm{r}}(f) \quad = \quad 1, \tag{14}$$

$$H_{\mathrm{i}}(f) \quad = \quad \begin{cases} \frac{1}{2(f_{\max}-f_{\min})}, & f_{\min} < |f| < f_{\max}, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

In case, the transmission gain G_{UWB} is given by

$$G_{\mathrm{UWB}} = \frac{c^2}{(4\pi d)^2 f_{\min} f_{\max}}, \tag{16}$$

which is identical to the result shown in Ref. [4].

3.2 Dipole Antennas

As a next example, we considered the dipole antennas with the length of 4.84 cm and the intrinsic impedance of 50 Ω. Figure 1 shows the actual gain at the broadside of the antenna considering the mismatch loss as well, which is obtained by using NEC2 simulator [5].

The transmission gain is computed for the full FCC band [6], i.e. $3.1 - 10.6$ GHz. By comparing with that for isotropic case Eq. (16), the relative transmission gain for the dipole pair is $2.94 = 4.68$ dB. It is noted, however, that it does not mean that each of the dipoles have the UWB gain of 2.34 dB, since the transmission gain can not be expressed as the simple products of the gain values.

4 Conclusion

In this paper, the UWB transmission gain has been presented, which is an extension of the Friis' transmission formula in order to take into account

the transmit waveform and the antenna characteristics, for the free space link budget evaluation of UWB-IR. This value gives the upper bound of the transmission gain of UWB signals. Some numerical examples give the insight of the UWB transmission gain.

In the realistic situation, the distortion of the waveform shall be taken into account as well as the receiver filter or the equivalent template waveform of the correlator. They shall be discussed in a separate paper.