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Stability of adiabatic circuit using asymmetric 1D-capacitor array between the power supply and ground

Shunji Nakata[a)]
NTT Microsystem Integration Laboratories
3–1 Morinosato Wakamiya, Atsugi-shi, Kanagawa 243–0198, Japan
a) *nakata@aecl.ntt.co.jp*

Abstract: The stability of a new adiabatic stepwise charging circuit with an asymmetric 1D capacitor array is discussed. SPICE simulation shows that this circuit, like the one with a symmetric array, is stable. For the analytical discussion, we derive a matrix that connects charge and voltage in the circuit with the asymmetric 1D-capacitor array and show that the matrix is the positive-definite symmetric one. Using matrix theory, it is proved that the eigen value of the matrix connecting the initial voltage deviation from the step value with that after the charge-recycling process is smaller than 1. Therefore, the voltage deviation becomes zero after many charge-recycling processes.
Keywords: adiabatic charging, charge recycling, asymmetric 1D capacitor array, a positive-definite symmetric matrix, stability
Classification: Integrated circuits

References

[1] L. "J." Svensson and J. G. Koller, "Driving a capacitive load without dissipating fCV^2," *Proc. IEEE Symposium on Low Power Electronics*, pp. 100–101, 1994.
[2] S. Nakata, "Adiabatic Charging Reversible Logic using a Switched Capacitor Regenerator," *IEICE Trans. Electron.*, vol. E87-C, no. 11, pp. 1837–1846, 2004.
[3] S. Nakata, "The stability of adiabatic reversible logic using asymmetric tank capacitors and its application to SRAM," *IEICE Electron. Express*, vol. 2, no. 20, pp. 512–518, 2005.
[4] S. Nakata, "Stability of adiabatic reversible charging using 1D-capacitor array between the power supply and ground," *IEICE Electron. Express*, vol. 4, no. 1, pp. 9–14, 2007.
[5] S. Amari and K. Kanatani, *Linear Algebra*, Kodansha, Tokyo, 1987.
[6] T. Imamura, *Physics and Matrix*, Iwanami Shoten, Tokyo, 1994.

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1 Introduction

Adiabatic logic [1, 2, 3] is the only way to reduce the energy consumption when the supplied voltage is fixed to a certain value. A regenerator using a switched capacitor circuit has been proposed and realized for adiabatic logic [1, 2, 3]. In a previous letter [4], we proposed a new switched capacitor circuit with a one-dimensional (1D) array of tank capacitors located between the power supply and ground. In this circuit, all of the capacitors have the same capacitance value.

In this article, we discuss the stability of a circuit with an asymmetric 1D array of tank capacitors, which have different capacitance values. The stability of this circuit is confirmed by SPICE simulation and is also proved by the analytical method.

2 Circuit structure and stability of the regenerator

The switched capacitor regenerator with the asymmetric 1D capacitor array is shown in Fig. 1 (a). For simplicity, we show a four-step waveform circuit. The circuit structure is almost the same as in our previous work [4]. The different point is that the capacitance values are not the same. As in the previous work, tank capacitors C_i are connected in series between the power supply and ground. C_L is load capacitance, V_{out} is output voltage, and V_{Ci} is the voltage of the node between tank capacitor C_i and C_{i+1}. The switching transistor is a parallel connection of pMOSFET and nMOSFET, as shown in Fig. 1 (b). We assume that the tank capacitors are not charged at the start time so that power supply voltage V is divided due to the capacitance value.

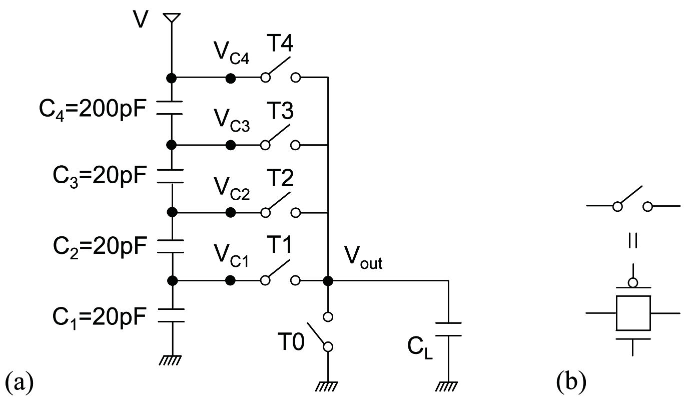

Fig. 1. Switched capacitor regenerator with the asymmetric 1D capacitor array. (a) Circuit. (b) Switching transistor circuit.

First, we discuss the circuit simulation results. C_1, C_2, and C_3 were 20 pF. C_4 was 200 pF, which is ten times larger than in the previous work. Other circuit parameters were the same as in [4]. Load capacitance C_L was 0.2 pF. We used the 0.25-μm design rule. For the pMOS and nMOS transistors, the gate length and the width were 0.25 and 6 μm, respectively. V was 2 V.

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Threshold voltages were 0.4 and −0.4 V in the nMOS and pMOS transistors, respectively. The period of the four-step waveform cycle was 0.25 μs.

When the capacitances were not charged at the start time, V_{C1}, V_{C2}, and V_{C3} values were $10\,V/31$, $20\,V/31$ and $30\,V/31$, respectively. Fig. 2 (a) shows the simulation results. At the start time, V_{Ci} is equal to the expected value. After 400 μs, V_{Ci} becomes $iV/4$ spontaneously. The time it takes to reach the $iV/4$ is longer than in the previous work. This is because C_4 is ten times larger than before.

Fig. 2 (b) shows the results when the initial V_{C1}, V_{C2}, and V_{C3} values were set to 0.1, 0.6, and 1.1 V, respectively. After 400 μs, V_{Ci} again becomes $iV/4$ spontaneously. In this case, curvatures of V_{Ci} are different from each other, which is different from the previous results [4]. This suggests that the eigen value is not the same because C_i is not the same. We discuss this point in more detail later.

From the results, it is clear that the operation of the switched capacitor regenerator is not dependent on the initial V_{Ci}, so that we can say that this circuit is very stable against external noise.

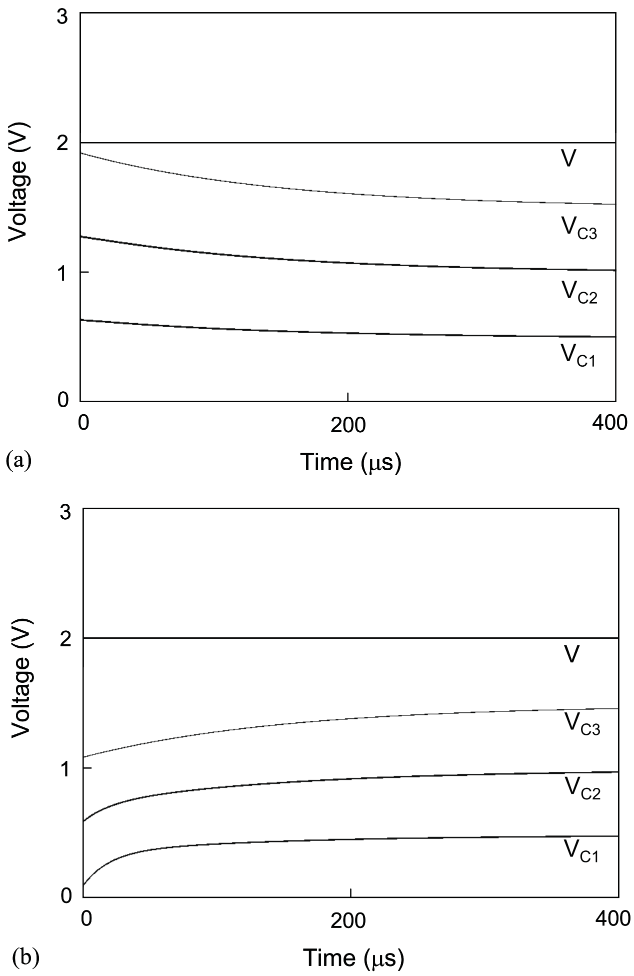

Fig. 2. Voltage change of V_{Ci} with initial states such that (a) V_{C1}, V_{C2}, and V_{C3} are $10\,V/31$, $20\,V/31$, and $30\,V/31$ and (b) 0.1, 0.6, and 1.1 V, respectively.

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Next, we investigate the reason for the stability analytically. Here, we assume that C_i is much larger than C_L. Let Q_{ti} be the transferred charge quantity from the tank capacitor to C_L at the ith step voltage [Fig. 3 (a)], and Q_{ri} be the restored charge from C_L to the tank capacitor [Fig. 3 (b)].

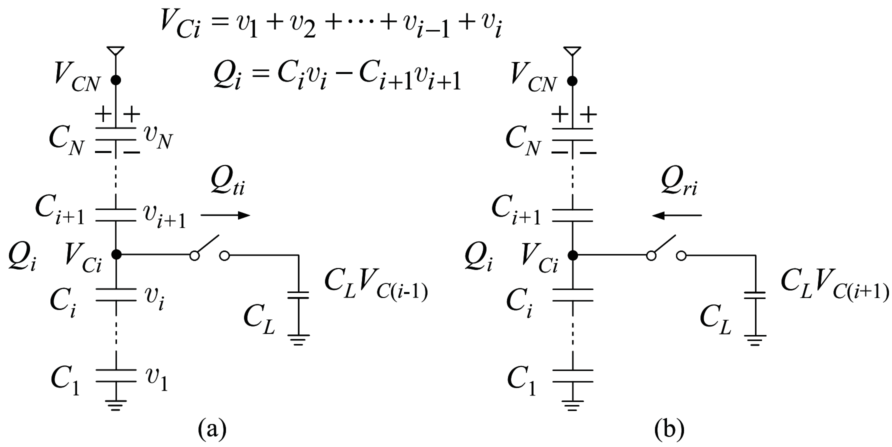

Fig. 3. Definitions of charge and voltage in the switched capacitor regenerator. (a) Q_{ti} is transferred from the tank capacitor to C_L at the ith step. (b) Q_{ri} is restored from C_L to the tank capacitor at the ith step.

We define Q_i as the amount of charge between C_i and C_{i+1}. Then, ΔQ_i (the change of Q_i) after one cycle of charging and restoring can be written as [4]

$$\Delta Q_i = -Q_{ti} + Q_{ri} = C_L(V_{C(i-1)} - 2V_{Ci} + V_{C(i+1)}), \ (1 \le i \le N-1). \quad (1)$$

Here, we define V_i as $V_i = V_{Ci} - iV/N$. Using V_i and (1), we have

$$\Delta \boldsymbol{Q} = -\boldsymbol{A}\boldsymbol{V},$$

$$\text{where} \quad \Delta \boldsymbol{Q} = \begin{bmatrix} \Delta Q_1 \\ \vdots \\ \Delta Q_{N-1} \end{bmatrix}, \ \boldsymbol{V} = \begin{bmatrix} V_1 \\ \vdots \\ V_{N-1} \end{bmatrix},$$

$$\text{and} \quad \boldsymbol{A} = C_L \begin{bmatrix} 2 & -1 & & 0 \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ 0 & & -1 & 2 \end{bmatrix}. \quad (2)$$

Next, we define v_i as the voltage difference between the capacitor plates [Fig. 3 (a)]. Then, we have

$$\begin{bmatrix} V_{C1} \\ V_{C2} \\ \vdots \\ V_{CN} \end{bmatrix} = \boldsymbol{B} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}, \text{ where } \boldsymbol{B} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 1 & 1 & \cdots & 1 \end{bmatrix}. \quad (3)$$

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Q_i can be written as

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{bmatrix} = \boldsymbol{D} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix},$$

$$\text{where } \boldsymbol{D} = \begin{bmatrix} C_1 & -C_2 & & & & 0 \\ & C_2 & -C_3 & & & \\ & & C_3 & -C_4 & & \\ & & & \ddots & \ddots & \\ & & & & C_{N-1} & -C_N \\ 0 & & & & & C_N \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \\ \vdots \\ \\ v_N \end{bmatrix}. \tag{4}$$

Then, using (3) and (4), we have

$$\begin{bmatrix} \Delta Q_1 \\ \vdots \\ \Delta Q_{N-1} \\ \Delta Q_N \end{bmatrix} = \boldsymbol{D}\boldsymbol{B}^{-1} \begin{bmatrix} \Delta V_{C1} \\ \vdots \\ \Delta V_{CN-1} \\ \Delta V_{CN} \end{bmatrix} = \boldsymbol{D}\boldsymbol{B}^{-1} \begin{bmatrix} \Delta V_{C1} \\ \vdots \\ \Delta V_{CN-1} \\ 0 \end{bmatrix}, \tag{5}$$

where ΔV_{Ci} is the change of V_{Ci} after one cycle of charging and restoring. Using a simple calculation, we can know easily that

$$\boldsymbol{B}^{-1} = \begin{bmatrix} 1 & & & 0 \\ -1 & 1 & & \\ & \ddots & \ddots & \\ 0 & & -1 & 1 \end{bmatrix}. \tag{6}$$

Therefore, we have

$$\boldsymbol{D}\boldsymbol{B}^{-1} = \begin{bmatrix} C_1+C_2 & -C_2 & & & & 0 \\ -C_2 & C_2+C_3 & -C_3 & & & \\ & -C_3 & C_3+C_4 & -C_4 & & \\ & & -C_4 & \ddots & \ddots & \\ & & & \ddots & C_{N-1}+C_N & -C_N \\ 0 & & & & -C_N & C_N \end{bmatrix}. \tag{7}$$

Using (5), (7) and $\Delta V_{Ci} = \Delta V_i$, we have two relations. One is

$$\boldsymbol{\Delta Q} = \boldsymbol{F} \cdot \boldsymbol{\Delta V}, \text{ where } \boldsymbol{\Delta V} = \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_{N-1} \end{bmatrix} \text{ and}$$

$$\boldsymbol{F} = \begin{bmatrix} C_1+C_2 & -C_2 & & & \\ -C_2 & C_2+C_3 & -C_3 & & \\ & -C_3 & \ddots & & \\ & & & C_{N-2}+C_{N-1} & -C_{N-1} \\ & & & -C_{N-1} & C_{N-1}+C_N \end{bmatrix}. \tag{8}$$

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The other is $\varDelta Q_N = -C_N \varDelta V_{CN-1}$, which is consistent with (4). Using (2) and (8), we have

$$\varDelta \boldsymbol{Q} = \boldsymbol{F} \cdot \varDelta \boldsymbol{V} = -\boldsymbol{A} \cdot \boldsymbol{V}. \tag{9}$$

Next, we investigate the characteristic of matrix $\boldsymbol{F}$ and $\boldsymbol{A}$. These are positive-definite symmetric matrices as follows [5, 6]:

$$\begin{aligned} \boldsymbol{x}^t \boldsymbol{F} \boldsymbol{x} &= C_1 x_1^2 + C_2 (x_1 - x_2)^2 + \cdots + C_{N-1}(x_{N-2} - x_{N-1})^2 + C_N {x_{N-1}}^2 \\ \boldsymbol{x}^t \boldsymbol{A} \boldsymbol{x} &= C_L [x_1^2 + (x_1 - x_2)^2 + \cdots + (x_{N-2} - x_{N-1})^2 + {x_{N-1}}^2], \end{aligned} \tag{10}$$

where $\boldsymbol{x}$ is one of any vectors. Then, $\boldsymbol{F}$ is a positive-definite symmetric matrix so that there exists a regular matrix $\boldsymbol{P}$ such that $\boldsymbol{P}^t \boldsymbol{F} \boldsymbol{P} = \boldsymbol{I}$. Next, we consider a matrix $\boldsymbol{P}^t \boldsymbol{A} \boldsymbol{P}$. This matrix is symmetric because $(\boldsymbol{P}^t \boldsymbol{A} \boldsymbol{P})^t = \boldsymbol{P}^t \boldsymbol{A}^t \boldsymbol{P} = \boldsymbol{P}^t \boldsymbol{A} \boldsymbol{P}$. Therefore, there exists orthogonal matrix $\boldsymbol{T}$ such that $\boldsymbol{A}$ can be transformed to a diagonal matrix $\breve{\boldsymbol{A}}$. Therefore, we have

$$\boldsymbol{T}^t \boldsymbol{P}^t \boldsymbol{A} \boldsymbol{P} \boldsymbol{T} = \breve{\boldsymbol{A}} \text{ and } \boldsymbol{T}^t \boldsymbol{P}^t \boldsymbol{F} \boldsymbol{P} \boldsymbol{T} = \boldsymbol{I}. \tag{11}$$

This means that $\boldsymbol{PT}$ can transform $\boldsymbol{A}$ and $\boldsymbol{F}$ into a diagonal matrix and identify matrix at the same time. Next, for the following discussion, we consider the generalized eigen value λ of the eigen equation such that $\boldsymbol{A}\boldsymbol{p} = \lambda \boldsymbol{F} \boldsymbol{p}$, where $\boldsymbol{p}$ is the eigenvector. Defining $\boldsymbol{q} = (\boldsymbol{PT})^{-1} \boldsymbol{p}$, we have

$$(\boldsymbol{PT})^t (\lambda \boldsymbol{F} - \boldsymbol{A}) \boldsymbol{PT} \boldsymbol{q} = (\lambda \boldsymbol{I} - \breve{\boldsymbol{A}}) \boldsymbol{q} = 0. \tag{12}$$

Using $\left| \lambda \boldsymbol{I} - \breve{\boldsymbol{A}} \right| = 0$, we have $\lambda_i = \breve{\boldsymbol{A}}_{ii}$. Regarding λ_i, we can derive the following other relation:

$$\boldsymbol{p}_i^t (\lambda_i \boldsymbol{F} - \boldsymbol{A}) \boldsymbol{p}_i = 0, \text{ where } \boldsymbol{p}_i \text{ is the eigenvector of } \lambda_i. \tag{13}$$

Using (10) and (13), we have

$$\lambda_i = (\boldsymbol{p}_i^t \boldsymbol{A} \boldsymbol{p}_i)/(\boldsymbol{p}_i^t \boldsymbol{F} \boldsymbol{p}_i) > 0. \tag{14}$$

Using (14) and $C_L \ll C_i$, we have $0 < \lambda_i \ll 1$.

Here, we define vector $\boldsymbol{W}$ as $\boldsymbol{W} = (\boldsymbol{PT})^{-1} \boldsymbol{V}$. Using (9), we have

$$(\boldsymbol{PT})^t (\boldsymbol{FPT} \varDelta \boldsymbol{W} + \boldsymbol{APTW}) = 0. \tag{15}$$

Using (11) and (15), we have

$$\varDelta \boldsymbol{W} + \breve{\boldsymbol{A}} \boldsymbol{W} = 0. \tag{16}$$

Here, we define W'_i as $W'_i = W_i + \varDelta W_i$. Then, we have

$$\begin{bmatrix} W'_1 \\ \vdots \\ W'_{N-1} \end{bmatrix} = \boldsymbol{G} \begin{bmatrix} W_1 \\ \vdots \\ W_{N-1} \end{bmatrix},$$

$$\text{where } \boldsymbol{G} = \boldsymbol{I} - \breve{\boldsymbol{A}} = \begin{bmatrix} 1-\lambda_1 & & 0 \\ & \ddots & \\ 0 & & 1-\lambda_{N-1} \end{bmatrix}. \tag{17}$$

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The eigen value of $\boldsymbol{G}$ is 1-λ_i, which is in the range of 0 to 1 because $0 < \lambda_i \ll 1$. This means W_i (or V_i) approaches zero as the charging and recycling process is repeated.

If C_i is the same as C, $\boldsymbol{F}$ is equal to $1/k \cdot \boldsymbol{A}$, where k is C_L/C. Using (11), we have $\breve{\boldsymbol{A}} = k\boldsymbol{I}$, which is consistent with [4]. On the other hand, if C_i is not the same, λ_i has different values. Therefore, 1-λ_i is not the same and the curvatures of V_i are therefore not the same, which was already discussed before in this article.

3 Conclusion

In summary, we analyzed an adiabatic circuit with an asymmetric 1D array of tank capacitors between the power supply and ground. We showed that this circuit is stable by SPICE simulation and proved its stability by an analytical method.

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