

# The study of the scale factor of micro-machined gyroscope

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**Abstract:** This paper reports the principle of an micro-machined gyroscope (angular rate sensor), whose output signal is fuse both roll rate and yaw rate two information. The output sine wave frequency is the rotating carrier roll rate, which is easily detected, but the amplitude depends on the roll rate and yaw rate. Otherwise, fabrication defects are always inevitably present, which gives rise to the scale factor of different gyroscope can not be identical. In order to solve these problems, a simple method is presented. The results show that the relationship between output signals and yaw rate is linearity and thus reduce the effect of the roll rate variety on the output signal. Further, different gyroscopes have the same scale factor.

**Keywords:** micro-machined gyroscope, angular rate sensor, scale factor

**Classification:** Micro- or nano-electromechanical systems

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## 1 Introduction

Gyroscopes are typically angular rate sensors, which have been widely used in inertial navigation systems, e.g. marine navigation and aeronautical navigation. Recently, the advent of MEMS technology led to the micro-machined gyroscope being able to be realized. Compared with the conventional gyroscope, although they are based on the Coriolis effect, micro-machined gyroscope, which is characterized by its small size, light weight, high reliability and low cost, has been receiving much more attention over the course of the past two decades [1, 2, 3]. In this paper, we reports a novel micro-machined gyroscope [4, 5], which is not only can solve the problems in others MEMS gyroscopes, such as structure complexity because the micro-machined gyroscope do not use driven device, but also can detect the rotation rate of rotating carrier and its yaw angles rate meanwhile. Furthermore, roll angles and yaw angles can be obtained by integrated the rotation rate and yaw angles rate with respect to time, respectively. Despite these advantages, however, on the one hand, rotating carrier rate variety significantly affect the stability of the gyroscope owing to its output signal depending on the both roll rate and yaw rate two information, on the other hand, fabrication defects are always inevitably present, which gives rise to the scale factor is different. That how to make the relationship between output signals and yaw rate is linearity is key problem.

There are three parts in this paper. We start with a brief introduction to the principle of the gyroscope and bring forward the problem. Then, we propose a method to solve the problem. Furthermore, we give the results. An additional discussion of is also provided. We end the study with the conclusion.

## 2 Principle

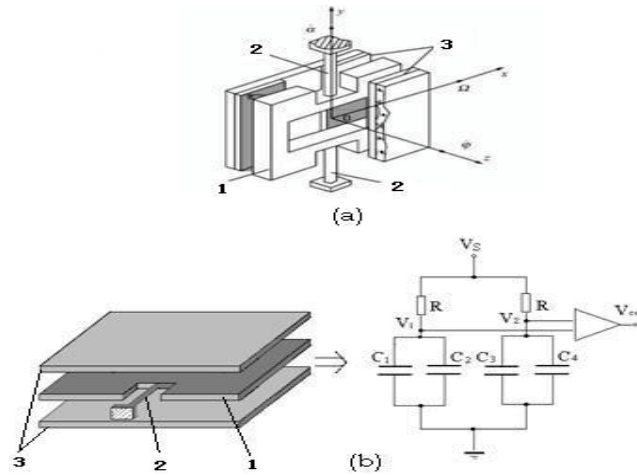
The structure of the silicon micro-machined gyroscope is shown in Fig. 1 (a). The number 1 denotes the silicon pendulum, the number 2 denotes the silicon elasticity torsion girder, and the number 3 denotes the electrode. Four electrodes and one silicon pendulum form four capacitors. The coordinates are fixed on the silicon pendulum, where  $\dot{\alpha}$  is the angular velocity about silicon pendulum vibrating around the  $oy$  axis,  $\dot{\varphi}$  is the carrier's spin angular velocity, and  $\Omega$  is the carrier's yawing or pitching angular velocity. The gyroscope fixed on the carrier rotates with the carrier at the velocity  $\dot{\varphi}$ , yawing or pitching with the velocity  $\Omega$  at the same time. The silicon pendulum is affected by Coriolis force that changes frequently (the frequency of the Coriolis force is equal to that of the carrier's rotation), and then the silicon pendulum oscillates along the  $oy$  axis.

The oscillation causes the capacitance variety of four capacitors ( $C_1, C_2, C_3$  and  $C_4$ ), all of which are formed by the silicon pendulum and four electrodes. The dynamic equation of silicon micro-machined gyroscope has been discussed systematically elsewhere [5]. According to the equation the angular

has the following from

$$\alpha = \frac{(C + B - A)\dot{\varphi}}{\sqrt{[(C - B - A)\dot{\varphi}^2 + K_T]^2 + (D\dot{\varphi})^2}} \Omega \sin(\dot{\varphi}t - \beta) \quad (1)$$

Where  $A, B, C$  are the moments of inertia of the silicon pendulum about  $ox, oy, oz$  axes respectively,  $D$  is damping coefficient,  $K_T$  is the torsions' elastic factor.



**Fig. 1.** (a) Structure of the micro-machined gyroscope The silicon pendulum oscillates under the Coriolis force caused by the carrier rotating and yawing at the same time. (b) Principle of the voltage detecting circuit Electrode 3 and silicon pendulum 1 forming four capacitors  $C_1, C_2, C_3$  and  $C_4$

From Eq. (1), we know the yaw angular rate  $\Omega$  of carrier directly proportional to amplitude of silicon pendulum, which directly lead to four capacitors ( $C_1, C_2, C_3$  and  $C_4$ ) changing, used the voltage detecting circuit (see Fig. 1 (b)) the variety of capacitance voltage is converted into that of voltage and then the voltage is amplified, so we can obtain the voltage in proportion to the angular velocity  $\Omega$  that we need detect.

The output voltage can be expressed as:

$$u = \frac{(C + B - A)\dot{\varphi}K_{TS}}{\sqrt{[(C - B - A)\dot{\varphi}^2 + K_T]^2 + (D\dot{\varphi})^2}} \Omega \sin(\dot{\varphi}t + \beta) \quad (2)$$

Where  $K_{TS}$  denotes the angle pick-up transmission factor.

Let

$$k = f(\dot{\varphi}) = \frac{(C + B - A)\dot{\varphi}K_{TS}}{\sqrt{[(C - B - A)\dot{\varphi}^2 + K_T]^2 + (D\dot{\varphi})^2}} \quad (3)$$

Where  $k$  denotes the scale factor of silicon micro-machined gyroscope. So we can gain virtual value:

$$U = \frac{(C + B - A)\dot{\varphi}K_{TS}}{\sqrt{[(C - B - A)\dot{\varphi}^2 + K_T]^2 + (D\dot{\varphi})^2}} \Omega = f(\dot{\varphi})\Omega = k\Omega \quad (4)$$

### 3 Methods

But from Eq. (4), we can see that the value of output voltage is depending on the two angular rates. To solve the above problem, the method is as follows.

Initially, the silicon micro-machined gyroscope was set on rotating carrier, Next, rotation frequency of the rotating carrier was set at  $\dot{\varphi}_1$  Hz and measured angular rate was changed from  $\Omega_1$  ( $^\circ$ /s) to  $\Omega_6$  ( $^\circ$ /s). Meanwhile, the output voltages were recorded. After that, the rotation frequency of the rotating carrier was set at  $\dot{\varphi}_2$  Hz,  $\dot{\varphi}_3$  Hz,  $\dot{\varphi}_4$  Hz, and  $\dot{\varphi}_5$  Hz, respectively, testing and recording and as shown in the Table I.

Step1: calculating scale factor.

**Table I.** The output signal at different rotating carrier rate and yaw rate

| $\Omega$ ( $^\circ$ /s) \ $\dot{\varphi}$ (Hz) | $\dot{\varphi}_1$ | $\dot{\varphi}_2$ | $\dot{\varphi}_3$ | $\dot{\varphi}_4$ | $\dot{\varphi}_5$ |
|------------------------------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| $\Omega_1$                                     | $U_{11}$          | $U_{12}$          | $U_{13}$          | $U_{14}$          | $U_{15}$          |
| $\Omega_2$                                     | $U_{21}$          | $U_{22}$          | $U_{23}$          | $U_{24}$          | $U_{25}$          |
| $\Omega_3$                                     | $U_{31}$          | $U_{32}$          | $U_{33}$          | $U_{34}$          | $U_{35}$          |
| $\Omega_4$                                     | $U_{41}$          | $U_{42}$          | $U_{43}$          | $U_{44}$          | $U_{45}$          |
| $\Omega_5$                                     | $U_{51}$          | $U_{52}$          | $U_{53}$          | $U_{54}$          | $U_{55}$          |
| $\Omega_6$                                     | $U_{61}$          | $U_{62}$          | $U_{63}$          | $U_{64}$          | $U_{65}$          |
|                                                | $k_1$             | $k_2$             | $k_3$             | $k_4$             | $k_5$             |

Step2. Making certain the relation between  $k \sim \dot{\varphi}$ .

According to the data shown in Table 1. Here we utilize software Origin6.0 fitting, according to the theory, we can fit the relation, which can be written as:

$$k = f(\dot{\varphi}) = A_2 + (A_1 - A_2)/(1 + \exp(\dot{\varphi} - \dot{\varphi}_0)/A_3) \quad (5)$$

$A_1, A_2, A_3$  And  $\dot{\varphi}_0$  is constant, which are changing with different gyroscope.

Step3: making certain the relation between output signal and rate

Let

$$U_o = \frac{U}{f_o(\dot{\varphi})} = k_o \Omega + a \quad (6)$$

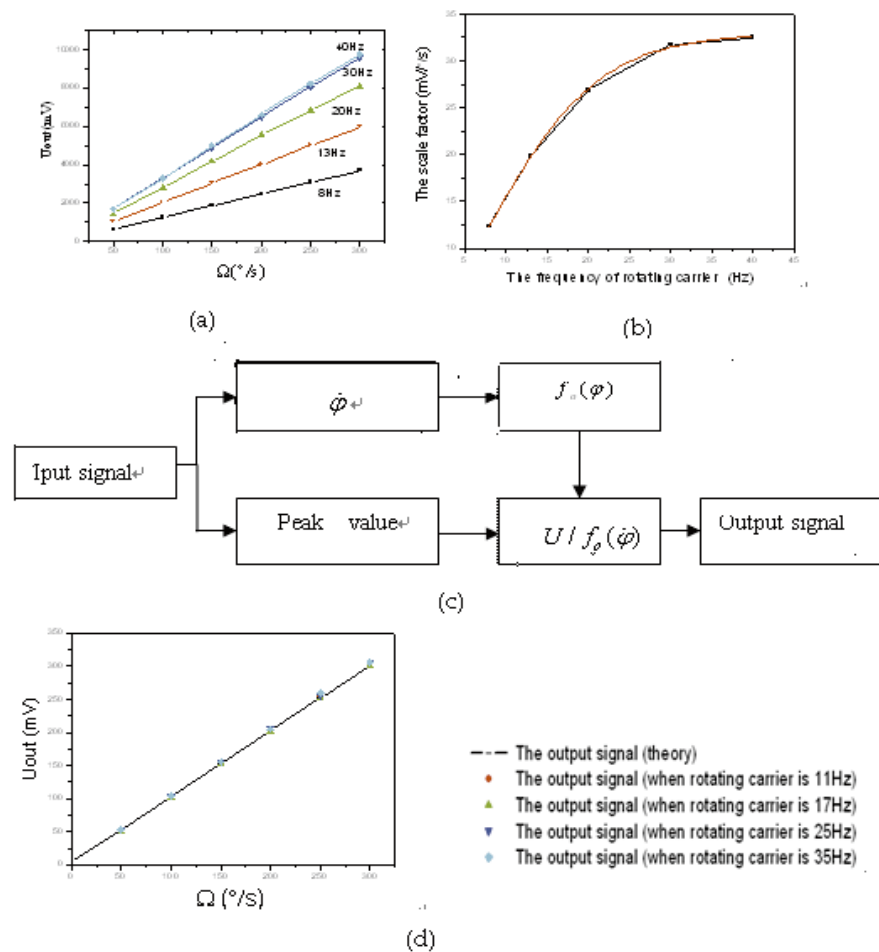
The  $U_o$  is defined as new output signal.  $k_o$  Is defined scale factor, i.e. the rate between output signal and the angular velocity, whose value is given by us discretionarily  $a$  is the zero voltage. So, whichever gyroscope and whatever roll rate, the gyroscope's scale factor is certain.

### 4 Results and analysis

For the sake of verifying validity of the method, the micro-machined gyroscope is fabricated and tested. Firstly, the roll rate of the rotating carrier was set at 8 Hz, 13 Hz, 20 Hz, 30 Hz and 40 Hz, respectively, when the yaw rate ranging from 50( $^\circ$ /s) to 300( $^\circ$ /s) changing, recording the output signal value.

Figure 2 (a) is the description of the dependence of the gyroscope output signal on the measured angular rate at random rotation frequencies of

rotating carrier. The relations between output signal and yaw rate is linearly when roll rate of rotating carrier remains constant but the scale factor is changing with the roll rate variety. we can obtain the scale factor of the micro-machined gyroscope at the different roll rate carrier by least-squares algorithm linear fitting calculated at different rotating carrier frequency, so the scale factor and different roll rate carrier is gained as shown in Fig. 2 (b). The relationship between different frequency of rotating carrier and scale factor shows a saturation tendency in Fig. 2 (b), because the influence of the frequency on the scale factor is decreased with the increase of the frequency of rotating carrier, which is in according with the Eq. (4). Moreover, we can make certain the relation between  $k \sim \dot{\varphi}$ .



**Fig. 2.** (a) Dependence of the gyroscope output signal on the measured angular rate at random rotation frequencies of rotating carrier (b) The scale factor and different frequency of rotating carrier (c) The flow chart of signal transforms (d) The output signal at different rotating carrier rate

The Fig. 2 (c) illustrates the flow chart of signal transform. Last, when  $\dot{\varphi}$  is 11, 17, 25, 35 Hz, respectively, we can get the scale factor ( $k$ ) through the Eq. (5), so we can obtain the new output signals by the Eq. (6). The results

are shown in Fig. 2 (d).

From the Fig. 2 (c), we can see that there is a good agreement between the actual and the theoretical results. This method is simple and feasible.

## 5 Conclusions

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The micromechanical gyroscope can detect the rotating carrier roll rate and yaw rate at the same time. By the method presented, on the one hand, we can make the relationship between output signals and yaw rate is linearity and thus reduce the effect of the roll rate variety on the output signal, on the other hand, different gyroscopes have the same scale factor. The experiment results show this method is easy and effective. This paper plays an important role in practicality of micro-machined gyroscope.

## Acknowledgments

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We would like to thank the Program of Beijing Key Lab for Sensor (No.KM200710772013) and National Nature Science Foundation (No.60627001) support.