

A novel inverse scattering approach for a stratified slab without explicit knowledge of incident fields

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Abstract: A novel inverse scattering approach in time domain is presented for reconstructing electrical parameters of a stratified slab. The approach based on the field equivalence principle does not need explicit information of incident fields. Considering a problem equivalent to the original scattering problem inside the region bounded by the measurement points, a cost functional of electrical parameters of the slab is introduced. The minimization of the functional is achieved by a genetic algorithm. Numerical simulations of reconstructing the relative permittivity of a stratified dielectric slab demonstrate the efficacy of the proposed method.

Keywords: inverse scattering, microwave imaging, time domain

Classification: Electromagnetic theory

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1 Introduction

A significant number of inverse scattering methods have been proposed last decades due to the theoretical interests and a variety of potential applications such as medical diagnosis, geophysical exploration, and nondestructive testing [1, 2, 3, 4, 5, 6]. In most inversion methods based on integral equations incident field information is assumed to be known or a reasonable model for the incident field which fits measured incident field data is used. In case that an object of interest is located near a transmitting antenna such as applications in medical imaging, it is preferable to model the incident field as accurately as possible since inverse scattering problems are ill-posed and nonlinear. He et al. [7] have shown that reconstruction of electrical parameter distributions can be done only from the measured total field data using wave-splitting techniques.

In this paper, we propose a novel inverse scattering method for reconstructing of electrical parameters of an inhomogeneous object only from the measured total field data without explicit use of the information of incident field. The basic idea of the proposed method is based on the field equivalence principle [8]. Although the idea is applicable to multidimensional case, we are focusing on one-dimensional case for simplicity. Reconstruction of a stratified medium from reflection and transmission data is examined. In order to assess the effectiveness of the approach, some numerical simulations are carried out.

2 Formulation

2.1 Original direct problem

Let us consider a plane wave normally incident on a stratified slab whose material properties depend only on the z -direction, as shown in Fig. 1(a). For simplicity, the material is assumed to be lossless and nondispersive. The background medium is assumed to be free space. The plane wave polarized in the x -direction is generated by a current source $J_x(z, t)$ flowing along the x -direction on the $z = z_s$ plane:

$$J_x(z, t) = I(t)\delta(z - z_s) \quad (1)$$

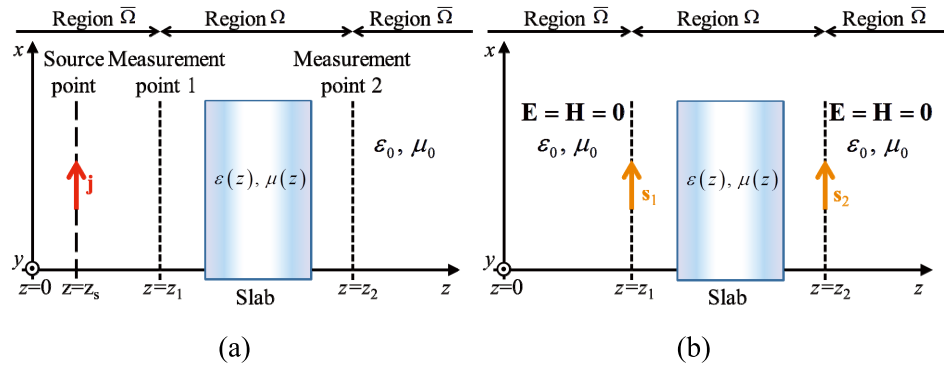


Fig. 1. (a) Original problem and (b) equivalent problem for the interior region.

where $\delta(z)$ is the Dirac delta function and the time factor $I(t)$ is assumed to be zero before time $t = 0$. Then, Maxwell's equations can be reduced to

$$\mathcal{L}\mathbf{v} = \mathbf{j} \quad (2)$$

where

$$\mathbf{v} = \begin{pmatrix} E_x(z, t) \\ \eta H_y(z, t) \end{pmatrix}, \quad \mathbf{j} = \begin{pmatrix} \eta J_x(z, t) \\ 0 \end{pmatrix}. \quad (3)$$

$\eta (= \sqrt{\mu_0/\varepsilon_0})$ is the intrinsic impedance of free space. The partial differential operator \mathcal{L} is defined by

$$\mathcal{L} = \mathbf{C} \frac{\partial}{\partial z} - \mathbf{F} \frac{\partial}{\partial(ct)} \quad (4)$$

where

$$\mathbf{C} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \varepsilon_r(z) & 0 \\ 0 & \mu_r(z) \end{pmatrix}. \quad (5)$$

$c (= 1/\sqrt{\varepsilon_0\mu_0})$ is the speed of light in free space, and ε_r, μ_r are the relative permittivity and permeability of the medium, respectively. Electromagnetic fields \mathbf{v} after time $t = 0$ are the solution of Eq. (2) under the initial condition of zero fields:

$$\mathbf{v}(z, t) = \mathbf{0}, \quad t < 0 \quad (6)$$

2.2 Equivalent problem

Let us consider a problem equivalent to the original problem inside a region $\Omega = \{z \mid z_1 < z < z_2\}$ [8]. The location $z = z_s$ of the impressed original source \mathbf{j} is assumed to be outside the region Ω , e.g. $z_s < z_1$. In the equivalent problem, the original source \mathbf{j} is removed and equivalent sources are placed on the boundary $\partial\Omega$ ($z = z_1$ and z_2) as shown in Fig. 1(b). The original fields exist inside the region Ω and a null field outside Ω . To support this field, surface electric and magnetic currents $\mathbf{J} = \hat{n} \times \mathbf{H}$, $\mathbf{M} = \mathbf{E} \times \hat{n}$ on the boundary $\partial\Omega$ ($z = z_1$ and z_2) are required where \hat{n} is a unit inward normal to $\partial\Omega$, i.e., $\hat{n} = \hat{z}$ at $z = z_1$ and $\hat{n} = -\hat{z}$ at $z = z_2$ (\hat{z} is the unit vector in the z -direction).

Electromagnetic fields of the equivalent problem are the solution of the following Maxwell's equations

$$\mathcal{L}\mathbf{u} = \mathbf{s}_1 + \mathbf{s}_2 \quad (7)$$

under the initial condition of zero fields:

$$\mathbf{u}(z, t) = \mathbf{0}, \quad t < 0 \quad (8)$$

The sources \mathbf{s}_1 and \mathbf{s}_2 consist of the equivalent surface currents $\mathbf{J} = \hat{n} \times \mathbf{H}$, $\mathbf{M} = \mathbf{E} \times \hat{n}$ corresponding to the tangential components of the original fields $\mathbf{v}(z_i, t)$ at the boundary $z = z_i$, $i = 1, 2$:

$$\mathbf{s}_1 = \begin{pmatrix} -\eta H_y(z, t) \\ -E_x(z, t) \end{pmatrix} \delta(z - z_1), \quad \mathbf{s}_2 = \begin{pmatrix} \eta H_y(z, t) \\ E_x(z, t) \end{pmatrix} \delta(z - z_2) \quad (9)$$

These sources $\mathbf{s}_1, \mathbf{s}_2$ produce the same fields internal to $\partial\Omega$ as do the original source while a null field external to $\partial\Omega$, i.e.,

$$\mathbf{u}(z, t) = \begin{cases} \mathbf{v}(z, t) & z_1 < z < z_2 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

2.3 Inverse problem

Let transient total fields be measured at two observation points $z = z_1$ and z_2 during a time interval $[0, T]$. The time-domain inverse scattering problem under consideration here is the estimation of $\varepsilon_r(z)$ and $\mu_r(z)$ with the knowledge of the measured transient field data $\mathbf{v}(z_i, t)$, $i = 1, 2$ but without the knowledge of the incident fields. According to the equivalence principle [8], when the estimated electrical parameters is identical with the true ones, the fields produced by the equivalent surface currents \mathbf{s}_1 and \mathbf{s}_2 cancel out the scattered fields by the slab outside the region Ω so that no fields are there during the time interval $[0, T]$. Incorrect estimated parameters, however, does not yield null fields outside the region Ω . Based on this observation, the inverse problem estimating medium parameters is cast as an optimization problem finding the minimizer $\mathbf{p} = \mathbf{p}^{\text{true}}$ of the following functional:

$$Q(\mathbf{p}) = \int_0^{cT} \int_{\bar{\Omega}} |\mathbf{u}(\mathbf{p}; z, t)|^2 dz d(ct) \quad (11)$$

where $\mathbf{p} = [\varepsilon_r(z), \mu_r(z)]$ is a parameter vector (\mathbf{p}^{true} is the parameter vector consisting of true electrical parameters of the unknown slab), and $\bar{\Omega}$ is the complementary region $(-\infty, z_1] \cup [z_2, \infty)$ of Ω . In numerical simulations, a bounded exterior region $\bar{\Omega}'$ is used in place of the region $\bar{\Omega}$. If a FDTD grid space is $[0, d]$, then $\bar{\Omega}' = [0, z_1] \cup [z_2, d]$.

3 Numerical results

Numerical simulations are performed for two types of dielectric slab: one is a homogeneous slab and the other is a layered slab. The width of the slabs is 0.4 m. The FDTD solution space $[0, d] = [0 \text{ m}, 4 \text{ m}]$ consists of 400 cells

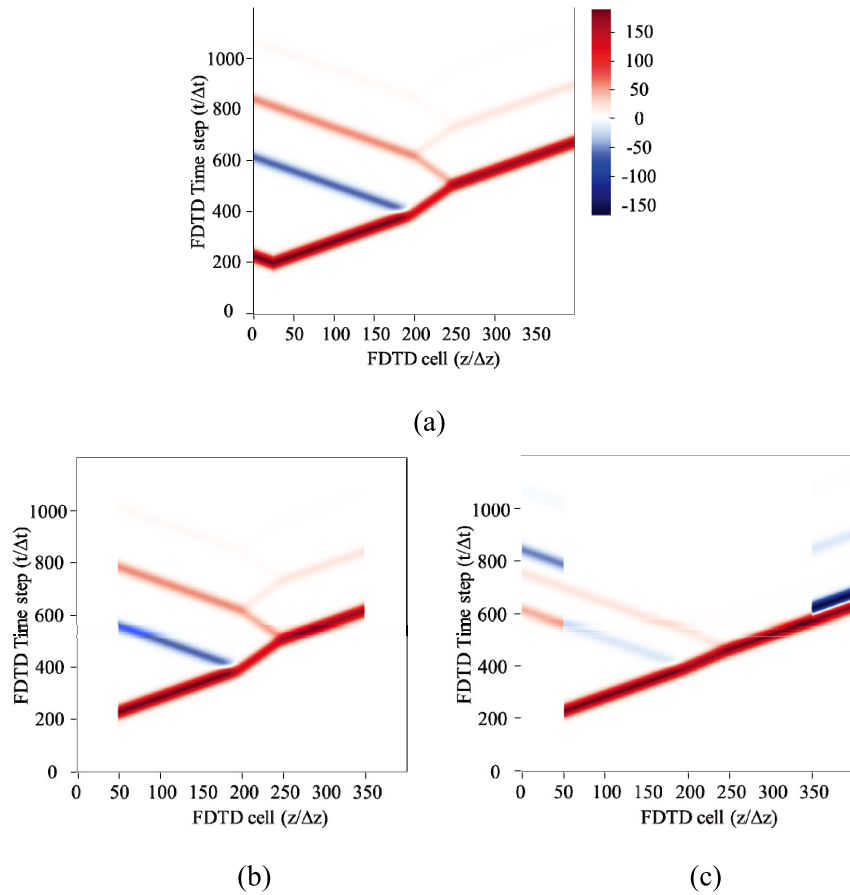


Fig. 2. Electric field distributions in time-space, (a) obtained by solving Eq. (2) with true permittivity, (b) obtained by solving Eq. (7) with true permittivity, and (c) obtained by solving Eq. (7) with an incorrect permittivity.

with cell size $\Delta z = 0.01$ m and is bounded by Mur's first-order absorbing boundary condition. Time duration T of the measurement is $1200\Delta t$ where the time step size is $\Delta t = 30$ ps. The background medium is assumed to be free space. An impressed source point is located at $z_s = 0.25$ m. The following Gaussian pulse is used:

$$I(t) = \exp \left\{ - \left[\frac{4(t - 2t_0)}{t_0} \right]^2 \right\} u(t) \quad (12)$$

where $t_0 = 100\Delta t$ and $u(t)$ is the unit step function. The observation points are located at $z_1 = 0.5$ m and $z_2 = 3.5$ m.

The first example is to show numerically the validity of the proposed method. Fig. 2 (a) shows the electric field distribution in time-space for a homogeneous slab with $\epsilon_r = 4.0$ obtained by solving Eq. (2) under the initial condition Eq. (6), while Fig. 2 (b) shows the electric field distribution based on Eqs. (7) and (8) for the correct permittivity $\epsilon_r = 4.0$. It is found that the field inside the region Ω in Fig. 2 (b) is identical to that in Fig. 2 (a), while there is no field outside Ω in Fig. 2 (b). The electric field distribution with the use of an incorrect permittivity ($\epsilon_r = 1.5$) in Eq. (7) is shown

Table I. The slab parameters estimated by GA.

| | $z_{\text{left}} [\text{m}]$ | ε_{r1} | ε_{r2} | ε_{r3} | ε_{r4} | ε_{r5} | ε_{r6} | ε_{r7} | ε_{r8} |
|-----------------|------------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| True value | 2.00 | 6.00 | 3.00 | 4.00 | 5.50 | 2.00 | 3.00 | 5.00 | 2.00 |
| Estimated value | 2.00 | 6.01 | 3.00 | 4.02 | 5.49 | 2.00 | 3.02 | 4.99 | 1.99 |

in Fig. 2(c). The scattered field by the slab with the incorrect permittivity cannot be completely cancelled out outside the region Ω by the field produced by the equivalent currents \mathbf{s}_1 and \mathbf{s}_2 , so that the remained nonzero field exists outside Ω .

Next example is reconstruction of a slab consisting of eight layers. Their relative permittivities are $\varepsilon_{r1} = 6.0$, $\varepsilon_{r2} = 3.0$, $\varepsilon_{r3} = 4.0$, $\varepsilon_{r4} = 5.5$, $\varepsilon_{r5} = 2.0$, $\varepsilon_{r6} = 3.0$, $\varepsilon_{r7} = 5.0$, and $\varepsilon_{r8} = 2.0$, respectively. A width of each layer is 0.1 m. A genetic algorithm (GA) [9], which is a global optimization technique, is applied to the minimization of the functional (11). The unknown parameters to be estimated are the position of the left face of the slab z_{left} and the relative permittivities of the eight layers ε_{r1} , ε_{r2} , ε_{r3} , ε_{r4} , ε_{r5} , ε_{r6} , ε_{r7} , and ε_{r8} . The width of the layers is assumed to be known. Since the parameter vector \mathbf{p} is a function of $\mathbf{q} = (z_{\text{left}}, \varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3}, \varepsilon_{r4}, \varepsilon_{r5}, \varepsilon_{r6}, \varepsilon_{r7}, \varepsilon_{r8})$ specifying the slab, the argument \mathbf{p} of the functional Q given by Eq. (11) is changed to \mathbf{q} . The population size is chosen to be 45 chromosomes. On the basis of a priori knowledge, the searching ranges of the dielectric constants are set to be $1 \leq \varepsilon_{ri} \leq 7$, $i = 1, 2, 3, 4, 5, 6, 7, 8$ at the beginning of the optimization process. Thirty runs of the GA search for the slab are made, because GAs are stochastic in nature and only one test of reconstruction does not validate the method. The evolutionary process was continued until a value of the fitness $\exp(-Q(\mathbf{p})/Q(\mathbf{p}_0))$ became more than 0.9999 where \mathbf{p}_0 is the parameter vector corresponding to the chromosome giving the largest value to the functional at the first generation. The average alternation number required to complete the evolution is about 288. Table I shows the average values of the estimated slab parameters. It is confirmed that the electrical parameters of the slab were successfully estimated by the proposed method.

4 Conclusions

We have presented an inverse scattering approach for a stratified slab without using explicit information of the incident field. Taking into account that fields outside the region bounded by two measurement points are null for a problem equivalent to the original one inside the bounded region, a functional of electrical parameter distributions has been introduced. A genetic algorithm has been applied to minimization of the functional in order to estimate material parameters of the slab. Numerical simulations for a homogeneous slab and a 8-layered one have demonstrated the validity of our inverse technique. The extension of the approach to multidimensional cases is straightforward.