

# Suboptimum custom-tailored model based on the pruned Volterra series for power amplifiers

Leyu Zhai<sup>1,2a)</sup>, Haitao Zhai<sup>1</sup>, Ziwei Zhou<sup>1</sup>, Eryang Zhang<sup>1</sup>,  
and Runzhou Zhang<sup>2</sup>

<sup>1</sup> School of Electronic Science and Engineering,

National University of Defense Technology, Changsha, 410073, China

<sup>2</sup> Xi'an Communications Institution, Xi'an, 710102, China

a) 4922250@qq.com

**Abstract:** This paper presents a suboptimum custom-tailored (SCT) model for power amplifiers (PAs). Based on the pruned Volterra series, the SCT model is developed by using a recursive optimum-term selecting (ROS) algorithm with an offline characterization process. The proposed model sacrifices accuracy in comparison with the custom-tailored (CT) model derived from the general Volterra series. But the former is still available, while the latter becomes unobtainable, as nonlinearity order and memory length increase. Further more, the proposed model inherits the merit of high efficiency from the base model. Simulation results show that it performs well in the time and frequency domains.

**Keywords:** behavioral model, power amplifiers, nonlinear systems, Volterra series, digital pre-distortion (DPD)

**Classification:** Microwave and millimeter wave devices, circuits, and systems

## References

- [1] M. Schetzen: *The Volterra and Wiener Theories of Nonlinear Systems* (Wiley, New York, 1980) 199.
- [2] L. Ding, G. T. Zhou, D. R. Morgan, Z. Ma, J. S. Kenney, J. Kim and C. R. Giardina: *IEEE Trans. Commun.* **52** (2004) 159. DOI:10.1109/TCOMM.2003.822188
- [3] D. R. Morgan, Z. Ma, J. Kim, M. G. Zierdt and J. Pastalan: *IEEE Trans. Signal Process.* **54** (2006) 3852. DOI:10.1109/TSP.2006.879264
- [4] A. Zhu, J. C. Pedro and T. J. Brazil: *IEEE Trans. Microw. Theory Tech.* **54** (2006) 4323. DOI:10.1109/TMTT.2006.883243
- [5] C. C. Carlos, J. Tosina, M. J. M. Ayora and J. M. Cruzado: *IEEE Trans. Signal Process.* **58** (2010) 2113. DOI:10.1109/TSP.2009.2039815
- [6] L. Zhai, H. Zhai, Z. Zhou, E. Zhang and K. Gao: *IEICE Electron. Express* **11** (2014) 20131030. DOI:10.1587/elex.11.20131030
- [7] A. Tehrani, H. Cao, S. Afsardoost, T. Eriksson, M. Eriksson and C. Fager: *IEEE Trans. Microw. Theory Tech.* **58** (2010) 1510. DOI:10.1109/TMTT.2010.2047920

## 1 Introduction

The Volterra series is a general tool to model wideband RF power amplifiers (PAs) which cause nonlinearity as well as memory effects. However, there are two basic difficulties associated with the practical application of the general Volterra series. The first difficulty concerns a large number of coefficients that should be extracted, and the second problem is the convergence of the general Volterra series [1]. So it is just suitable to the weak nonlinearity with short memory effects. A wide variety of methods have been proposed to circumvent these problems, such as the memory polynomial (MP) model [2], the generalized memory polynomial (GMP) model [3], the dynamic deviation reduction (DDR) model [4], and the radially pruned Volterra (RPV) model [5]. These models provide significant simplification by abandoning those unimportant terms to prune the general Volterra series. But all of them only perform well on some special occasions, as they actualize the simplification by fixed rules, which are appropriate for a class of PAs, but perhaps not for others. For instance, the terms which do not belong to the MP model sometimes identify nonlinearity more efficiently than those which do.

A novel approach is proposed in the literature [6], where a recursive optimum-term selecting (ROS) procedure is employed to develop custom-tailored (CT) model based on the general Volterra series. Simulation results show that the approach is effective and the achieved models are efficient. However, the general Volterra series results in too many parameters that it becomes impractical to apply the ROS algorithm as nonlinearity order and memory length increase, even though the procedure is executed by an offline procedure. Therefore, we propose a pruned Volterra model as the base model. By identifying the terms in the proposed base model recursively, we can obtain a suboptimum terms group, which could construct a new behavioral model with acceptable loss of accuracy.

This paper is organized as follows. In Section II, the suboptimum custom-tailored model is discussed and an demonstration is given. The simulation results are described in Section III. In Section V, conclusions are given.

## 2 The suboptimum custom-tailored model

The first step to develop a custom-tailored model is to select a right base model. Here, we propose a pruned Volterra model, the GMP model, as the base model. The GMP model, as well as the DDR and RPV models, originates from the general Volterra series by keeping the near-diagonal or radial-direction terms. However, it has been proved to be the most efficient model in terms of accuracy verse floating point operations (FLOPs) [7]. Therefore, without loss of generality, we mainly discuss the SCT model based on the GMP model in this study. It is accessible to the other pruned Volterra models by similar process. For convenience, we write the base model in terms of the discrete complex envelope equation as

$$\begin{aligned}\tilde{y}(n) = & \sum_{\substack{p=1 \\ p \text{ odd}}}^P \sum_{m=0}^M a_{pm} \tilde{x}(n-m) |\tilde{x}(n-m)|^{p-1} \\ & + \sum_{\substack{p=3 \\ p \text{ odd}}}^P \sum_{m=0}^M \sum_{g=1}^G b_{pmg} \tilde{x}(n-m) |\tilde{x}(n-m-g)|^{p-1} \\ & + \sum_{\substack{p=3 \\ p \text{ odd}}}^P \sum_{m=0}^M \sum_{g=1}^G c_{pmg} \tilde{x}(n-m-g) |\tilde{x}(n-m)|^{p-1}\end{aligned}\quad (1)$$

where  $\tilde{x}(n)/\tilde{y}(n)$  is the complex envelope of the input/output of the PA,  $a_{pm}$  is the complex coefficient of the main diagonal term,  $b_{pmg}$  and  $c_{pmg}$  are the complex coefficients of the cross term between  $\tilde{x}(n-m)$  and the  $g$ -order lagging  $\tilde{x}(n-m-g)$ ,  $P$  is the order of nonlinearity,  $M$  represents memory delay, and  $G$  denotes the order of lagging signal. We should note that  $P$  is an odd number, because the nonlinear regrowth induced by even terms is far away from the carrier frequency and could be filtered in a real system [2].

$$\Phi_x = \begin{pmatrix} \dots & \tilde{x}_{1-m} |\tilde{x}_{1-m}|^{p-1} & \tilde{x}_{1-m} |\tilde{x}_{1-m-g}|^{p-1} & \tilde{x}_{1-m-g} |\tilde{x}_{1-m}|^{p-1} & \dots \\ \dots & \tilde{x}_{2-m} |\tilde{x}_{2-m}|^{p-1} & \tilde{x}_{2-m} |\tilde{x}_{2-m-g}|^{p-1} & \tilde{x}_{2-m-g} |\tilde{x}_{2-m}|^{p-1} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & \tilde{x}_{n-m} |\tilde{x}_{n-m}|^{p-1} & \tilde{x}_{n-m} |\tilde{x}_{n-m-g}|^{p-1} & \tilde{x}_{n-m-g} |\tilde{x}_{n-m}|^{p-1} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & \tilde{x}_{N-m} |\tilde{x}_{N-m}|^{p-1} & \tilde{x}_{N-m} |\tilde{x}_{N-m-g}|^{p-1} & \tilde{x}_{N-m-g} |\tilde{x}_{N-m}|^{p-1} & \dots \end{pmatrix}\quad (2)$$

Given that the captured input and output data streams have  $N$  samples respectively, then the terms of the GMP model can be written in terms of matrix  $\Phi_x$  as (2), where  $|\cdot|^{p-1}$  denotes the absolute-value calculation. Define a column vector  $Y = (\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n, \dots, \tilde{y}_N)^T$ , where  $\tilde{y}_n$  indicates  $\tilde{y}(n)$ , and define another column vector  $B = (\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_s, \dots, \tilde{b}_S)^T$ , where  $\tilde{b}_s$  represents the  $s$ th complex coefficient, then we can obtain a matrix equation that is equivalent to (1)

$$Y = \Phi_x B \quad (3)$$

The parameters vector  $B$  can be extracted by using the least square (LS) methods [6]. For instance, suppose that  $P$  is 7,  $M$  is 2 and  $G$  is 2, then a SCT model can be developed by using the mentioned ROS approach based on the proposed GMP model, which could be named as the GMP-based SCT model. Similarly, the CT model based on the general Volterra series could be called the Volterra-based CT model. For comparative study, we make the number of terms in the CT/SCT model equal to that in the MP model artificially. It should be pointed out that the number of terms could be any, if only less than that of the base model. As illustrated in Table I, it is clear that the CT/SCT model contains some terms belonging to the MP model. But there are also some different terms. The GMP-based SCT model is suboptimum compared to the Volterra-based CT model, because the GMP model is a pruned version of the general Volterra series. To be

**Table I.** CT/SCT model terms ( $P = 7, M = 2, G = 2$ )

	Volterra-based CT Model	GMP-based SCT Model
1	$\tilde{x}_{n-0}$	$\tilde{x}_{n-0}$
2	$\tilde{x}_{n-1}$	$\tilde{x}_{n-1}$
3	$\tilde{x}_{n-2}$	$\tilde{x}_{n-2}$
4	$\tilde{x}_{n-0}\tilde{x}_{n-0}\tilde{x}_{n-0}^*$	$\tilde{x}_{n-0} \tilde{x}_{n-0} ^2$
5	$\tilde{x}_{n-0}\tilde{x}_{n-1}\tilde{x}_{n-0}^*$	$\tilde{x}_{n-1} \tilde{x}_{n-1} ^2$
6	$\tilde{x}_{n-0}\tilde{x}_{n-0}\tilde{x}_{n-0}\tilde{x}_{n-0}^*\tilde{x}_{n-0}^*$	$\tilde{x}_{n-2} \tilde{x}_{n-2} ^2$
7	$\tilde{x}_{n-0}\tilde{x}_{n-0}\tilde{x}_{n-1}\tilde{x}_{n-0}^*\tilde{x}_{n-0}^*$	$\tilde{x}_{n-1} \tilde{x}_{n-0} ^2$
8	$\tilde{x}_{n-0}\tilde{x}_{n-1}\tilde{x}_{n-1}\tilde{x}_{n-0}^*\tilde{x}_{n-1}^*$	$\tilde{x}_{n-4} \tilde{x}_{n-2} ^2$
9	$\tilde{x}_{n-0}\tilde{x}_{n-1}\tilde{x}_{n-2}\tilde{x}_{n-0}^*\tilde{x}_{n-2}^*$	$\tilde{x}_{n-0} \tilde{x}_{n-0} ^4$
10	$\tilde{x}_{n-0}\tilde{x}_{n-2}\tilde{x}_{n-2}\tilde{x}_{n-0}^*\tilde{x}_{n-1}^*$	$\tilde{x}_{n-3} \tilde{x}_{n-2} ^4$
11	$\tilde{x}_{n-0}\tilde{x}_{n-2}\tilde{x}_{n-2}\tilde{x}_{n-0}^*\tilde{x}_{n-2}^*$	$\tilde{x}_{n-0} \tilde{x}_{n-0} ^6$
12	$\tilde{x}_{n-0}\tilde{x}_{n-0}\tilde{x}_{n-0}\tilde{x}_{n-0}\tilde{x}_{n-0}^*\tilde{x}_{n-0}^*\tilde{x}_{n-1}^*$	$\tilde{x}_{n-1} \tilde{x}_{n-0} ^6$

specific, owing to the  $G$ -order lagging signal, the GMP-based SCT model sacrifices accuracy compared to the  $(M + G)$ -delay Volterra-based CT model. However, the GMP-based SCT model is more efficient than the others, as every term of the base model is constructed by a product of complex envelope and power of the absolute value. This means that the implementation of the digital pre-distortion (DPD) by the circuit in field-programmable gate arrays (FPGA) or the digital signal processor (DSP) procedure is simplified dramatically.

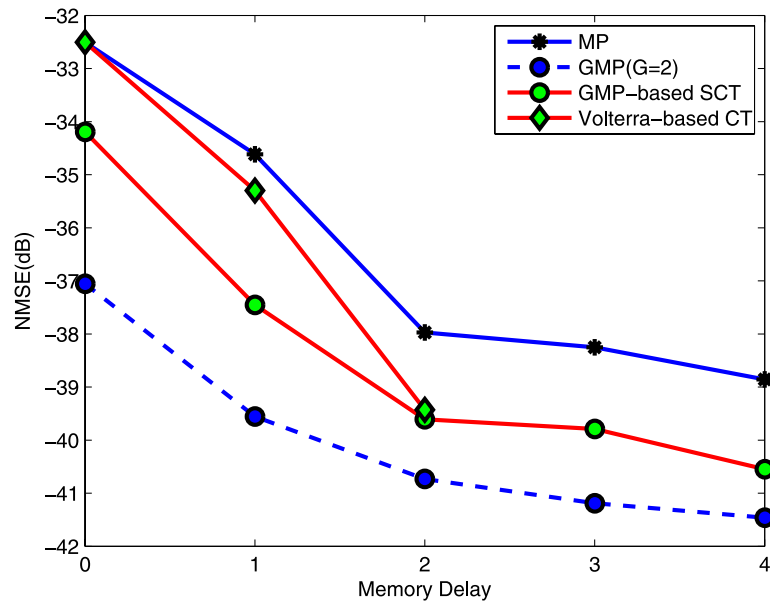
### 3 Model evaluation

In order to evaluate the proposed model, a test bench was set up, which was similar to that in [6]. On this occasion, a 20M-bandwidth orthogonal frequency-division multiple-access (OFDM) signal was adopted as the probing signal, which was a long term evolution like (LTE-like) signal with 8.9-dB peak-to-average ratio (PAR). The RF signal was directly generated by a vector signal generator (VSG) and with a mean power of 0-dBm. The device under test (DUT) was a solid wideband PA employed in the fourth-generation (4G) base station. A total of 8,192 baseband samples were captured by digital oscilloscope (DSO) and vector signal analysis (VSA) software. Of these, 4,096 samples were used for model extraction after alignment, and the remainder of the data were used for validation separately. Using the proposed approach, two models were developed, i.e. the Volterra-based CT and GMP-based SCT models. Two assessments were used to evaluate the performance of the proposed model: normalized mean square error (NMSE) in the time domain and normalized power spectral density (NPSD) in the frequency domain.

The NMSE and the number of coefficients comparisons are shown in Table II, where the NMSEs are listed on the left, and the numbers of coefficients are embraced by a pair of brackets on the right. According to Table II, the NMSE decreases evidently as  $P$  and/or  $M$  increase for all models but the general Volterra series. The NMSEs of the MP, GMP and GMP-based SCT models decrease about 0.7–1.0-dB as  $M$  increases from 2 to 4, while the NMSEs of the MP and GMP

**Table II.** NMSE (dB) and number of coefficients comparisons

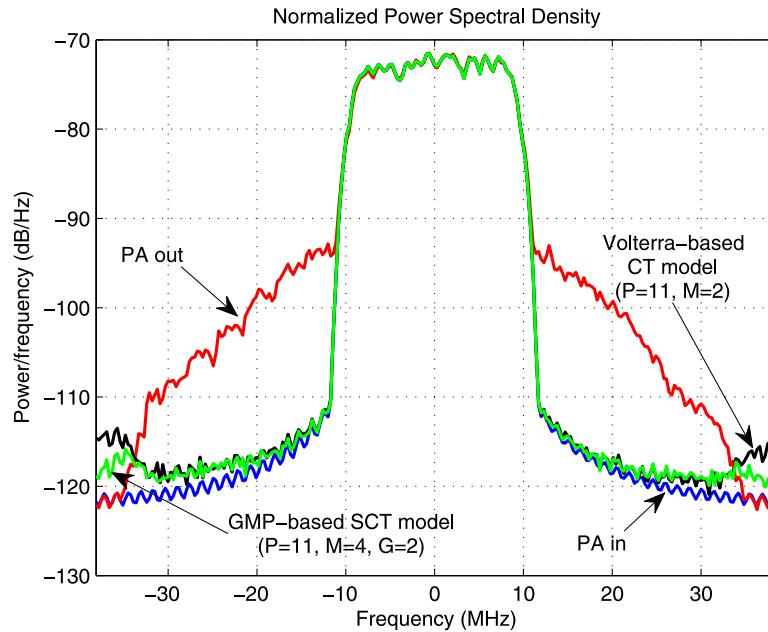
Model	P = 9		P = 11	
	M = 2	M = 4	M = 2	M = 4
MP	−37.9(15)	−38.8(25)	−38.0(18)	−38.9(30)
GMP (G = 2)	−40.6(63)	−41.3(105)	−40.7(78)	−41.5(130)
general Volterra	−17.0(546)	(11875)	−12.2(1134)	(38335)
GMP-based SCT	−39.2(15)	−40.2(25)	−39.6(18)	−40.6(30)
Volterra-based CT	−39.0(15)	-	−39.4(18)	-



**Fig. 1.** Effectiveness comparison in terms of NMSE when  $P = 11$ .

models decrease about 0.1–0.2-dB, the NMSEs of the CT/SCT models decrease about 0.4-dB, as  $P$  increases from 9 to 11. The performance of the general Volterra series is even worse than those pruned Volterra models, this is due to the fact that the general Volterra series diverges when too many nonorthogonal terms are involved [1, 4]. Since the comparisons are made on the basis of memory delay  $M$ , and the GMP-based SCT model employs  $(M + G)$ -delay signals in fact, the NMSE of the GMP-based SCT model is lower than that of the Volterra-based CT model when  $M = 2$ . Generally, in terms of NMSE, the calculation precision and the number of coefficients increase as  $P$  and  $M$  increase, this means that the accuracy can be improved by adding calculation quantity. But when concerning the general Volterra series, it becomes unfeasible when  $P$  and  $M$  increase to a big number. For instance, when  $M = 4$  and  $P = 9/11$ , the number of coefficients in the general Volterra series reaches 11875/38335. This is a too large number that it is impractical and insignificant to extract the coefficients. So the Volterra-based CT model is not available in these conditions.

When  $P = 11$ , the NMSE plots are shown in Fig. 1. It is evident that the CT/SCT model shows better performance than the MP model at every memory delay. And, the GMP-based SCT model shows a bit better than the Volterra-based CT



**Fig. 2.** Effectiveness comparison in the frequency domain.

model, because the  $G$ -order lagging signals are involved in the former. Despite that the decrease rate of the NMSE reduces after  $M = 2$ , the GMP-based SCT model is still available as  $M$  increases. As shown in Fig. 2, the CT/SCT model compensates distortion efficiently. They could reduce the adjacent channel spectrum regrowth about 20-dB. Moreover, the GMP-based SCT model ( $P = 11$ ,  $M = 4$ ,  $G = 2$ ) performs a little better than the Volterra-based CT model ( $P = 11$ ,  $M = 2$ ) when frequency offset is further.

#### 4 Conclusion

A GMP-based SCT model is presented in this paper. The proposed model sacrifices accuracy compared to the Volterra-based CT model. However, the former is still available and the latter becomes unobtainable as  $P$  and  $M$  increase. What's more, it inherits the character of high efficiency from the GMP model. So it is a more suitable model to PAs.