

Successive elimination algorithm for two-bit transform-based motion estimation

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Abstract: A successive elimination algorithm for two-bit transform (2BT) based motion estimation (ME) is proposed. By mathematically deriving the lower bound for 2BT-based matching criterion, we can discard the impossible candidates earlier and save computations substantially. Experimental results show that although the performance of the proposed algorithm is the same as that of the full search 2BT (FS-2BT) based ME algorithm, the computational complexity has been reduced significantly.

Keywords: motion estimation, block matching, full-search, video coding, two-bit transform

Classification: Science and engineering for electronics

References

- [1] *Information Technology — Coding of Audio Visual Objects — Part 2: Visual*, JTC1/SC29/WG11, ISO/IEC 14496-2 (MPEG-4 Visual), 2002.
- [2] “Advanced Video Coding for Generic Audiovisual Services,” *ITU-T Recommendation H.264*, May 2005.
- [3] W. Li and E. Salari, “Successive elimination algorithm for motion estimation,” *IEEE Trans. Image Process.*, vol. 8, pp. 105–107, Jan. 1995.
- [4] B. Natarajan, V. Bhaskaran, and K. Konstantinides, “Low-Complexity Block-based Motion Estimation via One-Bit Transforms,” *IEEE Trans. Circuits Syst. Video Technol.*, vol. 7, no. 5, pp. 702–706, Aug. 1997.
- [5] S. Erturk, “Multiplication-Free One-Bit Transform for Low-Complexity Block-Based Motion Estimation,” *IEEE Signal Process. Lett.*, vol. 14, no. 2, pp. 109–112, Feb. 2007.
- [6] A. Erturk and S. Erturk, “Two-Bit Transform for Binary Block Motion Estimation,” *IEEE Trans. Circuits Syst. Video Technol.*, vol. 15, no. 7, pp. 938–946, July 2005.
- [7] Y.-K. Wang and G.-F. Tu, “Successive Elimination Algorithm for Binary Block Matching Motion Estimation,” *IEE Electron. Lett.*, vol. 36, no. 24, pp. 2007–2008, Nov. 2000.
- [8] M. Mano, *Digital Design*, 3rd Ed., Prentice Hall.

1 Introduction

The block matching algorithm (BMA) for ME is the most popular and is deployed in many video compression standards because of its implementation simplicity and effectiveness [1, 2]. The full search algorithm (FSA) can give the optimal estimation of the motion in terms of minimal matching error by checking all the candidates within the search range, but the prohibitively huge computational complexity makes it impractical for the real-time video applications. Thus, many fast algorithms are proposed in the literature including successive elimination algorithm (SEA) [3]. In SEA, the sum norms of the blocks were used to decide whether the matching computation should be executed at a certain search position in the search range and to reduce the number of matching calculations in the FSA. Note that the ME accuracy of the SEA is the same as that of the FSA.

The techniques that exploit different matching criteria instead of the classical sum of absolute differences (SAD) such as one-bit transform (1BT), multiplication-free 1BT, and 2BT were proposed to make the faster computation of the matching criteria using Boolean exclusive-OR (XOR) operations [4, 5, 6]. In [4], 1BT-based ME where the reference frames and the current frames are transformed into one-bit representations by comparing the original image frame against a bandpass filtered output was proposed. Each frame I is filtered with a 17×17 kernel K which is given as (1). The filtered frame I_F is compared with the original frame I to create a one-bit frame B as in (2).

$$K(i, j) = \begin{cases} 1/25, & i, j \in [0, 4, 8, 12, 16] \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$B(i, j) = \begin{cases} 1, & I(i, j) \geq I_F(i, j) \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

After this transform, the matching error criterion between two one-bit image frames, which is called the number of non-matching points of 1BT ($NNMP_{1BT}$), is given by

$$NNMP_{1BT}(m, n) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \{B^t(i, j) \oplus B^{t-1}(i + m, j + n)\} \quad (3)$$

where $B^t(i, j)$ and $B^{t-1}(i, j)$ are the 1BT representations of the current and the previous image frames, respectively, \oplus denotes the Boolean XOR operation, the motion block size is $N \times N$, and $-s \leq m, n \leq s$ is the search range [4].

2BT-based ME was proposed to enhance the ME accuracy of 1BT-based ME algorithms [6]. In 2BT-based ME, the values of local mean μ , variance σ^2 , and the approximate standard deviation σ_a are used to convert frames into two-bit representations which are calculated as [6]:

$$\begin{aligned} \mu &= E[I_{tw}] \\ \sigma^2 &= E[I_{tw}^2] - E^2[I_{tw}] \\ \sigma_a &= 15 + 0.0125\sigma^2 \end{aligned} \quad (4)$$

where I_{tw} are the pixel values in the local threshold window around the transforming block. The 2BT representations can then be attained as:

$$\begin{aligned} B_1(i, j) &= \begin{cases} 1, & I(i, j) \geq \mu \\ 0, & \text{otherwise} \end{cases} \\ B_2(i, j) &= \begin{cases} 1, & I(i, j) \geq \mu + \sigma \text{ or } I(i, j) \leq \mu - \sigma \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (5)$$

where $I(i, j)$ are the pixel values of the transforming block and $B_1(i, j)$ and $B_2(i, j)$ are the 2BT representations. The 2BT-based ME uses the number of non-matching points ($NNMP_{2BT}$) as a matching criterion given as:

$$\begin{aligned} NNMP_{2BT}(m, n) &= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \{B_1^t(i, j) \oplus B_1^{t-1}(i+m, j+n)\} \\ &\quad \parallel \{B_2^t(i, j) \oplus B_2^{t-1}(i+m, j+n)\} \end{aligned} \quad (6)$$

where $B_{1,2}^t(i, j)$ and $B_{1,2}^{t-1}(i, j)$ are the 2BT representations of the current and the previous image frames, respectively, \parallel denotes the Boolean OR operation, the motion block size is $N \times N$, and $-s \leq m, n \leq s$ is the search range.

In this paper, we propose a SEA for 2BT-based ME to reduce the computational complexity. The experimental results show that the proposed algorithm reduces the computational complexity dramatically without affecting the prediction accuracy.

2 SEA for 1BT-based ME

In this Section, we derive the SEA for 1BT-based ME since it is one of the bases of our proposed algorithm. Although the SEA for 1BT-based ME is already proposed in [7], we derive the same result in somewhat different approach.

To derive a SEA for 1BT-based ME, we define the Boolean XOR-based correlations (BXC) which is the fundamental operation in the matching criterion of 1BT. We also define the Boolean OR-based correlation (BOC) which is the fundamental operation in the matching criterion of 2BT. Given two binary vectors \mathbf{x} and \mathbf{y} of length k , BXC and BOC are defined as follows:

$$\begin{aligned} BXC(\mathbf{x}, \mathbf{y}) &\triangleq \sum_{n=0}^{k-1} \{x(n) \oplus y(n)\} \\ &= w_H(\mathbf{x} \oplus \mathbf{y}) \end{aligned} \quad (7)$$

where $w_H(\cdot)$ denotes the Hamming weight which is the number of nonzero components. We identify a vector \mathbf{x} as a sequence $x(n)$ ($0 \leq n < k$). Note the connection between the Boolean operations [8] and the Hamming weight. To visualize the relationship between the Boolean operations and the Hamming weight, we define the index set \mathbf{X} whose elements are the indices of the 1's positions in vector \mathbf{x} . For example, if $\mathbf{x} = (0, 1, 1, 0, 1)$, then $\mathbf{X} = \{1, 2, 4\}$. Note that $w_H(\mathbf{x}) = \text{order}(\mathbf{X})$, where $\text{order}(\mathbf{X})$ denotes the number of elements of a set \mathbf{X} .

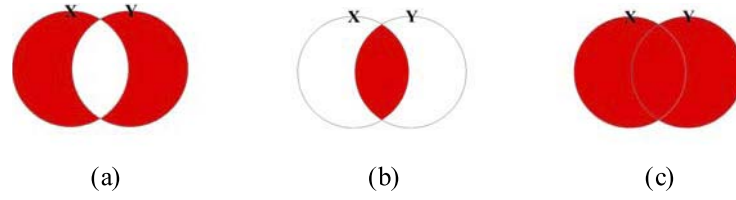


Fig. 1. Venn Diagrams of (a) $\mathbf{X} \oplus \mathbf{Y}$, (b) $\mathbf{X} \& \mathbf{Y}$ and (c) $\mathbf{X} \parallel \mathbf{Y}$

Then, the Venn diagrams of the $\mathbf{X} \oplus \mathbf{Y}$, $\mathbf{X} \& \mathbf{Y}$ and $\mathbf{X} \parallel \mathbf{Y}$ are given in Figure 1.

By using the Figure 1, we have the following equations:

$$w_H(\mathbf{x} \oplus \mathbf{y}) = w_H(\mathbf{x} \parallel \mathbf{y}) - w_H(\mathbf{x} \& \mathbf{y}) \quad (8)$$

$$w_H(\mathbf{x} \parallel \mathbf{y}) = w_H(\mathbf{x}) + w_H(\mathbf{y}) - w_H(\mathbf{x} \& \mathbf{y}). \quad (9)$$

By the property of the Boolean AND operation, the inequality (10) holds for arbitrary binary vectors \mathbf{a} and \mathbf{b} ,

$$w_H(\mathbf{a}) \geq w_H(\mathbf{a} \& \mathbf{b}) \quad (10)$$

where $\&$ denotes the Boolean AND operation. Therefore, we obtain the following inequalities:

$$w_H(\mathbf{x}) \geq w_H(\mathbf{x} \& \mathbf{y}) \quad (11)$$

$$w_H(\mathbf{y}) \geq w_H(\mathbf{x} \& \mathbf{y}). \quad (12)$$

By (9), (11), and (12), we can obtain the following inequalities:

$$w_H(\mathbf{x} \parallel \mathbf{y}) = w_H(\mathbf{x}) + (w_H(\mathbf{y}) - w_H(\mathbf{x} \& \mathbf{y})) \geq w_H(\mathbf{x}) \quad (13)$$

$$w_H(\mathbf{x} \parallel \mathbf{y}) = w_H(\mathbf{y}) + (w_H(\mathbf{x}) - w_H(\mathbf{x} \& \mathbf{y})) \geq w_H(\mathbf{y}) \quad (14)$$

Since $w_H(\mathbf{x} \parallel \mathbf{y})$ must satisfy both inequalities (13) and (14), we obtain the following inequality:

$$w_H(\mathbf{x} \parallel \mathbf{y}) \geq \max(w_H(\mathbf{x}), w_H(\mathbf{y})). \quad (15)$$

By (10) and Fig. 1 (b), we obtain

$$w_H(\mathbf{x} \& \mathbf{y}) \leq \min(w_H(\mathbf{x}), w_H(\mathbf{y})). \quad (16)$$

By (8), (15) and (16), we obtain

$$\begin{aligned} w_H(\mathbf{x} \oplus \mathbf{y}) &= w_H(\mathbf{x} \parallel \mathbf{y}) - w_H(\mathbf{x} \& \mathbf{y}) \\ &\geq \max\{w_H(\mathbf{x}), w_H(\mathbf{y})\} - \min\{w_H(\mathbf{x}), w_H(\mathbf{y})\} \\ &= |w_H(\mathbf{x}) - w_H(\mathbf{y})|. \\ &= BXC(\mathbf{x}, \mathbf{y}) \end{aligned} \quad (17)$$

Note that $NNMP_{1BT}$, which is the matching criterion of 1BT-based ME is nothing but a $BXC(\mathbf{B}^t, \mathbf{B}^{t-1})$, where we identify a vector \mathbf{B}^t as $B^t(i, j)$

($0 \leq i, j < N$) which is the 1BT representation of the current frame and a vector \mathbf{B}^{t-1} as $B^{t-1}(i, j)$ ($i+m, j+n$) ($0 \leq i, j < N$), which is that of the reference frame. Finally, we attain the SEA for 1BT-based ME which is the same result of [7] as follows:

$$\begin{aligned} NNMP_{1BT}(m, n) &= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \{B^t(i, j) \oplus B^{t-1}(i+m, j+n)\} \\ &\geq |w_H(\mathbf{B}^t) - w_H(\mathbf{B}^{t-1})|. \end{aligned} \quad (18)$$

3 Proposed algorithm

In this section, we derive the lower bound for 2BT matching criterion using the result in Section 2.

Let $\mathbf{x}_{mn} = \mathbf{B}_1^t \oplus \mathbf{B}_{1,mn}^{t-1}$ and $\mathbf{y}_{mn} = \mathbf{B}_2^t \oplus \mathbf{B}_{2,mn}^{t-1}$ be binary vectors of length $N \times N$. In this case, we identify a vector \mathbf{B}_l^t as $B_l^t(i, j)$ and a vector $\mathbf{B}_{l,mn}^{t-1}$ as $B_l^{t-1}(i+m, j+n)$ ($l = 1, 2$ and $0 \leq i, j < N$) for some fixed order. Applying the inequality (15) into the equation (9), we obtain the following inequality:

$$\begin{aligned} NNMP_{2BT}(m, n) &= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \{B_1^t(i, j) \oplus B_1^{t-1}(i+m, j+n)\} \\ &\quad \parallel \{B_2^t(i, j) \oplus B_2^{t-1}(i+m, j+n)\} \\ &= w_H(\mathbf{x}_{mn} \parallel \mathbf{y}_{mn}) \\ &\geq \max(w_H(\mathbf{B}_1^t \oplus \mathbf{B}_{1,mn}^{t-1}), w_H(\mathbf{B}_2^t \oplus \mathbf{B}_{2,mn}^{t-1})) \end{aligned} \quad (19)$$

By (18), we obtain

$$\begin{aligned} w_H(\mathbf{B}_1^t \oplus \mathbf{B}_{1,mn}^{t-1}) &\geq |w_H(\mathbf{B}_1^t) - w_H(\mathbf{B}_{1,mn}^{t-1})| \\ w_H(\mathbf{B}_2^t \oplus \mathbf{B}_{2,mn}^{t-1}) &\geq |w_H(\mathbf{B}_2^t) - w_H(\mathbf{B}_{2,mn}^{t-1})|. \end{aligned} \quad (20)$$

The following is the main result of the proposed algorithm:

$$NNMP_{2BT}(m, n) \geq \max(|w_H(\mathbf{B}_1^t) - w_H(\mathbf{B}_{1,mn}^{t-1})|, |w_H(\mathbf{B}_2^t) - w_H(\mathbf{B}_{2,mn}^{t-1})|). \quad (21)$$

In the search process, $\max(|w_H(\mathbf{B}_1^t) - w_H(\mathbf{B}_{1,mn}^{t-1})|, |w_H(\mathbf{B}_2^t) - w_H(\mathbf{B}_{2,mn}^{t-1})|)$ is compared with the $NNMP_{min}$ which is the up-to-date minimum $NNMP_{2BT}$ in the search process. When $\max(|w_H(\mathbf{B}_1^t) - w_H(\mathbf{B}_{1,mn}^{t-1})|, |w_H(\mathbf{B}_2^t) - w_H(\mathbf{B}_{2,mn}^{t-1})|)$ is less than $NNMP_{min}$, we calculate the $NNMP_{2BT}$ of eq. (6). Otherwise, we discard this search position and move on to the next search position. Note that the calculation of Hamming weights of binary vectors can be done efficiently using the method in [3] of calculating the sum norms.

4 Experimental results

Several experiments were conducted on the first 100 frames of 5 QCIF (176×144) sequences and 5 CIF (352×288) sequences. The motion block size is 16×16 and the search range is ± 16 , and all the searching processes were in spiral order.

Before we address the comparison of the computational reduction between the proposed algorithm and the FS-2BT based ME, we tested whether the ME accuracy of the proposed algorithm be the same as that of the FS-2BT based ME. Figure 2 shows the frame-wise PSNR performance of the sequence “Foreman” reconstructed by the FS-2BT based ME and the proposed algorithm, respectively. As can be seen from the figure, the PSNR performance of the FS-2BT based ME and that of the proposed algorithm is the same and their graphs overlap. From this figure and the mathematical derivation in Section 3, we can conclude that the ME accuracy of the proposed algorithm is perfectly same as that of the FS-2BT.

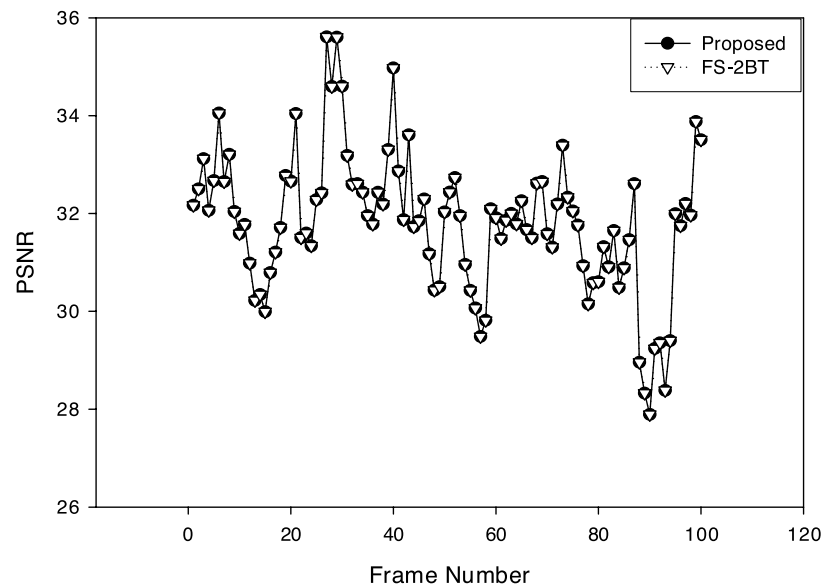


Fig. 2. Frame-wise PSNR Performance Comparison of the sequence “Foreman”

Table I. Experimental results of QCIF-size sequences

Sequences	FS-2BT		Proposed	
	Calculated Points	PSNR	Calculated Points	PSNR
Carphone	886	33.13dB	99.18	33.13dB
Silent	886	34.85dB	74.60	34.85dB
Container	886	43.44dB	89.80	43.44dB
Mother and Daughter	886	40.35dB	64.78	40.35dB
Salesman	886	39.76dB	53.81	39.76dB

We compared the performance of the proposed algorithm with that of FS-2BT based ME. Table I and II show the experimental results of the sequences of QCIF size and CIF size, respectively. The computational complexity is shown as the average number of checking positions per motion

Table II. Experimental results of CIF-size sequences

Sequences	FS-2BT		Proposed	
	Calculated Points	PSNR	Calculated Points	PSNR
Akiyo	984	42.43dB	35.12	42.43dB
Children	984	28.32dB	129.30	28.32dB
Stefan	984	25.25dB	414.81	25.25dB
Foreman	984	31.81dB	217.27	31.81dB
Flower	984	25.83dB	471.37	25.83dB

block. Note that the average PSNR performance of the proposed algorithm and the FS-2BT based ME is the same. The performance gains of the proposed algorithm are 91.37% and 74.23% in average over FS-2BT based ME when the test sequences are of QCIF size and of CIF size, respectively.

5 Conclusions

In this paper, a SEA for 2BT has been proposed to reduce the computational complexity of the typical 2BT-based ME. Using the mathematical derivation, the proposed algorithm can efficiently evaluate lower bounds for 2BT matching criterion and eliminate the impossible candidates earlier and save computations significantly. Experimental results show that although the performance of the proposed algorithm is the same as that of the FS-2BT based ME, the computational complexity has been reduced substantially.