

## MODELING OF GEOMATERIALS BEHAVIOR

KENICHI SOGA<sup>i)</sup> and CATHERINE O’SULLIVAN<sup>ii)</sup>

### ABSTRACT

The review of 50 years of papers published in *Soils and Foundations* demonstrates that significant effort has been made by many researchers to develop soil models. The contributions of *Soils and Foundations* in the area of soil modeling are subdivided into two topics: (a) “macro”-mechanics and (b) “micro”-mechanics. The attention to macro-mechanics is focused on continuum-based models, whereas that to micro-mechanics is focused on discrete based models. Soils are examples of complex systems, made up of basic units that interact to generate an emergent response that is more complex than the response of the units themselves. Despite the challenges posed by the complexity of the systems, our understanding of the link between the particles, their interactions and the overall “macro-scale” soil response has significantly been improved over the life of *Soils and Foundations*. In this review of contributions to soil modeling, many examples of macro-scale soil models developed with a real consideration of the mechanical response of the individual particles were found.

### Key words:

### INTRODUCTION

Soils are diverse, ranging from more common materials, such as clay, silt and sand to less familiar soils such as peat and methane hydrate soils. These materials share some common characteristics. In contrast to the materials manufactured in controlled industrial conditions that are often encountered in engineering, they are natural. They are also particulate, and this particular nature leads to response characteristics such as dilatancy, the importance of state, stress dependant strength and stiffness, non-linearity, etc. While all these geomaterials share common general trends in their mechanical behavior, differences in the size, mineralogy, and shape of their constituent particles result in real differences in the way the materials response to applied loads and deformations. There is, as yet, no single model that can describe the response of all types of soils to all possible loading and deformation conditions. Hence, it is not surprising that geotechnical engineers spend much of their time measuring the mechanical behavior of geomaterials and modeling them. Such tasks are becoming more important nowadays as the use of numerical methods like the finite element method has become standard engineering practice for solving geotechnical problems.

Soils are examples of complex systems, made up of basic units that interact to generate an emergent response that is more complex than the response of the units themselves (e.g., Watts, 2004). This means a reductional approach to soil mechanics that breaks the material down

into its individual particles will not easily provide answers about the overall material response. Consequently, while geotechnical engineers have always implicitly acknowledged the particulate nature of their materials, most geomechanics modeling has focused on the development of phenomenological models that capture the overall, macro-scale material response.

Despite the challenges posed by the complexity of the systems, our understanding of the link between the particles, their interactions and the overall “macro-scale” soil response has significantly been improved over the life of *Soils and Foundations*. In fact, it has been one of the primary sources of information on soil modeling from the early years of its publication. In this review of contributions to soil modeling, many examples of macro-scale soil models developed with a real consideration of the mechanical response of the individual particles were found. The quality and impact of the models published in *Soils and Foundations* is not be surprising as many authors of the journal papers were taught or inspired by the work of the academic ‘giants’, who had strong backgrounds in applied mechanics and led the early years of soil mechanics in Japan.

Takeo Mogami of the University of Tokyo was one of the pioneers in the development of the mechanics of granular materials. He applied statistical approaches to particle assembly and used the work equation to link the microscopic information to the macroscopic mechanical behavior (Mogami, 1965, 1968). It can be said that such work on micro-macro relationships of soils is one of the

<sup>i)</sup> Professor of Civil Engineering, University of Cambridge, Department of Engineering, Cambridge, UK (KS207@cam.ac.uk).

<sup>ii)</sup> Senior Lecturer in Soil Mechanics, Imperial College London, Department of Civil Engineering, London, UK.

The manuscript for this paper was received for review on August 23, 2010; approved on October 1, 2010.

important contributions made by the many papers published in *Soils and Foundations*. Sakuro Murayama of Kyoto University also investigated the discrete nature of soils and had a strong interest in the time-dependent behavior of soils (Murayama, 1983; Murayama et al., 1984). He organized a session on constitutive modeling at the 9th International Conference on Soil Mechanics and Foundation Engineering in Tokyo in 1977 (Murayama, 1977). This session was the first specialty session for an international conference to host in the area of soil constitutive modeling, and probably enhanced the reputation of *Soils and Foundations* as one of the leading journals in this area. Hakuju Yamaguchi of the Tokyo Institute of Technology introduced plasticity theories of soil mechanics and contributed to the academic community by providing basic theories of continuum mechanics to develop elasto-plastic soil models (Yamaguchi, 1975). It is these academics in particular who laid the foundations, in effect enabling *Soils and Foundations* to rapidly become one of the premier journals in soil modeling.

In this review paper, we have subdivided the contributions to *Soils and Foundations* in the area of soil modeling into two topics: (a) “macro”-mechanics and (b) “micro”-mechanics. The attention to macro-mechanics will focus on continuum based models, whereas consideration of micro-mechanics will focus on discrete based models. The number of papers on soil modeling exceeds 350. Hence, it is impossible to describe all these papers and the papers referenced in this review are our personal choices.

## “MACRO” MECHANICS

### *Early Years*

Although soil strength is an important factor in performing a stability analysis of geotechnical structures such as bearing capacity and slope stability, it is the work by Drucker (e.g., 1957) on elasto-plastic theory that initiated ‘modern’ soil mechanics. His elasto-plastic theory allows us now to model the stress-strain relationship of soils from small strain to large strain by coupling elasticity and plasticity. It became possible to evaluate the deformation and progressive failure of geotechnical structures using this theory rather than conducting separate analyses of deformation (elastic analysis) and stability (plastic analysis) using the ‘classical’ soil mechanics of Terzaghi (1948).

There is no doubt that the most widely known work on constitutive modeling of soils is the critical state soil mechanics framework proposed by Roscoe et al. (1958) and documented by Schofield and Wroth (1968). The Cam-clay model (Roscoe et al., 1963) starts from the following work equation, extending the expression originally proposed by Taylor (1948):

$$p' \dot{\epsilon}_v^p + q \dot{\epsilon}_s^p = M p' \dot{\epsilon}_s^p \quad (1)$$

where  $M$  is the critical state friction parameter,  $p'$  and  $q$  are the mean pressure and the deviator stress, and  $\dot{\epsilon}_v^p$  and  $\dot{\epsilon}_s^p$  are the plastic volumetric and deviator strains, respec-

tively. Using this equation and the Drucker stability postulate, they derived a yield surface function as well as an incremental stress-strain relationship.

Equation (1) can be rearranged to take the following form:

$$\dot{\epsilon}_v^p / \dot{\epsilon}_s^p = M - (q/p') \quad (2)$$

This equation is called the stress-dilatancy relationship (in this case it is in a linear form). The coupling of shear and volumetric strains expressed in this equation is one of the key concepts in soil modeling.

It is important to note here that a constitutive model very similar to the Cam-clay model was proposed independently by Ohta and his colleagues at Kyoto University at the same time as the Cam-clay model was developed (some call this the ‘Kamo’-clay model named after the River Kamo running through the middle of Kyoto, like the River Cam at Cambridge, UK). The model is based on the experimental finding that the volume change upon shearing is associated with the mean pressure change as well as shear induced dilatancy. Ohta and Hata (1971) demonstrated that there is a specific case where the model becomes equivalent to the Cam-clay model.

### *Stress-dilatancy—The Fundamental of Soil Modeling*

The stress-dilatancy relationship expressed in a form such as Eq. (2) is one of the key concepts of soil modeling, and this relationship has been the subject of extensive research from the birth of modern soil mechanics (e.g., Rowe, 1962; Horne, 1965, 1969). Although the relationship itself does not give a stress-strain relationship unless other models (such as deviator stress-deviator strain relationship) are proposed, it provides conceptualization of the coupled volumetric and deviator strain development under different stress states. Expressions other than Eq. (2) have been proposed for various conditions (anisotropy, cyclic loading, etc) by Oda (1974, 1975), Tatsuoka (1975, 1976, 1980), Tokue (1979), Nemat Nasser (1980), Miura and Toki (1984), Moroto (1987), Pradhan and Tatsuoka (1989) and Wan et al. (2005). Some of these papers also propose a constitutive model.

Equation (2) uses two stress invariants ( $p'$  and  $q$ ), but experimental evidence suggests that the stress-dilatancy relationship is also influenced by the Lode angle. To make the stress-dilatancy relation applicable for the general stress condition, there is a need to consider the effect of the third stress invariant or the intermediate principal stress on the relationship.

Matsuoka (1974a, 1976) proposed that a linear stress-dilatancy relationship could be applied on a plane called the “spatial mobilized plane (SMP)”. The SMP plane can be derived by considering that under a three dimensional stress condition, each of the three principal strain increments caused by shear deformation is obtained by a linear summation of two components which are produced by two different, idealized, two dimensional slippings. By transforming the stresses in conditions such as triaxial compression, extension and plane strain to the stress on the SMP plane, Matsuoka and his colleagues demonstrat-

ed that the different observed stress-dilatancy relationships were approximately equivalent when considered using this unified framework (Matsuoka and Nakai, 1985). Furthermore, Nakai and Matsuoka (1983) found that the principal stress directions may not coincide with the principal strain increment direction; they attributed this to the anisotropy of the soil fabric and subsequently modified their model.

Guo and Stolle (2004) extended Rowe's stress dilatancy equation from two dimensional triaxial and plane strain conditions to general three dimensional stress conditions. More recently Guo (2009) derived a stress-dilatancy relationship for general stress conditions based on a micromechanical deformation mechanism. The relationship takes account of non-coaxiality (*see next*) and the relationships proposed by Taylor, Rowe and Matsuoka can be recovered as special cases.

When deriving the stress-dilatancy relationship, Rowe (1962) assumed that the directions of principal strain increments coincide with those of principal stress. This is called the "coaxial" condition. Non-coaxiality refers to deviation of the principal stress directions from the principal plastic strain increment directions, which becomes evident when loading involves the rotation of the principal stress directions. The physical origin of non-coaxiality was discussed by Tobita (1989), who proposed that the introduction of a fabric tensor is essential to describe the non-coaxial and contractive behavior observed when the principal stress axes rotate. Setouchi et al. (2006) emphasized the importance of the strain increment induced by the stress rate component tangential to the yield surface, which is generated when the principal stress direction rotates.

Gutierrez et al. (1993) added a non-coaxial term  $c$  to the stress-dilatancy expression given in Eq. (2).

$$\dot{\epsilon}_v^p / \dot{\epsilon}_s^p = M^* - c(q/p') \quad (3)$$

A theoretical interpretation of parameter  $c$  was given by Gutierrez and Ishihara (2000). However, it is noted here that Vardoulakis and Georgopoulos (2005) pointed out that a limitation of the Gutierrez and Ishihara rule is encountered when there is an onset of localization accompanied by an imposed abrupt rotation of the principal axes. This in turn results in a dynamic instability where a dilatancy oscillation occurs in global undrained conditions.

#### *Yield Surface to Define the State of Full Plastic Deformation*

For the Cam-clay model, using Eq. (2) and adopting the associated flow rule, integration gives an expression for the yield surface and the size is controlled by the preconsolidation pressure (i.e., a hardening parameter). The Cam-clay model assumes that the preconsolidation pressure depends on plastic volumetric strain.

Although the Cam-clay model has been successfully used to model the mechanical behavior of clay, it has long been recognized that clays exhibit strong anisotropy both in elastic and plastic behavior. This anisotropy originates

from the preferential horizontal orientation of the platy clay particles during deposition and the subsequent post-depositional stress changes. To capture the effect of anisotropy, a rotating yield surface was first proposed by Sekiguchi and Ohta (1977). The details of the model, as well as the analytical solutions for undrained shear strength for triaxial compression and extension, are given by Ohta and Nishihara (1985). Other rotating yield surface models were proposed by Matsui and Abe (1981), Hirai (1989), Newson and Davies (1996), Muhunthan et al. (1996) and Guo and Stolle (2005).

Granular materials also yield in compression due to particle crushing. By conducting triaxial tests at very high confining pressures, Yasufuku et al. (1991a) showed nicely that the yield surface of sand is indeed nearly elliptical in shape (like clay), exhibiting isotropic hardening under increasing confining stress and kinematic hardening when anisotropically consolidated. Also, the stress-dilatancy relationship was presented, showing that the plastic strain increments are non-associated. Based on their experimental findings, they developed an elasto-plastic constitutive model for sands (Yasufuku et al., 1991b). Lade (1992) and Lade and Prabuki (1995) presented a similar asymmetric, bullet shaped yield surface. McDowell (2000) modified the Cam-clay work equation by adding a term that described the energy dissipation when particles fracture. Chavez and Alonso (2003) developed a constitutive model for crushed granular aggregates (i.e., rockfills), which involved particle rearrangement and particle crushing. Yao et al. (2008) proposed a constitutive model that considered particle crushing at high confining pressures and large shear stress ratios.

Conventionally sand behavior has been modeled using a non-associated flow rule. However, Hashiguchi (1989) gave theoretical objections to the use of non-associated flow rules. Anandarajah (1994) showed that the monotonic loading behavior of both dense and loose sands in drained and undrained conditions could be modeled by a bounding surface model (*see later*) with an associated flow rule and a distorted ellipse yield surface.

#### *Modified Stress Tensor—An Alternative Approach to Model Complex Soil Behavior*

To model the complex mechanical behavior of soils, one approach is to propose complicated mathematical functions for the yield and plastic functions in the general stress space. Another approach is to transform the stress tensor to another tensor and then adopt a more simple mathematical form for the yield and plastic potential functions. For example, rather than rotating the yield surface, Tobita (1988) and Tobita and Yanagisawa (1992) developed a yield criterion for anisotropic materials using a modified stress tensor in which the stress was modified by a fabric tensor that gave the spatial distribution of contact normals. In their model that captures the anisotropic behavior of soft rock, Oka et al. (2002) used a transversely anisotropic stiffness for the elastic strain component and a transformed stress tensor that considered anisotropic soil structure for the plastic strain com-

ponent.

The most notable soil modeling contribution that used the transformed stress tensor is the series of models proposed by Nakai and his colleagues. Nakai and Mihara (1984) extended the SMP concept by introducing a new tensor ( $t_{ij}$ ), which is a function of the stress tensor and a second transformation tensor that is related to the direction of the SMP. They then used  $t_{ij}$  rather than the stress tensor for the Cam-clay model. This tensor transformation allowed them to incorporate the effect of the intermediate principal stress implicitly. Nakai and Matsuoka (1986), Nakai et al. (1986), Matsuoka et al. (1990) and Pedroso et al. (2005) demonstrated the models capability. Their most notable contributions being (i) incorporating the subloading surface concept (*see later*) to predict the behavior of overconsolidated soils and the effect of density and pressure on the mechanical behavior including the cyclic behavior and (ii) splitting the plastic strains into one computed from the flow rule and the other with additional isotropic plastic component, which provides a stress path dependent strain increment (Nakai and Hinokio, 2004).

#### *Stiffness and Its Degradation—Elasticity and Smooth Transition from Elastic to Plastic*

It is well known that the elastic stiffness of soil is stress dependent. Houlsby and Wroth (1991) proposed a method for expressing the elastic shear modulus of clay as a power function of the applied pressure and the preconsolidation pressure. Hashiguchi and Collins (2001) formulated a pressure-dependent elastic constitutive equation in a rate form in which no energy dissipation or extraction occurred during any loading cycle. More generalized versions were given by Hashiguchi and Collins (2001) and Einav and Puzrin (2002).

When the stress increment direction coincides with the direction of initial fabric anisotropy, the material is cross-anisotropic throughout loading. The elastic properties remain cross-anisotropic and can be described by five independent parameters. Hoque and Tatsuoka (1998) proposed a comprehensive cross-anisotropic elastic model, in which the Young's modulus and Poisson's ratios are functions of the stress ratio. Puzrin and Tatsuoka (1998) demonstrated that an elastic strain energy potential could be derived from the proposed model.

Although the conventional elasto-plastic theory requires a clear separation of the elastic region and the elasto-plastic region in the stress space, in reality soils do not exhibit such behavior. Truly elastic behavior (i.e., where soil particles do not slide relative to each other) only occurs at very small strains (less than  $10^{-5}$ ) or when small stress cycles (up to a few kilopascals) are applied. At larger strain or stress increments, soil particles slide or roll relative to each other and the stress-strain relationship becomes nonlinear. This means that, even the stress state is inside the yield surface, plastic strains can start to develop from a strain level of  $10^{-4}$ . Jardine (1992) proposed a unified framework of stiffness degradation by having two kinematic sub-yield surfaces within a bound-

ing surface in the general stress space.

In the early days of soil testing, a resonant column apparatus was the only experimental apparatus that could provide reliable stiffness degradation data from a very small strain. The measured decrease in secant shear modulus with shear strain was then fitted to a simple mathematical form such as the hyperbolic equation (e.g., Hardin and Drnevich, 1972) or others (Iwasaki et al., 1978; Ishibashi and Zhang, 1993; Tatsuoka et al., 1993).

However, knowing that plastic strains develop from the early stages of deformation, it may be more appropriate to use a constitutive model that incorporates the plastic deformation inside the yield surface. The subloading surface concept (Hashiguchi and Ueno, 1977) and the bounding surface concept (Dafalias and Popov, 1975) are the two popular methods that do this. Although use of bounding surface plasticity is widely documented in the broader geomechanics literature, there have been few papers in Soils and Foundations that have used this concept (e.g., Papadimitriou et al., 2005). On the other hand, many papers have documented the use of the subloading surface concept. Notably, Asaoka and his colleagues have been strong advocates of the concept and they have published many interesting papers that have explained complex soil behavior using this approach. For example, Asaoka et al. (1997) incorporated a subloading surface into their finite deformation Cam-clay model and successfully simulated the unstable behavior of heavily overconsolidated clay specimen caused by local hardening and softening associated with local pore fluid migration. Hashiguchi (2000) argued that the continuity and smoothness conditions and the loading criterion are the most fundamental elements in constitutive models for reversible and irreversible deformations. Examples of problems associated with the violation of the continuity and smoothness conditions by some popular cyclic models were presented in his paper.

#### *Kinematic Hardening Models to Model Induced Anisotropy and to Simulate Cyclic Behavior*

Many constitutive models such as the Cam-clay model are based on isotropic hardening; that is, the size of the yield surface is controlled by a scalar variable such as preconsolidation pressure. Typically, the preconsolidation pressure increases when the soil contracts during yielding, whereas it decreases when the soil dilates during yielding. The major limitation of such isotropic hardening models is their inability to simulate soil deformation during cyclic loading. For instance, in real soils, there is often a certain amount of densification associated with the stress reversal imposed during a course of deformation even when the material state is such that it gives a dilatant response under monotonic loading. A physical explanation of the behavior is that the cyclic loading modifies the anisotropic soil fabric. To model induced anisotropy and complex volume change behavior during cyclic loading, there is a need to either (i) kinematically rotate the yield surface with plastic strains or (ii) develop anisotropic shear resistance.

Matsuoka and Geka (1983) and Matsuoka et al. (1985) coupled the SMP concept and the fabric change with strain to derive a model that simulated the stress-strain behavior under cyclic loading including liquefaction under undrained conditions. Poorooshasb and Pietruszczak (1986) developed a model using the bounding surface concept to model liquefaction and cyclic mobility. Topolnicki (1990) was an early adopter of the subloading surface model and examined use of the kinematic hardening parameter to model the induced anisotropy of clayey soils. Other kinematic hardening models were proposed by Hirai (1987), Matsui (1988), Nishi and Kanatani (1990), Zhang et al. (1997), Kiyama and Hasegawa (1998), Chowdhury et al. (1999), Matsuo et al. (2000), Hashiguchi (2001a), Kobayashi et al. (2003), and Zhang et al. (2007). Some of these models were capable of modeling cyclic mobility leading to liquefaction for medium dense sand and liquefaction by structure collapse for loose sand.

Soil does not deform along a preferential plane unless strain localization occurs; microscopically the motion of each soil particle depends on the interparticle contact load increments, and while this motion is subject to some kinematic constraints due the presence of other particles or boundaries, the particle deformations are not restricted to any particular direction. However, one may hypothesize that the soil deformation can be assumed to be the sum of sliding deformation on planes defined at multiple angles. By doing so, each plane defined at a particular angle will have its own stress/strain history and hence the summation produces anisotropic characteristics. This approach produces kinematic hardening implicitly and hence modeling of cyclic behavior becomes possible. Such models are called multi-directional, or multi-laminate models, and Calladine (1971) was probably the first to propose the application of this type of modeling to simulate soil behavior. These models are attractive because the physical meaning of the model parameters for the sliding model (shear displacement versus shear stress for example) is easier to comprehend than the model parameters used in kinematic hardening models.

A multi-directional sliding model to capture deformation during the rotation of principal stress axes has been proposed by Miura et al. (1986), Matsuoka and Sakakibara (1987), Matsuoka et al. (1990) and Gutierrez et al. (1993). Nishimura and Towhata (2004) developed a three dimensional multi-directional sliding model, which gave a good prediction of drained shear and cumulative volumetric strain after loading cycles. They demonstrated that the model could reproduce the widely used empirical relationship between cyclic stress ratio and the number of cycles to liquefaction. Iai et al. (1992) and Iai (1993a) developed a unique multi-directional model, in which the shear planes were defined in strain space rather than stress space. By controlling the cyclic mobility, they demonstrated that the model was stable even when the effective stress state became close to the failure line under cyclic loading. The micromechanical interpretation of the model was given by Iai (1993b, c). Galavi and Schweiger

(2009) developed a multi-laminate model, in which an anisotropic bonded structure and its degradation with deformation were considered.

#### *Time Dependent Deformation*

Soils and Foundations has been the primary source of models that simulate the time dependent behavior of soils. The paper by Adachi and Okano (1974) was the first publication that proposed a unified time-dependent constitutive model. They combined the elasto-viscoplasticity theory of Perzyna (1963) with the Cam-clay model and elegantly showed that the strain-rate dependent stress-strain curves observed in undrained shearing of clays can be simulated. This model is one of the pioneering time dependent models in the history of soil modeling and there has been a continuous effort to improve the model (Adachi and Oka, 1982; Adachi et al., 1987, 1998, 2005; Oka et al., 2003; Zhang et al., 2003, 2005).

Sekiguchi and Ohta (1977) proposed a time dependent model for normally consolidated soils, in which the size of the yield surface changed with time. The formulation of the model was described by Sekiguchi (1984) and the model effectively simulated undrained creep failure (or creep rupture). The model adopted the secondary compression index determined from conventional oedometer tests as a time dependent parameter; making the model attractive for engineering practice (Iizuka and Ohta (1987) outline the procedure used for the selection of the model parameters).

By allowing plastic strains to develop inside the yield surface, the time dependent behavior of overconsolidated soils can be modeled. Kaliakin and Dafalias (1990a, b) developed a rate dependent bounding surface model. Al-Shamrani and Sture (1998) developed an anisotropic, rotating bounding surface model and incorporated a microstructure based damage law in order to simulate creep rupture. Hashiguchi and Okayasu (2000) developed a time dependent elasto-plastic constitutive equation based on the subloading surface concept, which allowed the simulation of a quick response to the abrupt change of loading.

Soil's shear resistance at a given strain usually increases with strain rate. The classical time dependent models are often used to simulate the so-called "isotach" behavior, in which the stress-strain relationship is uniquely defined by a given strain rate. Through careful laboratory experiments, Tatsuoka and his colleagues have shown that there are other types of time dependent behavior of soils. For example, some soils exhibit an instantaneous change in stress when there is a sudden change in strain rate, but this change decays with additional straining (called TESRA behavior). For poorly graded materials with round and stiff particles, they even found negative "isotach" behavior (i.e., shear resistance decreases with increased strain rate).

To model such complicated time dependent behavior of soils, Di Benedetto and Tatsuoka (1997) proposed a rheological model that decomposed the strain into elastic and irreversible components and the stress into time de-

pendent and independent components. This approach was further extended by Di Benedetto et al. (2002) and Tatsuoka et al. (2002) to model more complicated time dependent behavior when the strain rate was changed stepwise or at a constant rate, at creep and relaxation stages, and immediately after loading was restarted at a constant strain rate following a creep stage. Tatsuoka et al. (2004) further proposed that the model should be formulated using a new stress parameter and a normalized irreversible shear strain energy. These developments allowed them to simulate the time dependent behavior observed at different stress paths in triaxial compression conditions. Further developments of the model are given by Tatsuoka et al. (2008a), Kongkitkul et al. (2008) and Peng et al. (2009).

#### *Soil Structure—Modeling Natural Soils and Cemented Soils*

The term soil structure is often used to account for differences between the properties of a soil in its natural state and the soil properties measured after remolding and reapplication of the original stress state (i.e., the soil in a destructured state). Every natural, undisturbed soil has a structure and any disturbance (natural or man-made) may degrade its structure, which has significant implications in engineering design. Hence modeling the behavior as the soil evolves from its natural state to a destructured state is an important subject. Similar arguments can be made for improved soil, in which soil structure is created by adding cementing agents to the soil. The development of soil structure (ageing) is time dependent. Tatsuoka et al. (2008b) defined “ageing effects” as time dependent changes in intrinsic material properties and “loading effects” as the rate dependent stress-strain relationships such as creep, stress relaxation and strain rate effects.

Oka et al. (1989) developed a model for natural soft clays and proposed a stress history tensor, which is a function of the current stress tensor and a strain related variable. A change in the stress history tensor gave a softening behavior associated with soil structure degradation. Kimoto and Oka (2005) developed an elasto-viscoplastic model with structural degradation by shrinking the bounding surface and the static yield surface, which allowed them to model unstable behavior during consolidation.

Asaoka et al. (2000a) introduced a three surface model, in which the outmost yield surface called the “super-subloading surface” was a function of soil structure. Using this concept, they demonstrated that the secondary compression observed in one-dimensional consolidation tests could be related to the delayed consolidation arising due to softening of soil structure expressed in the degradation of the super-subloading surface as plastic volumetric compression proceeded (Asaoka et al., 2000b). The model was extended by adding kinematic hardening (Asaoka et al., 2002) to model the decay of soil structure and evolution of anisotropy that they associated with compaction and densification of loose sand. Nakano et

al. (2005) verified the model for simulating the mechanical behavior of natural clays and Noda et al. (2005) used this model to evaluate sample disturbance effects.

Natural soils may have weak planes created by the depositional environment. Maekawa et al. (1992) examined the yield surface of intact diatomaceous mudstone, which had planes of weakness created by the depositional environment.

There have been a number of papers modeling the behavior of improved soils and various approaches to incorporate cement hardening or setting have been proposed. Hirai et al. (1989) proposed a double hardening elasto-plastic model with a yield surface that is increased by cementing, whereas Lee et al. (2004) developed a model using the concept of a bonding stress ratio, which decays by the breakage of cemented bonds by shearing. Matsuoka and Sun (1995) proposed an extended spatially mobilized plane by introducing a bonded stress parameter, which allows modeling of the mechanical behavior of cemented sands. In a similar manner, Kasama et al. (2000) developed a model of lightly cemented clay by transforming the stress to a stress variable that includes the tensile capacity of the clay. Yu et al. (1998) developed a constitutive model for soil-cement mixtures using the framework of continuous damage theory. The model separates the stress into two components; that the stress transmitted by the cement bonded structure and the stress transmitted by the frictional resistance.

#### *Unsaturated Soils*

The constitutive modeling of unsaturated soils is complex as the pore water (in suction) can act as an apparent cohesion by forming water meniscus at particle contacts as well as contributing to the pore pressure effects (or effective stress) acting on particles. It is the need to model these different effects of pore water that poses the main challenge to developing effective models for unsaturated soil.

Kohgo et al. (1993a, b, 2007) proposed a new definition of suction induced effective stress and developed an elasto-plastic constitutive model for unsaturated soils and rockfills. To model the apparent increase in shear resistance due to the capillary force that develops at particle contacts, the preconsolidation pressure defining the yield surface location was assumed to be a function of both plastic volumetric strain and suction. Sun et al. (2000) extended the SMP concept to include unsaturated soils by defining the effective stress to be a non-linear function of suction. Toyota et al. (2004) proposed a tensile failure criterion for unsaturated soils, in which the tensile strength was generated by the microscopic water menisci at the particle contacts.

#### *Large Deformations, Bifurcation, Localization and Fluidization*

In principle continuum soil models are developed to simulate homogeneous materials. The idea for models that utilize particular shear plane(s) in their formulation (such as Mohr-Coulomb model, SMP type models or

multi-directional models) has to remain conceptual and not real. That is, there is no one particular plane that governs the mechanical behavior of soils. However, as the deformation progresses, strain localization and bifurcation may occur, possibly leading to shear band formation. This bifurcation creates the following interesting phenomenon (e.g., Ikeda and Murota, 1996); (a) the stress-strain relationship becomes sample-size dependant, (b) soil deformations may differ depending on the pattern of imperfections, (c) local fluid movements cause different deformation patterns depending on loading rate, and (d) the soil deformation becomes stochastic or uncertain in nature due to variation in the magnitude of imperfections. Although most bifurcation analyses consider a localized deformation, Ikeda and Murota (1997) examined recursive patterns of bifurcation often seen in the earlier stage of soil sample shearing. This leads to localized shear band at the later stage.

Geometrical nonlinearities are important to develop the recursive bifurcation and hence a finite strain-deformation model is necessary to trigger bifurcation and Yatomi and Nishihara (1984) were the first to introduce finite deformation theories to soil modeling in Soils and Foundations. When this type of modeling is attempted, Hashiguchi (2003) emphasizes that the constitutive equation has to be formulated to be independent of the coordinate system. That is, it should be independent of the superposition of rigid body motion. This means that a constitutive equation needs to be described using rate tensors with objectivity.

Shear band formation can occur if the material exhibits strain-softening behavior or where a strain hardening model with a non-associated flow rule is used. Yatomi et al. (1989a, b) were the first to develop a finite deformation Cam-clay model and they demonstrated that shear band formation near the critical state can be modeled by incorporating a non-coaxial term. Asaoka et al. (1994) also derived a finite deformation Cam-clay model. This model was then used to understand the effect of localized pore water migration on the behavior of a soil specimen subjected to triaxial compression tests (Asaoka et al., 1995; Asaoka and Noda, 1995).

Han and Drescher (1993) investigated the formation and growth of shear bands and they compared their experimental results to the theoretical predictions made by equilibrium bifurcation theory using incremental constitutive models. They demonstrated that a yield function with a corner (vertex) could effectively simulate shear band formation.

For so-called “active” clays, the friction angle at the critical state is obtained when shear deformations occur at a constant volume and the particle orientations become random. But at larger displacements, particles can align in a preferred orientation, which in turn reduces the shear resistance to the residual friction angle. Modeling of residual strength at large strains require further investigation for understanding of the rapid deformation of clay-rich shear zones. Gerolymos et al. (2007) attempted to model this by incorporating strain-softening, viscoplastic

behavior, and frictional softening due to heat generated pore water pressures.

Large strains can develop when the effective stress reduces to be close to zero (post-liquefaction). The soil may become liquid-like and the use of elasto-plastic models may be limited. Towhata et al. (1992) and Moriguchi et al. (2005) assumed liquefied sand as Newtonian or Bingham fluid. In order to use this method, Gallage et al. (2005) showed experimentally that the equivalent viscosity of liquefied soil depended on shear strain rate, mean pressure and fine content. Shamoto et al. (1997) proposed a model, in which shear strain was decomposed into two components; (i) a component generated by the change in effective stress and (ii) a component triggered at zero effective stress and the magnitude was related to the preceding maximum shear strain.

### *Thermal Effects*

The applications driving the need to simulate thermal effects include the prediction of the influence of climate change on permafrost, nuclear waste containment, and geothermal energy structures. Unfortunately to date, efforts to develop thermal models for geomaterials have been limited. Hueckel and Pellegrini (1991) proposed a thermo-plastic model that simulated the development of pore pressure build-up in undrained conditions when soil temperature increases. Yashima et al. (1998) developed an elasto-thermo-viscoplastic model that extended the Adachi and Oka model (1982) by including temperature and strain rate dependent material parameters. Zhang and Zhang (2009) developed a new thermo-elasto-viscoplastic model for soft sedimentary rocks.

### *Generalized Theories*

Gudehas (1996), Bauer (1996) and Masin (2009) introduced their “hypoplastic” model, which has been under development by researchers working at the University of Karlsruhe for 30 years. The model does not use a yield surface with a flow rule and the objective rate of effective stress is defined as tensor-valued function of void ratio, effective stress and the stretching rate. The stiffness and strength are not defined, but the critical state concept is incorporated in their constitutive equation. The model is able to simulate nonlinear mechanical behavior at different densities and pressures in a unified framework.

One interesting development is the use of thermomechanical principles to develop constitutive models for soils starting from the first law of thermodynamics (Collins and Houlsby, 1997; Houlsby and Puzrin, 2006). In this framework, the free energy and dissipation functions are formulated to represent the elastic and plastic responses. The free energy is the sum of the internal energy and the reversible part of entropy. The irreversible part of the rate of entropy is defined as the dissipation function. The dissipation function gives the flow rule as well as the yield criteria. Imai and Xie (1990, 1991) were the first in Soils and Foundations to consider use of irreversible thermodynamics explicitly and used an “endochronic” theory to model the behavior of overconsoli-

dated clays. Rojas and Garnica (2000) presented a constitutive model for anisotropic soil using thermodynamics principles. The comments by Hashiguchi (2001b) on the applicability of the thermodynamics approach to develop elasto-plastic constitutive equations are worth noting.

### “MICROMECHANICS”

Soil particles are small, three dimensional and opaque; while resin impregnated thin sections can be used to study the micro-structure in two dimensions (e.g., Oda, 1972a, b), direct observation of the soil particle deformations during loading is difficult and measurement of the inter-particle forces is intractable. Particulate soil mechanics research has therefore used physical models, particle based numerical and analytical models, and micro-structural continuum theories.

For sand particles and some silt particles, the surface interaction forces are negligible in comparison with the particle inertia and their interactions are therefore considerably simpler than those of clay particles. Most of the documented particulate mechanics research within geomechanics, including research documented *Soils and Foundations*, has therefore focused on sand response.

Here we present a review of the particle scale contributions to *Soils and Foundations* considering the modeling approaches adopted, the tools used to analyze the experimental data and the key findings.

#### *Microstructure Modeling from Physical Models*

Two dimensional systems of rods or disks can be used to create physical, analogue models of soil to study particle kinematics, and where photo-elastic materials are used, particle stresses and inter-particle forces. Key contributions documenting use of these models to advance understanding of the influence of microstructure on soil response and microstructure evolution during loading have appeared in *Soils and Foundations*. The first contribution is probably by Mogami (1965); his contemporary Rowe (1962) was also using analogue soils to develop his stress dilatancy theory. Despite the advent of discrete element modeling (DEM), the use of physical particle scale models to study soil response continues, as documented in *Soils and Foundations* (Wan et al., 2005) and elsewhere (Ibrahim et al., 2010). The available technology for particle scale physical modeling has evolved over the life of *Soils and Foundations*, for example Matsushima et al. (2002) used laser added tomography to track displacements and rotations for 3D systems of particles.

The use of photoelastic particles was established in geomechanics in the 1950s, 1960s and early 1970s (e.g., Dantu, 1968; Drescher and de Josselin de Jong, 1972). Key micro-mechanics contributions including examples of use of photoelastic rods made by Oda and his colleagues were published in *Soils and Foundations* between 1974 and 1985 (Oda, 1972a, b; Oda and Konishi, 1974a, b; Oda, 1977; Oda et al., 1985). These papers document the evolution of understanding of the fundamental micro-mechanisms and the interpretative approaches.

Oda and Konishi (1974a, b) considered the response of dense and loose photoelastic circular disks in a two-dimensional simple shear apparatus and found the differences in the responses to be qualitatively similar to the differences between the response of loose and dense sand. While Drescher and de Josselin de Jong (1972) analyzed the development of the force network and calculated the stress and strain tensors, Oda and Konishi (1974a, b) considered the evolution of the micro-structure or fabric. Using rose diagrams which give the frequency distributions of the contact orientations, they found that the contact normals tend to be orientated in the major principal stress direction and that they rotate during shear, to be distributed around a line inclined at 45° in simple shear conditions. They found that sliding occurred only at a small number of contacts. Oda (1977) illustrated the significance of the coordination number (number of contacts per particle) for soil response using glass ballotini as a model soil. These publications in *Soils and Foundations* in the 1970s directly influenced the discussion on soil micromechanics given by Oda et al. (1980).

Oda (1982) used the ideas of Satake (1978) and Kanatani (1981) to calculate a fabric tensor for the contact normal orientations. He showed that principal values (eigenvalues) can be used to quantify anisotropy and that the eigenvectors give the orientation of the anisotropy. Oda's ideas on fabric quantification were applied to specimens of oval photoelastic rods subject to biaxial compression (Oda et al., 1985). Oda et al. proposed a very useful particle fabric tensor that combines the particle shape and orientation data and a void fabric tensor that quantifies void shape and orientation. The contact normal fabric showed the strongest correlation with the principal stress ratio. The void fabric data indicated the development of vertical elongated voids post peak, providing evidence of the formation of column like load paths orientated in the principal stress direction. These observations show that the failure of granular materials can be considered as a buckling mechanism (e.g., Tordesillas and Muthuswamy, 2009). Oda et al. (1985) demonstrated the physical relevance of the fabric tensor, influencing future interpretations of DEM simulations (e.g., Thornton, 2000) and thin section analysis (Oda and Kazama, 1998).

#### *Soil Modeling Using Numerical Models*

DEM as proposed by Cundall and Strack (1979) provided an additional tool for research into the fundamentals of granular material response facilitating 3D models, easier access to data on contact forces, etc. The ideal nature of the particle shapes and contact models mean that DEM simulates an analogue soil, but the simulations clearly capture many of the inherent features of soil response (e.g., Cundall, 2001). One example of the ability of DEM to provide micro-macro links is provided in the widely cited *Géotechnique* paper by Thornton (2000). However two papers published in *Soils and Foundations* a decade earlier (Chen and Ishibashi, 1990; Chen and Hung, 1991) are also particularly good illustrations of how DEM can be used in fundamental geomechanics



research. In both studies monotonic loading conditions and stress reversals were simulated. The papers differed in the contact model used; Chen and Hung demonstrated the benefits of using a Hertz Mindlin contact model instead of the linear model used by Chen and Ishibashi. The influence of stress history on the microstructure was clearly shown: upon unloading to the initial stress level the coordination number increases only slightly and upon unloading to an isotropic stress state the anisotropic fabric remained essentially intact. The study documented by O'Sullivan et al. (2008) was completed, unfortunately, without any reference to either Chen and Ishibashi or Chen and Hung. O'Sullivan et al. (2008) simulated physical strain controlled cyclic triaxial experiments on steel spheres. The evolution of coordination number and fabric (anisotropy and orientation) over 50 cycles was considered at different cyclic strain amplitudes.

In their Soils and Foundations papers, Lobo-Guerrero and Vallejo (2005, 2006) adopted an efficient approach to model particle crushing, where they replaced each "crushed" disk with a number of unbonded subparticles when the stress conditions meet a specified criterion. While this approach to simulate particle crushing is computationally efficient, the crushing model is much simpler than that of Robertson (2000) and Cheng et al. (2003) and the simulations are not calibrated against physical tests. Lobo-Guerrero and Vallejo (2005, 2006) are nice examples that illustrate the potential of DEM to allow us to assess hypotheses about the particle scale mechanics that govern the overall material response; they showed the ability of DEM to provide a continuous record of particle damage and its evolution, such information cannot be obtained in physical laboratory experiments.

The original DEM implementation by Cundall and Strack (1979) was based upon disk and sphere particles, the most widely used DEM codes Trubal and PFC use sphere and disk particles, and so most of the published geomechanics studies using DEM have used disks or spheres. Mirghasemi et al. (1997) described a 2D polygonal DEM that they used to simulate a series of biaxial compression tests at different confining pressures. A stress dependent response was observed with dilation reducing with stress level. The coordination number and anisotropy were found to be stress level dependant. DEM allows controlled variation in the particle characteristics and observation of the implications for the overall response. Matsushima et al. (2003) compared the particle rotations observed in DEM simple shear simulations using polygonal and disk shaped particles with the continuum rotation tensor. While a linear relationship was found between the particle rotations and the continuum rotation in both cases, the scatter was significantly larger for the disk shaped particles.

A number of alternative algorithms for particle scale simulation have been proposed and documented in Soils and Foundations, including the implicit approaches used by Ai (1985) and Takahara and Miura (1998) (who used Kishino's algorithm from 1989). Sakaguchi et al. (1996) proposed a method based on cellular automata and

Tamura and Yamada (1996) proposed a method based on rigid plastic analysis. A particularly unusual approach is the application of the Markov stochastic process documented by Kitamura (1981a, b). While these methods may not exhibit significant advantages over Cundall and Strack's algorithm, they are noteworthy as they may be amenable to future adaptation or may offer advantages over DEM for certain micro mechanics analyses.

DEM simulations provide a large amount of data that cannot always be used to simply explain basic mechanisms. It can be more worthwhile to create simpler models that can be analytically studied, a nice example of this approach is the work of Maeda et al. (1995) who considered the stability a chain of particles arranged around elliptically shaped voids using shell theory and the theory of micropolar elasticity and captured some of the inherent features of granular materials.

### *Micromechanics Models*

The micro-structural or micro-mechanics approach uses quantitative measures of particle packing to derive overall constitutive parameters. Key contributions to the micro-structural approach have been made by Chang and his colleagues as documented elsewhere (e.g., Chang and Liao, 1990). Yimsiri and Soga (2002) used the microstructural model developed by Yimisiri and Soga (2000) to develop a relationship between the fabric and the elastic stiffness parameters. A rough surface contact model was adopted to overcome the inability of the Hertzian theory to describe the typical relationship between stiffness and stress level observed in experiments on soil. In their contribution, McDowell and Bolton (2001) discuss micromechanical hypotheses for the experimentally observed relationship between stiffness and stress level. Yimsiri and Soga (2002) back-calculated the fabric anisotropy for a range of geomaterials and demonstrated that in triaxial tests, the measured elastic parameters  $G_{vh}$  and  $G_{hh}$  depend on the stress path direction.

While Oda et al. (1984), McDowell and Bolton (2001), and Yimsiri and Soga (2002), considered elastic stiffness, there have also been attempts to develop failure criteria derived from micromechanical principles. For example, Guo and Stolle (2005) derived an anisotropic failure criterion by decomposing the stress tensor into the normal and shear force components.

Another micro-structural type model is proposed by Oda et al. (1984), who developed an elastic model for cracked materials using a fabric tensor that characterizes crack geometry following the previous work of Oda (1984).

### **FUTURE OPPORTUNITIES**

As is evident from this review, there are a number of aspects of soil behavior that require advanced modeling at the macroscopic "continuum" level. Examples of these include deformation at residual state, tensile behavior, thermal effects, ageing, soil liquidisation and solidifi-

cation processes, granular flows, and erosion. Soil types that require more attention for modeling include fissured soils, mixed soils (clay, silt and sand mixtures), well graded (transitional) soils, very active clays (e.g., bentonite) from dry to wet state, layered soils, frozen soils, peat and organic soils, deep sea soils, methane hydrate soils and soils in deep oil/gas reservoirs.

Modeling the mechanical behavior of soils should be achieved based upon fundamental understanding of the underlying physical, chemical and sometimes biological mechanisms. Otherwise, the model parameters determined in the laboratory may not be applicable to the field, where the environment differs from the laboratory. For example, although many time dependent models have been proposed in the past, at the current time we still do not know the actual mechanisms of creep. Perhaps this is the main reason why time dependant models are seldom used in engineering practice even though we often see from field measurements that there is indeed time dependent behavior. Gudehus (2004) incorporated viscosity into the hypoplasticity concept to model creep, strain rate jumps and relaxation. By proposing that the hardness of the solid is strain rate dependent, he gave the physical mechanism of the time dependency as thermally activated cold shear melting.

Many geotechnical structures involve an interface between soil and a structure (like concrete). Compared to the amount of effort spent on modeling of the soil itself, there seems to be a limited work done in modeling the interface behavior.

It is evident looking across the micro-mechanics research that there is scope for better cross-linkages between the different approaches. DEM modelers need to reconsider their almost exclusive use of the Hertzian contact model, and collaboration between DEM modelers and researchers developing constitutive models that include fabric terms seems essential. For example, Oda's work provides tools for micro scale analysis that should be adopted by all DEM analysts.

DEM models can provide more guidance on how we model heterogeneities. Soil layering is often observed at different scales. Questions on this issue include (a) how does microscale layering affect measurements at the laboratory scale? and (b) how can we upscale the soil behavior observed at the soil specimen scale to the numerical grid scale for numerical analysis of layered soil profiles? The ability of DEM to include inhomogeneities needs to be utilized more so that a continuum model that considers heterogeneities from small strain to large strain can be developed. Modeling inhomogeneity can also be used to design soil improvement technologies.

There has been little consideration of fluid flow at the particle scale in Soils and Foundations. Various approaches for coupling DEM simulations with fluid flow are available (e.g., Tsuji et al., 1993). Analysis of mechanisms such as erosion that operate at the particle scale using DEM and upscaled "continuum" models are likely to be of interest in the future.

## CONCLUSIONS

This review of 50 years of papers published in *Soils and Foundations* demonstrates that significant effort has been made by many researchers to develop "macro" and "micro" soil models. However, it seems that we have not yet reached a point that we have developed the definitive soil model. Whether this can be achieved or not is not certain. It is clear that soil models are becoming more complex and/or made from new generalized theories. Often advanced models require many parameters and unless clear guidance is given on how to use them, the results obtained from the models will not be appreciated by the audience and it will be difficult to use them in engineering analysis. One approach that could be adopted to advance confidence in and acceptance of a model is to demonstrate the capability of the model by reproducing relevant empirical charts developed from experimental data and currently used in engineering practice. Model performance should also be demonstrated for undrained, partially drained and drained conditions. More active discussion on how a model can gain acceptance in engineering practice is needed within the geotechnical engineering community.

Although the success of geotechnical analysis depends on choice of an appropriate constitutive model for the problem, we should always be reminded that the use of an advanced soil model will not always give reliable predictions of displacements or the failures of geotechnical structures. This is because our ability to analyze and compute often exceeds our ability to understand, measure, and characterize the problem. Therefore, any geotechnical analysis has to be carried out with the right balance between model complexity and the available data. Great advances have been made recently in the area of experimental apparatus (e.g., high pressure-high temperature-multiphase element testing equipment, nonintrusive measurements such as micro computed tomography, micro-indentation, image analysis) and field instrumentation (e.g., geophysical methods, computer vision, fiber optics, miniature sensors, wireless sensor network). The efforts expended in developing soil models need to be coupled with measurements using these advanced techniques to find "robust" soil models that can be used in practice.

We appreciate that soil behavior is complex. Because of this, we are fascinated by new experimental findings and motivated by the challenge of modelling the observed complicated behavioural phenomena. It is our intellectual curiosity that drives this effort. However, we should not forget about other drivers. For example, economic drivers come from the fact that we need to develop and maintain geo-infrastructure to be safe but also less expensive than it was previously. There is a need to develop soil models that can provide good answers to this problem. There are also social drivers such as climate change, demographic growth, resource depletion, and energy security. We need to consider how the advance in soil modeling can help to solve problems related to these is-

sues.

## REFERENCES

- 1) Adachi, T. and Oka, F. (1982): Constitutive equations for normally consolidated clay based on elasto-viscoplasticity, *Soils and Foundations*, **22**(4), 57–70.
- 2) Adachi, T. and Okano, M. (1974): A constitutive equation for normally consolidated clay, *Soils and Foundations*, **14**(4), 55–73.
- 3) Adachi, T., Oka, F. and Mimura, M. (1987): Mathematical structure of an overstress elasto-viscoplastic model for clay, *Soils and Foundations*, **27**(3), 31–42.
- 4) Adachi, T., Oka, F. and Zhang, F. (1998): An elasto-viscoplastic constitutive model with strain softening, *Soils and Foundations*, **38**(2), 27–35.
- 5) Adachi, T., Oka, F. and Koike, M. (2005): An elasto-viscoplastic constitutive model with strain-softening for soft sedimentary rocks, *Soils and Foundations*, **45**(2), 125–133.
- 6) Ai, M. (1985): On mechanics of two dimensional rigid assemblies, *Soils and Foundations*, **25**(4), 49–62.
- 7) Al-Shamrani, M. A. and Sture, S. (1998): A time-dependent bounding surface model for anisotropic cohesive soils, *Soils and Foundations*, **38**(1), 61–76.
- 8) Anandarajah, A. (1994): Granular material model based on associated flow rule applied to monotonic loading behavior, *Soils and Foundations*, **34**(3), 81–98.
- 9) Asaoka, A., Kodaka, T. and Nozu, M. (1994): Soil-water coupled behaviour of saturated clay near/at critical state, *Soils and Foundations*, **34**(1), 91–105.
- 10) Asaoka, A. and Noda, T. (1995): Imperfection-sensitive bifurcation of Cam-clay under plane strain compression with undrained boundaries, *Soils and Foundations*, **35**(1), 83–100.
- 11) Asaoka, A., Nakano, M. and Noda, T. (1995): Annealable behaviour of saturated clay: An experiment and simulation, *Soils and Foundations*, **35**(4), 9–20.
- 12) Asaoka, A., Nakano, M. and Noda, T. (1997): Soil-water coupled behaviour of heavily overconsolidated clay near/at critical state, *Soils and Foundations*, **37**(1), 13–28.
- 13) Asaoka, A., Nakano, M. and Noda, T. (2000a): Superloading yield surface concept for highly structured soil behavior, *Soils and Foundations*, **40**(2), 99–110.
- 14) Asaoka, A., Nakano, M., Noda, T. and Kaneda, K. (2000b): Delayed compression/consolidation of natural clay due to degradation of soil structure, *Soils and Foundations*, **40**(3), 75–85.
- 15) Asaoka, A., Noda, T., Yamada, E., Kaneda, K. and Nakano, M. (2002): An elasto-plastic description of two distinct volume change mechanisms of soils, *Soils and Foundations*, **42**(5), 47–57.
- 16) Bauer, E. (1996): Calibration of a comprehensive hypoplastic model for granular materials, *Soils and Foundations*, **36**(1), 13–26.
- 17) Calladine, C. R. (1971): A microstructural view of the mechanical properties of saturated clay, *Géotechnique*, **21**, 391–415.
- 18) Chavez, C. and Alonso, E. E. (2003): A constitutive model for crushed granular aggregates which includes suction effects, *Soils and Foundations*, **43**(4), 215–227.
- 19) Chang, C. and Liao, C. (1990): Constitutive relations for particulate medium with the effect of particle rotation, *International Journal of Solids and Structures*, **26**(4), 437–453.
- 20) Chen, Y.-C. and Ishibashi, I. (1990): Dynamic shear modulus and evolution of fabric of granular materials *Soils and Foundations*, **30**(3), 1–10.
- 21) Chen, Y.-C. and Hung, H. Y. (1991): Evolution of shear modulus and fabric during shear deformation, *Soils and Foundations*, **31**(4), 148–160.
- 22) Cheng, Y., Nakata, Y. and Bolton, M. (2003): Discrete element-simulation of crushable soil, *Géotechnique*, **53**(7), 633–641.
- 23) Chowdhury, E. Q., Nakai, T., Tawada, M. and Yamada, S. (1999): A model for clay using modified stress under various loading conditions with the application of subloading concept, *Soils and Foundations*, **39**(6), 103–116.
- 24) Collins, I. F. and Houlsby, G. T. (1997): Application of thermomechanical principles to the modelling of geotechnical materials, *Proc. Royal Society of London*, Series A, 453.
- 25) Cundall, P. and Strack, O. (1979): A discrete numerical model for granular assemblies, *Géotechnique*, **29**(1), 47–65.
- 26) Cundall, P. A. (2001): A discontinuous future for numerical modelling in geomechanics? *Geotechnical Engineering Proceedings of the Institution of Civil Engineers*, **149**(1), 41–47.
- 27) Dafalias, Y. F. and Popov, E. P. (1975): A model of nonlinearly hardening materials for complex loading, *Acta Mech.*, **21**, 173–192.
- 28) Dantu, P. (1968): Etude statistique des forces intergranulaires dans un milieu pulvérulent, *Géotechnique*, **18**(1), 50–55.
- 29) Di Benedetto, H. and Tatsuoka, F. (1997): Small strain behavior of geomaterials: Modelling of strain rate effects, *Soils and Foundations*, **37**(2), 127–138.
- 30) Di Benedetto, H., Tatsuoka, F. and Ishihara, M. (2002): Time-dependent shear deformation characteristics of sand and their constitutive modelling, *Soils and Foundations*, **42**(2), 1–22.
- 31) Drescher, A. and de Josselin de Jong, G. (1972): Photoelastic verification of a mechanical model for the flow of a granular material, *Journal of the Mechanics and Physics of Solids*, **20**(5), 337–340.
- 32) Drucker, D. C., Gibson, R. E. and Henkel, D. J. (1957): Soil mechanics and work hardening theories of plasticity, *Transactions of the American Society of Civil Engineers*, **122**, 338–346.
- 33) Einav, I. and Puzrin A. M. (2002): Discussion on Hashiguchi K. and Collins I. (2002), Stress rate-elastic stretching relations in elastoplastic constitutive equations for soils, *Soils and Foundations*, **42**(1), 159–160.
- 34) Galavi, V. and H. Schweiger, F. (2009): A multilaminate model with destructuration considering anisotropic strength and anisotropic bonding, *Soils and Foundations*, **49**(3), 341–353.
- 35) Gallage, C. P. K., Towhata, I. and Nishimura, S. (2005): Laboratory investigation on rate-dependent properties of sand undergoing low confining effective stress, *Soils and Foundations*, **45**(4), 43–60.
- 36) Gerolymos, N., Vardoulakis, I. and Gazetas, G. (2007): A thermo-poro-visco-plastic shear band model for seismic triggering and evolution of catastrophic landslides, *Soils and Foundations*, **47**(1), 11–25.
- 37) Gudehus, G. (1996): Comprehensive constitutive equation for granular materials, *Soils and Foundations*, **36**(1), 1–12.
- 38) Gudehus, G. (2004): A visco-hypoplastic constitutive relation for soft soils, *Soils and Foundations*, **44**(4), 11–25.
- 39) Guo, P. J. (2009): Energy approach towards stress-dilatancy formulation based on double slip-rotation rate concept, *Soils and Foundations*, **49**(3), 443–457.
- 40) Guo, P. J. and Stolle, D. F. E. (2004): The extension of Rowe's stress-dilatancy model to general stress condition, *Soils and Foundations*, **44**(4), 1–10.
- 41) Guo, P. J. and Stolle, D. F. E. (2005): On the failure of granular materials with fabric effects, *Soils and Foundations*, **45**(4), 1–12.
- 42) Gutierrez, M., Ishihara, K. and Towhata, I. (1993): Model for the deformation of sand during rotation of principal stress directions, *Soils and Foundations*, **33**(3), 105–117.
- 43) Gutierrez, M. and Ishihara, K. (2000): Non-coaxiality and energy dissipation in granular materials, *Soils and Foundations*, **40**(2), 49–59.
- 44) Han, C. and Drescher, A. (1993): Shear bands in biaxial tests on dry coarse sand, *Soils and Foundations*, **33**(1), 118–132.
- 45) Hardin, B. O. and Drnevich, V. P. (1972): Shear modulus and damping in soils: design equations and curves, *Journal of the Soil Mechanics and Foundations Division*, American Society of Civil Engineers, **98**(7), 667–692.
- 46) Hashiguchi, K. and Ueno, M. (1977): Elastoplastic constitutive laws of granular materials, constitutive equations of soils, *Proc. 9th Int. Conf. Soil Mech. Found. Eng.*, Special Session 9, JSSMFE, Tokyo, 73–82.

- 47) Hashiguchi, K. (1989): Subloading surface model in unconventional plasticity, *Int. J. Solids Struct.*, **25**, 917–945.
- 48) Hashiguchi, K. (2000): Fundamentals in constitutive equation: Continuity and smoothness conditions and loading criterion, *Soils and Foundations*, **40**(4), 155–161.
- 49) Hashiguchi, K. and Okayasu, T. (2000): Time-dependent elastoplastic constitutive equation based on the subloading surface model and its application to soils, *Soils and Foundations*, **40**(4), 19–36.
- 50) Hashiguchi, K. and Collins, I. F. (2001): Stress rate-elastic stretching relations in elastoplastic constitutive equations for soils, *Soils and Foundations*, **41**(2), 77–87.
- 51) Hashiguchi, K. (2001a): Description of inherent/induced anisotropy of soils: Rotational hardening rule with objectivity, *Soils and Foundations*, **41**(6), 139–145.
- 52) Hashiguchi, K. (2001b): On the thermomechanical approach to the formulation of plastic constitutive equations, *Soils and Foundations*, **41**(4), 89–94.
- 53) Hashiguchi, K. (2003): On the replacement of material-time derivative to corotational rate of yield function: Mathematical proof, *Soils and Foundations*, **43**(5), 189–194.
- 54) Hirai, H. (1987): Modelling of cyclic behaviour of sand with combined hardening, *Soils and Foundations*, **27**(2), 1–11.
- 55) Hirai, H. (1989): Combined hardening model for anisotropically consolidated clays, *Soils and Foundations*, **29**(3), 14–24.
- 56) Hirai, H., Takahashi, M. and Yamada, M. (1989): Elastic-plastic constitutive model for the behavior of improved sandy soils, *Soils and Foundations*, **29**(2), 69–84.
- 57) Hoque, E. and Tatsuoka, F. (1998): Anisotropy in elastic deformation of granular materials, *Soils and Foundations*, **38**(1), 163–179.
- 58) Horne, M. R. (1965): The behaviour of an assembly of rotound, rigid, cohesionless particles, I and II, *Proc. Royal Society of London*, Series A, Mathematical and Physical Sciences, **286**(1404), 62–97.
- 59) Horne, M. R. (1969): The behaviour of an assembly of rotound, rigid, cohesionless particles, III, *Proc. Royal Society of London*, Series A, Mathematical and Physical Sciences, **310**(1500), 21–34.
- 60) Houlby, G. T. and Wroth, C. P. (1991): Variation of shear modulus of a clay with pressure and overconsolidation ratio, *Soils and Foundations*, **31**(3), 138–143.
- 61) Houlby, G. T. and Puzrin, A. M. (2006): *Principles of Hyperplasticity: An Approach to Plasticity Theory Based on Thermodynamic Principles*, Springer Verlag, London, p. 375.
- 62) Hueckel, T. and Pellegrini, R. (1991): Thermoplastic modeling of undrained failure of saturated clay due to heating, *Soils and Foundations*, **31**(3), 1–16.
- 63) Iai, S., Matsunaga, Y. and Kameoka, T. (1992): Strain space plasticity model for cyclic mobility, *Soils and Foundations*, **32**(2), 1–15.
- 64) Iai, S. (1993a): Three dimensional formulation and objectivity of a strain space multiple mechanism model for sand, *Soils and Foundations*, **33**(1), 192–199.
- 65) Iai, S. (1993b): Concept of effective strain in constitutive modeling of granular materials, *Soils and Foundations*, **33**(2), 171–180.
- 66) Iai, S. (1993c): Micromechanical background to a strain space multiple mechanism model for sand, *Soils and Foundations*, **33**(1), 102–117.
- 67) Ibraim, E., Lanier, J., Muir Wood, D. and Viggiani, G. (2010): Strain path controlled shear tests on an analogue granular material, *Géotechnique*, **60**(7), 545–559.
- 68) Iizuka, A. and Ohta, H. (1987): Determination procedure of input parameters in elasto-viscoplastic finite element analysis, *Soils and Foundations*, **27**(3), 71–87.
- 69) Ikeda, K. and Murota, K. (1996): Bifurcation as sources of uncertainty in soil shearing behavior, *Soils and Foundations*, **36**(1), 73–84.
- 70) Ikeda, K. and Murota, K. (1997): Recursive bifurcation as sources of complexity in soil shearing behaviour, *Soils and Foundations*, **37**(3), 17–29.
- 71) Imai, G. and Xie, C. (1990): Endochronic constitutive law for static shear behavior of overconsolidated clays, *Soils and Foundations*, **30**(1), 65–75.
- 72) Imai, G. and Xie, C. (1991): Endochronic modeling for cyclic behavior of overconsolidated clays, *Soils and Foundations*, **31**(4), 103–114.
- 73) Ishibashi, I. and Zhang, X. (1993): Unified dynamic shear moduli and damping ratios of sand and clay, *Soils and Foundations*, **33**(1), 182–191.
- 74) Iwasaki, T., Tatsuoka, F. and Takagi, Y. (1978): Shear moduli of sands under cyclic torsional shear loading, *Soils and Foundations*, **18**(1), 39–56.
- 75) Jardine, R. J. (1992): Some observations on the kinematic nature of soil stiffness, *Soils and Foundations*, **32**(2), 111–124.
- 76) Kaliakin, V. N. and Dafalias, Y. F. (1990a): Theoretical aspects of the elastoplastic-viscoplastic bounding surface model for cohesive soils, *Soils and Foundations*, **30**(3), 11–24.
- 77) Kaliakin, V. N. and Dafalias, Y. F. (1990b): Verification of the elastoplastic-viscoplastic bounding surface model for cohesive soils, *Soils and Foundations*, **30**(3), 25–36.
- 78) Kanatani, K. (1981): A theory of contact force distributions in granular materials, *Powder Technology*, **28**, 167–172.
- 79) Kasama, K., Ochiai, H. and Yasufuku, N. (2000): On the stress-strain behaviour of lightly cemented clay based on an extended critical state concept, *Soils and Foundations*, **40**(5), 37–47.
- 80) Kimoto, S. and Oka, F. (2005): An elasto-viscoplastic model for clay considering destructuration and consolidation analysis of unstable behavior, *Soils and Foundations*, **45**(2), 29–42.
- 81) Kishino, Y. (1989): Investigation of quasi-static behavior of granular materials with a new simulation method, *Proc. of JSCE*, **406**, 97–196 (In Japanese).
- 82) Kitamura, R. (1981a): A mechanical model of particulate material based on stochastic process, *Soils and Foundations*, **21**(2), 63–72.
- 83) Kitamura, R. (1981b): Analysis of deformation mechanism of particulate material at particle scale, *Soils and Foundations*, **21**(2), 85–98.
- 84) Kiyama, S. and Hasegawa, T. (1998): A two-surface model with anisotropic hardening and nonassociated flow rule for geomaterials, *Soils and Foundations*, **38**(1), 45–59.
- 85) Kobayashi, I., Soga, K., Iizuka, A. and Ohta, H. (2003): Numerical interpretation of a shape of yield surface obtained from stress probe tests, *Soils and Foundations*, **43**(3), 95–104.
- 86) Kohgo, Y., Nakano, M. and Miyazaki, T. (1993a): Theoretical aspects of constitutive modelling for unsaturated soils, *Soils and Foundations*, **33**(4), 49–63.
- 87) Kohgo, Y., Nakano, M. and Miyazaki, T. (1993b): Verification of the generalized elastoplastic model for unsaturated soils, *Soils and Foundations*, **33**(4), 64–73.
- 88) Kohgo, Y., Asano, I. and Hayashida, Y. (2007): An elastoplastic model for unsaturated rockfills and its simulations of laboratory tests, *Soils and Foundations*, **47**(5), 919–929.
- 89) Kongkitkul, W., Tatsuoka, F., Duttine, A., Kawabe, S., Enomoto, T. and Benedetto, H. D. (2008): Modelling and simulation of rate-dependent stress-strain behaviour of granular materials in shear, *Soils and Foundations*, **48**(2), 175–194.
- 90) Lade, P. V. (1992): Discussion: A general failure criterion and stress strain relation for granular materials to metals by Matsuo, H. et al., *Soils and Foundations*, **32**(1), 225–230.
- 91) Lade, P. V. and Prabucki, M. J. (1995): Softening and preshearing effects in sand, *Soils and Foundations*, **35**(4), 93–104.
- 92) Lee, K., Chan, D. and Lam, K. (2004): Constitutive model for cement treated clay in a critical state frame work, *Soils and Foundations*, **44**(3), 69–77.
- 93) Lobo-Guerrero, S. and Vallejo, L. E. (2005): Analysis of crushing of granular material under isotropic and biaxial stress conditions, *Soils and Foundations*, **45**(4), 79–88.
- 94) Lobo-Guerrero, S. and Vallejo, L. E. (2006): Modeling granular crushing in ring shear tests: experimental and numerical analyses, *Soils and Foundations*, **46**(2), 147–158.
- 95) Maeda, K., Miura, K. and Toki, S. (1995): Mechanical properties

- of elliptic microstructure formed in granular materials, *Soils and Foundations*, **25**(2), 1–13.
- 96) Maekawa, H., Miyakita, K. and Ohta, H. (1992): Yield locus of a diatomaceous mudstone with artificially made plane of separation, *Soils and Foundations*, **32**(2), 21–34.
  - 97) Masin, D. (2009): Comparison of predictive capabilities of selected elasto-plastic and hypoplastic models for structured clays, *Soils and Foundations*, **49**(3), 381–390.
  - 98) Matsui, M. (1988): Constitutive model for cyclic viscoplasticity of soils, *Soils and Foundations*, **28**(4), 19–37.
  - 99) Matsui, T. and Abe, N. (1981): Multi-dimensional elasto-plastic consolidation analysis by finite element method, *Soils and Foundations*, **21**(1), 79–95.
  - 100) Matsuo, O., Shimazu, T., Uzuoka, R., Mihara, M. and Nishi, K. (2000): Numerical analysis of seismic behavior of embankments founded on liquefiable soils, *Soils and Foundations*, **40**(2), 21–39.
  - 101) Matsuoka, H. (1974a): A microscopic study on shear mechanism of granular materials, *Soils and Foundations*, **14**(1), 29–43.
  - 102) Matsuoka, H. (1976): On the significance of the “spatial mobilized plane”, *Soils and Foundations*, **16**(1), 91–100.
  - 103) Matsuoka, H. and Geka, H. (1983): Stress-strain model for granular materials considering mechanism of fabric change, *Soils and Foundations*, **23**(2), 83–97.
  - 104) Matsuoka, H. and Nakai, T. (1985): Relationship among Tresca, Mises, Mohr-Coulomb and Matsuoka and Nakai criteria, *Soils and Foundations*, **25**(4), 123–128.
  - 105) Matsuoka, H., Koyama, H. and Yamazaki, H. (1985): Constitutive equation for sands and its application to analyses of rotational stress paths and liquefaction resistance, *Soils and Foundations*, **25**(1), 27–42.
  - 106) Matsuoka, H. and Sakakibara, K. (1987): Constitutive model for sands and clays evaluating principal stress rotation, *Soils and Foundations*, **27**(4), 73–88.
  - 107) Matsuoka, H., Suzuki, Y. and Murata, T. (1990): Constitutive model for soils evaluating principal stress rotation and its application to some deformation problems, *Soils and Foundations*, **30**(1), 142–154.
  - 108) Matsuoka, H. and Sun, D. A. (1995): Extension of Spatially Mobilized Plane (SMP) to frictional and cohesive materials and its application to cemented sands, *Soils and Foundations*, **35**(4), 63–72.
  - 109) Matsushima, T., Ishii, T. and Konagai, K. (2002): Observation of grain motion in the interior of a PSC test specimen by laser aided tomography, *Soils and Foundations*, **42**(4), 27–36.
  - 110) Matsushima, T., Saomoto, H., Tsubokawa, Y. and Yamada, Y. (2003): Grain rotations versus continuum rotation during shear deformation of a granular assembly, *Soils and Foundations*, **43**(4), 95–106.
  - 111) McDowell, G. R. (2000): A family of yield loci based on micro mechanics, *Soils and Foundations*, **40**(6), 133–137.
  - 112) McDowell, G. R. and Bolton, M. D. (2001): Micro mechanics of elastic soil, *Soils and Foundations*, **41**(6), 147–152.
  - 113) Mirghasemi, A. A., Rothenburg, L. and Matyas, E. L. (1997): Numerical simulations of assemblies of two-dimensional polygon shaped particles and effects of confining pressure on shear strength, *Soils and Foundations*, **37**(3), 43–52.
  - 114) Miura, S. and Toki, S. (1984): Elastoplastic stress-strain relationship for loose sands with anisotropic fabric under three-dimensional stress conditions, *Soils and Foundations*, **24**(2), 43–57.
  - 115) Miura, K., Toki, S. and Miura, S. (1986): Deformation prediction for anisotropic sand during the rotation of principal stress axes, *Soils and Foundations*, **26**(3), 42–56.
  - 116) Mogami, T. (1965): A statistical approach to the mechanics of granular materials, *Soils and Foundations*, **5**(2), 26–36.
  - 117) Mogami, T. (1968): On the critical state line proposed by Roscoe et al., *Soils and Foundations*, **8**(4), 1–9.
  - 118) Moriguchi, S., Yashima, A., Sawada, K., Uzuoka, R. and Ito, M. (2005): Numerical simulation of flow failure of geomaterials based on fluid dynamics, *Soils and Foundations*, **45**(2), 155–165.
  - 119) Moroto, N. (1987): On deformation of granular material in simple shear, *Soils and Foundations*, **27**(1), 77–85.
  - 120) Muhunthan, B., Chameau, J. L. and Masad, E. (1996): Fabric effects on the yield behavior of soils, *Soils and Foundations*, Japanese Society of Geotechnical Engineering, **36**(3), 85–97.
  - 121) Murayama, S. (1977): Constitutive equations of soils, *Proc. 9th International Conference on Soil Mechanics and Foundation Engineering*, **3**, 537–544.
  - 122) Murayama, S. (1983): Formulation of stress-strain-time behavior of soils under deviatoric stress condition, *Soils and Foundations*, **23**(2), 43–57.
  - 123) Murayama, S., Michihiro, K. and Sakagami, T. (1984): Creep characteristics of sands, *Soils and Foundations*, **24**(2), 1–15.
  - 124) Nakai, T. and Matsuoka, H. (1983): Constitutive equation for soils based on the extended concept of ‘spatial mobilized plane’ and its application to finite element analysis, *Soils and Foundations*, **23**(4), 87–105.
  - 125) Nakai, T. and Mihara, Y. (1984): New mechanical quantity for soils and its application to elastoplastic constitutive models, *Soils and Foundations*, **24**(2), 82–94.
  - 126) Nakai, T. and Matsuoka, H. (1986): Generalized elastoplastic constitutive model for clay in three-dimensional stresses, *Soils and Foundations*, **26**(3), 81–98.
  - 127) Nakai, T., Matsuoka, H., Okuno, N. and Tsuzuki, K. (1986): True triaxial tests on normally consolidated clay and analysis of the observed shear behavior using elastoplastic constitutive models, *Soils and Foundations*, **26**(4), 67–78.
  - 128) Nakai, T. and Hinokio, M. (2004): A simple elastoplastic model for normally and over consolidated soils with unified material parameters, *Soils and Foundations*, **44**(2), 53–70.
  - 129) Nakano, M., Nakai, K., Noda, T. and Asaoka, A. (2005): Simulation of shear and one-dimensional compression behavior of naturally deposited clays by Super/subloading Yield Surface Cam-clay model, *Soils and Foundations*, **45**(1), 141–151.
  - 130) Nemat-Nasser, S. (1980): On behavior of granular materials in simple shear, *Soils and Foundations*, **20**(3), 59–73.
  - 131) Newson, T. A. and Davies, M. C. R. (1996): A rotational hardening constitutive model for anisotropically consolidated clay, *Soils and Foundations*, **36**(3), 13–20.
  - 132) Nishi, K. and Kanatani, M. (1990): Constitutive relations for sand under cyclic loading based on elasto-plasticity theory, *Soils and Foundations*, **30**(2), 43–59.
  - 133) Nishimura, S. and Towhata, I. (2004): A three-dimensional stress-strain model of sand undergoing cyclic rotation of principal stress axes, *Soils and Foundations*, **44**(2), 103–116.
  - 134) Noda, T., Yamada, S. and Asaoka, A. (2005): Elasto-plastic behavior of naturally deposited clay during/after sampling, *Soils and Foundations*, **45**(1), 51–64.
  - 135) Oda, M. (1972a): The mechanism of fabric changes during compressional deformation of sand, *Soils and Foundations*, **12**(2), 1–18.
  - 136) Oda, M. (1972b): Initial fabrics and their relations to mechanical properties of granular materials, *Soils and Foundations*, **12**(1), 17–36.
  - 137) Oda, M. (1974): A mechanical and statistical model of granular material, *Soils and Foundations*, **14**(1), 13–27.
  - 138) Oda, M. and Konishi, J. (1974a): Microscopic deformation mechanism of granular material in simple shear, *Soils and Foundations*, **14**(4) 25–38.
  - 139) Oda, M. and Konishi, J. (1974b): Rotation of principal stresses in granular material during simple shear, *Soils and Foundations*, **14**(4), 39–53.
  - 140) Oda, M. (1975): On stress-dilatancy relation of sand in simple shear test, *Soils and Foundations*, **15**(2), 17–29.
  - 141) Oda, M. (1977): Coordination number and its relation to shear strength of granular material, *Soils and Foundations*, **17**(2), 29–42.
  - 142) Oda, M., Konishi, J. and Nemat-Nasser, S. (1980): Some experimentally based fundamental results on the mechanical behaviour of granular materials, *Géotechnique*, **30**(4), 479–495.
  - 143) Oda, M. (1982): Fabric tensor for discontinuous geological materi-

- als, *Soils and Foundations*, **22**(4), 96–108.
- 144) Oda, M. (1984): Similarity rule of crack geometry in statistically homogeneous rock masses, *Mechanics of Materials*, **3**(2), 119–120.
- 145) Oda, M., Suzuki, K. and Maeshibu, T. (1984): Elastic compliance for rock like materials with random cracks, *Soils and Foundations*, **24**(3), 27–40.
- 146) Oda, M., Nemat-Nasser, S. and Konishi, J. (1985): Stress-induced anisotropy in granular masses, *Soils and Foundations*, **25**(3), 88–97.
- 147) Ohta, H. and Hata, S. (1971): A theoretical study of the stress-strain relations for clays, *Soils and Foundations*, **11**(3), 195–219.
- 148) Ohta, H. and Nishihara, A. (1985): Anisotropy of undrained shear strength of clays under axis-symmetric loading conditions, *Soils and Foundations*, **25**(2), 73–86.
- 149) Oka, F., Leroueil, S. and Tavenas, F. (1989): Constitutive model for natural soft clay with strain softening, *Soils and Foundations*, **29**(3), 54–66.
- 150) Oka, F., Kimoto, S., Kobayashi, H. and Adachi, T. (2002): Anisotropic behavior of soft sedimentary rock and a constitutive model, *Soils and Foundations*, **42**(5), 59–70.
- 151) Oka, F., Kodaka, T., Kimoto, S., Ishigaki, S. and Tsuji, C. (2003): Step-changed strain rate effect on the stress-strain relations of clay and a constitutive modeling, *Soils and Foundations*, **43**(4), 189–202.
- 152) O'Sullivan, C., Cui, L. and O'Neill, S. C. (2008): Discrete element analysis of the response of granular materials during cyclic loading, *Soils and Foundations*, **48**(4), 51–530.
- 153) Papadimitriou, A. G., Dafalias, Y. F. and Yoshimine, M. (2005): Plasticity modeling of the effect of sample preparation method on sand response, *Soils and Foundations*, **45**(2), 109–123.
- 154) Pedroso, D. M., Farias, M. M. and Nakai, T. (2005): An interpretation of subloading  $t_{ij}$  model in the context of conventional elastoplasticity theory, *Soils and Foundations*, **45**(4), 61–77.
- 155) Peng, F., Siddiquee, M. S. A., Tatsuoka, F., Yasin, S. J. M. and Tanaka, T. (2009): Strain energy-based elasto-viscoplastic constitutive modelling of sand for numerical simulation, *Soils and Foundations*, **49**(4), 611–629.
- 156) Perzyna, P. (1963): The constitutive equations for rate-sensitive plastic materials, *Quart. Appl. Math.*, **20**, 321–332.
- 157) Poorooshasb, H. B. and Pietruszczak, S. (1986): Generalized flow theory for sand, *Soils and Foundations*, **26**(2), 1–15.
- 158) Pradhan, T. B. S. and Tatsuoka, F. (1989): On stress-dilatancy equations of sand subjected to cyclic loading, *Soils and Foundations*, **29**(1), 65–81.
- 159) Puzrin, A. and Tatsuoka, F. (1998): Elastic strain energy potential for uncemented granular materials, *Soils and Foundations*, **38**(4), 267–274.
- 160) Robertson, D. (2000): Numerical simulations of crushable aggregates. *PhD dissertation*, University of Cambridge.
- 161) Rojas, E. and Garnica, P. (2000): Thermomechanical anisotropic model for soils, *Soils and Foundations*, **40**(2), 61–75.
- 162) Roscoe, K. H., Schofield, A. N. and Wroth, C. P. (1958): On the yielding of soils, *Géotechnique*, **8**(1), 22–52.
- 163) Roscoe, K. H., Schofield, A. N. and Thurairajah, A. (1963): Yielding of clays in states wetter than critical, *Géotechnique*, **13**(3), 211–240.
- 164) Rowe, P. W. (1962): The stress-dilatancy relation for static equilibrium of an assembly of particles of contact, *Proc. Roy. Soc. London*, **A269**, 500–527.
- 165) Sakaguchi, H., Murakami, A., Hasegawa, T. and Shirai, A. (1996): Connected lattice cellular automaton particle: a model for pattern formation in vibrating granular media, *Soils and Foundations*, **36**(1), 105–110.
- 166) Satake, K. (1978): Constitution of mechanics of granular materials through graph representation, *Theoretical and Applied Mechanics*, **26**, 257–266.
- 167) Schofield, A. N. and Wroth, C. P. (1968): *Critical State Soil Mechanics*, McGraw-Hill, London.
- 168) Sekiguchi, H. and Ohta, H. (1977): Induced anisotropy and time dependency in clays, *Proc. of Specialty Session 9, 9th International Conference Soil Mechanics and Foundation Engineering*, Tokyo, 475–484.
- 169) Sekiguchi, H. (1984): Theory of undrained creep rupture of normally consolidated clay based on elasto-viscoplasticity, *Soils and Foundations*, **24**(1), 129–147.
- 170) Setouchi, H., Hashiguchi, K., Shikanai, T. and Okayasu, T. (2006): Extended tangential-subloading surface model for general loading behavior of soils: Application to nonproportional loadings, *Soils and Foundations*, **46**(4), 449–463.
- 171) Shamoto, Y., Zhang, J. M. and Goto, S. (1997): Mechanism of large post-liquefaction deformation in saturated sand, *Soils and Foundations*, **37**(2), 71–80.
- 172) Sun, D. A., Matsuoka, H., Yao, Y. P. and Ichihara, W. (2000): An elasto-plastic model for unsaturated soil in three-dimensional stresses, *Soils and Foundations*, **40**(3), 17–28.
- 173) Takahara, T. and Miura, K. (1998): Mechanical characteristics of EPS block fill and its simulation by DEM and FEM, *Soils and Foundations*, **38**(1), 97–110.
- 174) Tamura, T. and Yamada, Y. (1996): A rigid-plastic analysis for granular materials, *Soils and Foundations*, **36**(3), 113–121.
- 175) Tatsuoka, F. (1975): Dilatancy characteristics of soil, *Soils and Foundations*, **15**(2), 73–78.
- 176) Tatsuoka, F. (1976): Stress-dilatancy relations of anisotropic sands in three dimensional stress condition, *Soils and Foundations*, **16**(2), 1–18.
- 177) Tatsuoka, F. (1980): Stress-strain behavior of an idealized anisotropic granular material, *Soils and Foundations*, **20**(3), 75–90.
- 178) Tatsuoka, F., Siddiquee, M. S. A., Park, C. S., Sakamoto, M. and Abe, F. (1993): Modelling stress-strain relations of sand, *Soils and Foundations*, **33**(2), 60–81.
- 179) Tatsuoka, F., Ishihara, M., Di Benedetto, H. and Kuwano, R. (2002): Time-dependent shear deformation characteristics of geomaterials and their simulation, *Soils and Foundations*, **42**(2), 103–129.
- 180) Tatsuoka, F., Nawir, H. and Kuwano, R. (2004): A modelling procedure for shear yielding characteristics affected by viscous properties of sand in triaxial compression, *Soils and Foundations*, **44**(6), 83–89.
- 181) Tatsuoka, F., Dibenedetto, H., Enomoto, T., Kawabe, S. and Kongkitkul, W. (2008a): Various viscosity types of geomaterials in shear and their mathematical expression, *Soils and Foundations*, **48**(1), 41–60.
- 182) Tatsuoka, F., Dibenedetto, H., Kongkitkul, W., Kongsukprasert, L., Nishi, T. and Sano, Y. (2008b): Modelling of ageing effects on the elasto-viscoplastic behaviour of geomaterial, *Soils and Foundations*, **48**(2), 155–174.
- 183) Taylor, D. W. (1948): *Fundamentals of Soil Mechanics*, John Wiley, New York.
- 184) Terzaghi, K. and Peck, R. B. (1948): *Soil Mechanics in Engineering Practice*, John Wiley, New York.
- 185) Thornton, C. (2000): Numerical simulations of deviatoric shear deformation of granular media, *Géotechnique*, **50**(1) 43–53.
- 186) Tobita, Y. (1988): Yield condition of anisotropic granular materials, *Soils and Foundations*, **28**(2), 113–126.
- 187) Tobita, Y. (1989): Fabric tensors in constitutive equations for granular materials, *Soils and Foundations*, **29**(4), 91–104.
- 188) Tobita, Y. and Yanagisawa, E. (1992): Modified stress tensors for anisotropic behavior of granular materials, *Soils and Foundations*, **32**(1), 85–99.
- 189) Tokue, T. (1979): Stress-dilatancy model of granular material under general stress condition, *Soils and Foundations*, **19**(1), 63–80.
- 190) Topolnicki, M. (1990): Elasto-plastic subloading surface model for clay with isotropic and kinematic mixed hardening parameters, *Soils and Foundations*, **30**(2), 103–113.
- 191) Tordesillas, A. and Muthuswamy, M. (2009): On the modeling of confined buckling of force chains, *Journal of the Mechanics and Physics of Solids*, **57**(4), 706–727.
- 192) Towhata, I., Sasaki, Y., Tokida, K., Matsumoto, H., Tamari, Y.

- and Yamada, K. (1992): Prediction of permanent displacement of liquefied ground by means of minimum energy principle, *Soils and Foundations*, **32**(3), 97–116.
- 193) Toyota, H., Nakamura, K. and Sramoon, W. (2004): Failure criterion of unsaturated soil considering tensile stress under three-dimensional stress conditions, *Soils and Foundations*, **44**(5), 1–13.
  - 194) Tsuji, Y., Kawaguchi, T. and Tanaka, T. (1993): Discrete particle simulation of two-dimensional fluidized bed, *Powder Technology*, **77**(1), 79–87.
  - 195) Vardoulakis, I. and Georgopoulos, I. O. (2005): The “stress-dilatancy” hypothesis revisited: Shear-banding related instabilities, *Soils and Foundations*, **45**(2), 61–76.
  - 196) Wan, R., Guo, P. and Al-Mamun, M. (2005): Behaviour of granular materials in relation to their fabric dependencies, *Soils and Foundations*, **45**(2), 77–86.
  - 197) Watts, D. J. (2004): *Six Degrees: The Science of a Connected Age*. Vintage Books.
  - 198) Yamaguchi, H. (1975): *Dan-Sosei Rikigaku* (Elasticity and Plasticity), Morikita-Shuppan, in Japanese.
  - 199) Yao, Y. P., Yamamoto, H. and Wang, N. D. (2008): Constitutive model considering sand crushing, *Soils and Foundations*, **48**(4), 603–608.
  - 200) Yashima, A., Leroueil, S., Oka, F. and Gunter, I. (1998): Modelling temperature and strain rate dependent behavior of clays: One dimensional consolidation, *Soils and Foundations*, **38**(2), 63–73.
  - 201) Yasufuku, N., Murata, H. and Hyodo, M. (1991a): Yield characteristics of anisotropically consolidated sand under low and high stresses, *Soils and Foundations*, **31**(1), 95–109.
  - 202) Yasufuku, N., Murata, H., Hyodo, M. and Hyde, A. F. L. (1991b): Stress strain relationship for anisotropically consolidated sand over a wide stress region, *Soils and Foundations*, **31**(4), 75–92.
  - 203) Yatomi, C. and Nishihara, A. (1984): Principles for constitutive equations and expressions of anisotropy in soil materials, *Soils and Foundations*, **24**(3), 15–26.
  - 204) Yatomi, C., Yashima, A., Iizuka, A. and Sano, I. (1989a): General theory of shear bands formation by a non-coaxial Cam-clay model, *Soils and Foundations*, **29**(3), 41–53.
  - 205) Yatomi, C., Yashima, A., Iizuka, A. and Sano, I. (1989b): Shear bands formation numerically simulated by a non-coaxial Cam-clay model, *Soils and Foundations*, **29**(4), 1–13.
  - 206) Yimsiri, S. and Soga, K. (2000): Micromechanics-based stress-strain behaviour of soils at small strains. *Géotechnique*, **50**(5), 559–571.
  - 207) Yimsiri, S. and Soga, K. (2002): Application of micromechanics model to study anisotropy of soils at small strain, *Soils and Foundations*, **42**(5), 15–26.
  - 208) Yu, Y., Pu, J. and Ugai, K. (1998): A damage model for soil-cement mixture, *Soils and Foundations*, **38**(3), 1–12.
  - 209) Zhang, J., Shamoto, Y. and Tokimatsu, K. (1997): Moving critical and phase-transformation stress state lines of saturated sand during undrained cyclic shear, *Soils and Foundations*, **37**(2), 51–59.
  - 210) Zhang, F., Yashima, A., Ye, G. L., Adachi, T. and Oka, F. (2003): An elastoplastic strain-hardening and strain-softening constitutive model for soft rock considering the influence of intermediate stress, *Soils and Foundations*, **43**(5), 107–117.
  - 211) Zhang, F., Yashima, A., Nakai, T., Ye, G. L. and Aung, H. (2005): An elasto-viscoplastic model for soft sedimentary rock based on  $t_{ij}$  concept and subloading yield surface, *Soils and Foundations*, **45**(1), 65–73.
  - 212) Zhang, F., Ye, B., Noda, T., Nakano, M. and Nakai, K. (2007): Explanation of cyclic mobility of soils: Approach by stress-induced anisotropy, *Soils and Foundations*, **47**(4), 635–648.
  - 213) Zhang, S. and Zhang, F. (2009): A thermo-elasto-viscoplastic model for soft sedimentary rock, *Soils and Foundations*, **49**(4), 583–595.