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CORRELATING CYCLIC STRENGTH WITH FINES CONTENTS THROUGH STATE PARAMETERS

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ABSTRACT

Attempts have been made in the past few decades to establish an equivalent void ratio indicative of the effective density state of sands that contain fines. The concept of an equivalent void ratio is most useful when the grain size distribution of the sand/silt mixture is similar to that of binary packing. Due to its grain characteristics and the wide range of compressibility when the fines content changes, correlating the cyclic strength to fines content via the equivalent void ratio for Mai Liao Sand (MLS) is not a straightforward procedure. Earlier studies have demonstrated the effectiveness of relating cyclic strength to state parameters for sands. Using a series of cyclic triaxial test results as a database, the authors demonstrated the effectiveness of relating cyclic strength to fines contents (f_c) through a state parameter for MLS with an f_c ranging from 0 to 30% that takes advantage of the equivalent void ratio. In addition to the density and stress states, the state parameter based correlations indirectly reflect the effects of soil grain characteristics. The drawback of requiring a separate critical state line when f_c changes can be circumvented by using a unified critical state line based on the equivalent void ratio.

Key words: cyclic strength, sand, fines, state parameter, void ratio (**IGC**: D7)

INTRODUCTION

Studies on the effects of fines (soil particles passing #200 sieve) on the cyclic behavior of granular soils have included a wide spectrum of gradations that span from clean sand, silty sand, clayey sand, sandy silt to pure silt. The unified soil classification of these soils can vary from poorly graded sand (SP), well graded sand to silty sand (SW-SM), silty sand (SM), silty sand to clayey sand (SM-SC) to low plastic silt (ML), depending on the amount and characteristics of the fines. The term silt/sand (M/S) is proposed to serve as an abbreviated term to describe granular soils with various contents of nonplastic and plastic fines with a wide range of gradations, inclusively.

Been and Jefferies (1985) proposed the state parameter Ψ as a semi-empirical normalizing parameter for sand behavior. The state parameter Ψ defines the void ratio (e) and effective mean normal stress (p') level of a sand relative to a reference state (the steady state) as shown in Fig. 1. The steady state of deformation for any mass of particles is that state in which the mass is continuously deforming at constant volume, constant effective mean normal stress, constant shear stress, and constant velocity (Poulos, 1981). The effective mean normal stress, p' shown in Fig. 1 is defined as

$$p' = \frac{(\sigma'_v + 2\sigma'_h)}{3} \quad (1)$$

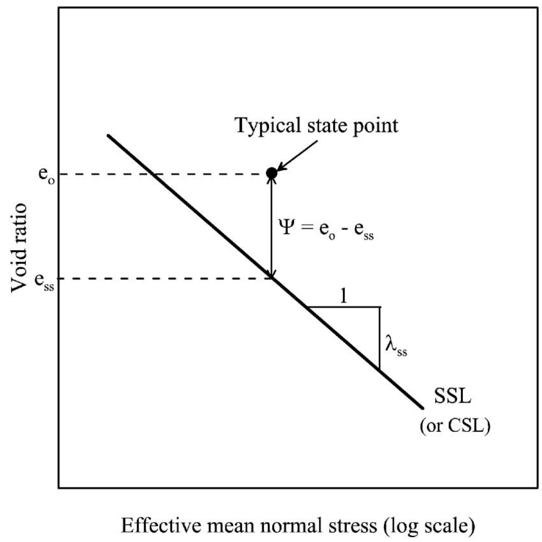

Fig. 1. Definition of a state parameter (after Been et al., 1991)

where
σ'_v and σ'_h = effective vertical and horizontal stress, respectively.

The steady state line is the locus of steady state points in the void ratio/stress space. The projection of the steady state line (SSL) on the e-p' plane is used for data presentation and the definition of the state parameter Ψ

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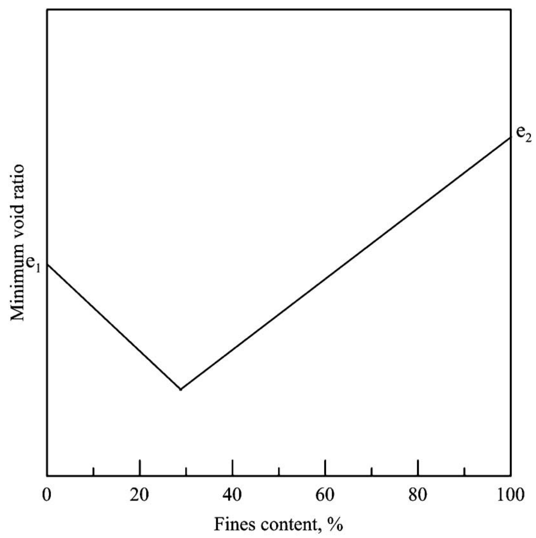

Fig. 2. A conceptual description of the effects of fines on minimum void ratios of binary packing (after Lade et al., 1998)

as shown in Fig. 1. The steady state has traditionally been measured using undrained tests on loose sand samples. Further studies have indicated that the same relationship between e_{ss} and p'_{ss} exists if it is determined using drained tests (Been et al., 1991). In this case the e_{ss}-p'_{ss} relationship is referred to as the critical state line (CSL). It should be noted that the CSL is the same as SSL according to Been et al. (1991). For simplicity, the term critical state line will be used hereinafter.

Studies on M/S soils have mostly been concentrated on laboratory tests using reconstituted specimens. The M/S specimens tend to have been made of mixtures of clean quartz (e.g., Ottawa) sand with crushed silica, kaolin or other types of natural silt. These mixtures of sand and fines, or gap graded artificial soils have been compared to those of coarse and fine spherical grains (Lade et al., 1998), or spherical binary packing. As the diameter ratio of the coarse grains over that of the fine grains exceeded approximately 7, one can expect a bilinear, V shaped correlation between the minimum void ratio (e_{min}) and fines content, as is conceptually described in Fig. 2.

The e_{min} reaches its lowest value as the fines content approaches 30%. The amount of decrease in e_{min}, or depth of the V shaped correlation depicted in Fig. 2, is affected by the diameter ratio of the coarse grains over that of the fine grains. The V shape becomes less obvious as this ratio drops below 7 (Lade et al., 1998). Figure 2 implies that as fines content reaches 30%, the binary packing becomes unstable unless the grain mixture is in a denser state (hence the lower void ratio). Thus, a threshold fines content should correspond to that when the packing is at its least stable state, provided the M/S soil gradation is close to binary packing. The cyclic strength of M/S soils with the same void ratio and a fines content below the threshold value decreases with the fines content. This trend is reversed when the fines content exceeds the threshold. Sand particles may be round in shape, but they are rarely exactly spherical. There can be various degrees of grain size differences or distribution, even for uniformly graded clean sand. For these reasons, natural or artificially mixed M/S soils may not meet the description of binary packing in a strict sense and the threshold fines content may therefore not be 30%. Some of the studies on artificial silty sand specimens, mostly silica in nature, have demonstrated that the threshold fines content generally ranged from 25 to 45% (Koester, 1994; Polito, 1999; Xenaki and Athanasopoulos, 2003; Papadopoulou and Tika, 2008; Rahman et al., 2008).

A series of void ratio indices that relate the active grain contacts (i.e., the soil skeleton) to fines contents have been proposed and evolved in the past few decades (Rahman et al., 2008). These indices are based on the concept of spherical binary packing. The intergranular contact index void ratio, e^* (Thevanayagam, 2000) or simply the equivalent granular void ratio is a relatively recent development. It is defined as:

$$e^* = \frac{e + (1-b)f_c}{1-(1-b)f_c} \qquad (2)$$

where
e = void ratio
b = parameter reflects the fraction of fines participating in the force structure of the solid skeleton
f_c = fines content in decimal

The value of b ranges from 0 to 1. A higher b corresponds to an increase in the contribution of fines in the force structure of the soil skeleton. Predicting the b value was considered to be problematic and controversial by Rahman et al. (2008). Earlier methods generally involved the selection of a case-specific, fines content independent b value by a back-analysis so that the steady state or cyclic strength for a given sand-fines type can be correlated with an equivalent granular void ratio irrespective of the fines content (Ni et al., 2004). Thevanayagam et al. (2002) suggested that b depends on $U_c U_f^2/\chi_T$, where U_c is the uniformity coefficient of the coarse matrix, U_f is the uniformity coefficient of fines and χ_T is the diameter ratio defined as D_{50}/d_{50} (the ratio of the medium coarse grain size over that of the fines). Ni et al. (2004) reported that b depends on the grain size ratio between the lower 10% fractile of the coarse (D_{10}) and d_{50} of fine particles ($\chi = D_{10}/d_{50}$).

Rahman et al. (2008) proposed a semi-empirical equation that defines b as:

$$b = \{1 - \exp[-2.5(f_c)^2/(1-r^{0.25})]\}(rf_c/f_{thre})^r \qquad (3)$$

where
$r = \chi^{-1}$
f_{thre} = threshold fines content

According to Eq. (3), b increases with the fines content. Most of the cases reported in the literature involve the use of round shaped coarse particles artificially mixed with fines. These reports showed promising results in the use of an equivalent granular void ratio as an index to reflect the density state of an M/S soil. Rahman et al. (2008) reported that for certain M/S soils, a series of void ratio (e) based critical state lines that correspond to various fines contents can be collapsed into one by replacing e with e^*. This unified e^* based critical state line should be close to or the same as that of zero fines content where $e^* = e$.

The concept of an equivalent granular void ratio covers only the aspect of the density state. The cyclic or undrained strength of sands is generally dependent on the stress and density states. Relating the fines content to the cyclic strength through a state parameter is a more desirable approach as it considers both density and stress states. Jefferies and Been (2006) have demonstrated a clear trend between the state parameter and cyclic strength of clean quartz sands. Huang et al. (2004) and Papadopoulou and Tika (2008) demonstrated consistent relationships between the cyclic strength and state parameter for silty sands based on their laboratory tests.

Mai Liao Sand (MLS) represents a typical M/S soil deposit in Central Western Taiwan. The term Mai Liao Sand is used loosely, and only because most of the tests reported in the paper involve a mixture of sand and silt which came from Mai Liao. Due to the nature of its parental rock, the coarse sand grains of MLS are mostly platy. Because of its mineral content, MLS is significantly more compressible than the typical clean quartz sand reported in the literature (Huang et al., 1999, 2004). The authors have performed a series of monotonic and cyclic triaxial tests on MLS with fines contents of 0, 15 and 30%. These data allow the concept of using the state parameter as a medium for relating the fines content and cyclic strength to be explored. The majority of properties of Mai Liao Sand and the results of laboratory triaxial tests have been reported (Huang et al., 1999, 2004, 2005) earlier. Additional cyclic triaxial tests have been performed to supplement the available data. The physical and mechanical properties of Mai Liao Sand as they relate to the subject of the paper are summarized first. The feasibility of using an equivalent granular void ratio and a state parameter as a means to correlate cyclic strength with fines contents is then evaluated.

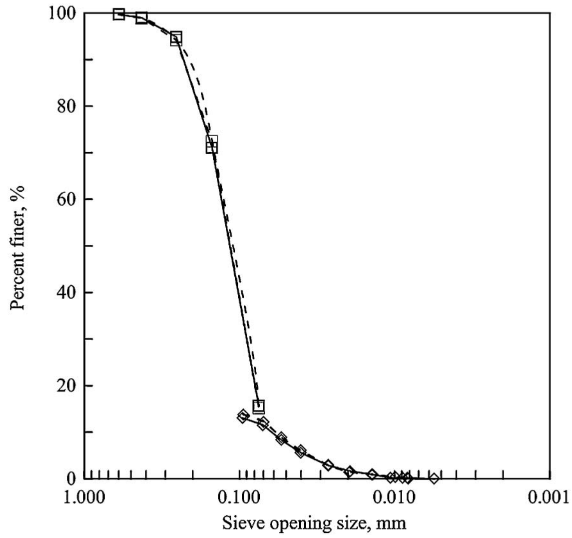

Fig. 3. Grain size distribution of MLS

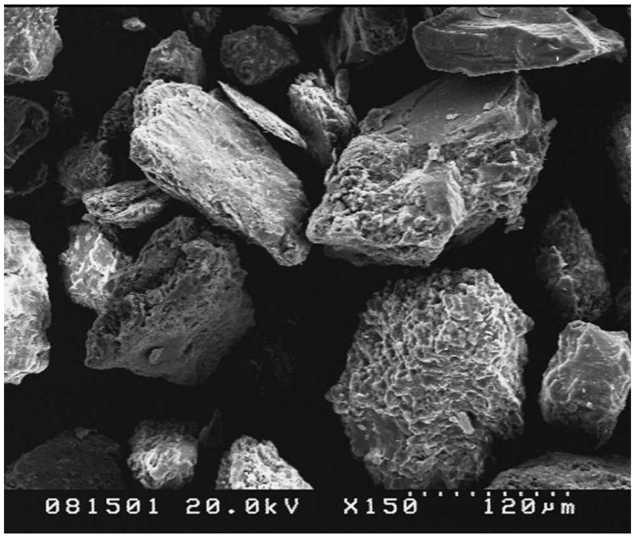

Fig. 4. Scanning electron microscope photograph of MLS particles retained on #200 sieve

PHYSICAL PROPERTIES OF MAI LIAO SAND/SILT

Mai Liao Sand originated from the weathering of sedimentary and metamorphic rocks such as sandstone, shale and slate in the Central Mountain Range of Taiwan. The process of transportation ground the fractured rock into sand and silt particles. Figure 3 shows typical grain size distribution curves of MLS samples taken from a land reclamation site. The ratio of D_{10} (=0.08 mm) over d_{50} (=0.044 mm) (χ) was 1.82. The ratio of medium size grains of the coarse grain (D_{50}=0.125 mm) over d_{50} (χ_T) was 2.84. These values are significantly less than the ideal minimum ratio of 7 for binary packing. A few metric tones of MLS was acquired from a land reclamation site. The sand was washed and sieved through a #200 sieve to separate the fines from the sand particles. The specific gravity of clean MLS sand particles had an average value of 2.69 and the fines had an average specific gravity of 2.71. An X-ray diffraction analysis of MLS showed significant amounts of muscovite and chlorite in addition to quartz. The relatively soft muscovite is more prevalent in the fines. The sand particles retained on the #200 sieve were angular and platy as indicated in the scanning electron microscope photograph shown in Fig. 4. The fines which passed through the No. 200 sieve had a liquid limit of 32 and a plasticity index of less than 8. The processed fines and sand particles were mixed according to required proportions to reach the desired fines content for soil specimens. Figure 5 shows the variation of minimum (e_{min}) and maximum (e_{max}) void ratios of MLS as the fines contents changed from 0 to 80%. The e_{min} (maximum density) was determined according to ASTM D4253 method 1A using a standard 152.4 mm inside diameter compaction mold (total volume = 2830 cm^3). The dry sand placed in the compaction mold was subjected to a

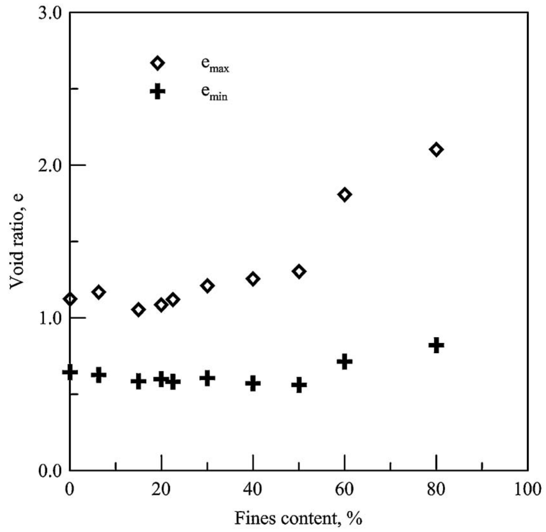

Fig. 5. Minimum and maximum void ratios versus fines contents

surcharge of 14 kPa, and then electromagnetically vibrated under a 60 Hz frequency for 8 minutes. The e_{max} (minimum density) was obtained according to ASTM D4254 method C, using a 2000 mL glass graduated cylinder. Approximately 1000 g of dry sand was placed in the glass cylinder and then plugged with a stopper. The total volume under the loosest state was determined according to the height of the sand after swiftly tipping the cylinder upside down twice. It should be noted that none of the above ASTM standards were applicable for sand with fines in excess of 15%. Also, as will be described later, due to the compressibility of MLS, the relative density could only serve the purpose of initial specimen setup. The analysis of test data used void ratios to represent the soil density state.

Apparently, due to the characteristics of its grain size distribution (i.e., low grain size ratio) and grain shapes (i.e., platy sand grains), the relationship between e_{min} and fines content for MLS shows little obvious similarity to a V shape as shown in Fig. 2.

MECHANICAL PROPERTIES OF MAIL LIAO SAND/SILT

The mechanical property tests included isotropic consolidation, and isotropically consolidated undrained monotonic and cyclic triaxial tests on reconstituted MLS. The consolidation and monotonic triaxial tests were performed on specimens prepared by the moist tamping (MT) method. The MT method was used to create specimens with a wide range of initial densities for the specimens. For cyclic triaxial tests, the specimens were made by water sedimentation (WS) and dry deposition (DD), in addition to the MT method. The additional specimen preparation methods were chosen to evaluate the effects of soil fabric. Readers are referred to Huang et al. (2004) for details on the specimen preparation and triaxial test procedures for this series of laboratory tests. Soil specimens with f_c of 0, 15, 30 and 50% by weight were used in the consolidation and monotonic triaxial tests in this study. No cyclic triaxial tests on specimens with f_c of 50% will be reported herein.

The main purpose of the monotonic triaxial tests was to establish the critical state lines for MLS with various fines contents. The term ''critical state line'' is used in a general way as it can be a curve or equally referred to as ''steady state line''. Figure 6 depicts the consolidation curves and critical state lines resulted from the isotropic consolidation and isotropically consolidated undrained monotonic axial compression triaxial (CIU) tests.

For consolidation tests, p' was the effective confining stress applied during consolidation. For critical state lines, p' represents the effective mean normal stress at the critical state. The initial void ratio (e_o) marked in Fig. 6 represents the void ratio when the specimen was prepared, prior to saturation and consolidation. The void ratios (e) included in Fig. 6 are back calculated, post-consolidation void ratios determined from the water content of the specimen after the test was completed. Details of obtaining the post-consolidation void ratio for triaxial specimens are given in Huang et al. (2004). The e_{min} and e_{max} are marked on the figure. In general, the compressibility increased with fines content and p', and decreased as the specimen became denser. The void ratio, e, after application of the initial consolidation pressure of 10 kPa may be significantly less than e_o. When the fines contents reached 50%, the application of the initial confining stress and the saturation process apparently caused enough compression that the consolidation curves were almost indistinguishable among specimens with different e_o values. It is thus imperative that the post-consolidation void be used in presenting M/S data that involve density states. Since no meaningful critical state line could be obtained from the monotonic triaxial tests for $f_c = 50\%$, none is reported in Fig. 6.

The specimens were sheared monotonically by axial compression to strains well in excess of 20% to reach or be close to critical state (Been et al., 1991). The critical state lines for each of the three types of fines contents were more or less parallel to each other but moved downward as the fines content increased. This phenomenon is similar to the observation reported by Kikumoto et al. (2009). For the densities and confining stresses applied, the specimens rarely demonstrated dilatant behavior, even in the case of dense specimens. The relationships among deviator stress, excess pore pressure and axial strain, as well as the effective stress paths in terms of q ($= \sigma'_v - \sigma'_h$) and p' ($= (\sigma'_v + 2\sigma'_h)/3$) from all the consolidated undrained triaxial tests are depicted in Figs. 7(a) to 7(c). A combination of deviator stress, q and pore pressure was chosen for each undrained triaxial test according to the definition of critical state where the sand deformed continuously at relatively constant pore pressure and constant q. The critical state points are marked in Figs. 7(a) to 7(c). A straight line was fitted to the data set of (p', q) that corresponded to the critical state of each triaxial test, as shown in Fig. 7(c). The slope of this fitted line was referred to as M_s. For the tests performed, $M_s = 1.24$ for

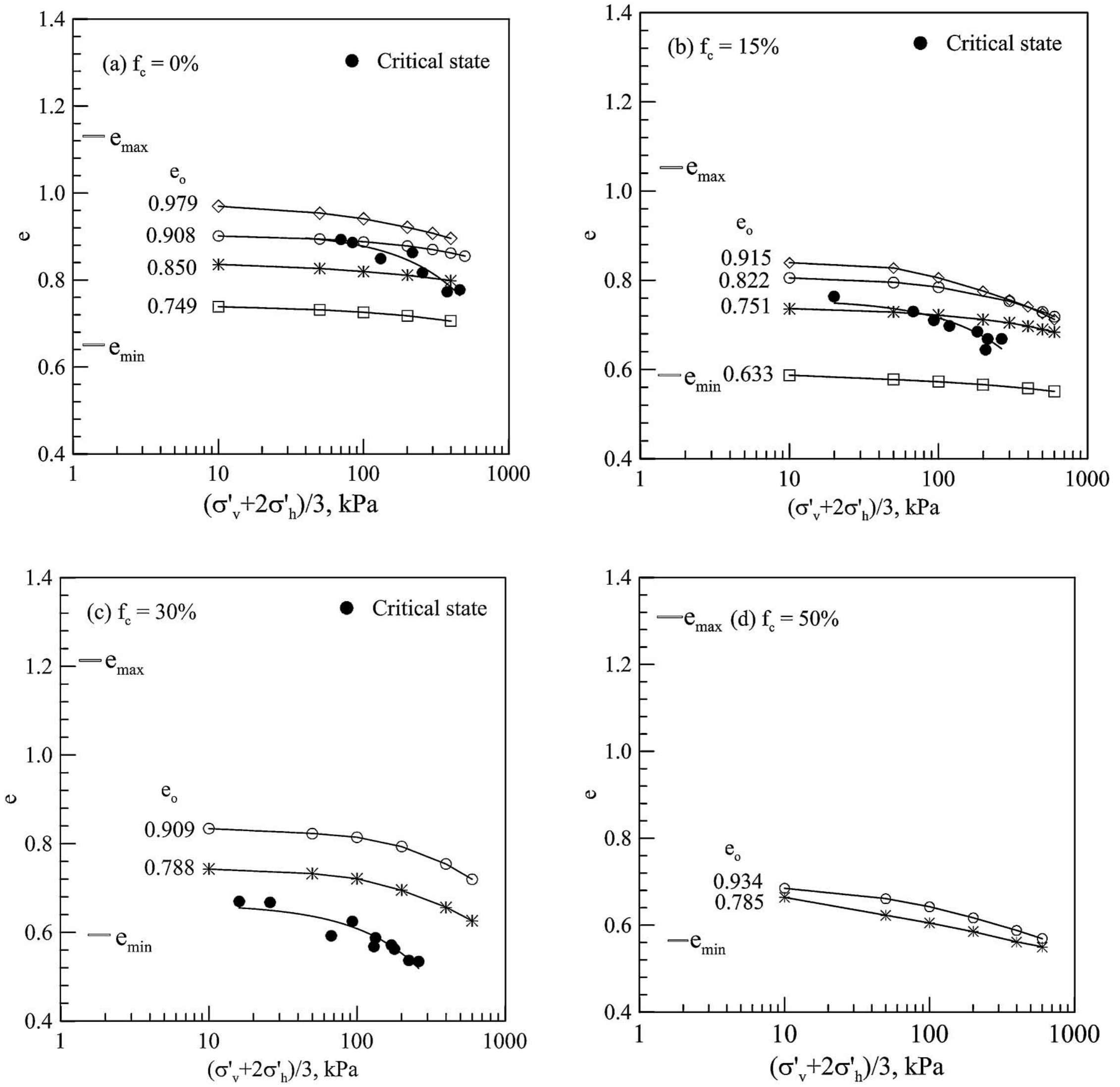

Fig. 6. Isotropic consolidation curves and critical state lines for MLS with various fines contents (after Huang et al., 2004)

FC = 0 to 30%, and M_s = 1.21 as FC reached 50%. These M_s ($= 6 \sin \phi_s'/(3 - \sin \phi_s')$) values most likely correspond to the interparticle friction angles (ϕ_s') of 30.9° and 30.2°, respectively. The similarity in ϕ_s' regardless of the fines content, indicates that the coarse and fine MLS particles had similar grain to grain frictional behavior.

Table 1 compiles all of the related cyclic triaxial tests performed in the past few years on MLS specimens. These tests provided a database to evaluate the relationships among e, e^*, cyclic strength and the state parameter (Ψ). In the series of cyclic triaxial tests on specimens prepared by the water sedimentation (WS) method, all specimens were consolidated and sheared under a p' of 100 kPa. In cyclic triaxial tests on dry deposition (DD) specimens, p' values of 50, 100 and 200 kPa were used. The advantage of the moist tamping (MT) method is that it creates specimens with a wide range of densities, which is why it has been used extensively in cyclic triaxial tests. In this series of tests, specimens with an over consolidation ratio (OCR) of 4 were used. For over consolidated specimens, the p' values shown in Table 1 are the effective mean normal stress after the unloading stage, just prior to the cyclic triaxial tests. For MT specimens, p' varied from 80 to 200 kPa. Collectively, the cyclic triaxial test series made it possible to consider the effects of density state, stress state, stress history and soil fabric.

The cyclic strength in terms of the cyclic resistance ratio (CRR) was measured using stress controlled cyclic triaxial tests. The soil specimen was consolidated under an isotropic effective confining stress σ_c' and then subjected to a cyclic deviator stress, σ_d in the axial direction. Three to five cyclic triaxial tests were performed with various $\sigma_d/2\sigma_c'$ values. The CRR was defined as the $\sigma_d/2\sigma_c'$ that produced an axial strain of 5% in double amplitude in 20 cycles (N_c) of uniform load application. Figure 8 depicts the CRR versus N_c from selective cyclic triaxial tests on MLS with f_c of 0, 15 and 30% performed under σ_c' = 100 kPa. The specimens used in this series of cyclic triaxial tests were fabricated using the moist tamping (MT) method. According to Fig. 8, the CRR consistently decreased as the fines contents changed from 0 to 15 and 30% for tests conducted under similar void ratios in the

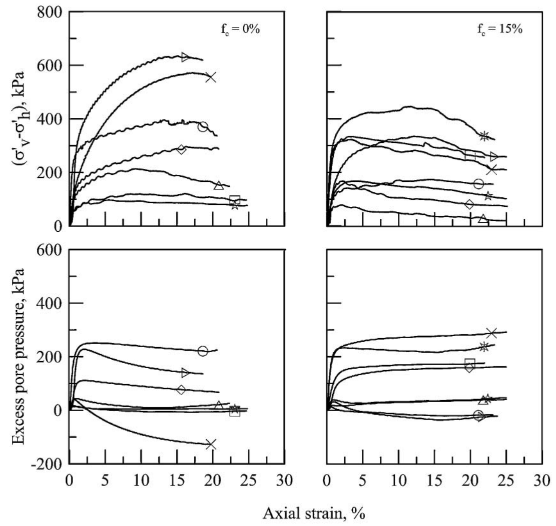

Fig. 7a. Stress, excess pore pressure and strain relationships from CIU tests with $f_c = 0$ and 15% (after Huang et al., 2004)

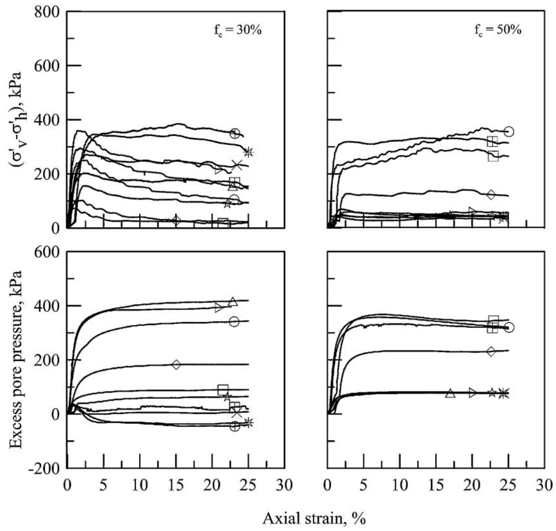

Fig. 7b. Stress, excess pore pressure and strain relationships from CIU tests with $f_c = 30$ and 50% (after Huang et al., 2004)

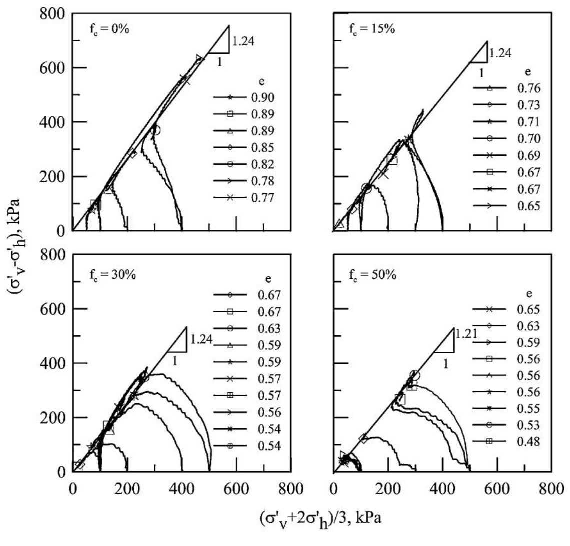

Fig. 7c. Effective stress paths from triaxial tests (after Huang et al., 2004)

Table 1. Summary of cyclic triaxial tests

SPM+	f_c (%)	OCR	p' (kPa)	CRR	e	Ψ	e^*
MT	0	4	100	0.469	0.820	−0.060	0.820
MT	0	4	80	0.412	0.940	0.060	0.940
MT	15	4	100	0.356	0.850	0.131	1.115
MT	15	4	100	0.506	0.810	0.091	1.070
MT	30	4	100	0.312	0.800	0.190	0.185
MT	30	4	80	0.288	0.830	0.220	0.215
MT	0	1	100	0.180	0.954	0.094	0.954
MT	0	1	100	0.210	0.914	0.054	0.914
MT	0	1	100	0.340	0.834	−0.027	0.834
MT	0	1	100	0.440	0.784	−0.076	0.784
MT	0	1	100	0.520	0.714	−0.147	0.714
MT	15	1	100	0.320	0.812	0.093	1.071
MT	15	1	100	0.360	0.760	0.041	1.012
MT	15	1	100	0.440	0.730	0.011	0.978
MT	30	1	100	0.230	0.636	0.026	0.867
MT	30	1	100	0.191	0.666	0.056	0.901
MT	30	1	100	0.167	0.766	0.156	1.016
MT	30	1	100	0.146	0.816	0.206	1.073
MT	30	1	100	0.149	0.816	0.206	1.073
MT	30	1	100	0.162	0.766	0.156	1.016
MT	30	1	100	0.210	0.736	0.126	0.981
MT	30	1	200	0.162	0.694	0.144	0.867
MT	30	1	200	0.150	0.724	0.174	0.975
MT	30	1	200	0.139	0.764	0.214	1.020
MT	30	1	200	0.135	0.804	0.254	1.066
MT	30	1	200	0.247	0.715	0.165	0.964
MT	30	1	175	0.258	0.631	0.081	0.868
MT	30	1	100	0.229	0.724	0.114	0.974
DD	0	1	50	0.248	0.850	−0.047	−0.047
DD	0	1	50	0.330	0.780	−0.117	−0.117
DD	0	1	100	0.329	0.760	−0.120	0.760
DD	0	1	100	0.261	0.810	−0.070	0.810
DD	0	1	100	0.231	0.850	−0.030	0.850
DD	0	1	100	0.210	0.870	−0.007	0.870
DD	0	1	200	0.270	0.780	−0.070	0.780
DD	0	1	200	0.213	0.830	−0.020	0.830
DD	0	1	200	0.181	0.870	0.020	0.870
DD	15	1	50	0.230	0.730	−0.005	0.978
DD	15	1	100	0.290	0.720	0.001	0.967
DD	15	1	100	0.202	0.790	0.071	1.047
DD	15	1	100	0.192	0.850	0.131	1.115
DD	15	1	200	0.202	0.710	0.035	0.955
DD	15	1	200	0.195	0.780	0.105	1.035
DD	30	1	100	0.138	0.725	0.115	0.975
DD	30	1	100	0.165	0.710	0.100	0.958
DD	30	1	100	0.166	0.677	0.067	0.920
WS	0	1	100	0.340	0.760	−0.120	0.760
WS	0	1	100	0.300	0.800	−0.080	0.800
WS	0	1	100	0.260	0.860	−0.020	0.860
WS	15	1	100	0.300	0.710	−0.009	0.955
WS	15	1	100	0.270	0.750	0.031	1.001
WS	15	1	100	0.230	0.770	0.051	1.024
WS	30	1	100	0.200	0.680	0.070	0.924
WS	30	1	100	0.170	0.730	0.120	0.981
WS	30	1	100	0.130	0.760	0.150	1.016

+: SPM = specimen preparation method

range of 0.7 and 0.8. Note that possible range of void ratios under various fines contents is wide, as shown in Fig. 6, due to differences in compressibilities. Void ratios of approximately 0.82 (close to the maximum applicable e for specimens with FC = 30%) and 0.71 (close to the minimum applicable e for specimens with FC = 0%) represent the maximum and minimum void ratio, respectively, that

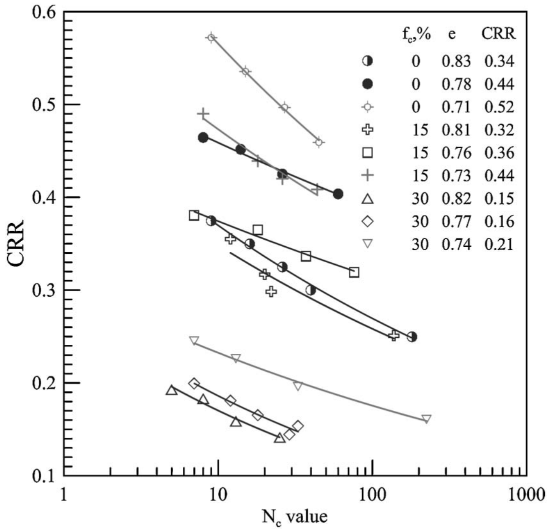

Fig. 8. CRR versus N_c from selective cyclic triaxial tests

could practically be used in fabricating the cyclic triaxial test specimens.

CORRELATING CRR WITH FINES CONTENT THROUGH DENSITY STATES

Comparisons of the CRR obtained from specimens with different fines contents can be useful in evaluating the effects of fines. A valid comparison should be made among specimens prepared by the same method and under a common density state. The database included in Table 1 allows such comparisons to be made. The comparisons are grouped according to the three specimen preparation methods. For specimens made by the WS method, all cyclic triaxial tests were performed under normally consolidated conditions and an effective confining stress (p') of 100 kPa. For DD specimens, p' also included 50 and 200 kPa for certain parts of the cyclic triaxial tests. The MT database included a limited number of cyclic triaxial tests performed on specimens with an over consolidation ratio (OCR) of 4. The p' values in this case correspond to the unloaded final effective confining stress just prior to the cyclic shearing. The relative density is likely to be a poor index to represent the density state of MLS due to the drastic differences in compressibility and the possible range of post-consolidation void ratios among specimens with different fines contents, as indicated in Fig. 6. The following comparisons are made based on e and e^* to evaluate the effectiveness of equivalent granular void ratio to represent the density state for MLS. As described above, the grain size distribution of MLS deviates significantly from ideal binary packing. There was no clear trough in the e_{min}-f_c correlation in Fig. 5 that corresponds to typical binary packing. The value of f_{thre} was assumed as 30% to facilitate computations of b parameters and e^*. According to the grain size distribution of MLS shown in Fig. 3, assumed f_{thre} and Eq. (3), $b = 0.164$ and 0.577 for $f_c = 15$ and 30%, respectively.

Correlations among e, e^* and CRR are presented in

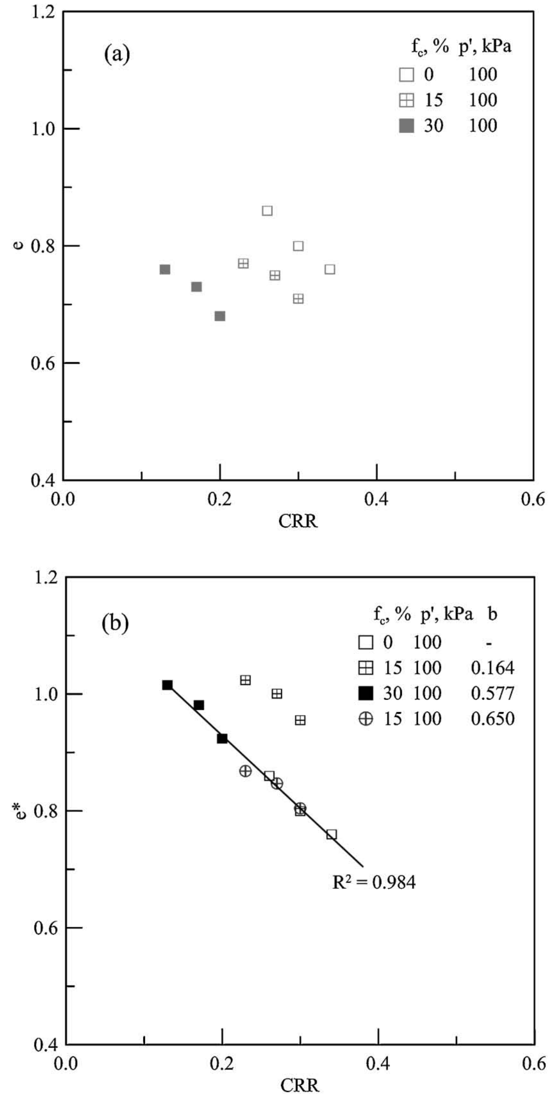

Fig. 9. Relationship between cyclic strength and (a) void ratio, e; (b) equivalent granular void ratio, e^* for specimens prepared by WS method

Figs. 9 to 11. Due to the high compressibility of the fines, a post consolidation void ratio of higher than 0.75 can be assumed to be excessively loose, and it is very difficult to make for specimens with an f_c larger than 15%. On the other hand, specimens with an f_c of less than 15% are significantly less compressible, and a void ratio less than 0.75 would be rather dense. For these reasons, only a narrow band of common void ratios can be found in Figs. 9(a), 10(a) and 11(a) where a comparison of the cyclic strength with different fines contents can be made. According to these figures, void ratios of around 0.75 are best suited for selection as a common ground for cyclic strength comparisons. The results show that regardless of the specimen preparation method, for void ratios around 0.75, CRR decreases with f_c. For specimens with the same f_c, the lower void ratio corresponds to a denser state and, hence, a higher CRR. For the range of confining stress applied, there was generally a trend of slightly lower CRR from tests performed at higher confining stress. This is to

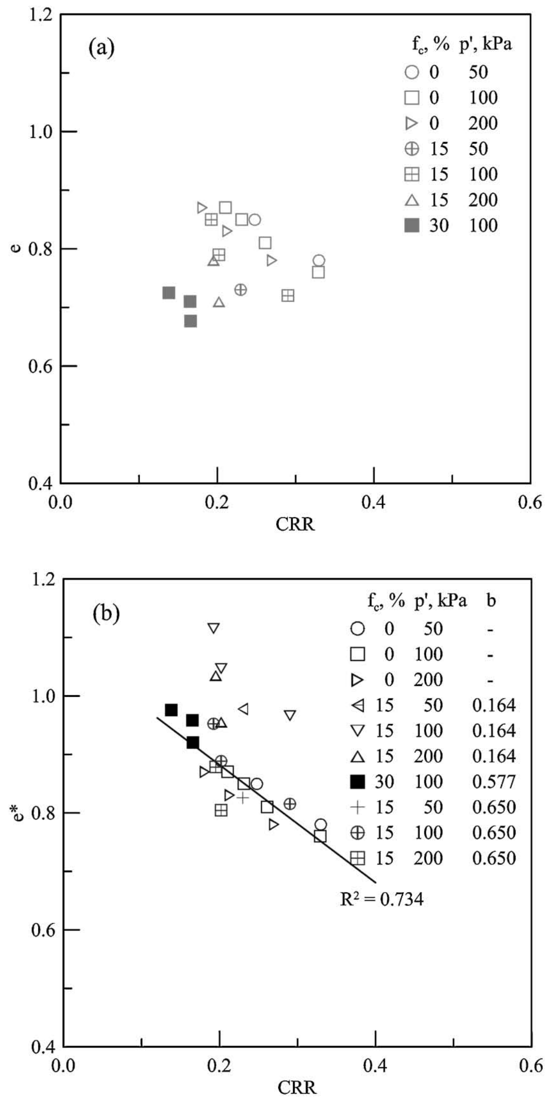

Fig. 10. **Relationship between cyclic strength and (a) void ratio, *e*; (b) equivalent granular void ratio, *e** for specimens prepared by DD method**

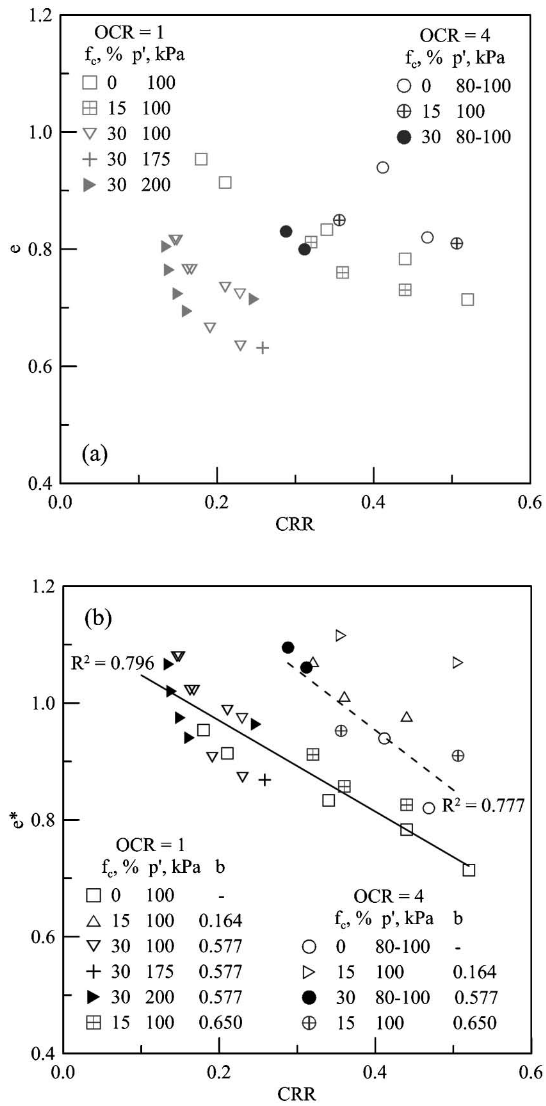

Fig. 11. **Relationship between cyclic strength and (a) void ratio, *e*; (b) equivalent granular void ratio, *e** for specimens prepared by MT method**

be expected since granular soil is less dilatant under higher confining stress.

The use of e^* to represent density state shifts all data close to a more unified e^*-CRR correlation among three types of fines contents, as shown in Figs. 9(b), 10(b) and 11(b). The results indicate that there appears to be an over adjustment from e to e^* when $f_c = 15\%$, regardless of the specimen preparation method. The fines may play a much more active role (b value needs to be larger) than predicted in Eq. (3) in the force structure of the MLS specimens at $f_c = 15\%$. If the e^* values were computed using Eq. (2) according to the b parameter ($=0.164$) from Eq. (3), the e^*-CRR data of $f_c = 15\%$ would deviate significantly away from a linear e^*-CRR correlation established from the remainder of the e^*-CRR data within each group of the specimen preparation method. The e^*-CRR data would have a coefficient of correlation less than 0.4 with a linear regression, unless the data of $f_c = 15\%$ are excluded. As stated above, f_{thre} had to be assumed when using Eq. (3) due to a lack of clear trough in the e_{min}-f_c correlation for MLS. Other factors such as its platy grain shape and high compressibility mean that MLS behaves significantly differently from typical spherical binary packing. Under these circumstances, it was decided to adjust b empirically to enhance the e^*-CRR correlation. This is similar to the approach used by Ni et al. (2004). The b value obtained from Eq. (3) was used as an initial trial value. Following an iterative procedure, it was determined that a revised b value of 0.65 (b remains to be 0.577 for $f_c = 30\%$) for the case of $f_c = 15\%$ would result in a much improved e^*-CRR correlation for specimens within each group of preparation method. As shown in Figs. 9(b), 10(b) and 11(b), using the revised b value, the e^*-CRR data fits a linear regression with coefficients of correlations (R^2) better than 0.734. The same linear regression covers the range of stress conditions applied for each group of specimen preparation method. Less scattering can be expected if all tests of the same specimen preparation method were conducted under the same confining stress. For the MT specimens, a separate linear regression can be made for the normally (OCR = 1) and over consolidated (OCR = 4) specimens.

It is evident that for MLS and at least for f_c between 15 and 30%, b is close to a constant as suggested by Ni et al. (2004). The fines play a much more significant role in the sand-fines force structure than predicted by Eq. (3) in the case of $f_c = 15\%$. This relatively high and constant b value is believed to be a reflection of the mild differences in grain size ($\chi = 1.82$) and mineral contents between sand and fines in MLS. For f_c between 15 and 30%, a first order estimation of b can be made by linear interpolation. For practical purposes, a constant b value selected between 0.577 and 0.65 may also be assumed for f_c in the range of 15 and 30%, with a less desirable e^*-CRR correlation.

CORRELATING CRR WITH FINES CONTENT THROUGH THE STATE PARAMETER

Jefferies and Been (2006) showed that a clear trend between the state parameter (Ψ) and CRR_{15} (cyclic strength determined at 15 cycles of loading) can be established as shown in Fig. 12, for clean quartz sands from different parts of the world. The state parameter is soil fabric independent. The cyclic strength, however, is soil fabric dependent. The effects of soil fabric are evident when specimens prepared by different methods results in different cyclic strength (Huang et al., 2004). The scatter of test data in Fig. 12 is believed to have been caused by variations in soil fabrics originated in different specimen preparation methods involved in the data of Fig. 12 (Jefferies and Been, 2006). There are clear advantages to extending the idea of correlating cyclic strength with state parameter into sands with fines. The strength, including the cyclic strength, is a function of density and the stress states which are both covered in state parameter. The equivalent granular void ratio on the other hand, considers the density state only. The use of the state parameter does not require a gradation similar to that of binary packing. For the case of MLS, the state parameter (Ψ) in Table 1 was determined following the procedure shown in Fig. 1. Similar trends between CRR and Ψ can be identified for MLS from data of the same specimen preparation group, as shown in Fig. 13. Only the data from normally consolidated MT specimens are included in Fig. 13. The test data generally have coefficients of correlations in excess of 0.75 with their corresponding exponential curve fits. The state parameters of MLS extend into the positive side ($\Psi > 0$) much more so than those of the clean quartz sands reported by Jefferies and Been (2006). This is a reflection of the compressive nature of MLS grains and the mostly contractive behavior during shear, as shown in Fig. 7. For data of the same specimen preparation group, those with high fines contents are clustered further towards the positive side of the state parameter axis. This is again a reflection of the higher compressibility associated with M/S mixtures with higher fines contents. For specimens with the same state parameter, the CRRs of MT specimens are the highest and those of the DD specimens are the lowest. This is consistent with the earlier findings reported by Huang et al. (2004) which indicated that for specimens with the same void ratio, the CRRs from MT specimens are the highest and those from DD specimens are the lowest. Therefore, in addition to the consideration of the density state, the use of the state parameter has the additional advantage of reflecting soil grain characteristics. These are desirable features that the concept of equivalent granular void ratio lacks.

To use the state parameter derived from the void ratio, e has an important drawback. Ideally, a separate critical state line is required for M/S mixtures with different fines contents. For the tests on MLS reported herein, only three types of fines contents are involved. The three critical state lines from each of the three types of fines con-

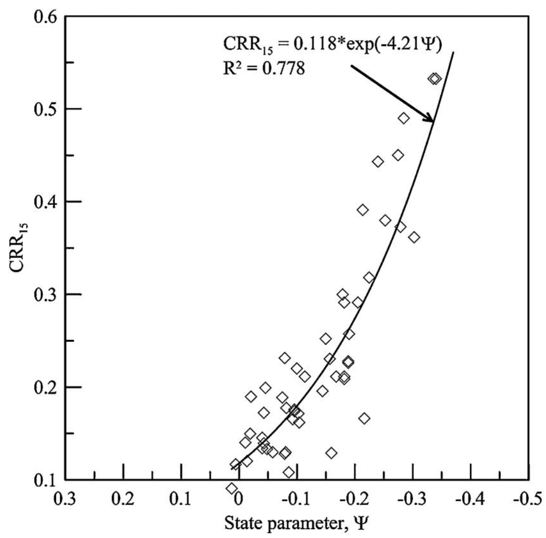

Fig. 12. Correlation between cyclic strength and state parameter for clean quartz sands (after Jefferies and Been, 2006)

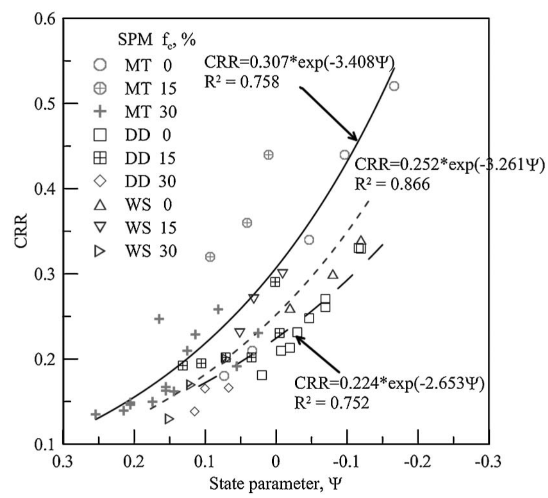

Fig. 13. Correlation between cyclic strength and state parameter, Ψ for MLS test data

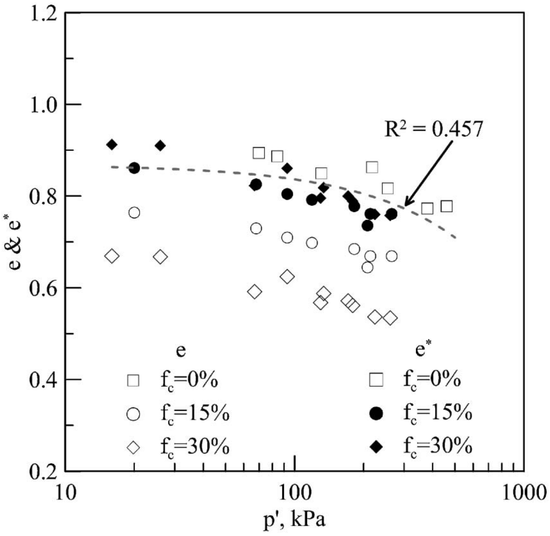

Fig. 14. The *e* based critical state loci and the critical state line derived using *e**

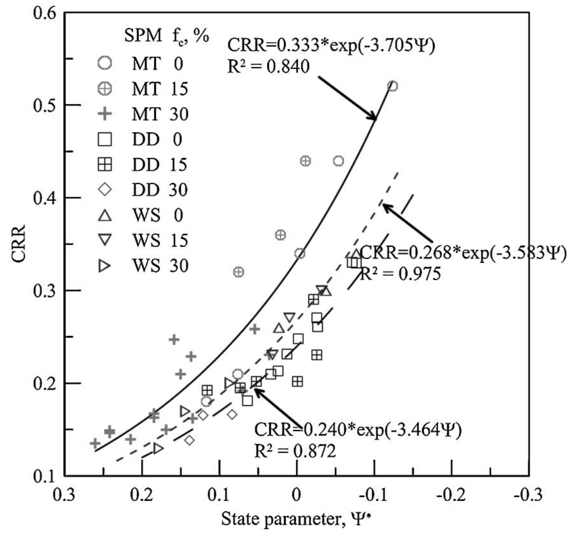

Fig. 15. Correlation between cyclic strength and equivalent granular state parameter, *Ψ** for MLS test data

tents given in the paper are sufficient. For natural M/S soils, however, there can be numerous possibilities of fines contents. Thus, it is conceivable that an impractically large number of laboratory tests is required to provide the series of critical state lines that correspond to all given fines contents unless a fines content based on an interpolation scheme can be effectively used to determine these critical state lines. This drawback may be reduced or minimized by invoking the equivalent granular void ratio in the determination of critical state line. Figure 14 shows the void ratio, e, based on the critical state loci that correspond to each of the three types of fines contents included in Fig. 6 along with the those computed based on e^*. In establishing a unified e^*-p' critical state line and e^* based state parameters, $e^* = e$ for $f_c = 0\%$, the revised b parameter of 0.65 was used for $f_c = 15\%$, $b = 0.577$ as determined from Eq. (3) was used for the case of $f_c = 30\%$. The e^* based critical state loci shown in Fig. 14 are concentrated in a relatively narrow range. A unified critical state line fitted to these e^* based critical state loci has a marginal R^2 value of 0.457. The state parameter determined based on the e^* of the soil specimen and the unified e^*-p' critical state line is referred to as the equivalent granular state parameter (Ψ^*). A plot of Ψ^* versus CRR for specimens prepared by the three methods is shown in Fig. 15. Again, only those of normally consolidated MT specimens are included in Fig. 15. A similar trend between the CRR and Ψ^* as those observed in Fig. 13 can be found in Fig. 15. Despite of the marginal R^2 associated with the determination of the e^* based critical state line, the CRR-Ψ^* data, show much improved R^2 values in fitting with the corresponding exponential curves. Apparently, the determination of Ψ^* based on e^* and a single, unified e^*-p' critical state line is much less noisy than deriving Ψ using e and separate e-p' critical state lines for each type of fines content.

CONCLUDING REMARKS

A series of monotonic and cyclic triaxial tests have been performed on reconstituted MLS specimens. The cyclic triaxial tests included specimens prepared by water sedimentation (WS), dry deposition (DD) and moist tamping (MT) methods with a wide a variety of confining stresses and densities. The fines contents used included 0, 15 and 30%. Some of the cyclic triaxial tests were performed on over-consolidated specimens. Collectively, the cyclic triaxial test series considered the effects of density state, stress state, stress history and soil fabric. These tests provided a database to evaluate the relationships among e, e^*, cyclic strength and state parameter (Ψ).

According to the grain size distribution of MLS and an assumed f_{thre} of 30%, $b = 0.164$ and 0.577 for $f_c = 15$ and 30%, respectively, following the method by Rahman et al. (2008). However, a b value of 0.65 was required for the case of $f_c = 15\%$ to establish a more consistent e^*-CRR correlation, for the combined data of $f_c = 0$, 15 and 30%. The relatively high and constant b value is believed to be a reflection of the mild differences in grain size ($\chi = 1.82$) and mineral contents between sand and fines in MLS. Because of the grain characteristics of MLS, the fines play a much more active role in the granular force structure than most of the cases reported earlier. For f_c between 15 and 30%, a first order estimation of b can be made by linear interpolation. For practical purposes, a constant b between 0.577 and 0.65 may also be assumed for f_c in the range of 15 and 30%, with a less desirable e^*-CRR correlation.

The state parameter considers the combined effects of density and the stress states. The characteristics, notably the compressibility of the sand grains, are reflected in the general range of the state parameters. Because of these advantages, the state parameter is a superior choice over the equivalent granular void ratio as a means of correlating the CRR with fines contents. The drawback and impracticality of requiring a series of critical state lines for each type of fines contents involved can be circumvented by invoking the equivalent granular void ratio in the determination of state parameters. With properly calibrated b parameters or a function that determines b parameters, a unified e^*-p' critical state line can be established for the range of fines contents involved. For MLS and the tests performed, the equivalent granular state parameter Ψ^* can be an effective means to correlate cyclic strength with fines contents.

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