

# Performance enhancement of OFDM-SQ<sup>2</sup>AM in distorted channel environments

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**Abstract:** In this letter, a new pulse shaped OFDM-Superposed Quadrature Quadrature Amplitude Modulation (OFDM-SQ<sup>2</sup>AM) system is proposed to reduce the inter-channel interference (ICI) as well as to maintain the orthogonalities between the OFDM subcarriers in the presence of the subcarrier frequency offset. Time and frequency dispersion properties of the proposed SQ<sup>2</sup>AM pulse shape are analyzed using the Heisenberg parameter and in-band energy function. Such properties of the proposed pulse are compared with those of a rectangular pulse and IOTA pulse. The BER performance of the pulse shaped OFDM-SQ<sup>2</sup>AM is compared with those of conventional OFDM-QAM and IOTA/OFDM-OQAM in the 3GPP ITU-R environment. Our simulation results indicate that the proposed OFDM-SQ<sup>2</sup>AM achieves higher spectral efficiency and power efficiency as compared to the other schemes and gives better robustness of the frequency dispersion.

**Keywords:** SQ<sup>2</sup>AM, OFDM, pulse shaping filter, OFDM-OQAM, time-frequency localization, IOTA

**Classification:** Wireless circuits and devices

## References

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## 1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has been considered as a promising candidate technology that can transmit effectively wide-band signals over multipath fading channels. OFDM technology has been applied for many wireless standards such as digital audio broadcasting (DAB), digital video broadcasting (DVB-T), IEEE 802.11a local area network (LAN), IEEE 802.16a metropolitan area network (MAN), and is also under consideration as a potential candidate of the fourth-generation (4G) mobile wireless systems. Nonetheless, the conventional OFDM has shown several problems such as a loss in the spectral efficiency and an increase of power consumption due to the addition of the cyclic prefix. OFDM is also very sensitive to the carrier frequency offset and Doppler spread in the frequency dispersive channel. Such imperfections will destroy the orthogonalities of the subcarriers and introduce ICI among the subcarriers [1]. Furthermore, the unfiltered rectangular pulse has large sidelobes, which would cause serious ICI problems into the adjacent channels.

In order to overcome such drawbacks in the conventional OFDM system, OFDM-offset QAM (OFDM-OQAM) technique was introduced [2]. In OFDM-OQAM each subcarrier is modulated by a real-valued symbol. A well localized pulse shaping is employed in order to reduce both the ICI and out-of-band energy and also to increase the spectral efficiency by eliminating the guard interval. The well designed pulse known as IOTA/OFDM-OQAM was introduced in 3GPP UTRAN enhancement [3] and has been considered for use in Cognitive Radio (IEEE 802.22) [4]. However, since IOTA pulse is Gaussian type, its time-frequency localization (TFL) is not fully optimized in the doubly dispersive channel from the view of Heisenberg parameter which measures the TFL property. Therefore, the out-of-band energy of the IOTA pulse is still high. In this letter, OFDM-SQ<sup>2</sup>AM modulation technique is proposed to depress the out-of-band energy in the doubly dispersive channel.

## 2 System model

Let us consider an OFDM Offset-QAM (OQAM) system employing  $M$  subcarriers each spaced by  $F = 1/T$ . Each subcarrier transmits one QAM symbol  $a_m[n] = a_m^I[n] + ja_m^Q[n]$  every  $T$  seconds on the  $m$ -th subcarrier at  $n$ -th symbol, where  $a_m^I[n]$  and  $a_m^Q[n]$  are the real part and the imaginary part of the symbol, respectively. OQAM symbols are obtained by shifting the imaginary part  $a_m^Q[n]$  by  $T/2$ . Pulse shaping filter which has good localization properties in the time and frequency domains is required in order to limit the inter-symbol interference and inter-channel interference. The transmit

OFDM signal can be expressed as

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{M-1} \left[ a_m^I[n]g(t - nT) + ja_m^Q[n]g(t - nT - T/2) \right] e^{jm\pi/2} e^{j2\pi mFt} \quad (1)$$

where,  $g(t)$  is the impulse response of the pulse shaping filter. Note that the phase factor  $e^{jm\pi/2}$  in the subcarrier modulator is important to maintain the orthogonalities between the subcarriers. The pulse shaping function  $g_{m,n}(t)$  can be obtained by taking the time-frequency translated version of  $g(t)$  in the following way

$$g_{m,n}(t) = e^{j(m+n)\pi/2} e^{j2\pi mFt} g(t - nT), \quad TF = 1/2. \quad (2)$$

To maintain the orthogonality between the pulse shaping filters in the transmitter and receiver, following notation must be satisfied

$$\Re \left\{ \int_{\mathbb{R}} g_{m',n'}^*(t) g_{m,n}(t) dt \right\} = \delta_{m,m'} \delta_{n,n'}, \quad (3)$$

where  $\Re\{\cdot\}$  is the real value operator. The well designed pulse known as Isotropic Orthogonal Transform Algorithm (IOTA) pulse guarantees the orthogonality between the pulse shaping filter, and has been used for OFDM-OQAM system. IOTA function modifies Gaussian filter  $g_{\alpha}(t) = (2\alpha)^{1/4} e^{-\pi\alpha t^2}$  as follows

$$g_{iota} = \frac{1}{2} \sum_{k=0}^{\infty} d_{k,\alpha,F} \left[ g_{\alpha}(t + \frac{k}{F}) + g_{\alpha}(t - \frac{k}{F}) \right] \times \sum_{l=0}^{\infty} d_{l,1/\alpha,T} \cos \left( 2\pi l \frac{t}{T} \right), \quad (4)$$

where  $d_{k,\alpha,F}$  is real coefficient given numerically in [5].

### 3 Proposed pulse shaping filter

In order to improve the spectral and power efficiency of QAM system, SQ<sup>2</sup>AM was proposed and its performance was analyzed in a nonlinearly amplified single carrier channel [6]. SQ<sup>2</sup>AM produces a power spectrum having a compact main lobe and minimal sidelobes. SQ<sup>2</sup>AM signal can be generated by using the pulse overlapping method in conjunction with the multi-dimensional signalling method [7]. The impulse response of SQ<sup>2</sup>AM baseband signal is defined as

$$g(t) = \begin{cases} \frac{(1+A)}{2} \cos \frac{\pi t}{T} + \frac{(1-A)}{2} \cos \frac{3\pi t}{T} & , |t| \leq \frac{T}{2} \\ 0 & , \text{elsewhere} \end{cases} \quad (5)$$

where ‘A’ is an amplitude parameter of SQ<sup>2</sup>AM signal that determines the main-lobe bandwidth, side-lobe levels, and the envelope fluctuation of a modulated carrier. The Fourier Transform of SQ<sup>2</sup>AM signal is derived as

$$G(f) = \frac{T}{\pi} \left[ \frac{(1+A)}{1-4(fT)^2} - \frac{\frac{1}{3}(1-A)}{1-\frac{4}{9}(fT)^2} \right] \cos(\pi fT). \quad (6)$$

The normalized power spectral density (PSD) of SQ<sup>2</sup>AM signal is

$$\left| \frac{G(f)}{G(0)} \right|^2 = \left| \frac{1}{2(1+2A)} \left[ \frac{3(1+A)}{1-4(fT)^2} - \frac{(1-A)}{1-\frac{4}{9}(fT)^2} \right] \cos(\pi fT) \right|^2, \quad (7)$$

where  $G(0) = \frac{2}{3}\pi(1+2A)$ . The proposed filter is an even function since  $g(t) = g(-t)$  and time-limited with in  $-T/2 \leq t < T/2$ . This pulse shaping function obtained by the time-frequency translation as shown in (2) satisfies the orthogonal condition (3). Therefore, SQ<sup>2</sup>AM can be used for OFDM-OQAM system.

## 4 Properties of the proposed pulse shaping in time-frequency plane

### 4.1 Time-frequency localization

The time-frequency translated version of the pulse shaping filter as shown in (2) constitutes 2-dimensional plane in the time and frequency domains. The less power the pulse shaping function spreads to the neighboring points in the time-frequency plane, the higher demodulation gain of the pulse shaped signal can be achieved. Quantitative measure of such time-frequency localization is the Heisenberg parameter  $\xi$  [2] which is defined as a second moment in time and frequency domains given by

$$\xi = \frac{1}{4\pi\Delta t\Delta f} \leq 1, \quad (8)$$

where  $\Delta t$  is the time dispersion and  $\Delta f$  is the frequency dispersion of the pulse shaping function, respectively. This indicates the energy distribution of the pulse shaping function in the time-frequency plane. The larger  $\Delta t$  or  $\Delta f$ , the wider the energy is distributed in time or in frequency. These two parameters can be calculated as follows

$$\begin{cases} \Delta t^2 = \frac{1}{E} \int_{\mathbb{R}} t^2 |g_{m,n}(t)|^2 dt \\ \Delta f^2 = \frac{1}{E} \int_{\mathbb{R}} f^2 |G_{m,n}(f)|^2 df \end{cases} \quad (9)$$

where  $E = \int_{\mathbb{R}} |g_{m,n}(t)|^2 dt$  is the energy of the pulse shaping function. The larger  $\xi$  is, the better time-frequency localization the pulse shaping function has. To compare the localization property of various pulses, the Heisenberg parameter  $\xi$  of the corresponding pulse is calculated in Table I. The Heisenberg parameter of IOTA pulse shows a well-balanced localization in both the time and frequency domains. The proposed SQ<sup>2</sup>AM pulse produces more concentrated time dispersion than IOTA and its Heisenberg parameter is closer to the unity. This means that our proposed pulse will avoid the transmitted signal energy spreading out in the time-frequency dispersive channel and perturbing symbols at neighboring points in time-frequency plane.

**Table I.** Heisenberg Parameters

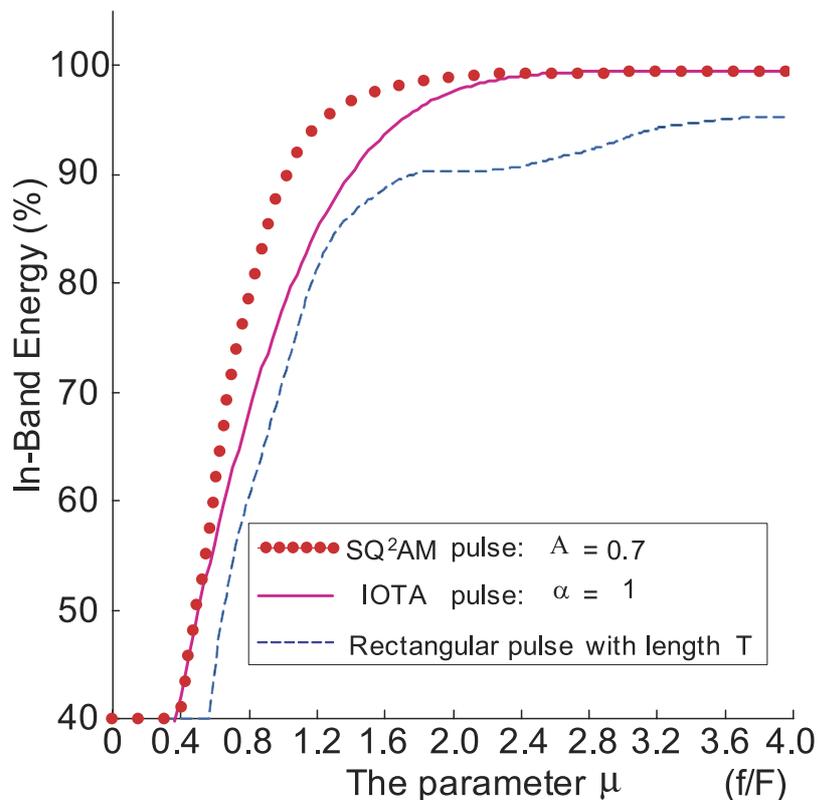
Pulse shaping function $g(t)$	Time dispersion $\Delta t$	Frequency dispersion $\Delta f$	Heisenberg Parameter $1/(4\pi\Delta t\Delta f)$
Rectangular	0.7493	1.1	0.096
IOTA	0.2821	0.2821	0.9769
Proposed SQ <sup>2</sup> AM	0.1446	0.5570	0.9885

**4.2 In-band energy**

Low out-of-band energy is desirable in OFDM systems in order to reduce the spectral overlap between adjacent subcarriers and to produce compact power spectrum. Minimizing the out-of-band energy can be done by maximizing the in-band energy. The in-band energy of a modulated subcarrier is defined as

$$E_{in} = \int_{-\mu F/2}^{\mu F/2} |G(f)|^2 df \tag{10}$$

The parameter  $\mu$  defines the width of the considered frequency band.  $\mu = 1$  corresponds to a frequency band as wide as the subcarrier spacing  $F$ . The in-band energy of rectangular, IOTA and SQ<sup>2</sup>AM pulse shaping functions are shown in Fig. 1. When  $\mu = 1$ , the in-band energy of SQ<sup>2</sup>AM is 0.8813 and that of IOTA is 0.7842. It is found that SQ<sup>2</sup>AM pulse produces more



**Fig. 1.** In-band energy of rectangular, IOTA and SQ<sup>2</sup>AM pulse shaping functions

compact power spectrum than IOTA pulse in a given frequency bandwidth. This indicates that SQ<sup>2</sup>AM pulse can reduce the adjacent interferences in multi-carrier modulation systems such as OFDM systems.

### 5 BER performances of OFDM-SQ<sup>2</sup>AM

The coded BER performance of the proposed OFDM-SQ<sup>2</sup>AM system has been evaluated and compared with that of IOTA/OFDM-OQAM system in the ITU-R vehicular A channel(120 km/h) and pedestrian B channel(3 km/h). OFDM with 512 subcarriers and convolutional code are used in the systems and the channel estimation is assumed to be perfect. Fig. 2 shows the BER performances of the proposed OFDM-SQ<sup>2</sup>AM, IOTA/OFDM-OQAM and OFDM-QAM in the mobile wireless channel. OFDM-SQ<sup>2</sup>AM and IOTA/OFDM-OQAM outperform OFDM-QAM, since they can adapt a lower code rate (code rate = 3/4) to maintain the same spectral efficiency as OFDM-QAM (code rate = 4/5). Such a coding gain comes from the fact that the guard interval is not required in OFDM-SQ<sup>2</sup>AM and IOTA/OFDM-OQAM. In the Pedestrian B channel, OFDM-SQ<sup>2</sup>AM outperforms IOTA/OFDM-OQAM and OFDM-QAM by 0.4 dB and 3.8 dB, respectively at BER = 1 × 10<sup>-3</sup>. In the Vehicular A channel, OFDM-SQ<sup>2</sup>AM outperforms IOTA/OFDM-OQAM and OFDM-QAM by 0.3 dB and 3.0 dB, respectively at BER = 1 × 10<sup>-3</sup>.

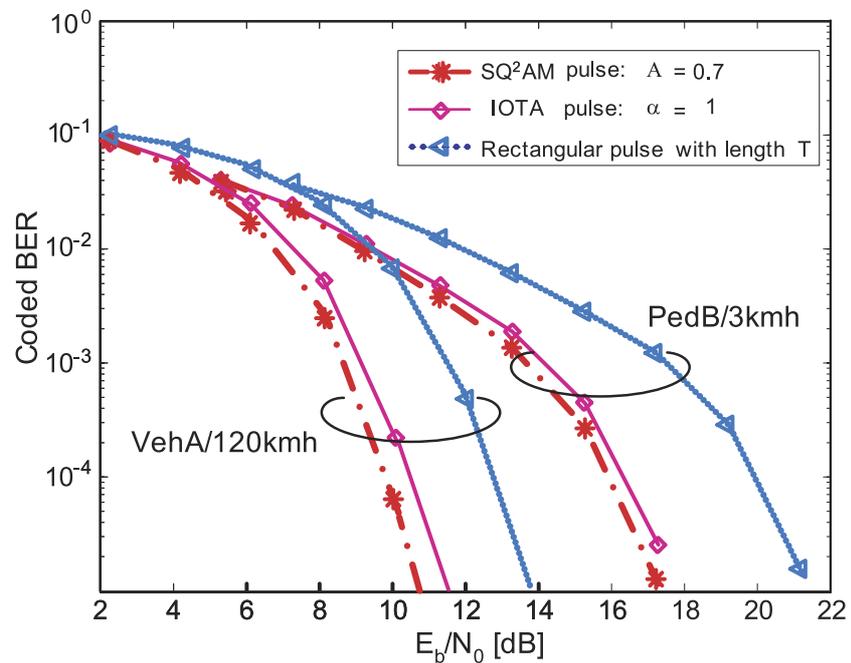


Fig. 2. BER performances in ITU-R mobile wireless channel

## 6 Conclusions

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In this paper, an efficient pulse shaping technique for OFDM-OQAM has been proposed and analyzed in the time and frequency dispersive environment. The simulation results demonstrate that the proposed pulse shaping can achieve higher spectral and power efficiencies than IOTA/OFDM-OQAM. It is also found that the proposed OFDM-SQ<sup>2</sup>AM provides an improved robustness to the time-frequency dispersion especially to the large frequency offsets.

## Acknowledgments

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This work was supported by the IT R&D program of MKE / KEIT [KI002091, Research on Multiple Antenna and Multi-hop Relay Transmission Technologies for Next Generation Mobile Broadcasting Service].