SOILS AND FOUNDATIONS Vol. 46, No. 4, 427–435, Aug. 2006
Japanese Geotechnical Society

ON SHALLOW PAD-FOUNDATIONS FOR FOUR-LEGGED PLATFORMS

ROY BUTTERFIELD[i)]

ABSTRACT

There are well-established procedures for assessing the combined horizontal and vertical load capacity of isolated, shallow-depth pad foundations. However, when the pads act together in an interconnected group, as they do, for example, under an offshore platform, some of them may be unloading whilst the loads increase on others. How the pads in such groups respond to a monotonically increasing horizontal load up to failure; how their diameter and spacing might best be assessed for specific design loads; how their safety, with respect to load, might be improved by shallow-depth burial and how things change if the horizontal load is not distributed equally between the pads, are less well understood questions. The paper presents a simple, new methodology for investigating these problems together with numerical examples of its application.

Key words: bearing capacity, failure, footing, interaction, load-path, safety factor (**IGC**: E3)

INTRODUCTION

Whereas the design and assessment of 'safety', with respect to load, of shallow individual pad foundations is relatively well understood, the process is much less well defined for interconnected groups of pads such as, for example, those which are sometimes used for offshore structures. Two, philosophically different, methods are commonly used to assess the safety of a specific foundation: in what follows the possibility that the acceptability of the foundation might be governed by service-state displacements is not considered.

The traditional approach is to design the pad so that it will 'fail' under a loading system comprising the expected service-state loads at which it is intended to operate scaled up by a 'load factor' F. In fact, this is a much more complicated process than it might appear since F is strongly dependent on the load-path to failure imposed on the pad beyond the service-state loads. Unless all the loads applied to the pad increase monotonically, whilst maintaining their service-state proportions, the simple concept of a unique value for F breaks down (Butterfield, 1993). In practice, this condition is most likely to be met under monotonically increasing, vertical, centreline loading.

Figure 1 is a loading diagram for a typical foundation supporting both vertical and horizontal loads. In this case its self-weight locates A on the vertical-load V axis and the load-path AB, B being the service-state load point, indicates that a horizontal load H has been added to the self-weight. The curved line represents a plausible 'failure load' envelope-the locus of load points at which the pad will fail-and V^* is the vertical centreline load capacity, shown here as unit load. Four possible load paths from B to failure (1,2,3,4) are also shown.

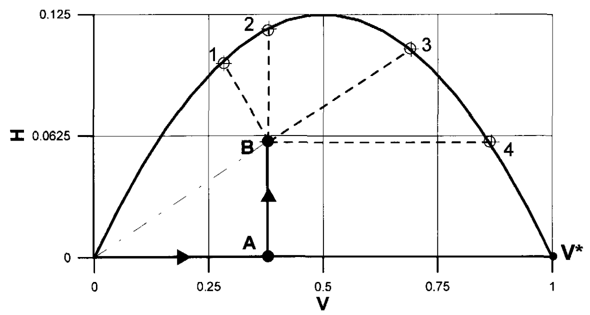

Fig. 1. Load-path dependence of Factor of Safety

On path 1, the pad is taken to failure by increasing H whilst decreasing V. Along path 2 H is increasing at constant V, whereas along path 4, V is increasing at constant H. On path 3, both H and V are increasing whilst maintaining their original ratio. This is the only case for which the simple notion of F is adequate ($F \approx 1.8$). In case 2, a load safety factor on $H = H_2/H_B$ can be defined although its value will depend upon V_A. It is clearly impossible to establish a unique load factor for the foundation applicable to all load paths.

An alternative approach, essentially that adopted in Eurocode 7, is derived from the concept of 'partial factors' under which 'service-state' loads are factored *up* (to become the 'design loads') and 'expected' ground strength parameters factored *down* (to become the

i) University of Southampton, UK (rb@soton.ac.uk).
The manuscript for this paper was received for review on July 11, 2005; approved on May 29, 2006.
Written discussions on this paper should be submitted before March 1, 2007 to the Japanese Geotechnical Society, 4-38-2, Sengoku, Bunkyo-ku, Tokyo 112-0011, Japan. Upon request the closing date may be extended one month.

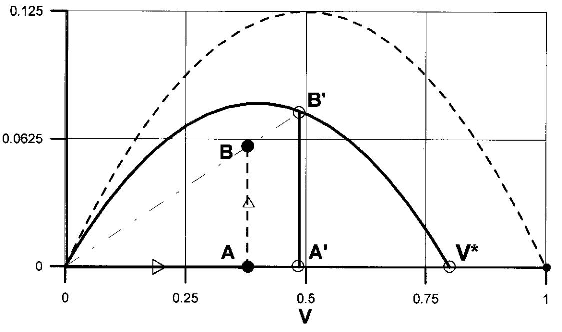

Fig. 2. Partial factors applied to Fig. 1

'mobilised strengths'). The foundation is then designed to fail under this combination of increased loads and decreased strengths. (In principle, different factors can be applied to different types of load and/or strength components to reflect the uncertainty in their magnitudes). Figure 2 interprets the load system of Fig. 1 in this way. The two loads have been factored up, by 1.3, and the relevant soil-strength parameter (e.g. the bearing capacity factor N_γ) factored down; divided by 1.25.

The factored loads plot at (A′, B′) and for the pad to fail at B′ it would need to have a vertical load capacity $V_d^* \approx 0.8$. It is clear that this method of including a measure of safety in a design also assumes monotonic, proportional loading and takes no account of alternative load-paths to failure. It is also clear that, for any specified load-path and suitably selected partial factors, both methods can lead to a footing design with identical dimensions.

It is of considerable practical importance for the safe design of interconnected pad footings, such as those supporting offshore platforms, to investigate: the manner in which these concepts might be applied to them; the sequential failure of the pads under increasing monotonic loads; the relationship between the diameter of the pads, their spacing and the loading regime, and the importance of the manner in which the horizontal load is distributed between them. The problem is further complicated by the fact that such a platform may also fail by overturning, which depends on the spacing of the pads but not on their diameter. These questions are addressed in the paper.

THE LOAD CAPACITY OF SHALLOW PAD FOUNDATIONS

It is now widely accepted that the failure-envelope with (V, H) axes, for a shallow, rigid pad-foundation is very closely a parabola (as shown in Figs. 1, 2). This is also true with (V, M/B) axes for a footing of breadth B loaded by a moment M, leading to the conclusion that a cigar-shaped three-dimensional failure envelope, with elliptic cross-sections in V= constant planes, might be applicable in (V, M/B, H) space (Butterfield, 1985). Experimental evidence supporting these results and extensive discussion of them can be found in Ticof (1977), Butterfield and Ticof (1979), Butterfield (1980, 1993), Gottardi and

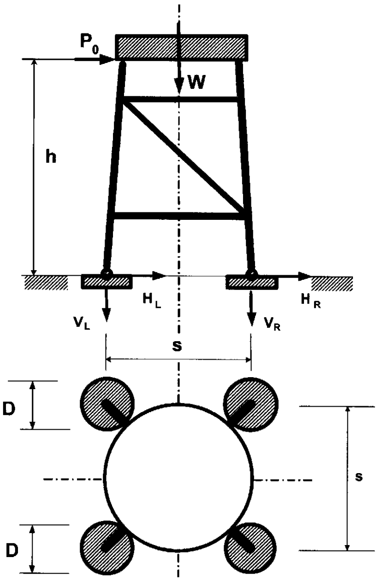

Fig. 3. A four-legged platform pin-jointed to rigid, circular pad-foundations

Butterfield (1993).

We shall be concerned here solely with a parabolic section of this envelope in the (V, $H+$) plane. The parabola is identical to the one shown in Figs. 1, 2. If its axes are normalized by plotting (H/V^*) against (V/V^*), its apex will always be at $(V/V^*)=1$.

The equation to the parabolic (V, H) failure-envelope is,

$$H = tV(1 - V/V^*) \tag{1}$$

or

$$(H/V^*) = t(V/V^*)(1 - V/V^*) \tag{2}$$

This equation predicts, correctly, that the maximum value of (H/V^*) occurs at $(V/V^*)=0.5$. Very many experiments, using dense sand, loose sand, Kaolin and brass-rod analogue material, have established that $(H/V^*)_{\max}=0.125$ very closely, consequently $t \approx 0.5$.

We shall also need to calculate the vertical, centreline load capacity Q_0 of a submerged, sea-bed supported, circular pad-foundation, diameter D. In terms of conventional bearing capacity and shape factors (N_γ, s_γ) this is

$$Q_0 = \gamma'.(D/2).\ N_\gamma.\ s_\gamma.(\pi D^2/4) = \pi.\gamma'.N_\gamma.s_\gamma.(D^3/8) \tag{3}$$

LOADING TO FAILURE OF A TYPICAL FOUR-LEGGED PLATFORM

Figure 3 shows an idealized, four-legged platform with its legs connected, via pin-joints, to shallow, rigid, circular pad-foundations. Its effective weight is W and it is subjected to wave and wind loads which generate a horizontal force P (with expected maximum value P_0), acting at height h above the top of a square array of pads, spaced at s centre to centre.

The loads generate vertical forces (V_L, V_R) and

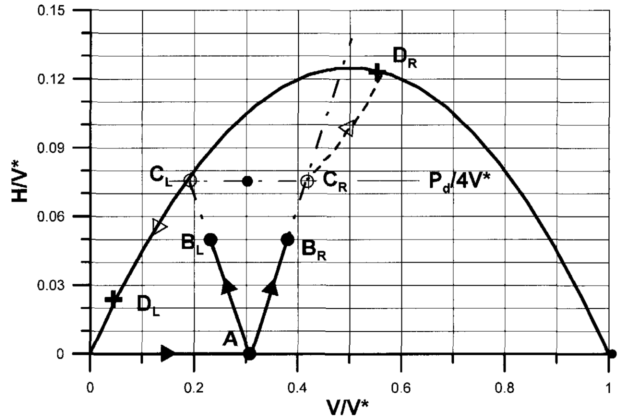

Fig. 4. Load paths of left-hand and right-hand pads to failure

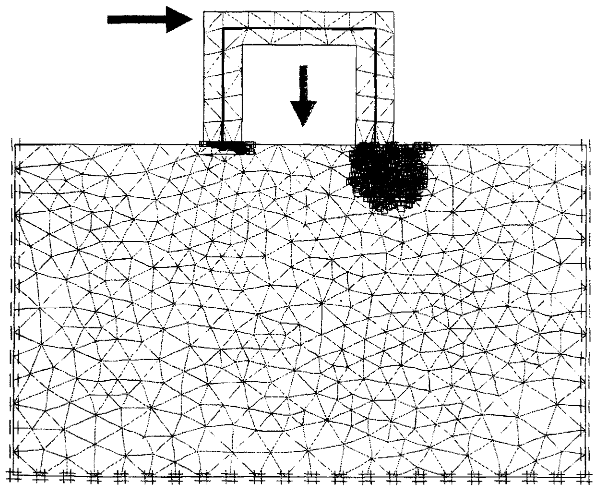

Fig. 5. FEM analysis showing plastic-zone development (Fisher and Cathie, 2003)

horizontal forces (H_L, H_R) on each pad in the 'left hand' and 'right hand' pairs, respectively, as shown in the figure. Initially, W will be distributed equally between all pads with $V_L = V_R = (W/4)$. It is not necessarily true that H will be distributed equally between them. A more general possibility is that $H_L = aP/2$ and $H_R = (1-a)P/2$, where ($0 \leq a \leq 1$); the requirement being that, for a four-pad group loaded as shown, $H_L + H_R = P/2$ always.

In what follows (W, P) will refer to 'global' forces applied to the platform and (V, H) the consequential 'local', forces supported by specific pads.

Any increment dP of P, will generate a moment ($dP.h$) at base level, which, because the legs are pinned to the pads, will be resisted solely by changes (dV) in the vertical forces acting on them. Hence,

$$dV_L = -(dP/2)(h/s) = -dP(r/2)$$

and

$$dV_R = (dP/2)(h/s) = dP(r/2), \text{ where } r = h/s. \quad (4)$$

Integration of Eq. (4) provides the general loading regime on the platform,

$$V_L = W/4 - Pr/2 = W/4 - H_L\, r/a, \text{ since } H_L = aP/2 \quad (5a)$$

$$V_R = W/4 + Pr/2 = W/4 + H_R\, r/(1-a),$$
$$\text{since } H_R = (1-a)P/2 \quad (5b)$$

The slopes of the (H, V) load paths followed as P is applied are therefore ($-a/r$, $(1-a)/r$). When $a = 1/2$, $H_L = H_R = P/4$, the gradients are $\pm(1/2r)$ and P is shared equally between the pads.

The (V, H) diagram of Fig. 4 illustrate this situation where, again, the vertical dead-load establishes point A and (B_L, B_R) are specific points on load-paths for left-hand and a right-hand pads, respectively, when a horizontal load P_0 is applied.

The diagram follows the loading sequence of the platform pad-foundations as they proceed, in (V/V^*, H/V^*) space, from a vertical self-weight load at A (here $V_0/V^* = 0.3$), via an increasing wave load P along load paths determined by Eq. (4) (gradient ≈ 0.75, hence $r = 2/3$) to points (B_L, B_R) at which $H_0/V^* = 0.05$. Hence $n_0 = P_0/W = 1/6$ (i.e. the applied horizontal load is one sixth of the weight of the platform).

When the horizontal load increases, the vertical load on the left-hand pair decreases whilst that on the right-hand pair increases until, if $a = 1/2$ is maintained, the 'upwind' pads reach failure at point C_L (H/V^* then being 0.075, accompanied by a decrease in V/V^* to about 0.2). The 'downwind' pads are now at point C_R, well away from the failure envelope. If P increases further the left-hand pair of pads will unload and both V_L and H_L will be decreasing whilst traversing the failure envelope towards D_L. The right-hand pads would follow the path (C_R, D_R), which reflects (C_L, D_L), until it intersects the failure envelope at D_R, simultaneously locating D_L: at which stage all pads have failed. During the latter process P can no longer be distributed equally between all pads.

The 'safety' for such a system (for the specified load-path) is governed by the onset of 'sliding' failure of the 'upwind' pads. A load safety-factor for P can be defined in this case-as the ratio of $P(C_L) + P(C_R)$ to that of $P(B_L) + P(B_R) \approx 0.15/0.1 = 1.5$. When total failure occurs the corresponding P ratio will be that for the D and B points, i.e. $(0.026 + 0.124)/0.1 = 1.5$ again. The ratio has therefore not increased perceptibly and, once started, total collapse of the complete foundation follows at an essentially constant value of P.

The highly unsymmetrical manner in which pads respond to horizontal loading is well illustrated in Fig. 5, which shows finite element output of plastic zones at failure under an idealised four-legged platform (Fisher and Cathie, 2003), in which the 'upwind' pad has failed by sliding and the 'downwind' one in the classical plunging mode

PAD DESIGN FOR A FOUR-LEGGED PLATFORM

In the 'partial factors' method, well-defined loads are factored up less than those whose values are uncertain. Thus the lateral wave loading would be increased from the estimated value P_0 to P_d, whereas the platform weight W, known more precisely, would not be factored. (In fact, a reduction in W would decrease the horizontal load capacity of the pads, leading to an increase in their diameter, whereas an increase in W would improve the overall stability of the platform against overturning.) The soil strength parameters $\tan\phi'$ (and thereby N_γ) would be reduced to 'mobilised' values but the shape-factor s_γ and the submerged soil density γ' would not be factored.

Eurocode 7 case C recommends a load factor of 1.3 "for unfavourable, variable actions" and a "strength mobilisation" (reduction) factor of 1.25.

There are two failure modes to be considered.

a. Failure by overturning: Since this is determined solely by the applied loads and the platform geometry and not at all by the subsoil properties, it can best be catered for by an independent load factor F_0 on P_0. The platform will therefore overturn when $F_0\ (P_0\ h) = W(s/2)$, a relationship which establishes the minimum pad spacing s as

$$s = 2F_0\ h(P_0/W) = 2F_0 h n_0 \qquad (6)$$

b. Foundation failure: From the previous analysis it is evident that, ideally, all pads should fail simultaneously. The design values of (V, H) would then lie on a single parabolic, mobilised-strength failure-envelope and the design value of V^*_d for each pad in the factored system would be identical.

If (W, P_d) represent the factored 'design' loads, and n the P_d/W ratio, then, from Eq. (5), the load coordinates $(V, H)_L$ for a left-hand pad at failure are, writing $r = h/s$.

$$(V, H)_L = \{(W/4 - nWr/2),\ naW/2\}$$
$$= (nW/2)\{(1/2n - r),\ a\} \qquad (7)$$

From Eq. (2), the equation to the parabola can be written as $V^*_L = tV^2/(tV\text{-}H)$ and, since $(V, H)_L$ lies on it, the value of V^*_L corresponding to a C_L failure-point can be found by substituting $(V, H)_L$ values from Eq. (7). This leads to,

$$V^*_L = (Wtc_1^2)/4(c_1\ t - 2na)\text{: where } c_1 = (1 - 2rn) \qquad (8)$$

The value of V^*_R, corresponding to the point C_R can be deduced similarly to be,

$$V^*_R = (Wtc_2^2)/4(c_2 t - 2n(1 - a))\text{: where } c_2 = (1 + 2rn) \quad (9)$$

By equating V^*_L and V^*_R in Eqs. (8), (9) we obtain the specific value of 'a' which will achieve simultaneous failure of all four pads,

$$a = \{1 - 2nr + 4n^2r^2 - 8n^2r^3t + 2(n^2r^2 - 2nr^2t$$
$$+ r^2t^2)^{0.5}\}/\{2(1 + 4n^2r^2)\} \qquad (10)$$

For a surface pad ($t = 0.5$) and symmetrical load paths ($a = 0.5$) this equation reduces to $r^2 = (1\text{-}4n)/n^2$ a condition which can only be satisfied by unrealistically small values of n, with r becoming zero when $n = 0.25$.

Inserting the Fig. 4 parameters $\{t = 0.5,\ r = 2/3,$

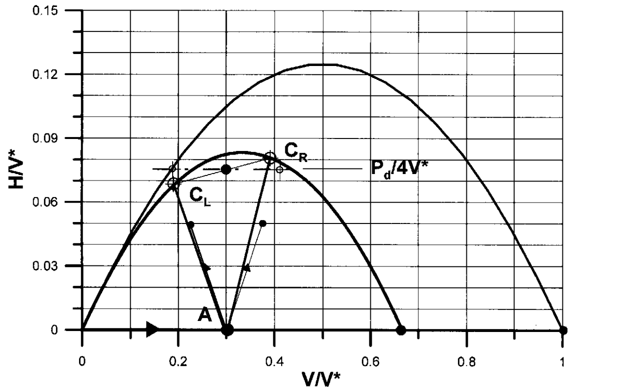

Fig. 6. Failure envelopes and load paths for surface pads ($a = 0.467$)

$n = 0.25\}$ into Eq. (10) generates $a = 0.467$ and $V^*_L = V^*_R = 0.667$. Figure 6 shows the resulting failure envelope together with that from Fig. 4. Although the difference in the slopes of the load paths involved is quite small, simultaneous failure of all the pads has been achieved, the design value of V^*_d has been reduced by one third and therefore (from Eq. (3)) the pad diameter reduced by about 13%.

The philosophy of this approach, which provides an alternative model for the load paths followed by the pads from first loading to their simultaneous failure, is that: even if $a = 1/2$ at the start of loading from A there is no reason why it should remain constant throughout the complete loading process; 'a' will, more likely, change progressively as P increases, distributing the horizontal load between the pads in such a way as to ensure that they fail simultaneously when $P = P_d$ (i.e. although the actual load paths may be curved lines the mean paths correspond to $a = 0.467$).

NUMERICAL EXAMPLE 1

The parameters used in the following example are similar to those used by Georgiadis (1985) and Powrie (2004). The platform (weight $W = 48$ MN) has an effective height $h = 98$ m, at which the expected horizontal wind/wave load $P_0 = 12$ MN acts. Therefore $n_0 = 1/4$ and, if a factor of safety $F_0 = 2$ against overturning under P_0 is adopted, Eq. (6) establishes $s = 98$ m and $r = h/s = 1$. Using the Eurocode 7 load factor, $P_d = (1.3\ P_0) = 15.6$ MN, hence $n_d = 0.325$ (considerably larger than the value of 0.25 used in Figs. 4, 6). The objective is to determine the safe diameter for the unburied pad foundations supporting the platform.

If the Eurocode 7 reduction factor of 1.25 is applied to tan(36°), the tangent of the expected soil friction angle ϕ'_0, the mobilised value $\phi'_{mob} = \tan^{-1}(0.5812) \approx 30°$.

For $\phi' = (30°, 36°)$ and circular pads, reasonable [(bearing-capacity factor) x (shape factor)] products are $(N_\gamma.\ s_\gamma) = (20,\ 60)$. Equation (3) can then be used to calculate Q_0 for any pad. Whence, if $\gamma' = 10$ kN/m^3, $Q_0 = (78.54,\ 235.62)\ D^3$ kN for the two values of $(N_\gamma.\ s_\gamma)$.

Equation (10) then provides the required value of

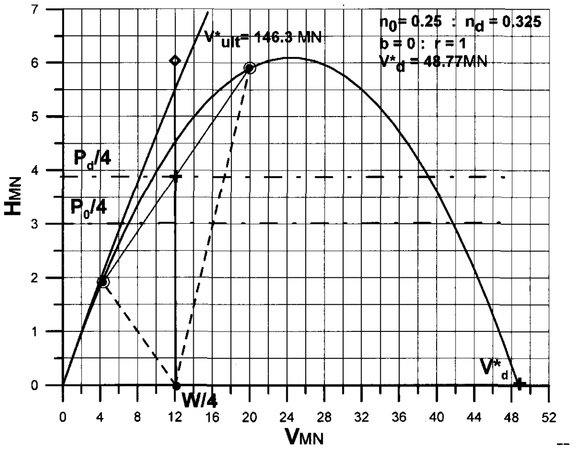

Fig. 7. Numerical example 1: failure envelopes and load paths for surface pads ($a = 0.246$)

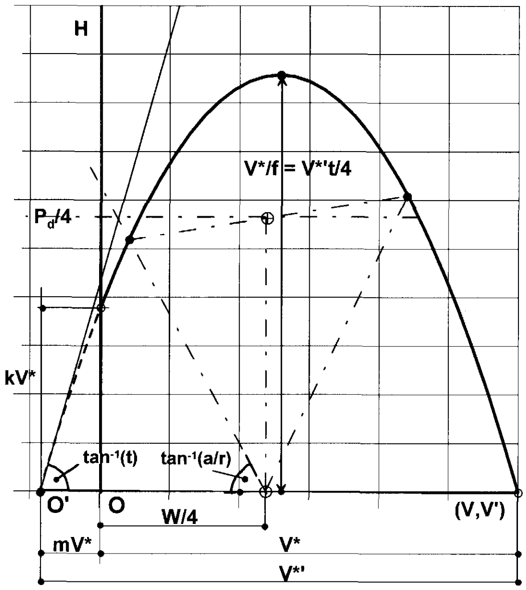

Fig. 8. Failure envelope and terminology for a buried pad

$a = 0.246$ and Eq. (8) (or (9)) the associated load-capacity of each pad $V^* = 48.8$ MN. Equating this to Q_0 (for $\phi' = 30°$) establishes the pad diameter $D = 8.5$ m. Figure 7 shows the relevant failure envelope, load paths and failure points. The point marked by a diamond on the $V = W/4 = 12$ line at $H = F_0 P_0/4 = 6$ represents the $P/4 = (H_L + H_R)/2$ value at which the platform will overturn (in this case, overturning would occur at $2P_0 = 24$ MN $\approx 1.5\ P_d$).

A portion of the failure envelope for the fully realised strength ($\phi'_0 = 36°$, $Q_0 = 146$ MN) is also shown. Whereas, at failure, the load on a right hand pad is always far removed from this envelope this is not so for the left hand pads which fail at essentially identical load points on both envelopes. If the safety factor of the whole set of pads is defined as the ratio of their mean horizontal load at failure to ($P_0/4$) it is clearly only about 1.3 for both full and reduced soil strengths. This is an unacceptably low figure, which highlights the vulnerability of surface pad foundations to horizontal loads (Butterfield, 1993).

Surface pads will not, in general, provide adequate safety against horizontal failure—they need to be embedded. The following sections extend the foregoing methodology to encompass pads buried at shallow depth d, where $0 \le d/D \le 1$.

LOAD CAPACITY AND FAILURE ENVELOPES FOR SHALLOW PADS ($0 \le d/D \le 1$)

Figure 8 illustrates the extended notation which becomes necessary to include, not only actual (H, V) load points with origin O, but also (H, V') load points relating to a coordinate origin O' at the 'zero' of a shifted parabola such that $V' = (V + mV^*)$. The maximum vertical load capacity of the buried pad is still at V^* but the maximum span of the parabola, referred to O', becomes $V^{*\prime} = (1 + m)\ V^*$.

If the maximum value of $H = V^*/f$, then, since $V^*/f = V^{*\prime} t/4 = V^*(1 + m)\ t/4$, m and f are related via,

$$(1 + m) = 4/ft \tag{11}$$

If, also, kV^* is the value of H at the origin O, at which $V' = mV^*$, then, since the point (mV^*, kV^*) lies on the failure parabola passing through O',

$$kV^* = mV^* t\ (1 - mV^*/V^{*\prime}) = mV^* t/(1 + m)$$

i.e.
$$(1 + m) = mt/k \tag{12}$$

Consequently the value of k determines both f and m from,

$$f = 4(t - k)/t^2 \text{ and } m = k/(t - k) \tag{13}$$

It is well known (e.g. Sokolowski, 1956) that, for $25° \le \phi' \le 40°$, the bearing capacity factors N_γ and N_q are almost equal. The vertical centreline load capacity Q_b of a footing buried at shallow depth can then be expressed in terms of the surface pad capacity Q_0 as,

$$Q_b = Q_0\ (1 + 2d/D) = Q_0(1 + 2b), \text{ where } b = d/D \tag{14}$$

Hence, by embedding a pad at $(d/D) = (0, 0.5, 1.0)$ its vertical load carrying capacity V^* increases in the ratios (1, 2, 3) and in our example the 'expected' $(N_\gamma s_\gamma)_b$ products become (60, 120, 180) with corresponding 'mobilised' values (20, 40, 60).

If, in addition, the lateral resistance of the buried pad, under zero vertical load, is equated to the passive resistance of an equivalent skirted or solid pier then, with K_p the relevant passive earth pressure coefficient,

$$kV^* = K_p\ (\gamma' d^2/2) D = \gamma' D^3 (K_p b^2/2) \tag{15}$$

For a surface-pad the theoretical value of N_γ is directly proportional to K_p. By adopting $K_p = N_\gamma/5$ we obtain $K_p = (4, 12)$ for $\phi' = (30°, 36°)$, respectively, values very close to those given by Sokolowski (1956) for rough walls with $\delta \approx 2\phi'/3 = (20°, 24°)$.

The bearing capacity Eq. (3), when extended to allow

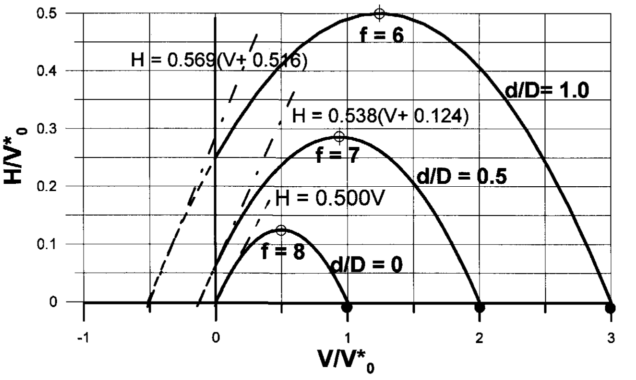

Fig. 9. Failure envelopes for pads, diameter *D*, buried at shallow depth *d*

for shallow burial, and re-arranging the terms, becomes,

$$\gamma'.D^3 = 8V^*/(\pi.N_\gamma.s_\gamma.(1+2b)) \quad (16)$$

By eliminating $\gamma'.D^3$ between Eqs. (15, 16), approximating $s_\gamma = 1$ and inserting $K_p = N_\gamma/5$ we obtain a relationship between k and b,

$$k = 4b^2/(5\pi(1+2b)) \approx (b^2/4)/(1+2b)) \quad (17)$$

There is evidence, from small, brass-rod model tests (Butterfield, 1985), which supports this lateral resistance assumption and also demonstrates a decrease in f with increasing burial depth to rather less than 6 for $b=1$. A numerical analysis of a pad buried in granular material at $b/D = 0.5$ (Gottardi et al., 2005) concluded that $f \approx 7$. We shall incorporate these results by assuming that,

$$f = (8-2b) \quad (18)$$

Using this relationship to eliminate f from the first of Eq. (13) connects (b, k, t) in an equation that can then be used in conjunction with Eq. (17), to eliminate k. The result is the following (b, t) expression that reconciles all the foregoing assumptions,

$$2(4\text{-}b)t^2 - 4t + b^2/(1+2b) = 0 \quad (19)$$

Equation (17) predicts that t will increase slowly with b; for example, $b = (0,\ 1/2,\ 1)$ leads to $k = \{0,\ 1/32,\ 1/12\}$ from Eq. (17), $t = \{0.500,\ 0.538,\ 0.569\}$ from Eq. (19) and $m = \{0,\ 0.062,\ 0.172\}$ from the second of Eq. (13). The outcome of this new formulation is a set of 'shifted' parabolic envelopes as shown in Fig. 9.

By using these diagrams, the surface-pad design methodology, developed earlier, can be extended to include shallow-depth pads ($0 \le d/D \le 1$) as follows.

The equation to the parabola in Fig. 6, referred to origin O' is,

$$H' = H = V't\,(1 - V'/V^{*\prime}) = (V + mV^*)t\{1 - (V + mV^*) \\ /(V^* + mV^*)\} \quad (20)$$

Equation (20) can be re-arranged to provide the value of V^*

$$V^* = \{c_3 + (4\,mt^2V^2 + c_3^2)^{0.5}\}/2\,mt$$

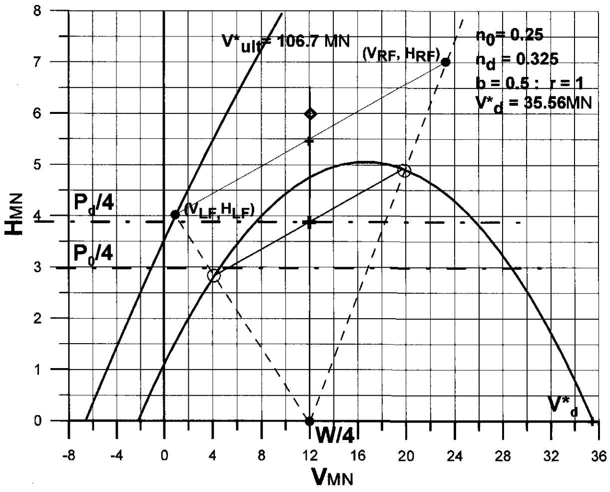

Fig. 10. Numerical example 2: failure envelopes and load paths for buried pads ($a = 0.366$, $b = 0.5$)

where

$$c_3 = (H + Hm + mtV - tV) \quad (21)$$

Equation (7) still applies and it can be used, now in conjunction with Eq. (21), to establish equations corresponding to (8, 9) which provide the values of V^*_L and V^*_R in terms of (W, r, t, n, a). Setting $V^*_L = V^*_R$ generates the required expression for 'a', in terms of (m, n, r, t) that will again ensure simultaneous failure, under the specified loads, of the four buried pads supporting the platform. As before back-substitution for 'a' determines V^* and the pad diameter via Eq. (3). These equations are cumbersome but easily handled by an algebraic computer package. (They are provided in APPENDIX A together with a short Mathematica procedure that derives and processes them, calculates key parameter values and plots the failure envelopes and load-paths.)

NUMERICAL EXAMPLE 2

The data for this example is identical to that used in example 1. The results provided are the failure envelopes, load paths, pad dimensions and 'safety factors' obtained using the buried-pad procedure for $b = \{0.5, 1.0\}$.

a. Burial depth $b = d/D = 0.5$

The values of (a, V^*_d) generated by the algorithm are (0.366, 35.56) MN with the latter leading to $D = 6$ m and, for the full-soil-strength parameters $(N_\gamma s_\gamma)_b = 40$, $V^*_{ult} = 106.7$ MN.

Figure 10 shows the relevant failure envelopes and load paths. If the left-hand pad failure-load coordinates at the intersection of its load path (Eq. (5a)) with the ultimate strength parabola (Eq. 20), are (V_{LF}, H_{LF}), then the value of H_{RF} for the right hand pad will lie symmetrically on its load path (Eq. (5b)) at $V_{RF} = (W/2 - V_{LF})$ as in Fig. 10. (The expressions for (V_{LF}, H_{LF}, V_{RF}, H_{RF}) are derived in the penultimate paragraph of the Mathematica code in APPENDIX A.)

The 'full soil strength' load factor of the pad relative to

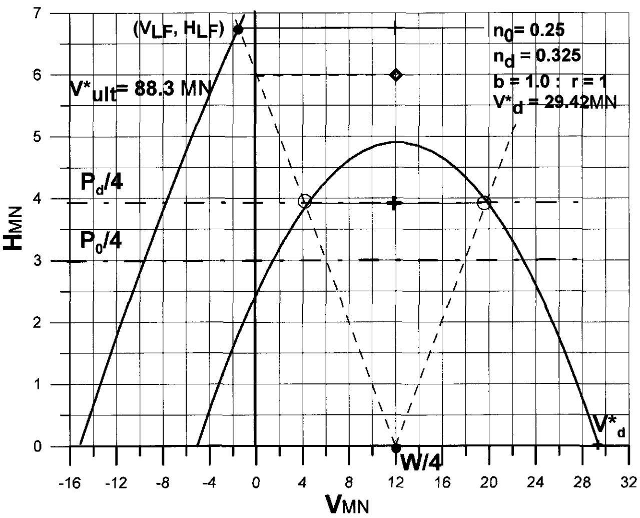

Fig. 11. Numerical example 2: failure envelopes and load paths for buried pads ($a=0.494$, $b=1.0$)

P_0 is, as previously, $(H_{LF}+H_{RF})/(Po/4)$; in this case 1.79. Embedment at 3 m depth therefore reduces the required pad diameter from 8.5 to 6 m and increases their 'factor of safety' against failure under horizontal loading by about 35%.

If the pad is embedded one diameter deep the design values of (a, V^*_d) become (0.494, 29.42 MN). Since $(N_\gamma s_\gamma)_b=60$ when $b=1$, the required pad diameter reduces further to 5 m. The 'full soil strength' load factor calculated as above is 2.26. However, to achieve this the value of V_{LF} has to become negative (Fig. 11). Since the platform overturns when $V_{LF}=0$ (indicated by the 'diamond' at (12, 6)) and a factor $F_0=2$ against overturning under Po has been incorporated, the 'full soil strength' load factor cannot exceed this value.

These results illustrate the proposed methodology and the economy in pad size and improvement in lateral load-capacity that can be achieved by modest depth burial (or skirting) of pad foundations.

A PLATFORM AS A THREE-DIMENSIONAL STRUCTURE

Since the lateral wave/wind load P_0 can strike the platform from any angle all four pads will be identical. However, it is still necessary to check whether or not pads designed to resist P_0 acting perpendicular to a side of the platform will be adequate when P_0 is directed along a platform-diagonal.

The resistance of the platform to overturning will nominally increase by a factor of $\sqrt{2}$ as the line of action of P swings from being perpendicular to its side to being diagonally aligned. In practice this will be offset by the possibility of lateral instability of the in-line pads and the true increase in stability is likely to be less.

Although the effective pad-spacing is increased, to $s'=s\sqrt{2}$, the overturning moment $P.h$ is now only resisted by changes in V on the two pads aligned with the diagonal. Consequently $\Delta V=\pm(P)(h/s')=\pm P\ (h/s\sqrt{2})=\pm(P/2)$ $(r\sqrt{2})$ and the pads themselves will be more vulnerable under diagonal loading.

If the pad spacing were to be increased to $s''=s\sqrt{2}$ the length of the diagonal becomes 2s. The ΔV generated in the diagonal load case would then be reduced to $\pm(P/2)$r again and, if it is assumed that the intermediate pads each support $(P/4)$, and the distribution of the horizontal loads between the new left-hand and 'right-hand' pads is again $\{a,\ (1-a)\}P/2\}$, the results of the previous 'side load' analyses using r would apply to the 'diagonally' loaded enlarged platform. The 'r' ratio to be used for loads directed perpendicular to the sides of this platform then becomes $r''=(h/s\sqrt{2})=r/\sqrt{2}$.

If all the previous analyses are to apply to a diagonal plane of a platform it is one with an actual 'r' value of $r/\sqrt{2}$-achieved by increasing the platform width to $s\sqrt{2}$. This is, of course, a major practical disadvantage of a 2×2 array of pad foundations-and an argument in favour of multiple pads or 'ring' foundations. The enlarged system will be intrinsically more secure against loads perpendicular to its sides and its factor of safety against overturning increased by a factor of $\sqrt{2}$; the pad diameter required will still be that provided by an analysis using r.

CONCLUDING REMARKS

The well-established concept, that the failure-load envelope for a shallow-depth pad foundation, under combined vertical and horizontal loading, can be represented satisfactorily by a parabola, has been extended to generate a new design and analysis methodology for a square group of pad-foundations interconnected by an idealised structural platform.

1. The procedure is simple to implement, either numerically or graphically to provide a rational prediction of the necessary pad diameter, for a specified set of soil parameters and design values of the static vertical and horizontal loads acting on the structure.
2. By allowing for the fact that the horizontal load may not be distributed equally between the 'upwind' and 'downwind' pads, a pad diameter can be deduced satisfying the ideal requirement that all four pad foundations should fail simultaneously under the specified design loading; thereby providing a minimum value.
3. New parabolic failure envelopes for pads embedded up to one diameter deep are presented and the analysis is extended to include buried pads. Both pad failure, predominantly of the 'upwind' pads, and platform overturning are incorporated with the latter becoming significant for the more deeply embedded foundations.
4. The methodology presented is thought to provide a novel and useful way of exploring the fundamental static load capacity, and overturning stability, of idealized groups of shallow pad-foundations under horizontally loaded, four-legged structures.

NOTATION

a: horizontal load-share factor, $(H_L, H_R)=(a, 1\text{-}a)(P/2)$
b: footing burial-depth ratio $=d/D$
d: footing burial depth
D: pad diameter
(f, m, k): dimensionless parameters defined in Fig. 8
F: general load factor
F_0: load factor against failure by overturning
h: height above (pinned) platform leg base to line of action of P
H: horizontal load on a pad
(H_L, H_R): horizontal loads on (left, right) or (upwind, downwind) pads
K_p: passive earth pressure coefficient
n: load ratio P/W
n_0: load ratio P_0/W
n_d: load ratio P_d/W
(N_γ, s_γ): bearing capacity and shape factors for a rigid, circular, surface-pad
P: horizontal load on platform
P_0: service state value of P
P_d: factored 'design load' value of P_0
Q_0: vertical, centreline load, capacity of a specific rigid, circular, surface-pad foundation
Q_b: vertical, centreline load, capacity of a specific rigid, circular pad foundation at depth-ratio b
r: platform aspect ratio $=h/s$
s: centre to centre spacing of pads in a square array
t: slope of a parabolic failure-envelope at its intersection with the vertical load axis
V: vertical load on a pad
V^*: vertical centreline load capacity of a circular, surface-pad
V^*_b: vertical centreline load capacity of a circular, pad at depth ratio b
V^*_d: design value of V^* at failure under the design loads
(V_L, V_R): vertical loads on (left, right) or (upwind, downwind) pads respectively
(V^*_L, V^*_R): values of (V_L, V_R) for a pad failing under the design loads
(V_{LF}, H_{LF}): intersection point of a left-hand pad load path with the 'full soil strength' failure envelope
(V_{RF}, H_{RF}): point on a right-hand load path corresponding to (V_{LF}, H_{LF})
W: effective platform weight = total vertical load on platform

REFERENCES

1) Butterfield, R. (1980): A simple analysis of the load capacity of rigid footings on granular materials, *Geotech.*, **30**, 128–137.
2) Butterfield, R. (1985): Load-path dependent stability of shallow footings, Discussion, *Soils and Foundations*, **25**(3), 150–154.
3) Butterfield, R. (1993): A new approach to safety factors for shallow foundations: load combination factors as a basis for risk assessment, *Risk and Reliability in Ground Engineering*, ICE, London, 112–125.
4) Butterfield, R. and Ticof, J. (1979): The use of physical models in design, Discussion, *Proc. 7th ECSMFE, Brighton*, **4**, 259–261.
5) Fisher, R. and Cathie, D. (2003): Optimisation of gravity based design for subsea applications, *Foundations: Innovations, Observations, Design and Practice*, Thomas Telford, London, 283–296.
6) Georgiadis, M. (1985): Load -path dependent stability of shallow foundations, *Soils and Foundations,* **25**(1), 84–88.
7) Gottardi, G. and Butterfield, R. (1993): On the bearing capacity of surface footings on dense sand under general planar loading, *Soils and Foundations*, **33**(3), 68–79.
8) Gottardi, G., Govoni, L. and Butterfield, R. (2005): Yield loci by 'swipe' testing, *ISFOG Conf.*, Perth, Australia (Accepted for publication).
9) Powrie, W. (2004): *Soil Mechanics-Concepts and Applications*, 2nd ed., London, E & F N Spon.
10) Sokolowski, V. V. (1956): *Statics of Soil Media*, London, Butterworth.
11) Ticof, J. (1977): Surface footings on sand under general planar loads, *PhD Thesis*, University of Southampton.

APPENDIX A

(i) The procedure for determining 'a' to ensure simultaneous failure of a set of four embedded pads leads to,

$$V^* = -\frac{-\mathrm{h}-\mathrm{hm}+\mathrm{tv}-\mathrm{mtv}-\sqrt{4\mathrm{mt}^2\mathrm{v}^2+(\mathrm{h}+\mathrm{hm}-\mathrm{tv}+\mathrm{mtv})^2}}{2\mathrm{mt}} \tag{22}$$

In these equations (v, h) represents (V, H). Substituting for (v, h) from Eqs. (8, 9) provides (V^*_L, V^*_R) at pad failure

$$\begin{aligned} V^*_L = -\frac{1}{2\mathrm{mt}}\Bigg(&-\frac{1}{2}\mathrm{anw}-\frac{1}{2}\mathrm{amns}+\frac{1}{4}(1-2\mathrm{nr})\mathrm{tw}-\frac{1}{4}\mathrm{m}(1-2\mathrm{nr})\mathrm{tw} \\ &-\sqrt{\frac{1}{4}\mathrm{m}(1-2\mathrm{nr})^2\mathrm{t}^2\mathrm{w}^2+\left(\frac{\mathrm{anw}}{2}+\frac{1}{2}\mathrm{amnw}-\frac{1}{4}(1-2\mathrm{nr})\mathrm{tw}+\frac{1}{4}\mathrm{m}(1-2\mathrm{nr})\mathrm{tw}\right)^2}\Bigg) \end{aligned} \tag{23a}$$

$$\begin{aligned} V^*_R = -\frac{1}{2\mathrm{mt}}\Bigg(&-\frac{1}{2}(1-\mathrm{a})\mathrm{nw}-\frac{1}{2}(1-\mathrm{a})\mathrm{mnw}+\frac{1}{4}(1+2\mathrm{nr})\mathrm{tw}-\frac{1}{4}\mathrm{m}(1+2\mathrm{nr})\mathrm{tw} \\ &-\sqrt{\left(\frac{1}{4}\mathrm{m}(1+2\mathrm{nr})^2\mathrm{t}^2\mathrm{w}^2+\left(\frac{1}{2}(1-\mathrm{a})\mathrm{nw}+\frac{1}{2}(1-\mathrm{a})\mathrm{mnw}-\frac{1}{4}(1+2\mathrm{nr})\mathrm{tw}+\frac{1}{4}\mathrm{m}(1+2\mathrm{nr})\mathrm{tw}\right)^2\right)}\Bigg) \end{aligned} \tag{23b}$$

Setting $V^*_L = V^*_R$ and selecting the relevant root provides the value of 'a' as

$$\begin{aligned} \mathrm{a} = (&1+2\mathrm{m}+\mathrm{m}^2-2\mathrm{nr}-4\mathrm{mnr}-2\mathrm{m}^2\mathrm{nr}+4\mathrm{n}^2\mathrm{r}^2+8\mathrm{mn}^2\mathrm{r}^2+4\mathrm{m}^2\mathrm{n}^2\mathrm{r}^2-8\mathrm{n}^2\mathrm{r}^3\mathrm{t}+8\mathrm{m}^2\mathrm{n}^2\mathrm{r}^3\mathrm{t}+ \\ &2\sqrt{}(\mathrm{n}^2\mathrm{r}^2+4\mathrm{mn}^2\mathrm{r}^2+6\mathrm{m}^2\mathrm{n}^2\mathrm{r}^2+4\mathrm{m}^3\mathrm{n}^2\mathrm{r}^2+\mathrm{m}^4\mathrm{n}^2\mathrm{r}^2-2\mathrm{nr}^2\mathrm{t}-4\mathrm{mnr}^2\mathrm{t}+4\mathrm{m}^3\mathrm{nr}^2\mathrm{t}+2\mathrm{m}^4\mathrm{nr}^2\mathrm{t}+\mathrm{r}^2\mathrm{t}^2+4\mathrm{mr}^2\mathrm{t}^2+ \\ &6\mathrm{m}^2\mathrm{r}^2\mathrm{t}^2+4\mathrm{m}^3\mathrm{r}^2\mathrm{t}^2+\mathrm{m}^4\mathrm{r}^2\mathrm{t}^2+16\mathrm{mn}^2\mathrm{r}^4\mathrm{t}^2+32\mathrm{m}^2\mathrm{n}^2\mathrm{r}^4\mathrm{t}^2+16\mathrm{m}^3\mathrm{n}^2\mathrm{r}^4\mathrm{t}^2))/(2(1+2\mathrm{m}+\mathrm{m}^2+4\mathrm{n}^2\mathrm{r}^2+8\mathrm{mn}^2\mathrm{r}^2+4\mathrm{m}^2\mathrm{n}^2\mathrm{r}^2)) \end{aligned} \tag{24}$$

These very cumbersome expressions are clearly best generated in, and handled by, an algebraic computing package.

(ii) The following is a list of Mathematica code that does this, processes the data used in the examples and generates a diagram in the form of Figs. 9, 10 and 11.

```
b=1.; (* the selected burial depth ratio *)
(* an example of input data*)
w=48; hh=98; po=12; pd=1.3*po; n=pd/w; fot=2; r=w/(2*po*fot);
gamdash=10; ngamsgam=20*(1+2*b); (* using Nγsγ=60 for φ'=30° *)
f=(8-2*b); k=(b^2/4)/(1+2*b);
Solve[2*(4-b)*t^2-4*t+b^2/(1+2*b)==0, t]//N;
tt=t/.%[[2]]; (* root providing 't' *)
m=k/(tt-k)+ .001; (* avoids the singularity when m=0*)
Simplify[Solve[h==(v+m*vstar)*tt*(1-(v+m*vstar)/(vstar+m*vstar)), vstar]]//N; vvstar=vstar/.%[[2]];
(* root providing 'vstar' *) vstarleft=%/. {v→(n*w/2)*(1-2*r*n)/(2*n), h→n*w*a/2}; vstarright=%%/.
{v→(n*w/2)*(1+2*r*n)/(2*n), h→n*w*(1-a)/2}; Simplify[Solve[vstarleft==vstarright, a]]//N; aa=a/.%
[[1,1]]; (* root providing 'a' *) vvstarleft=vstarleft/. a→aa//N; (* V* at apex of parabola *)
diam=10*(8*vvstarleft/(Pi*gamdash*ngamsgam))^(1/3); (* pad D*)
g1=Plot[(v+m*vvstarleft)*tt*(1-(v+m*vvstarleft)/(vvstarleft*(1+m))), {v,0,vvstarleft}, DisplayFunc-
tion→Identity];
g2=Plot[(v+m*vvstarleft*3)*tt*(1-(v+m*vvstarleft*3)/(vvstarleft*3*(1+m))), {v,0,1.5*vvstarleft}, Dis-
playFunction→Identity];
g3=Plot[-(aa/r)*(v-w/4), {v, 0, w/4},DisplayFunction→Identity];
g4=Plot[(1-aa)*(v-w/4),{v, w/4,1.5*vvstarleft}, DisplayFunction→Identity];
g5=ParametricPlot[{w/4,y},{y, 0, 3*vvstarleft/f}, DisplayFunction→Identity];
Show[{g1, g2, g3, g4, g5},DisplayFunction→$DisplayFunction]; (* figure *)

Solve[{h==-(aa/r)*(v-w/4),     h==(v+m*vvstarleft*3)*tt*(1-(v+m*vvstarleft*3)/(3*vvstarleft*(1+
m)))},{h,v}]//N;
(* (HFL,VFL) coords of failure point on full-strength parabola *)
failh1=h/.%[[2,1]]; failv1=v/.%%[[2,2]];
failv2=(w/2-failv1); failh2=(1-aa)*(failv2-w/4);
loadfactor=(failh1+failh2)/(po/2); (* load factor F0 of pads rel.to Po *)

If[loadfactor >fot, maxloadfactor=fot];
(* this condition arises when failv1< 0, i.e. overturning will occur before pad failure *)
Print[StringForm[''a=", vstar=", pad diameter=", maximum load factor on Po=", overturning load factor
="'', aa, vvstarleft, diam, loadfactor, fot]];
```