

# Multimode precoder design for STBC with limited feedback in MIMO based wireless communication system

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**Abstract:** In this paper, a multimode precoder design for space-time block code (STBC) is investigated, which varies the number of streams depending on the channel condition. We develop a design criterion of minimizing the vector symbol error rate and derive an efficient offline algorithm to generate precoders. Simulation results show that the multimode precoded STBC (MM-STBC) provides substantial performance improvements compared with the single mode precoded orthogonal STBC (SM-STBC) for a fixed data-rate. It also outperforms the multimode precoded multiple-input multiple-output (MM-MIMO) system when feedback rate is very limited.

**Keywords:** space time block code, multimode precoder, limited feedback, MIMO based wireless communication system

**Classification:** Science and engineering for electronics

## References

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## 1 Introduction

Orthogonal space-time block codes (OSTBC) have been demonstrated to be a powerful diversity technique to combat channel fading in wireless multiple-input multiple-output (MIMO) systems when there is no channel knowledge at the transmitter. Generally, OSTBC can be only designed for certain numbers of transmit antennas and do not provide array gain [1,2]. When channel information is available at the transmitter, a linear precoded OSTBC [3] was proposed so that it can support different numbers of transmit antennas and provide performance improvement, where it uses a low rate feedback path from the receiver to the transmitter to carry limited feedback information about channel which represent a precoder index in a finite codebook of precoders. When the perfect channel state information (CSI) is known to the transmitter in the MIMO system, optimal transmission scheme is a spatial multiplexing with waterfilling power allocation. For finite feedback, a multimode precoded spatial multiplexing MIMO system (MM-MIMO) is proposed for achieving full diversity and large multiplexing gain [4]. It varies the number of bit streams and the precoder based on current channel condition. However, when the number of feedback bits is very small, the scheme suffers performance degradation because different modes can not have sufficient number of precoders to guarantee diversity and multiplexing gain simultaneously.

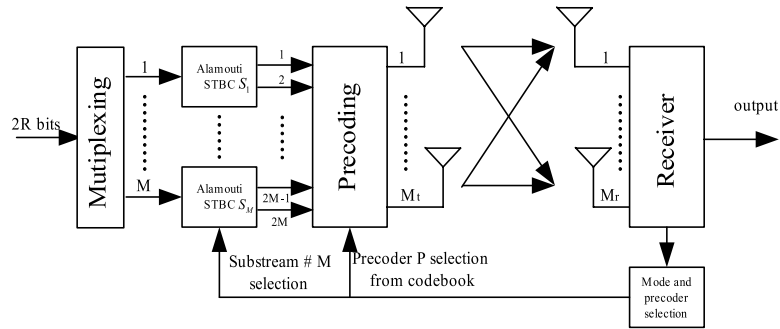
In this paper, a multimode precoded STBC system (MM-STBC) based on a multiple number of  $2 \times 2$  Alamouti OSTBC [1] is proposed. Based on observation that STBC can provide the diversity gain without channel information, when the finite feedback is very limited we can expect that the MM-STBC may perform better than the MM-MIMO. Furthermore, the codebook of precoders for the MM-STBC is designed such that it can minimize the average error probability directly rather than minimize the mean squared error (MSE) between the optimal precoder and codebooks packed in Grassmann manifold [3].

## 2 System model

A  $M_t$  transmit and  $M_r$  receive antennas multimode precoded STBC (MM-STBC) system is illustrated in Fig. 1. For each channel,  $2R$  bits are multiplexed into  $M (\leq M_t/2)$  different bit streams. The value of  $M$  is referred to the *mode* of the precoder and varies between 1 and  $M_t/2$ . Each bit stream is modulated with the same constellation and transformed to a  $2 \times 2$  matrix  $S_i$  by a Alamouti OSTBC, implying that each input stream carries  $2R/M$  bits of information over 2 time slots, i.e., the overall transmission data-rate is  $R$ . The final STBC transmission matrix is defined as

$$\mathbf{S} = [S_1, S_2, \dots, S_M]^T, S_i = \begin{bmatrix} s_{2i-1} & s_{2i} \\ s_{2i}^* & -s_{2i-1}^* \end{bmatrix} \quad (1)$$

The signal correlation matrix is  $\mathbf{S}^H \mathbf{S} = (\sum_{j=1}^{2M} |s_j|^2) \mathbf{I}_2$  and  $E_{s_j}[|s_j|^2] = 1$ ,  $j \in [1, \dots, 2M]$ , where the Hermitian of matrix  $\mathbf{A}$  is denoted by  $\mathbf{A}^H$ ,  $*$  stands



**Fig. 1.** System model of a limited feedback multimode precoding STBC

for conjugate and the  $n$ -dimension identity matrix is defined as  $\mathbf{I}_n$ .

Then, the STBC codeword  $\mathbf{S}$  is transmitted through over  $M_t$  antennas after being multiplied by a  $M_t \times 2M$  precoding matrix  $\mathbf{P}$  with  $\text{tr}(\mathbf{P}^H \mathbf{P}) \leq 2M$ . The matrix  $\mathbf{P}$  is selected from a set of precoder matrices  $\mathcal{P}_M$  that is known by both the transmitter and receiver, where  $M \in \mathcal{M}$  and  $\mathcal{M}$  denotes a set of supported mode values, e.g.,  $\mathcal{M} = \{1, 2\}$  for dual-mode precoding. If the number of precoders of mode  $M$  in  $\mathcal{P}_M$  is denoted as  $N_M$ , the total of feedback bits  $B$  is  $\lceil \log_2 (\sum_{M \in \mathcal{M}} N_M) \rceil$ , where  $\lceil x \rceil$  is a function for getting the smallest integer which is larger than  $x$ .

With assumption of perfect synchronization and channel estimation, the received signal can thus be written as

$$\mathbf{R} = \sqrt{\frac{\rho}{2M}} \mathbf{H} \mathbf{P} \mathbf{S} + \mathbf{W} \quad (2)$$

where  $\mathbf{R}$  is  $M_r \times 2$  matrix,  $\mathbf{H}$  is the  $M_r \times M_t$  channel matrix with independent entries distributed as a circular symmetric Gaussian random variable with zero mean and unit variance, i.e.,  $CN(0, 1)$ ,  $\mathbf{W}$  is an  $M_r \times 2$  noise matrix with independent entries distributed according to  $CN(0, 1)$ , and  $\rho$  is the signal-to-noise ratio (SNR).

By conjugation of the received signals at the second time slot, the input/output relation is represented as follows:

$$[r_1^1, \dots, r_1^{M_r}, (r_2^1)^*, \dots, (r_2^{M_r})^*]^T = \sqrt{\frac{\rho}{2M}} H_{eff}(\mathbf{H}, \mathbf{P}) \cdot \mathbf{s}_1 + \tilde{\mathbf{W}} \quad (3)$$

where  $r_m^n$  is the received signal at the  $n$ th receive antenna on the  $m$ th time slot and  $\mathbf{s}_1 = [s_0, s_1, \dots, s_{2M-1}, s_{2M}]^T$  is the first column vector of  $\mathbf{S}$ , and

$$H_{eff}(\mathbf{H}, \mathbf{P}) = \begin{pmatrix} \mathbf{H} \mathbf{P} \\ \mathbf{H}^* \mathbf{P}^* B \end{pmatrix} \quad (4)$$

where the coefficient matrix  $B = I_M \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$  and  $\otimes$  stands for the kronecker product.

### 3 Precoder design and selection criterion

A vector symbol error rate (VSER) is often seen a potential criterion for selection of the mode and precoder and leads to a fair comparison between modes that transmit symbol vectors of different dimensionality. Since a closed-form expression for the VSER would be extremely difficult to obtain. Taking approach in [4], the upper bound of VSER is

$$P_r(\text{Error}|\mathbf{H}) \leq C \cdot Q \left( \sqrt{SNR_{\min} \frac{d_{\min}^2(M, R)}{2}} \right) \quad (5)$$

where  $C$  is a scale factor related to the constellation and the number of streams.  $d_{\min}(M, R)$  is the minimum Euclidean distance of the constellation.  $SNR_{\min}$  denotes the minimum stream received SNR after linear processing. For zero forcing (ZF), it is given by

$$SNR_{\min} = \min_{1 \leq k \leq 2M} \frac{\rho}{2MN_0 \left[ H_{eff}^H H_{eff} \right]_{kk}^{-1}} \quad (6)$$

and bounded by [5]

$$SNR_{\min} \geq \lambda_{\min}^2(H_{eff}(\mathbf{H}, \mathbf{P})) \frac{\rho}{2MN_0} \quad (7)$$

where  $\lambda_{\min}(\cdot)$  is the minimum eigenvalue of the effective channel.

From this bound (7), we observe that minimizing VSER is equivalent to looking for an optimal precoder which maximizes the minimum eigenvalue of the effective channel.

#### 3.1 Precoder design

In [3], the precoder design for a single mode precoded OSTBC ended up with one that maximizes the minimum chordal distance between any pair of codebook column spaces resulting in the minimization of upper bound of average distortion. So the performance of a precoder set depends on how tight the upper bound is. In order to obtain a better precoder set, minimization of upper bound of average error probability is used for precoder design criterion and a two-step procedure is applied for producing precoders for each mode. Therefore, the optimization problem is defined as follows:

*Problem:* Find a set of precoders  $\{\mathbf{P}_1, \dots, \mathbf{P}_{N_M}\}$  given a channel partition  $\{\mathcal{H}_1, \dots, \mathcal{H}_{N_M}\}$  for mode  $M$ , such that  $\text{tr}(\mathbf{P}_q^H \mathbf{P}_q) \leq 2M$  and the average distortion  $D_{ave}$  is minimized

$$D_{ave} = \frac{1}{N_M} \sum_{q=1}^{N_M} E_{\mathbf{H} \in \mathcal{H}_q} [d(\mathbf{H}, \mathbf{P}_q)] \quad (8)$$

where  $d(\mathbf{H}, \mathbf{P}_q) = -[\lambda_{\min}^2(H_{eff}(\mathbf{H}, \mathbf{P}_q))]$  and  $E_{\mathbf{H} \in \mathcal{H}_q}[x]$  denotes the conditional expectation on  $\mathbf{H} \in \mathcal{H}_q$ .

We randomly generate a set of  $K$  precoders with  $M_t \times 2M$  dimensions, which is denoted as  $\{F_i, i \in [1, \dots, K]\}$  and each column vector of  $F_i$  is unit normalized for power constraint. Then Lloyd's algorithm [6] for solving this problem is outlined below.

**Step 1** Determine the precoder set  $\{\mathbf{P}_1, \dots, \mathbf{P}_{N_M}\}$  given a certain partition  $\{\mathcal{H}_1, \dots, \mathcal{H}_{N_M}\}$ .

$$\mathbf{P}_q = \arg \min_{\{F_i\}} E_{\mathbf{H} \in \mathcal{H}_q} [d(\mathbf{H}, F_i)] \quad (9)$$

**Step 2** Determine the partition  $\{\mathcal{H}_q\}$  given a precoder set  $\{\mathbf{P}_q\}$  by the nearest neighbor rule

$$\mathcal{H}_q = \{\mathbf{H} : d(\mathbf{H}, \mathbf{P}_q) \leq d(\mathbf{H}, \mathbf{P}_j), \forall j, q \in [1, \dots, N_M], j \neq q\} \quad (10)$$

Step 1 and Step 2 are repeated until convergence.

### 3.2 Selection criterion

The similar selection criterion as that in [4] can be applied and described as follows:

$$\mathbf{P}_m = \arg \max_{\mathbf{P} \in \mathcal{P}_m} \lambda_{\min}^2 (H_{\text{eff}}(\mathbf{H}, \mathbf{P})), m \in \mathcal{M} \quad (11)$$

$$M = \arg \max_{m \in \mathcal{M}} \frac{\lambda_{\min}^2 (H_{\text{eff}}(\mathbf{P}_m, \mathbf{H}))}{2 \cdot m} \cdot d_{\min}^2(m, R) \quad (12)$$

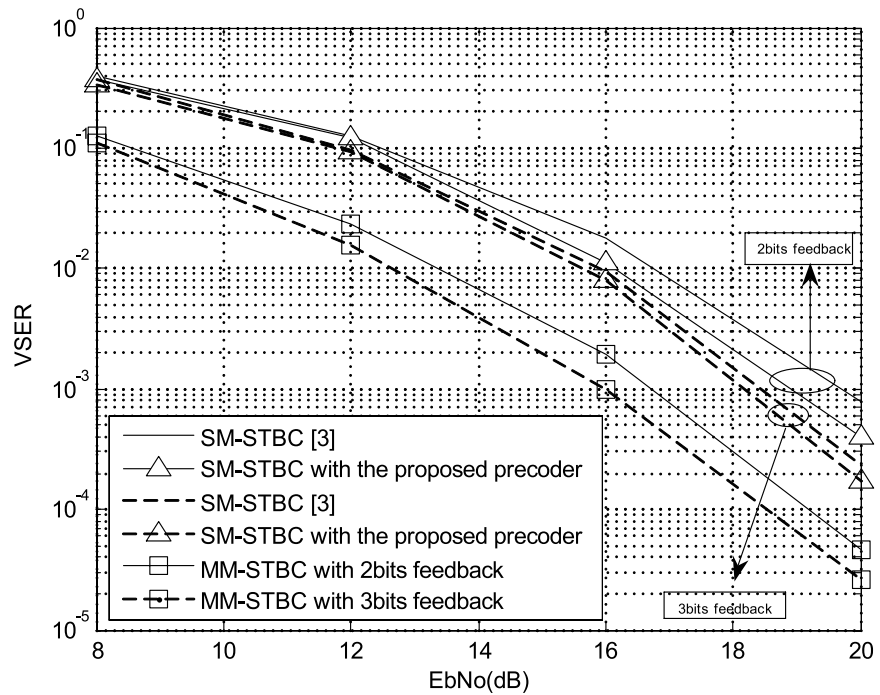
This criterion is computed by first searching for the precoder  $\mathbf{P}_m$  in each mode's codebook  $\mathcal{P}_m$  that maximizes minimum eigenvalue of the effective channel. Then, we select the optimal mode by computing (12). The receiver would send both the selected mode number and the optimal precoder back to the transmitter.

## 4 Simulation results

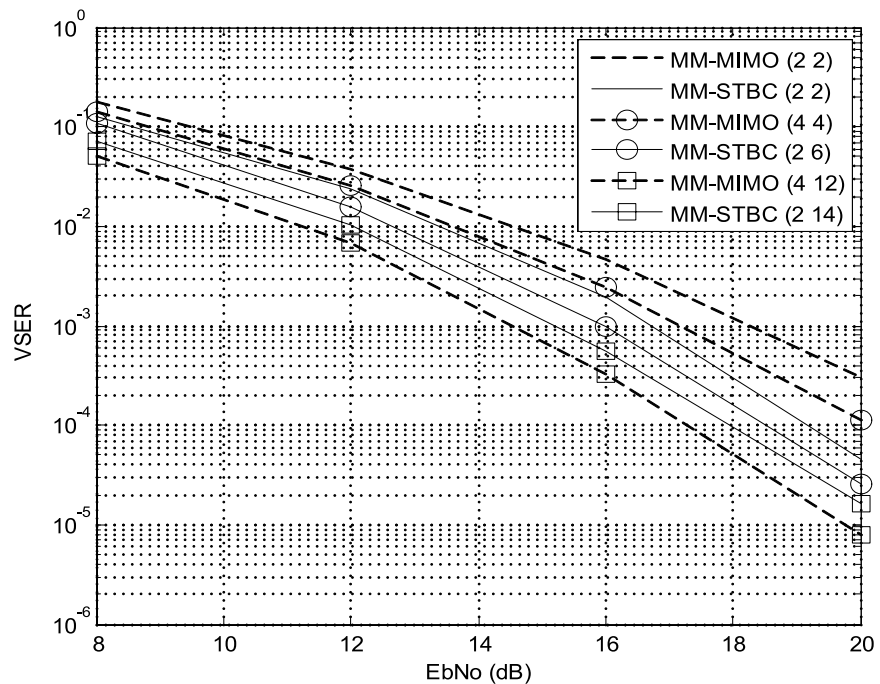
We consider only a case of four transmit and two receive antennas, where a dual-mode ( $\mathcal{M} = \{1, 2\}$ ) for STBC system with a ZF receiver is applied and the overall data-rate  $R$  is fixed at 8 bits such that the corresponding constellations are 256QAM and 16QAM for mode one and two, respectively. The larger number of transmit antennas and modes can be readily extended.

Fig. 2 shows the performance of the proposed codebook and one based on chordal distance criterion [3] for single mode (mode one) precoded OS-TBC (SM-STBC). It is observed that precoders generated by the proposed algorithm perform better than precoders based on subspace packing in the Grassmann manifold using chordal distance. The performance gain is about 1 dB at VSER  $10^{-3}$  for 2 bits feedback. When feedback bits are increased to three, the proposed precoder still achieves better performance gain than the codebook generated by using Grassmann manifold about 0.5 dB at VSER  $10^{-3}$ . Furthermore, we observe that the MM-STBC with the proposed precoder achieves significantly better performance than the single mode (mode one) precoded OSTBC with a fixed data rate, where feedback allocations for the MM-STBC are  $(N_1, N_2) = (2, 2)$  and  $(N_1, N_2) = (2, 6)$  for 2 bits and 3 bits feedback respectively and  $N_i$  denotes the number of precoders for mode  $i$ .

For comparison purpose, the same dual-mode precoded MIMO (MM-MIMO) [4] with a rate  $R = 8$  bits is considered in Fig. 3. For 2 bits feedback, two precoders are allocated to each mode, i.e.,  $(N_1, N_2) = (2, 2)$ , the



**Fig. 2.** Probability of vector symbol error comparison for MM-STBC and SM-STBC with different codebooks



**Fig. 3.** Probability of vector symbol error comparison for MM-STBC and MM-MIMO with different feedback

MM-STBC provides approximately 1.5 dB performance improvement over the MM-MIMO at  $VSEr = 10^{-3}$ . If the number of feedback bits is assumed to be three, we only consider the MM-MIMO with  $(N_1, N_2) = (4, 4)$  because it

achieves the best performance compared to any other feedback allocations for modes. Simulation results show that the MM-STBC with  $(N_1, N_2) = (2, 6)$  still achieve around 1.2 dB better performance than the MM-MIMO with  $(N_1, N_2) = (4, 4)$ . However, it is expected that the MM-MIMO can benefit from more beamforming gain for large feedback bits due to the intrinsic nature of STBC as an open-loop MIMO scheme. Hence, it is observed that the MM-MIMO with  $(N_1, N_2) = (4, 12)$  shows performance gains over the MM-STBC with  $(N_1, N_2) = (2, 14)$  for feedback of 4 bits.

## 5 Conclusion

In this paper, we proposed a multimode precoder design for STBC system with limited feedback, which allows the system to adaptively vary both the number of streams and the precoder using current channel conditions. It was numerically shown that the proposed design based on minimization of upper bound of VSER achieves better performance than the Grassmannian design for a fixed mode [3]. Furthermore, the numerical results showed that the multimode precoded STBC improves performance gain compared with the single mode precoded STBC and outperforms the multimode precoded MIMO [4] with very limited feedback. However, the criterion for the distribution of the number of codewords for each mode is still not known, which will be of future research.

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