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DFT-based interpolation with simple leakage suppression

Jeongwook Seo[1a)] **and Dong Ku Kim**[2]

[1] *Advanced Mobile Technology Research Center, Korea Electronics Technology Institute, Seoul 121–835, Korea*

[2] *School of Electrical and Electronic Engineering, Yonsei University, Seoul 120–749, Korea*

a) *jwseo@keti.re.kr*

Abstract: For pilot-aided channel estimation in OFDM systems with a large number of subcarriers, an efficient discrete Fourier transform (DFT)-based interpolator is proposed. To suppress the leakage in a conventional DFT-based interpolator, the proposed method simply reuses the channel frequency response (CFR) estimates at leftmost and rightmost pilot subcarriers.

Keywords: DFT, interpolation, leakage, OFDM, channel estimation

Classification: Science and engineering for electronics

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1 Introduction

The leakage caused by virtual subcarriers in an OFDM symbol yields serious degradation of mean square error (MSE) performance of pilot-aided channel estimation (PACE) based on discrete Fourier transform (DFT)-based interpolation [1, 2, 3]. In order to suppress the leakage, non-uniform pilot subcarrier allocation and extra pilot subcarrier insertion methods were presented

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in [1], [2], respectively. However, they cannot be used for any standardized OFDM systems where pilot subcarrier locations are predefined. Recently, the linear minimum mean square error (LMMSE) approach was investigated in [3], where the channel frequency responses (CFRs) at equally spaced virtual subcarriers were estimated prior to the DFT-based interpolation. It shows as good performance as no leakage case, but it is too complex to be used in OFDM systems with a large number of subcarriers such as the DVB-T2 system [4] which can support 1 K, 2 K, 4 K, 8 K, 16 K, and 32 K subcarriers. In this paper, therefore, we propose an efficient DFT-based interpolator with simple leakage suppression through the analysis of the leakage effect on MSE performance according to the number of subcarriers. The proposed method simply reuses the CFR estimates at leftmost and rightmost pilot subcarriers to provide the CFR estimates at equally spaced virtual subcarriers.

Henceforth, we shall use the following notations: $E[\cdot]$, $(\cdot)^H$, $(\cdot)^{-1}$, and $tr(\cdot)$ denote expectation, Hermitian transpose, inverse, and trace, respectively. Also, $[\cdot]_{m,n}$, $\mathbf{I}_m$, and $\mathbf{0}_{m\times n}$ denote the (m,n) entry of a matrix, the $m\times m$ identity matrix, and the $m\times n$ zero matrix, respectively.

2 Proposed DFT-based interpolation

The proposed DFT-based interpolation using simple leakage suppression is described as follows. Let the number of total subcarriers be $N = N_u + N_v + 1$ where $N_u + 1$ is the number of useful pilot and data subcarriers, and N_v is the number of virtual subcarriers. As shown in [1], the received pilot vector is given by

$$\mathbf{Y} = \mathbf{XFh} + \mathbf{W} \tag{1}$$

where $\mathbf{X}$ is a diagonal matrix with pilot symbols on its diagonal, and $\mathbf{F}$ is a DFT matrix with entries

$$[\mathbf{F}]_{m,n} = e^{-j2\pi i_m n/N} \tag{2}$$

where $0 \le m \le N_p - 1$, $0 \le n \le L-1$, $i_m = -N_u/2 + mD_f$ is the pilot subcarrier index, and $D_f = N/M$ is the minimum pilot spacing. Note that $M = N_b + N_p + N_f$ is the number of pilot subcarriers in the case of no virtual subcarriers, which consists of the number of actual pilot subcarriers (N_p), left-side and right-side D_f-spaced virtual subcarriers (N_b and N_f). Also, $\mathbf{h}$ is an $L\times 1$ zero mean circularly symmetric complex Gaussian (ZMCSG) random channel impulse response (CIR) vector with covariance matrix $\mathbf{C}_h$, and $\mathbf{W}$ is a ZMCSG random noise vector with covariance matrix $\sigma^2\mathbf{I}_{N_p}$.

Generally, the PACE scheme with the conventional DFT-based interpolation is performed by two phases. First, the observation vector at pilot subcarriers is given by the least squares (LS) estimation

$$\mathbf{Z} = \mathbf{X}^H\mathbf{Y} = \mathbf{Fh} + \tilde{\mathbf{W}} \tag{3}$$

where $\mathbf{X}^H\mathbf{X} = \mathbf{I}_{N_p}$, and $\tilde{\mathbf{W}} = \mathbf{X}^H\mathbf{W}$. Second, the final CFR vector is given by the conventional DFT-based interpolation

$$\hat{\mathbf{H}}_{conv} = \frac{1}{M}\mathbf{GF}^H\mathbf{Z} \tag{4}$$

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where $\mathbf{G}$ is a DFT matrix with entries

$$[\mathbf{G}]_{m,n} = e^{-j2\pi mn/N} \tag{5}$$

where $-N_u/2 \le m \le N_u/2$, and $0 \le n \le L-1$. However, the proposed DFT-based interpolation exploits a simple leakage suppression matrix. To represent the proposed method, $\mathbf{Z}$ is replaced with $\bar{\mathbf{Z}}$ without loss of generality

$$\bar{\mathbf{Z}} = (\mathbf{U}_1 + \mathbf{U}_2\mathbf{K})\,\mathbf{Z} \tag{6}$$

where $\mathbf{U}_1$ and $\mathbf{U}_2$ are rearrangement matrices given by

$$\mathbf{U}_1 = \begin{bmatrix} \mathbf{0}_{N_b \times N_p} \\ \mathbf{I}_{N_p} \\ \mathbf{0}_{N_f \times N_p} \end{bmatrix} \tag{7}$$

$$\mathbf{U}_2 = \begin{bmatrix} \mathbf{I}_{N_b} & \mathbf{0}_{N_b \times N_f} \\ \mathbf{0}_{N_p \times N_b} & \mathbf{0}_{N_p \times N_f} \\ \mathbf{0}_{N_f \times N_b} & \mathbf{I}_{N_f} \end{bmatrix} \tag{8}$$

Note that $\mathbf{K}$ denotes the simple leakage suppression matrix

$$\mathbf{K} = \begin{bmatrix} \mathbf{1}_{N_b \times 1} & \mathbf{0}_{N_b \times (N_p - N_b)} \\ \mathbf{0}_{N_f \times (N_p - N_f)} & \mathbf{1}_{N_f \times 1} \end{bmatrix} \tag{9}$$

Then, as shown in Fig. 1, the final CFR vector is given by the proposed DFT-based interpolation

$$\hat{\mathbf{H}}_{prop} = \frac{1}{M}\mathbf{G}\bar{\mathbf{F}}^H\bar{\mathbf{Z}} = \frac{1}{M}\mathbf{G}\bar{\mathbf{F}}^H(\mathbf{U}_1 + \mathbf{U}_2\mathbf{K})\,\mathbf{Z} \tag{10}$$

where $\bar{\mathbf{F}}$ is a DFT matrix with entries

$$[\bar{\mathbf{F}}]_{m,n} = e^{-j2\pi i'_m n/N} \tag{11}$$

where $0 \le m \le M-1$, $0 \le n \le L-1$, and $i'_m = -N_u/2 + (m - N_b)\,D_f$.

In (10), the leakage suppression matrix reuses the CFR estimates at leftmost and rightmost pilot subcarriers to provide the CFR estimates at equally spaced virtual subcarriers. Also, if $\mathbf{K} = \mathbf{0}_{(N_b+N_f)\times N_p}$, the proposed method in (10) is equivalent to the conventional method in (4) because of $\mathbf{F}^H = \bar{\mathbf{F}}^H\mathbf{U}_1$.

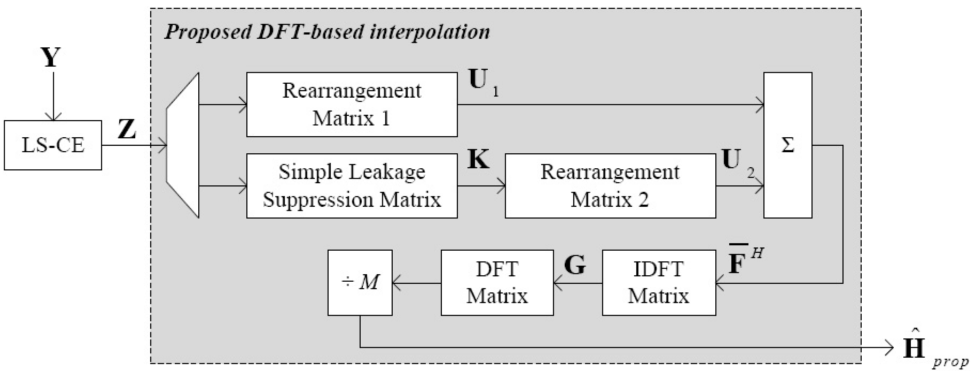

Fig. 1. Proposed DFT-based interpolation with simple leakage suppression.

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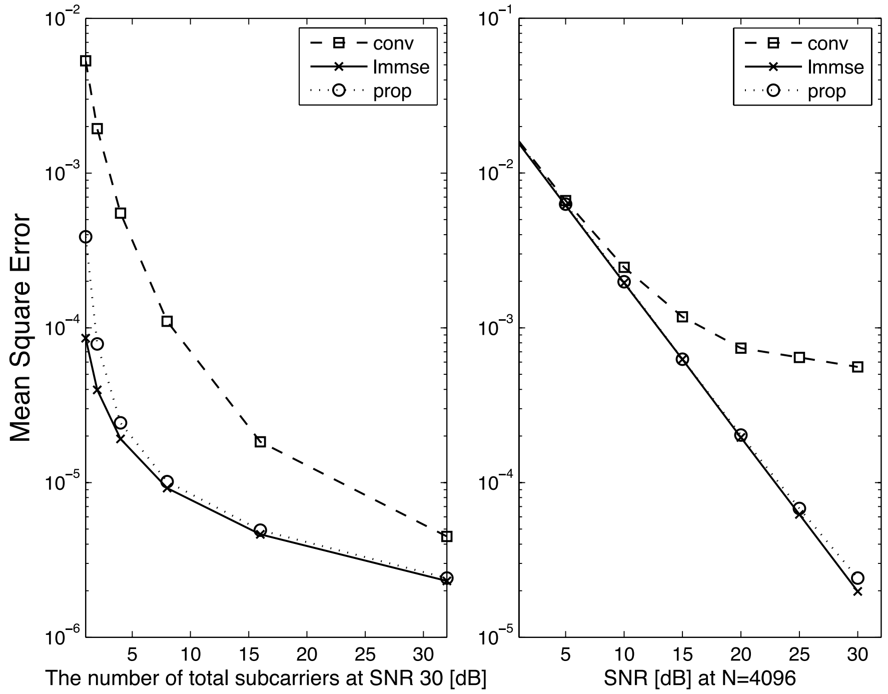

Fig. 2. MSE performances versus (a) number of subcarriers normalized by 1 K (SNR 30 [dB]) and (b) SNR [dB] (N=4096).

3 Simulation results

A QPSK-OFDM system is considered in 1.25 MHz band at 2.3 GHz with $D_f = 8$ and $N_v = 31$ under the exponential decaying multipath fading with covariance matrix $\mathbf{C}_h$ which is a diagonal matrix with $\sigma_l^2 = e^{-l/10}$ $(L = 20)$ on its diagonal.

To analyze the leakage effect according to the number of subcarriers, the MSE performance is defined as

$$\Gamma = \frac{1}{N_u + 1} tr\left(\mathbf{C}_{\hat{H}}\right) \tag{12}$$

where the error covariance matrix is represented by

$$\mathbf{C}_{\hat{H}} = E\left[\left(\hat{\mathbf{H}}_{prop} - \mathbf{H}\right)\left(\hat{\mathbf{H}}_{prop} - \mathbf{H}\right)^H\right] \tag{13}$$

where $\mathbf{H}$ is the ideal CFR vector. As shown in Fig. 2 (a), the MSE performance of the conventional method can be slightly improved by increasing the number of subcarriers ($N_u + 1$ and N_p), whereas it is much worse than that of the LMMSE method in [3], in which $\mathbf{K} = \mathbf{F}_v \mathbf{C}_h \mathbf{F}_v^H \left(\mathbf{F}\mathbf{C}_h\mathbf{F}^H + \sigma^2 \mathbf{I}_{N_p}\right)^{-1}$ and $\mathbf{F}_v$ is a sub-matrix consisting of the first N_b and the last N_f rows of $\bar{\mathbf{F}}$. However, it should be noted that the LMMSE method becomes too complicated due to the enlarged $\mathbf{K}$ as the number of subcarriers is increased. For $N \geq 4096$, the proposed method provides good performance, but the computational complexity of $\mathbf{K}$ is not increased at all. In Fig. 2 (b), the proposed method has almost the same performance as the LMMSE method. At the MSE of 10^{-3}, it provides about 5 dB signal-to-noise ratio (SNR) gain compared to the conventional method.

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4 Conclusion

This paper presented an efficient DFT-based interpolator using simple leakage suppression for OFDM channel estimation when the number of subcarriers is sufficiently large.

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