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Robust estimators for frequency offset of OFDM in non-Gaussian noise

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Abstract: This paper addresses robust estimation for the frequency offset of orthogonal frequency division multiplexing in non-Gaussian noise channels. We first propose a maximum-likelihood (ML) estimator in non-Gaussian noise modeled as a complex isotropic Cauchy process, and then present a simpler, yet still robust, estimator based on the ML estimator. From numerical results, it is confirmed that the proposed estimators offer a substantial performance improvement over the conventional estimators in non-Gaussian noise channels.

Keywords: frequency offset estimation, non-Gaussian noise, OFDM
Classification: Wireless communication hardware

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1 Introduction

Due to its high spectral efficiency and immunity to multipath fading, orthogonal frequency division multiplexing (OFDM) has been widely used as a modulation technique for wireless communication systems. However, OFDM is very sensitive to the frequency offset (FO) caused by a Doppler shift or oscillator instabilities: Thus, the FO estimation is one of the most important technical issues in OFDM systems [1]. Specifically, we are concerned about the FO estimation based on training symbols, which provides a better performance than that based on the blind approach [2].

Conventionally, the FO estimation techniques have been proposed under the assumption that the ambient noise is a Gaussian process [3, 4, 5], which is generally justified with the central limit theorem. However, it has been observed that the ambient noise often exhibits non-Gaussian nature in wireless channels, mostly due to the impulsive nature originated from various sources such as car ignitions, moving obstacles, lightning in the atmosphere, and reflections from sea waves [6, 7]. The conventional estimators developed under the Gaussian assumption on the ambient noise could suffer from severe performance degradation under such non-Gaussian noise environments.

In this paper, our goal is to obtain robust FO estimators for OFDM systems in non-Gaussian noise environments. We first derive a maximum-likelihood (ML) estimator in non-Gaussian noise modeled as a complex isotropic Cauchy noise, and then derive a simpler estimator with a lower complexity. From numerical results, the proposed estimators are confirmed to provide a substantial performance improvement over the conventional estimators in non-Gaussian noise environments.

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2 Signal model

In the presence of FO, the kth received OFDM sample can be expressed as

$$r(k) = \sum_{l=0}^{L-1} h(l)x(k-l)e^{j2\pi k\varepsilon/N} + n(k) \tag{1}$$

for $k = 0, 1, \cdots, N-1$, where $h(l)$ is the lth channel coefficient of a multipath channel with length L, $x(k)$ is the kth sample of the transmitted OFDM symbol generated by the inverse fast Fourier transform (IFFT) of size N, ε is the FO normalized to the subcarrier spacing $1/N$, and $n(k)$ is the kth sample of additive noise.

In this paper, we adopt the complex isotropic symmetric α stable (CISαS) model for the independent and identically distributed noise samples $\{n(k)\}_{k=0}^{N-1}$; this model has been widely employed due to its strong agreement with experimental data [8, 9]. The probability density function (pdf) of $n(k)$ is then given by [8]

$$f_n(\rho) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left[-\gamma \left(u^2+v^2\right)^{\frac{\alpha}{2}} - j\Re\left\{\rho\left(u-jv\right)\right\}\right] dudv, \tag{2}$$

where $\Re\{\cdot\}$ denotes the real part, the dispersion $\gamma > 0$ is related to the spread of the pdf, and the characteristic exponent $\alpha \in (0, 2]$ is related to the heaviness of the tails of the pdf: A smaller value of α indicates a higher degree of impulsiveness, whereas a value closer to 2 indicates a more Gaussian behavior.

A closed-form expression of (2) is not known to exist except for the special cases of $\alpha = 1$ (complex isotropic Cauchy) and $\alpha = 2$ (complex isotropic Gaussian). In particular, we have

$$f_n(\rho) = \begin{cases} \frac{\gamma}{2\pi}\left(|\rho|^2+\gamma^2\right)^{-\frac{3}{2}}, & \text{when } \alpha = 1 \\ \frac{1}{4\pi\gamma}\exp\left(-\frac{|\rho|^2}{4\gamma}\right), & \text{when } \alpha = 2. \end{cases} \tag{3}$$

Due to such a lack of closed-form expressions, we concentrate on the case of $\alpha = 1$: We shall see in Section 4 that the estimators obtained for $\alpha = 1$ are not only more robust to the variation of α, but they also provide a better performance for most values of α, than the conventional estimators.

3 Proposed estimators

3.1 Cauchy maximum-likelihood estimator

In estimating the FO, we consider a training symbol $\{x(k)\}_{k=0}^{N-1}$ with two identical halves as in [3], i.e., $x(k) = x(k+N/2)$ for $k = 0, 1, \cdots, \frac{N}{2}-1$. Then, from (1), we have

$$r(k+N/2) - r(k)e^{j\pi\varepsilon} = n(k+N/2) - n(k)e^{j\pi\varepsilon} \tag{4}$$

for $k = 0, 1, \cdots, \frac{N}{2}-1$. Observing that $n(k+N/2)-n(k)e^{j\pi\varepsilon}$ obeys the complex isotropic Cauchy distribution with dispersion 2γ (since the distribution

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of $-n(k)e^{j\pi\varepsilon}$ is the same as that of $n(k)$), we obtain the pdf

$$f_{\boldsymbol{r}}(\boldsymbol{r}|\varepsilon) = \prod_{k=0}^{\frac{N}{2}-1} \frac{\gamma}{\pi\left(\left|r(k+N/2)-r(k)e^{j\pi\varepsilon}\right|^2+4\gamma^2\right)^{\frac{3}{2}}} \tag{5}$$

of $\boldsymbol{r} = \{r(k+N/2) - r(k)e^{j\pi\varepsilon}\}_{k=0}^{N/2-1}$ conditioned on ε. The ML estimation is then to choose $\hat{\varepsilon}$ such that

$$\begin{aligned}\hat{\varepsilon} &= \arg\max_{\tilde{\varepsilon}}[\log f_{\boldsymbol{r}}(\boldsymbol{r}|\tilde{\varepsilon})] \\ &= \arg\min_{\tilde{\varepsilon}} \Lambda(\tilde{\varepsilon}),\end{aligned} \tag{6}$$

where $\tilde{\varepsilon}$ denotes the candidate value of ε and the log-likelihood function $\Lambda(\tilde{\varepsilon}) = \sum_{k=0}^{N/2-1}\log\left\{\left|r(k+N/2)-r(k)e^{j\pi\tilde{\varepsilon}}\right|^2+4\gamma^2\right\}$ is a periodic function of $\tilde{\varepsilon}$ with period 2: The minima of $\Lambda(\tilde{\varepsilon})$ occur at a distance of 2 from each other, causing an ambiguity in estimation. Assuming that ε is distributed equally over positive and negative sides around zero, the valid estimation range of the ML estimator can be set to $-1 < \varepsilon \le 1$, as in [3].

From the fact that $\left|r(k+N/2)-r(k)e^{j\pi\tilde{\varepsilon}}\right|^2 = 4\left|\sum_{l=0}^{L-1}h(l)x(k-l)\right|^2 \sin^2(\pi(\tilde{\varepsilon}-\varepsilon)/2)$ in the absence of noise, and that the logarithm is an increasing function, it is straightforward to see that $\Lambda(\tilde{\varepsilon})$ in the absence of noise is convex in each interval $\varepsilon+2z-1 < \tilde{\varepsilon} \le \varepsilon+2z+1$, where z is an integer. Thus, the ML estimate $\hat{\varepsilon}$ in (6) can be found by solving $\frac{d\Lambda(\tilde{\varepsilon})}{d\tilde{\varepsilon}}\Big|_{\tilde{\varepsilon}=\hat{\varepsilon}} = 0$ for $\hat{\varepsilon}$: After some algebraic manipulations, we get

$$\hat{\varepsilon} = \frac{1}{\pi}\angle\left(\sum_{k=0}^{\frac{N}{2}-1}\frac{r^*(k)r(k+\frac{N}{2})}{4\gamma^2+|r(k)|^2+|r(k+\frac{N}{2})|^2-2|r(k)r(k+\frac{N}{2})|\cos(\pi\hat{\varepsilon}+\theta_k)}\right), \tag{7}$$

where $\theta_k = \angle\left(r(k)r^*(k+N/2)\right)$ with $\angle$ denoting the phase angle in $(-\pi,\pi]$ of a complex number. The estimator (7) will be called the Cauchy ML estimator (CME): The ML estimate $\hat{\varepsilon}$ can be acquired, for example, via an iterative procedure.

3.2 Simplified Cauchy maximum-likelihood estimator

As the signal-to-noise ratio (SNR) gets smaller, the CME would require more iterations to produce a reliable estimate. To avoid the iteration, by assuming that $\hat{\varepsilon}$ is uniformly distributed over $(-1,1]$ (note that such an assumption is the 'worst-case' when the distribution of $\hat{\varepsilon}$ is unknown) and then taking the expectation of the argument of $\angle$ in (7) with respect to $\hat{\varepsilon}$, we can obtain a simplified estimator

$$\begin{aligned}\hat{\varepsilon}_{\mathrm{S}} &= \frac{1}{\pi}\angle\left(\sum_{k=0}^{\frac{N}{2}-1}\frac{1}{2}\int_{-1}^{1}\frac{r^*(k)r(k+N/2)}{A+B\cos(\pi\hat{\varepsilon}+\theta_k)}d\hat{\varepsilon}\right) \\ &= \frac{1}{\pi}\angle\left(\sum_{k=0}^{\frac{N}{2}-1}\frac{r^*(k)r(k+N/2)}{\sqrt{\left\{4\gamma^2+(|r(k)|+|r(k+N/2)|)^2\right\}\left\{4\gamma^2+(|r(k)|-|r(k+N/2)|)^2\right\}}}\right),\end{aligned} \tag{8}$$

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where $A = 4\gamma^2 + |r(k)|^2 + |r(k+N/2)|^2$, $B = -2|r(k)r(k+N/2)|$, and the second equality is obtained using $\int_{-\pi}^{\pi} \frac{1}{a+b\cos x}dx = \frac{2\pi}{\sqrt{a^2-b^2}}$ [10]. The estimator (8) will be referred to as the simplified Cauchy ML estimator (SCME).

4 Numerical results

The mean square error (MSE) performances of the proposed estimators CME and SCME are compared with those of the conventional estimators [3, 4, 5]. We assume the following parameters: The IFFT size $N = 64$, FO $\varepsilon = 0.25$, iteration number of 20 with an initial value zero for the CME, and a multi-path Rayleigh fading channel with length $L = 8$ and an exponential power delay profile of $\mathbf{E}[|h(l)|^2]$=$\exp(-l/L)/\left\{\sum_{l=0}^{L-1}\exp(-l/L)\right\}$ for $l = 0, 1, \cdots, 7$, where $\mathbf{E}[\cdot]$ denotes the statistical expectation. Since a CISαS noise with $\alpha < 2$ does not assume a variance, the standard SNR becomes meaningless for such a noise. We thus employ the geometric SNR (GSNR) [11] defined as $\mathbf{E}[|x(k)|^2]/(4C^{-1+2/\alpha}\gamma^{2/\alpha})$, where $C = \exp\{\lim_{m\to\infty}(\sum_{i=1}^{m}\frac{1}{i} - \ln m)\} \simeq 1.78$ is the exponential of the Euler constant. The GSNR indicates the relative strength between the information-bearing signal and the CISαS noise with $\alpha < 2$. Clearly, the GSNR becomes the standard SNR when $\alpha = 2$. Since γ can be easily and exactly estimated using only the sample mean and variance of the received samples [12], it may be regarded as a known value: Thus, γ is set to 1 without loss of generality.

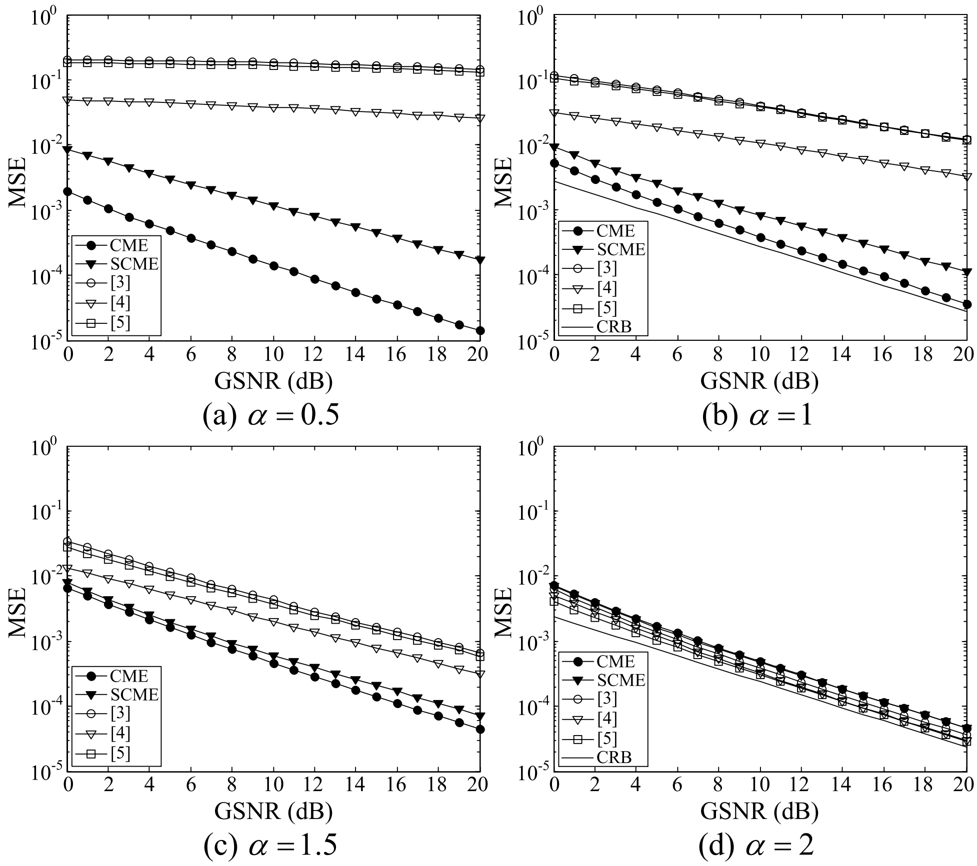

Fig. 1. The MSE performances of the proposed and conventional estimators as a function of the GSNR.

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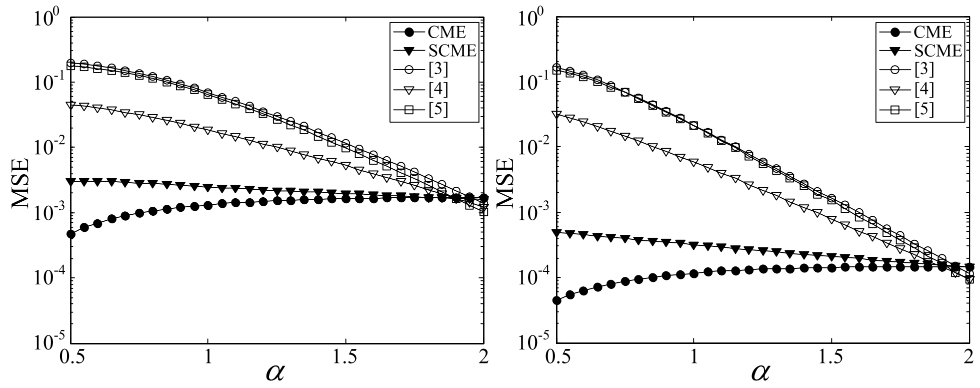

(a) GSNR = 5 dB (b) GSNR = 15 dB

Fig. 2. The MSE performances of the proposed and conventional estimators as a function of α.

Table I. Correlation operations of the received OFDM samples in the proposed and conventional estimators.

Estimator	Correlation operation
CME	$\sum_{k=0}^{\frac{N}{2}-1} \frac{r^*(k)r(k+N/2)}{4\gamma^2+\lvert r(k)\rvert^2+\lvert r(k+N/2)\rvert^2-2\lvert r(k)r(k+N/2)\rvert\cos(\pi\hat{\varepsilon}+\theta_k)}$
SCME	$\sum_{k=0}^{\frac{N}{2}-1} \frac{r^*(k)r(k+N/2)}{\sqrt{\{4\gamma^2+(\lvert r(k)\rvert+\lvert r(k+N/2)\rvert)^2\}\{4\gamma^2+(\lvert r(k)\rvert-\lvert r(k+N/2)\rvert)^2\}}}$
[3]	$\sum_{k=0}^{\frac{N}{2}-1} r^*(k)r(k+N/2)$
[4]	$\sum_{k=\frac{mN}{4}}^{N-1} r^*(k-mN/4)r(k), \text{ for } m=0,1,2$
[5]	$\sum_{z=1}^{\frac{N}{16}-1} \left\lvert \sum_{m=0}^{\frac{N}{16}-z-1} \sum_{k=0}^{\frac{N}{4}-1} zr(k+mN/4)r^*(k+(m+z)N/4) \right\rvert$

Figure 1 shows the MSE performances of the proposed and conventional estimators as a function of the GSNR when $\alpha = 0.5$, 1, 1.5, and 2. Similarly, Figure 2 shows the MSE performances as a function of α when the GSNR is 5 and 15 dB. The Cramér-Rao bounds (CRBs) $\frac{15N^2}{32\pi^4 C(N^2-1)(\mathrm{GSNR})}$ and $\frac{3N}{2\pi^2(N^2-1)(\mathrm{GSNR})}$ [4, 8] for FO estimation when $\alpha = 1$ and 2, respectively, are shown in Figures 1 (b) and (d) for reference.

From the figures, we can clearly observe that the proposed estimators not only outperform the conventional estimators for most values of α, except for those close to 2, but also provide a robustness to the variation of the value of α. This can be explained from Table I showing the correlation operations

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of the received OFDM samples in the proposed and conventional estimators. The correlation operations of the received OFDM samples in the proposed estimators involve sample-by-sample normalization by the terms including the magnitude of the received OFDM samples. As it is well-known in signal detection theory [13, 14], such normalization can effectively alleviate the erroneous increase of the correlation due to noise samples of large magnitudes arising frequently in non-Gaussian noise environments, and allows the proposed estimators to yield more reliable estimates than the conventional estimators in impulsive (non-Gaussian) environments.

5 Conclusion

In this paper, we have proposed robust FO estimators for OFDM systems in non-Gaussian noise environments. Modeling the non-Gaussian noise as a complex isotropic Cauchy process, an ML estimator using an iterative procedure has been first derived. A simpler estimator has then been proposed based on the ML estimator. From numerical results, we have confirmed that the proposed estimators not only outperform the conventional estimators, but also provide a robustness in non-Gaussian noise environments.

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