

# Sliding mode control of uncertain parameter for a matrix rectifier

Zhiping Wang<sup>1,2</sup>, Yunshou Mao<sup>1a)</sup>, and Zhanhu Hu<sup>2</sup>

<sup>1</sup> School of Automation, Guangdong University of Technology,  
Guangzhou 510006, China

<sup>2</sup> Guangdong Institute of Automation, Guangzhou 510007, China  
a) [maoyunshou@163.com](mailto:maoyunshou@163.com)

**Abstract:** The external uncertainties and disturbance affect the DC output of a matrix rectifier (MR) system. This paper presents a sliding mode control (SMC) of uncertain parameters for a MR. Adding a value of anti-voltage into the control function can effectively mitigate the chattering of the sliding motion. Besides, the method of calculating the minimum value of anti-voltage are presented, which can ensure better dynamic performance of a MR. Theoretical and experimental results are presented to verify the correctness and feasibility of the novel control scheme.

**Keywords:** matrix rectifier, uncertain parameter, sliding mode control

**Classification:** Power devices and circuits

## References

- [1] D. G. Holmes, *et al.*: IEEE Trans. Power Electron. **7** (1992) 240 (DOI: 10.1109/63.124596).
- [2] P. W. Wheeler, *et al.*: IEEE Trans. Ind. Electron. **49** (2002) 276 (DOI: 10.1109/41.993260).
- [3] P. C. Loh, *et al.*: IEEE Trans. Ind. Appl. **49** (2013) 1374 (DOI: 10.1109/TIA.2013.2252319).
- [4] M. Pahlevaninezhad, *et al.*: IEEE Trans. Power Electron. **27** (2012) 2085 (DOI: 10.1109/TPEL.2011.2170098).
- [5] G. Q. Si, *et al.*: IEEE CCDC (2014) 542 (DOI: 10.1109/CCDC.2014.6852217).
- [6] S. Pinto and J. Silva: IET Electr. Power Appl. **1** (2007) 439 (DOI: 10.1049/iet-epa:20060257).
- [7] M. Hamouda, *et al.*: IEEE IECON (2006) 1727 (DOI: 10.1109/IECON.2006.347326).
- [8] X. N. Lu, *et al.*: Trans. China Electrotech. Soc. **25** (2010) 108.
- [9] Z. P. Wang, *et al.*: IEICE Electron. Express **12** (2015) 20150818 (DOI: 10.1587/elex.12.20150818).
- [10] Z. P. Wang, *et al.*: IEEE IPEMC (2012) 2002 (DOI: 10.1109/IPEMC.2012.6259148).

## 1 Introduction

For the past years, the MR, a derivation of three-phase AC-AC converter, is still an attractive topology for scientific research, because it has several advantages: operating in all four quadrants of operation, no large energy storage elements, sinusoidal current input, and flexible input power factor regulation [1, 2]. Consequently, the MR has a broad application prospect in electrical traction, uninterruptible power supplies and power supplies for telecommunication system, etc [3, 4].

However the power network is often with uncertainty in external disturbances. Especially the AC supply usually becomes distorted or unbalanced, which leads to the abnormal DC output. Besides, the parameters of a rectifier, such as input capacitor or output induction, is inevitable varying with environmental conditions, which results in undesired performance of a power converter [5]. Recently sliding mode control (SMC) is often applied on the matrix rectifier. SMC is a nonlinear control method and it has several advantages like a quick response, strong robustness, simple implementation and etc [6, 7]. In [8], the author simply focused on the existence of sliding surface when a MR operates in sliding mode. In [9], the author analyzed the DQ model of a MR and presented a sliding mode controller based on reaching law to achieve both DC output voltage and input power factor. But previous works on a MR are not involved in the value of anti-disturbance.

## 2 Nonlinear modeling of a MR system

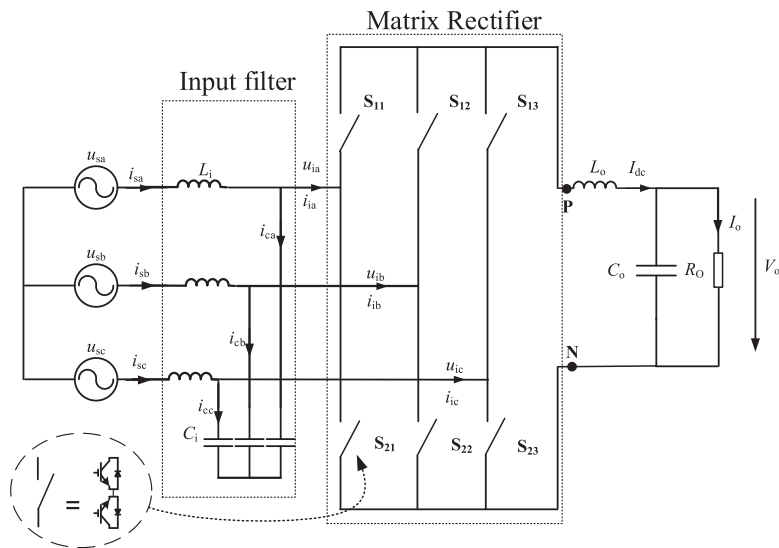


Fig. 1. Topology of a MR system.

Fig. 1 shows the topology of a MR system, and it consists of six-switch array. Supposing the switches are modeled as ideal switching function, and the three-phase input voltage is balanced, the averaged DC output voltage can be shown as:

$$V_{PN} = 1.5mV_{im} \cos \varphi_i \quad (1)$$

where  $m$  is the modulation index,  $V_{im}$  is the amplitude of the input phase voltage and  $\varphi_i$  is the desired input current displacement angle. However, when the grid side

input voltage is unbalanced, the averaged output voltage of a MR can be calculated as [10]:

$$V_{PN} = 1.5mV_{im} \cos \varphi_i + 1.5mV_{\max} \cos(n\omega t \pm \varphi_i) \quad (2)$$

where  $V_{\max}$  is the amplitude of AC component and  $\omega$  is the source angular frequency. In order to simplify the calculation, Eq. (2) can be simplified as

$$V_{PN} = 1.5mV_{im} \pm 1.5mV_{\max} \quad (3)$$

### 3 Design of the sliding mode controller

What influence the dc output voltage of a MR is the stochastic external disturbance and inevitable unknown factor of internal parameters. Those uncertainties leads to the bad performance of a MR. Thus, in this section, a sliding mode controller is given to solve such problems.

We define the error  $e_V$  and the switching function of DC output voltage as

$$e_V = V_{ref} - V_o \quad (4)$$

$$S_1 = e_V + c_1 \dot{e}_V \quad (5)$$

where,  $c_1$  sliding mode coefficient.

When the three-phase input voltage is unbalanced, the state equation is given as following:

$$\begin{aligned} \begin{bmatrix} \dot{e}_V \\ \dot{I}_{dc} \end{bmatrix} &= \begin{bmatrix} -\frac{1}{R_L C_O} & \frac{1}{C_O} \\ -\frac{1}{L_O} & 0 \end{bmatrix} \begin{bmatrix} e_V \\ I_{dc} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_L C_O} \\ -\frac{1}{L_O} \end{bmatrix} V_{ref} \\ &+ \begin{bmatrix} 0 \\ \frac{1}{L_O} \end{bmatrix} (1.5mV_{im} \pm 1.5mV_{\max}) \end{aligned} \quad (6)$$

From Eq. (2), when the reference voltage is  $V_{ref}$  and the three-phase input voltage is balanced, the modulation index  $m$  can be shown as

$$m = \frac{V_{ref}}{1.5V_{im} \cos \varphi_i} \quad (7)$$

Consequently the control function is design as:

$$m = \begin{cases} m_{ref} + \sigma & S_1(e_V) > 0 \\ m_{ref} - \sigma & S_1(e_V) < 0 \end{cases} \quad (8)$$

where  $m_{ref} = \frac{V_{ref}}{1.5V_{im} \cos \varphi_i}$  is the pre-modulation index and  $\sigma$  is the value of voltage anti-disturbance.

In order to ensure that the initial point  $S_1 = 0$  in the state space of any position can reach the switching surface in a limited period of time. The sliding mode control for the DC output voltage must satisfy the inequality:

$$S_1 \dot{S}_1 < 0 \quad (9)$$

The uncertain term of the internal parameters influence the dc output voltage is relatively small, therefore it can be eliminated by adjusting  $c_1$ . In this part, we

mainly focus on the external uncertainties which leads to the undesired performance of a MR system.

$$S_1 \dot{S}_1 = \left[ (V_{ref} - V_O) - \frac{c_1}{C_O} \left( I_{dc} - \frac{V_O}{R_L} \right) \right] \dot{S}_1 \quad (10)$$

When  $c_1$  is small enough, comparing to  $V_{ref} - V_O$ ,  $\frac{c_1}{C_O} (I_{dc} - \frac{V_O}{R_L})$  can be ignored. Consequently, substituting Eq. (2) into Eq. (10),

$$S_1 \dot{S}_1 = -\frac{1}{C_O} (V_{ref} - V_O) \left[ \left( I_{dc} - \frac{V_O}{R_L} \right) + \frac{c_1}{L_O} (1.5m(V_{im} \pm V_{max}) - V_O) \right] \quad (11)$$

The followings are separated into two parts to discuss under different switching manifold, whether the sliding mode motion exists or not.

$$1) \text{ when } S_1 > 0, m = m^+(e_v) = m_{ref} + \sigma = \frac{V_{ref}}{1.5V_{im}} + \sigma$$

In this moment,  $(V_{ref} - V_O) > 0$ . Since  $m^+(e_v) = \frac{V_{ref}}{1.5V_{im}} + \sigma$ , there will be  $1.5m(V_{im} \pm V_{max}) = 1.5(V_{im} \pm V_{max}) \left( \frac{V_{ref}}{1.5V_{im}} + \sigma \right) = V_{ref} + 1.5V_{im}\sigma \pm \frac{V_{ref}}{V_{im}} V_{max} \pm 1.5V_{max}\sigma$  and obviously in this switching cycle  $\left( I_{dc} - \frac{V_O}{R_L} \right) > 0$ . If  $1.5V_{im}\sigma \pm \frac{V_{ref}}{V_{im}} V_{max} \pm 1.5V_{max}\sigma \geq 0$ , there will be  $\sigma \geq \mp \frac{V_{ref} V_{max}}{1.5V_{im}(V_{im} \pm V_{max})}$ . Because  $\frac{V_{ref} V_{max}}{1.5V_{im}(V_{im} \pm V_{max})} > 0$ , as long as the inequality  $\sigma \geq \frac{V_{ref} V_{max}}{1.5V_{im}(V_{im} \pm V_{max})}$  holds, the sliding mode motion exists. Generally speaking,  $V_{max} = V_{im}/10$ . That is to say, if  $\sigma \geq \frac{V_{ref}}{13.5V_{im}}$ , the inequality (9) holds.

$$2) \text{ when } S_1 < 0, m = m^-(e_v) = m_{ref} - \sigma = \frac{V_{ref}}{1.5V_{im}} - \sigma$$

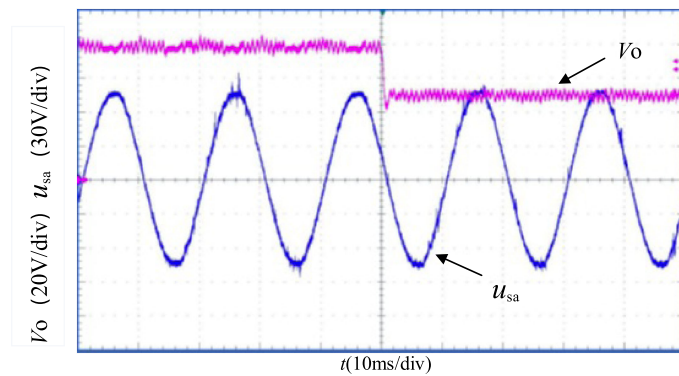
In this moment,  $(V_{ref} - V_O) < 0$ . Since  $m^-(e_v) = \frac{V_{ref}}{1.5V_{im}} - \sigma$ , there will be  $1.5m(V_{im} \pm V_{max}) = 1.5(V_{im} \pm V_{max}) \left( \frac{V_{ref}}{1.5V_{im}} - \sigma \right) = V_{ref} - 1.5V_{im}\sigma \pm \frac{V_{ref}}{V_{im}} V_{max} \mp 1.5V_{max}\sigma$  and similarly  $\left( I_{dc} - \frac{V_O}{R_L} \right) < 0$ . If  $-1.5V_{im}\sigma \pm \frac{V_{ref}}{V_{im}} V_{max} \mp 1.5V_{max}\sigma \leq 0$ , there will be  $\sigma \geq \mp \frac{V_{ref} V_{max}}{1.5V_{im}(V_{im} \pm V_{max})}$ . Because  $\frac{V_{ref} V_{max}}{1.5V_{im}(V_{im} \pm V_{max})} > 0$ , as long as the inequality  $\sigma \geq \frac{V_{ref} V_{max}}{1.5V_{im}(V_{im} \pm V_{max})}$  holds, the sliding mode motion exists. Generally speaking,  $V_{max} = V_{im}/10$ . That is to say, if  $\sigma \geq \frac{V_{ref}}{13.5V_{im}}$ , the inequality (9) holds.

No doubt, under the condition of  $|\sigma| \geq \frac{V_{ref}}{13.5V_{im}}$ , the DC output voltage of a MR system within the range of uncertainty are complete robustness.

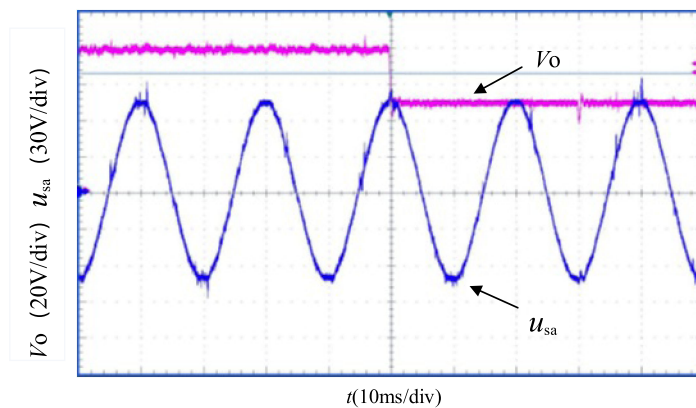
#### 4 Experimental results

Experiments are carried out to illustrate and validate the proposed control function. The parameters used in a MR system are as: three-phase input voltage and frequency 50 V/50 Hz; inductance and capacitor of input filter  $L_i = 2$  mH and  $C_i = 20$   $\mu$ F; inductance and capacitor of output filter  $L_o = 5$  mH and  $C_o = 20$   $\mu$ F; Load resistance  $R_L = 30$   $\Omega$ .

Two comparative experiments are to be shown in this section: 1) adapting the space vector modulation based on SMC to regulate the DC output without adding voltage anti-disturbance. 2) adapting the space vector modulation based on SMC with adding the appropriate value of voltage anti-disturbance.



**Fig. 2.** Experimental waveform of using space vector modulation based on conventional SMC when the three-phase input voltage is unbalanced.



**Fig. 3.** Experimental waveform of using space vector modulation based on SMC with adding voltage anti-disturbance when the three-phase input voltage is unbalanced.

Fig. 2 and Fig. 3 both show that applying space vector modulation based on SMC can regulate desirable DC output voltage. However, without adding the voltage anti-voltage (Fig. 2), the sliding mode chattering is much more obviously than Fig. 3 shows. Thus, adding the anti-disturbance can restrain the external uncertain disturbance. In the practical application, correctly setting the value of voltage anti-disturbance according to the power supply qualities can obtain satisfactory DC output results of a MR system.

## 5 Conclusion

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In this paper, a sliding mode control of uncertain parameters are proposed to achieve DC output regulation of a MR system. Theory and experiment are proved that adding the value of voltage anti-disturbance can successfully regulate the DC output voltage under unbalanced power supply.

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